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# IMPACT OF CHOICE OF RISK ASSESSMENT TIME HORIZONS ON DEFINED BENEFIT PENSION SCHEMES

By Douglas Andrews†, Stephen Bonnar†, Lori J. Curtis†,
Jaideep S. Oberoi‡, Aniketh Pittea‡ and Pradip Tapadar‡

## ABSTRACT

Years of high inflation and good investment returns during the 1970s and 1980s created the perception that defined benefit (DB) pension schemes were easily affordable. Over the past few decades however, increasing life expectancy and steady fall in interest rates have meant that pension costs have increased. This has led to the closure of many DB schemes and widespread implementation of so-called de-risking strategies, such as moving away from predominantly equity investments to bonds. We assess the importance of both asset allocation and contribution rates for the risk of a UK and a stylised US DB scheme, using both a run-off and a shorter 3-year horizon. Using the 3-year horizon, which is typically preferred by regulators, a high bond allocation does reduce the spread of the distribution of surplus. However, this result is reversed when examined on a run-off basis. Furthermore, under both the 3-year horizon and the run-off, the higher bond allocation reduces the median level of surplus. If the short-term risk assessment approach explains why DB schemes have been moving towards a higher bond allocation strategy, it might actually be detrimental to the long-term financial health of the schemes. Contribution rates have relatively lower impact on the risk.

## KEYWORDS

Defined benefit pension schemes; Risk assessment; Economic capital; Time horizon; Asset allocation strategies; Contribution rates.

## CONTACT ADDRESS

†University of Waterloo, Waterloo, ON, Canada N2L 3G1.
‡University of Kent, Canterbury, UK, CT2 7FS.
Corresponding author: Pradip Tapadar, P.Tapadar@kent.ac.uk.

# 1. INTRODUCTION

## 1.1 MOTIVATION

The main objective of a defined benefit (DB) pension scheme is to ensure that all of the promised benefits are paid. Whether the scheme has a surplus or deficit of assets is only known with certainty in the long run after the final benefit payment has been made. However, interim estimates of the scheme's finances are necessary in order to avoid runaway surpluses or deficits, and are also required for regulatory purposes.

In this article, we examine the implications of using different time horizons for quantifying and analysing pension scheme risks on the long-term financial health of a DB scheme. We use both a long-term run-off approach and a short 3-year time horizon to quantify pension scheme risks and compare and contrast the resulting impact. In our analysis we consider different asset allocation strategies and contribution rates, the two main levers available to pension scheme managers and trustees to alter the risk profile of a DB scheme. We also look at two different pension benefit structures, one based on a representative UK DB scheme and the other a stylised US scheme reflecting US-style pension benefits, to check the quantum of impact for different levels of guarantees inherent in different kinds of pension obligations.

A long-term run-off analysis involves projecting the scheme's cash flows, assets and liabilities over the entire term until the last of the current scheme members will leave the scheme through death or withdrawal. It is clearly a complex exercise. However, it arguably provides a truer reflection of the overall risk profile of a DB scheme, especially because the horizons faced by such scheme are typically much longer than the average investment fund. There is a large literature highlighting the differences between investing for the long term versus the short term (see, *e.g.* Campbell and Viceira, 2002; Ang and Kjaer, 2012), and it stands to reason that assessment of risks to portfolios with long horizons would be more accurate when assessed over a similar horizon.

On the other hand, interested parties like regulators, policymakers, pension scheme sponsors and scheme members, would justifiably require periodic updates on the financial health of a DB scheme. Typically, this can be simply obtained through a snapshot of current assets and liabilities, and the resulting funding surplus or deficit. However, obtaining point estimates of long-term liabilities comes with its own challenges. For example, if the estimate of liability is too prudent, there is a risk of overstating the underlying risk and any unnecessary follow-on remedial action can be detrimental for the scheme sponsor and can actually jeopardise the viability of an otherwise financially healthy DB scheme.

A slightly more sophisticated approach is to use a short, but non-zero, time horizon for risk quantification; *e.g.* the EU's Solvency II regulations for insurers employ a 1-year horizon. In the UK, the regulator requires triennial valuations of DB schemes, suggesting that a 3-year horizon would be a reasonable choice for risk assessment. However, any short horizon suffers from the same technical difficulty of arriving at a point estimate of the value of liabilities at the end of the period. We find that the valuation actuary's approach to setting the valuation discount rate at the end of 3 years has a material impact on the quantified risk for the 3-year risk assessment horizon. We outline a few different approaches that the valuation actuary might use to determine the valuation discount rate and analyse the impact of these approaches.

We find that regardless of the approach taken to setting the discount rate, the difference in time horizon generates different conclusions regarding the best approach to manage risk through changes in asset allocation. Over the long term, increasing a scheme's allocation to long-term bonds worsens the risk profile, both reducing the median level of surplus and increasing the spread of deficit. When examined over the shorter time horizon, increasing a scheme's allocation to long-term bonds reduces the median surplus but also reduces the spread from the median. This particular finding suggests that there may be a significant distortion created by taking a short-term view of a scheme's financial status. It is important to clarify, however, that this is not a paper about optimal asset allocation, but about the impact of the risk assessment horizon on potential misjudgements in asset allocation and scheme management.

We also find that the impact on a scheme's financial status of changing contribution rates is much smaller than the impact of changes to the scheme's asset allocation. Qualitatively all of our results and conclusions are unaffected by the choice of economic scenario generator.

### 1.2 Literature Review

There is a growing literature on quantifying risks of DB pension schemes. The regulators, policymakers, pension scheme sponsors and scheme members are all keenly interested in obtaining a better understanding of the underlying risks, particularly given the background of increasing life expectancies and a steady fall of interest rates over the last few decades.

A large proportion of the research is geared towards analysing the interactions between financial and mortality risks, and how pension scheme risk quantification can be formulated using a framework similar to existing insurance regulations, like Solvency II or the Swiss Solvency Test. However, as far as we are aware, there is limited evidence on the impact that a risk assessment approach based on a short time horizon can have on the long-term financial health and viability of a DB scheme. Moreover, the implications might be very different for DB schemes in different countries with different benefit structures.

Previous research on long-term economic capital risk assessments of UK DB schemes can be found in Porteous et al. (2012) and Yang and Tapadar (2015). Porteous et al. (2012) assessed risk for UK's Universities Superannuation Scheme (USS) based on the 2008 valuation report and estimated an economic capital requirement of 61% of liabilities. Yang and Tapadar (2015) updated the model of Porteous et al. (2012) to quantify economic capital for representative UK DB schemes and also UK's Pension Protection Fund (PPF). The authors found that economic capital for individual UK DB schemes ranges from 66% to 134% of liabilities, while for PPF it is only 10% of the aggregate economic capital of all individual UK DB schemes. This risk reduction is achieved because PPF benefits from pooling of risks of a large number of DB schemes.

There are also a number of papers attempting to quantify the solvency capital requirement for DB schemes in line with Solvency II regulations for insurers. For example, Devolder and Piscopo (2014) modelled a simple final salary DB scheme with a single model point to find that a geometric Brownian motion model for asset returns gives a probability of default of 0%-40% and the solvency capital requirement between 0%-30% of liabilities. Ai et al. (2015) also used the Solvency II framework over a 1-year time horizon to arrive at a solvency capital charge of around 50% of estimated reserves. Butt (2012) used stochastic economic and mortality models to simulate assets and liabilities of a pension scheme over a 30 year time horizon to find that changes in discount rates, investment returns and demographic variables accounted for 46.4%, 33.3% and less than 1% of the funding risk respectively. Liu (2013) also found similar results for an annuity portfolio, where an equal 40% change in the interest rate model parameter and mortality model parameter separately, leads to approximately 18% and 5% changes to solvency capital respectively.

There is relatively little research on the impact of time horizon on pension risk quantification. One recent attempt is Devolder and Lebegue (2016), who used ruin theory to estimate the solvency capital requirement for long-term life insurance and pension products, arguing that the 1-year horizon of Solvency II framework may not be appropriate for products with long terms. Using a simple model, the authors showed that under the Solvency II framework, solvency capital is understated at shorter durations and overstated at longer durations, citing inadequacy of the Solvency II framework to account for benefits of long-term equity investments. Although not specifically aimed at pension schemes, Karabey et al. (2014) estimated that among the different risk factors involved, for a 25 year and a 45 year annuity, investment risk accounts for approximately, 89% and 63.5% of the total underlying risk, respectively.

### 1.3 Outline of the Article

The rest of this article is set out as follows. Section 2 describes the economic capital framework that we use to analyse DB pension scheme risks. Section 3 describes the base case stochastic economic and demographic models that we employ. Section 4 describes the pension scheme provisions and valuation assumptions used. The results for the UK scheme is presented in Section 5. Section 6 contains the results for the stylised US scheme. Section 7 concludes. An illustration of the robustness of the results to a different stochastic economic model is contained in the Appendix.

## 2. Economic Capital Framework

The concept of economic capital is used extensively for the purpose of risk quantification of financial services firms and conglomerates, see for example Porteous and Tapadar (2005, 2008a,b). Porteous et al. (2012) and Yang and Tapadar (2015) extended the definition of economic capital to cover risk assessment of pension schemes. We adopt a modified version of the definition for the purposes of this article as stated below:

> **Definition:** *The economic capital of a pension scheme is the proportion by which its existing assets would need to be augmented in order to meet net benefit obligations with a prescribed degree of confidence. A pension scheme's net benefit obligations are all obligations in respect of current scheme members, including future service, net of future contributions to the scheme.*

Note that this definition is generic in nature and flexible in terms of time horizons and liability valuation methods.

For a mathematical formulation of economic capital, we need the following notation:

$A_t$**:** Value of pension scheme assets at time $t$.

$L_t$**:** Value of pension scheme liabilities at time $t$, i.e. the point estimate as quantified by the valuation actuary at time $t$.

$X_t$**:** Net cash (in)flow of the pension scheme at time $t$ (excluding investment returns).

$I_{(s,t)}$**:** Accumulation factor (accumulated value at time $t$, of a unit amount invested at time $s$, where $s \leq t$).

$D_{(s,t)}$**:** Discount factor, i.e. $D_{(s,t)} = I^{-1}_{(s,t)}$.

Given the history up to time 0, all the above entities $(A_t, L_t, X_t, I_{(s,t)}, D_{(s,t)})$ at future times $s$ and $t$ are random variables. In the first instance, we consider one single realisation, or simulation, of the future, over a specified time horizon $[0, T]$. For simplicity, we assume a discrete time framework with annual time intervals, i.e. all cash flows and valuation of assets and liabilities happen at annual intervals.

Based on this assumption, we define the profit vector, $P_t$ for $t = 0, 1, 2, \ldots T$ as:

$$P_t = L_{t-1}\, I_{(t-1,t)} - X_t - L_t, \text{ for } t > 0 \text{ and } P_0 = A_0 - X_0 - L_0. \tag{1}$$

We then define the present value of future profits over time horizon $[0, T]$ as:

$$V_0^{(T)} = \sum_{t=0}^{T} P_t\, D_{(0,t)}. \tag{2}$$

This can be equivalently expressed as:

$$V_0^{(T)} = A_0 - \sum_{t=0}^{T} X_t \, D_{(0,t)} - L_T \, D_{(0,T)}. \tag{3}$$

Intuitively, $V_0^{(T)}$ represents the amount of excess assets, on top of $A_0$, that is required to meet net cash outflow over $[0, T]$ and also to set up reserves, $L_T$, as quantified by the valuation actuary at time $T$, to meet future outstanding obligations beyond time $T$.

Under a run-off approach, the horizon $T$ is set until the time when the last of the current scheme members leaves the scheme either through death or withdrawal. We denote the run-off approach by $T = \infty$. Clearly, $L_\infty = 0$, and hence:

$$V_0^{(\infty)} = A_0 - \sum_{t=0}^{\infty} X_t \, D_{(0,t)}. \tag{4}$$

If, instead, a shorter time horizon is used for economic capital calculation, Equation 3 needs to be used. To illustrate the implications of the choice of time horizon for economic capital calculations, we use both the run-off approach and a shorter 3-year time horizon approach. We choose a time horizon of 3 years, because in the UK there is a regulatory requirement to carry out triennial valuations, so risk assessment over a 3-year time horizon would be a natural choice. For this horizon, the expression for present value of future profits becomes:

$$V_0^{(3)} = A_0 - \sum_{t=0}^{3} X_t \, D_{(0,t)} - L_3 \, D_{(0,3)}. \tag{5}$$

Clearly, $V_0^{(3)}$ depends on the assumptions used by the valuation actuary at the end of year 3 to determine $L_3$. For a prescient actuary, with perfect foresight, her future liability calculation for $L_3$ would be exact and so in that case $V_0^{(3)} = V_0^{(\infty)}$. However, in reality, a valuation actuary can only produce an estimate of $L_3$, based on historical evidence up until that time and also the future economic and mortality outlook.

One of the key drivers in this calculation is the valuation discount rate assumption. The choice of discount rate has a material impact on the estimated value of $L_3$, and consequently $V_0^{(3)}$. The code of practice, issued by The Pensions Regulator of the UK, provides practical guidance to trustees and employers of UK DB schemes on how to comply with the scheme funding requirements under the Pensions Act 2004. In terms of setting valuation discount rates, the code of practice stipulates in item 125[1]:

> *Discount rates used in setting technical provisions must be chosen prudently, taking into account either:*

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[1] https://www.thepensionsregulator.gov.uk/en/document-library/codes-of-practice/code-3-funding-defined-benefits-

- *the yield on assets held by the scheme to fund future benefits and the anticipated future investment returns; and/or*
- *the market redemption yields on government or high quality bonds.*

In Sections 5 and 6, we discuss a few different approaches that might be taken to set valuation discount rates based on the principles set out in the code of practice and the implications of adopting these.

We use $V_0^{(T)}$ to measure the risk of a DB scheme. However, as we will be assessing risks of pension schemes in two different countries, with different currencies and different benefit structures, it will be useful to express $V_0^{(T)}$ as a percentage of initial assets $A_0$, so that there is a standardisation in terms of currency and scale. This is also consistent with our economic capital definition and can be interpreted as the proportional increase in assets required to meet all future benefit obligations in a particular scenario.

$V_0^{(T)}$ is defined above for a single specimen realisation of the future. As the future is unknown at time 0, all future components in $V_0^{(T)}$ are random variables, and hence $V_0^{(T)}$ is itself a random variable. The actual economic capital risk measure can be mathematically defined in terms of the distribution of $V_0^{(T)}$. Two common measures used in the literature are:

- **Value-at-risk**, $VaR$, is defined as $\mathbb{P}\left[V_0^{(T)} \leq VaR\right] = p$, for a given probability $p$. Typically $VaR$ is negative and $(-VaR)$ represents the amount of additional initial assets required at time 0 (on top of existing assets) for the pension scheme to meet all its future obligations with probability, or confidence level, $(1-p)$.
- **Expected shortfall**, $ES$, is defined as the average of all losses that are greater than or equal to the value of $VaR$ for a given probability level $p$, i.e. $\mathbb{E}\left[V_0^{(T)} \middle| V_0^{(T)} \leq VaR\right]$. In other words, $ES$ provides an estimate of the expected value of losses in the worst $p$ proportion of cases.

These definitions of $VaR$ and $ES$ are based on McNeil et al. (2015). For all our results, we present the entire distribution of $V_0^{(T)}$ in conjunction with $VaR$ and $ES$ at certain probability levels.

## 3. Stochastic Economic and Demographic Models

To determine the distribution of $V_0^{(T)}$, we need to simulate future economic and demographic scenarios. In Section 3.1 we discuss the Economic Scenario Generators (ESGs) we have used for our analysis. Section 3.2 provides details of the stochastic mortality models we have used to project mortality of UK and US pension scheme members. For generating the distributions of $V_0^{(T)}$ for different scenarios, we have used 10,000 simulation runs in each case.

### 3.1 ECONOMIC SCENARIO GENERATOR

Oberoi et al. (2020) propose a graphical model approach to develop an ESG suitable for use by a life insurance company or a pension scheme in the UK that invests in equities and bonds. Andrews et al. (2019) extend this approach to develop an ESG for the US economy. In this section, we provide a brief outline of the graphical model methodology and the relevant parameterisation for the UK and the US ESGs, which we will use in this article.

A minimal ESG, which can be used to analyse DB scheme risks, requires data for price inflation ($I$), salary inflation ($J$), stock returns and bond returns. Following the approach of Wilkie (1986), Oberoi et al. (2020) use dividend yield ($Y$), dividend growth ($K$) and Consols yield ($C$) to construct stock and bond returns. The dataset used by Oberoi et al. (2020) consists of annual values of these economic variables from 1926 to 2017 as at the end of June each year.

Andrews et al. (2019) obtain US data from two sources. The first is Robert Shiller, who provides online data for price inflation, S&P 500 Index, S&P 500 High Dividend Index and bond yields. The second source is Emmanuel Saez, who provides online data for average wages in the US. The US data extend from 1913 to 2015.

In a graphical ESG, first a univariate time series model, typically an AR(1) process, is fitted to each individual economic variable, $X_t$ as follows:

$$\mu_x = \mathbb{E}\left[X_t\right], \tag{6}$$

$$Z_t = X_t - \mu_x, \tag{7}$$

$$Z_t = \beta_x \, Z_{t-1} + e_{x,t}, \quad \text{where } e_{x,t} \sim N\left(0, \sigma_x^2\right). \tag{8}$$

The parameter estimates of $\mu$, $\beta$ and $\sigma$ for the chosen economic variables in the UK and the US are given in Table 1.

Table 1: ESG time series parameter estimates.

| | UK | | | US | | |
|---|---|---|---|---|---|---|
| | $\mu$ | $\beta$ | $\sigma$ | $\mu$ | $\beta$ | $\sigma$ |
| $I_t$ | 0.0404 | 0.6102 | 0.0387 | 0.0328 | 0.6211 | 0.0392 |
| $J_t$ | 0.0528 | 0.7801 | 0.0282 | 0.0464 | 0.4908 | 0.0643 |
| $Y_t$ | 0.0468 | 0.6718 | 0.0085 | 0.0413 | 0.8293 | 0.0100 |
| $K_t$ | 0.0527 | 0.4263 | 0.0852 | 0.0507 | 0.2746 | 0.1084 |
| $C_t$ | 0.0617 | 0.9674 | 0.0083 | 0.0489 | 0.9346 | 0.0091 |

Once the residuals, $e_{x,t}$ are obtained for each variable $X_t$, a Gaussian graphical model is fitted to the multivariate residuals:

$$\mathbf{e_t} = (e_{I_t}, e_{J_t}, e_{Y_t}, e_{K_t}, e_{C_t}) \sim \mathcal{N}(\mathbf{0}, \Sigma), \tag{9}$$

separately for the UK and the US data. Estimation is carried out based on maximum likelihood and the resulting partial correlations are shown in Table 2.

Table 2: Partial correlations of residuals.

| | UK | | | | | US | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $I_t$ | $J_t$ | $Y_t$ | $K_t$ | $C_t$ | $I_t$ | $J_t$ | $Y_t$ | $K_t$ | $C_t$ |
| $I_t$ | 1 | | | | | 1 | | | | |
| $J_t$ | 0.48 | 1 | | | | 0.42 | 1 | | | |
| $Y_t$ | 0.16 | 0.11 | 1 | | | 0.20 | -0.47 | 1 | | |
| $K_t$ | 0.18 | 0.15 | -0.06 | 1 | | 0.17 | 0.10 | 0.28 | 1 | |
| $C_t$ | 0.20 | -0.09 | 0.37 | 0.06 | 1 | 0.19 | 0.04 | 0.12 | -0.06 | 1 |

Clearly, some of the partial correlations in the matrices are small. Graphical models provide a means to identify the minimum number of correlations which describe the underlying data adequately. In other words, it is a dimension reduction tool whereby correlations of all pairs of residuals need not be stipulated directly. Instead, in this approach, only the correlations between variables that are not conditionally independent are used.

Oberoi et al. (2020) use a number of statistical criteria, namely AIC, BIC and simultaneous $p$-values, to determine the optimal graphical model for the UK data. Model $E6$, shown in the top panel of Figure 1, is identified by Oberoi et al. (2020) as a model that satisfies certain desirable features and also possesses an intuitively appealing structure. For the purposes of this article, we use Model $E6$ as our ESG for the UK economy.

Andrews et al. (2019) use the same approach to determine the graphical structure for the US economic data. Interestingly, for the US data, all three statistical criteria, i.e. AIC, BIC and simultaneous $p$-values, produce the same graphical structure, which is shown in the bottom panel of Figure 1. We use this graphical ESG to model the US economy.

We have also checked that qualitatively, all the results and conclusions of our analysis are broadly very similar, regardless of the ESG employed. For completeness, the results for the UK scheme, using the well-known Wilkie model (Wilkie, 1986, 1995; Wilkie et al., 2011; Wilkie and Şahin, 2016, 2017a,b,c, 2018) are also included in the Appendix. We find that although, as expected, there are some quantitative differences between the numbers obtained from the Wilkie model and the graphical ESG, qualitatively the results point to very similar conclusions.

### 3.2 STOCHASTIC MORTALITY MODELS

Future projections of pension scheme cash flows also depend on the mortality assumptions. Cairns et al. (2009) provide a quantitative comparison of eight stochastic mortality models using data from England and Wales and the US, of which Model M7 provides a good fit for both UK and US data. For the purposes of this article, we use model M7 to project forward stochastic mortality rates of the UK and the US.

The structure of model M7, which models $q(t, x)$, the probability that an individual aged $x$ at time $t$ will die between times $t$ and $t + 1$, is as follows:

$$\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)} \left[(x - \bar{x})^2 - \sigma^2\right] + \gamma_{(t-x)}^{(4)}, \tag{10}$$

where $x$ is the age, $\bar{x}$ is average age, $\sigma^2$ is the average of $(x-\bar{x})^2$, $\kappa_t^{(i)}$ are the period effects and $\gamma_{t-x}^{(i)}$ is the cohort effect.

We parameterise model M7 using UK and US data from the Human Mortality Database for both males and females from 1961 to 2014 for ages 30–100.

Projecting future mortality rates involves projecting the time series $\kappa_t^{(i)}$ and $\gamma_{t-x}^{(i)}$ forward. Systematic risk arises from the uncertainty involved in projecting these time series. For example, if the mortality rates improve faster than expected then future $q(t, x)$ will be lower, which in turn will result in lower deaths. Cairns et al. (2011) suggest possible approaches to project mortality parameters forward based on the historical estimates of these parameters. For our purpose, we project $\kappa_t^{(i)}$ linearly over time. Given that we are not interested in future cohorts (but only in existing ones), we do not project $\gamma_{t-x}^{(i)}$ forward.

## 4. PENSION SCHEME ASSUMPTIONS

As a representative UK pension scheme, we use the model of UK's USS as developed by Andrews et al. (2019). For the US, we use a stylised US scheme using the same membership profile as the UK scheme, to ensure a certain degree of comparability in our analysis. However, to capture US-specific pension scheme features, we have incorporated a number of changes to the benefit structure for the stylised US scheme, as discussed in Section 4.2.

### 4.1 UK PENSION SCHEME ASSUMPTIONS

In this section, we provide a brief overview of our assumptions relevant to the UK scheme. Andrews et al. (2019) provide a detailed description. The UK scheme model is based on the USS, which is one of the largest open DB schemes operating in the UK, with more than 350 participating employers and approximately 400,000 scheme members. The assumptions presented are based on the actuarial valuation carried out for the scheme on March 31, 2014.

### 4.1.1 *Membership Profile*

The broad membership profile for the UK scheme is shown in Table 3. To capture the overall risk characteristics and the inter-generational risk dynamics of the scheme, we need a range of model points specifically for active members. So we use an age distribution for active members, as shown in Table 4. Table 4 also provides past service and salary assumptions for each model point, which have been set so that the average past service and average salary of active members broadly match the figures from Table 3. For deferred members and pensioners, we use single model points, based on the values given in Table 3, to represent each of these membership categories. We do not use a range of model points for deferred members and pensioners, as this does not have a material impact on our results and conclusions. We also assume a 50:50 gender split and no salary differential between genders for all membership categories.

Table 3: UK scheme membership profile.

| | | |
|---|---|---|
| Active members | Number | 167,545 |
| | Average pensionable salary | £42,729 |
| | Average age | 43.8 |
| | Average past service | 12.5 |
| Deferred members | Number | 110,430 |
| | Average deferred pension | £2,373 |
| | Average age | 45.1 |
| Pensioners | Number | 70,380 |
| | Average deferred pension | £17,079 |
| | Average age | 71.1 |

### 4.1.2 *Benefit Structure*

The pension benefits are comprised of an annual pension and a cash lump sum at retirement, calculated as follows:

$$\text{Annual pension} = \text{Pensionable salary} \times \text{Pensionable service} \times \text{Accrual rate};$$
$$\text{Lump sum} = 3 \times \text{Annual pension}.$$

To capture some of the USS-specific features of the benefit structure, while keeping the model simple, we assume that all members accrue benefits on a *final salary* basis up

Table 4: UK scheme model points, past service and salary of active members.

| | Age | Proportion | Number | Past service | Salary |
|---|---|---|---|---|---|
| | 30 | 30% | 50,264 | 7 | £25,500 |
| | 40 | 30% | 50,264 | 11 | £42,500 |
| | 50 | 20% | 33,509 | 15 | £52,500 |
| | 60 | 20% | 33,509 | 19 | £58,500 |
| Total | | 100% | 167,545 | | |
| Average | 43 | | | 12.2 | £42,600 |

to March 31, 2014; and from April 1, 2014 onward all members move to a *career revalued* basis. The accrual rate for the final salary and career revalued schemes are 1/80th and 1/75th respectively.

Annual pension is assumed to increase in line with price inflation, subject to a 5% limit. Members' salaries increase in line with salary inflation. In addition to salary inflation, there is an explicit age-based promotional salary scale. For our analysis, future projections of price inflation and salary inflation are generated by the ESG described in Section 3.1.

For members who withdraw from the scheme, a deferred inflation-linked pension is provided based on accrued service. Inflation indexation of salary is provided between the date the member withdraws from the scheme and the date of retirement.

On the death of an active member, a lump sum payment of three times the annual salary is paid at the time of death, along with a spouse's pension of half the amount of pension the member would have received if he or she had survived until normal retirement. On the death of a deferred pensioner, a lump sum equal to the present value of the deferred lump sum payable at normal retirement is provided along with a spouse's pension of half the amount of the deferred pension payable at normal retirement. On the death of a pensioner, a spouse's pension of half the amount of the member's pension is payable. All spousal pensions commence on the date of the scheme member's death.

#### 4.1.3 *Assets, Liabilities and Contributions*

On March 31, 2014, USS's total value of assets was £41.6 billion, of which approximately 70% was invested in real assets and 30% in fixed assets. For the purposes of this article, we assume an asset allocation of 70% in equities and 30% in bonds.

The 2014 valuation report also provides an estimate of the scheme liabilities as calculated by the valuation actuary using the projected unit method, which is a prospective valuation method in which liabilities are estimated based on the past service accrued on the valuation date, taking into account future salary inflation. The estimated value of liabilities was £46.9 billion, giving an initial valuation deficit of £5.3 billion.

We assume a contribution rate of 22.5% of salary.

### 4.2 Stylised US pension scheme

For our analysis, we use a hypothetical stylised US DB scheme with the same membership profile as for the UK scheme. In this section, we outline only those features which are specific to the stylised US scheme. All other assumptions are the same as that for the UK scheme, except monetary amounts, which are in US dollars.

#### 4.2.1 *Benefit Structure*

For the stylised US scheme, there is no lump sum paid on retirement, while pensionable salary is calculated on a *final salary* basis. The accrual rate is set at 1/66th (or 1.5%). Crucially, there is no inflation indexation of pension during the payment period.

For members who withdraw from the scheme, a deferred pension is provided based on accrued service. Again, no inflation indexation of salary is provided between the date the member withdraws from the scheme and the date of retirement. There is also no inflation indexation during the payment period.

On the death of an active member, a lump sum equal to the present value of the pension the member would have received if he or she had survived until normal retirement is paid at the time of death. On the death of a deferred pensioner, a lump sum equal to the present value of the pension the member would have received if he or she had survived until normal retirement is paid at the time of death. On the death of a pensioner, a spouse's pension of half the amount of the member's pension is payable.

#### 4.2.2 *Assets, Liabilities and Contributions*

To ensure comparability, we assume the same asset allocation strategy of 70% equities and 30% for the stylised US scheme. On this assumption, we obtain a future contribution rate of 6.5% of salary, calculated using the projected unit standard contribution rate.

The projected unit method can also be used to obtain an estimate of the actuarial liability of the stylised US scheme. Using the prevailing long-term US bond yield on the valuation date, we get an estimated value of liabilities of $47.5 billion.

We further assume that the stylised US scheme has assets worth $44.5 billion on March 31, 2014, to give an initial valuation deficit of $3 billion. We will see in Section 6 that this particular assumption leads to the same median value for the distributions of $V_0^{(\infty)}$ (as a percentage of $A_0$) for both the UK scheme and the stylised US scheme, providing us with a common frame of reference.

## 5. UK Pension Scheme Results

In this section, we examine the results for the UK scheme using the graphical ESG. The results for the UK scheme using the Wilkie model are contained in Appendix A. Qualitatively, the results are the same, regardless of the ESG employed, though some of the details are a little different.

We consider three different cases separately for the run-off and 3-year time horizon:

- the base case, in order to establish a baseline for the distributions of $V_0^{(\infty)}$ and $V_0^{(3)}$;
- the impact of changing the asset allocation to high bond content (30/70 in equity/bond) from the baseline allocation of high equity content (70/30 in equity/bond); and
- the impact of changing the contribution rate to 30% (high) and 15% (low), relative to the baseline contribution rate of 22.5% of salary.

### 5.1 Base case

The top panel of Figure 2 shows the density of $V_0^{(\infty)}$ (as a percentage of $A_0$) for the UK scheme. The second and third panels show the sensitivity of this distribution to changes in the asset allocation and the contribution rate, which we discuss in detail in Sections 5.2 and 5.3, respectively. Table 5 provides all the summary statistics.

In this section, we first focus on the base case run-off result shown in the top panel of Figure 2 and the corresponding summary statistics in Table 5. We observe:

1. The distribution has a long left skew, indicating the possibility of a significant deficit, albeit with a diminishing probability.
2. The median of the distribution is 24% of $A_0$, indicating that on *average* the scheme has more than sufficient assets to meet all its future obligations on a run-off basis.
3. The $10^{th}$ percentile of the distribution is $-36\%$ of $A_0$, which implies that if the scheme had 36% additional assets, it would meet all its future obligations with a 90% probability. The probability of a shortfall is 27%.
4. Going further into the left tail, we find that the scheme would require 148% additional assets to ensure a 99.5% chance of meeting all its future obligations.

Continuing with the base case, but now focusing on a shorter 3-year time horizon, Figure 3 shows the density of $V_0^{(3)}$ based on a number of alternative approaches that a valuation actuary might take to value the liabilities, $L_3$, at the end of year 3. In any specific valuation, the valuation actuary will use their judgement to select a particular

Table 5: Summary statistics of the results for the UK scheme using the graphical ESG.

| | | $50^{th}$ percentile | | $10^{th}$ percentile | | $0.5^{th}$ percentile | |
|---|---|---|---|---|---|---|---|
| Scenarios | $P[V_0 \leq 0]$ | VaR | ES | VaR | ES | VaR | ES |
| $V_0^{(\infty)}$ | | | | | | | |
| Base case | 0.27 | 24 | -14 | -36 | -74 | -148 | -192 |
| High bond allocation | 0.68 | -23 | -73 | -103 | -146 | -230 | -258 |
| High contribution | 0.19 | 35 | -1 | -22 | -58 | -129 | -172 |
| Low contribution | 0.37 | 13 | -27 | -50 | -89 | -168 | -211 |
| No contribution | 0.58 | -9 | -53 | -79 | -121 | -205 | -251 |
| $V_0^{(3)}$ **(Valuation rate based only on long-term bond yields)** | | | | | | | |
| Base case | 0.92 | -94 | -197 | -257 | -364 | -606 | -734 |
| High bond allocation | 0.92 | -99 | -197 | -252 | -349 | -550 | -673 |
| High contribution | 0.91 | -89 | -193 | -252 | -359 | -601 | -729 |
| Low contribution | 0.94 | -98 | -202 | -262 | -368 | -608 | -739 |
| $V_0^{(3)}$ **(Valuation rate based on returns on backing assets)** | | | | | | | |
| Base case | 0.42 | 22 | -153 | -267 | -468 | -880 | -1103 |
| High bond allocation | 0.59 | -15 | -127 | -191 | -326 | -608 | -755 |
| High contribution | 0.40 | 27 | -148 | -262 | -463 | -875 | -1097 |
| Low contribution | 0.43 | 18 | -157 | -272 | -473 | -885 | -1109 |
| $V_0^{(3)}$ **(Valuation rate converging to long-term returns after 50 years)** | | | | | | | |
| Base case | 0.38 | 26 | -70 | -129 | -227 | -411 | -508 |
| High bond allocation | 0.56 | -9 | -83 | -128 | -203 | -353 | -423 |
| High contribution | 0.36 | 31 | -65 | -124 | -221 | -406 | -501 |
| Low contribution | 0.39 | 22 | -74 | -134 | -232 | -418 | -514 |
| $V_0^{(3)}$ **(Valuation rate converging to long-term returns after 15 years)** | | | | | | | |
| Base case | 0.30 | 30 | -25 | -58 | -110 | -206 | -262 |
| High bond allocation | 0.52 | -2 | -55 | -87 | -133 | -227 | -264 |
| High contribution | 0.27 | 35 | -20 | -53 | -104 | -199 | -255 |
| Low contribution | 0.32 | 26 | -29 | -63 | -115 | -211 | -268 |

discount rate based on the code of practice, current data (such as on bond yields and inflation expectations), and long-term expected returns. In our stochastic projections, it is necessary to select an appropriate discount rate at several points in the future, and for 10,000 different scenarios. As such, whatever approach is taken will need to be mechanistic in nature and, perhaps, somewhat arbitrary.

We outline four possible approaches to select the valuation discount rate assumption and present the results based on these assumptions. In each panel of Figure 3, we also include the density of $V_0^{(\infty)}$, as a grey curve, for reference.

1. Firstly, we assume a valuation discount rate based solely on the prevalent bond yield at the end of 3 years. For the USS 2014 valuation, a discount rate of 5.2% was used, when the long-term bond yield was approximately 3.4%, i.e. a spread of 1.8%. In the top-left panel of Figure 3, we show the density of $V_0^{(3)}$ (as a percentage of $A_0$), when $L_3$ is calculated using the prevalent (simulated) long-term bond yield at the end of 3 years plus a spread of 1.8%. Clearly, this approach produces a very different distribution of $V_0^{(3)}$, as compared to $V_0^{(\infty)}$, with a large median loss and a large negative $10^{th}$ percentile.

   One characteristic that stands out is that the left tail of the distribution is much fatter than the one for the full run-off. The reason for the fatter left tail of $V_0^{(3)}$, as compared to $V_0^{(\infty)}$, is the use of a constant valuation discount rate to calculate $L_3$. If the long-term bond yield in the third year is low, this low yield is assumed to continue as a constant, indefinitely into the future for calculating $L_3$, which is unrealistic and is also unlikely to happen in the simulated realisations of the ESG, hence $V_0^{(\infty)}$ does not have such fat tails as $V_0^{(3)}$ in this case.

2. A more realistic valuation rate to calculate $L_3$ would be to use the returns earned on the backing pension scheme assets, reflecting the actual asset allocation strategy. The top-right panel of Figure 3 shows the result if the third year's returns (weighted by asset allocation) are used as the discount rate for all future years. Now the distribution of $V_0^{(3)}$ has approximately the same median as $V_0^{(\infty)}$, but has a very large dispersion (in both tails), as is expected, because any fluctuations in returns in the third year get magnified due to the use of a constant valuation discount rate to calculate $L_3$.

3. However, using returns earned only in the third year as a discount rate for all future years is also unrealistic. It would be more appropriate to use a time varying (but deterministic) discount rate to calculate $L_3$, where the discount rate increases (or decreases) from its value at the end of the third year to its long-term mean over a certain period of time. The bottom-left and bottom-right panels of Figure 3 show the result if the time period of convergence to the long-term mean is 50 years and

15 years, respectively. If the convergence to the long-term mean is slow (50 years), the dispersion is still large, but smaller than before. If the convergence to long-term mean is quicker (15 years), the distributions of $V_0^{(3)}$ and $V_0^{(\infty)}$ get closer.

In Figure 4, we show the scatter plots, or joint densities, of $V_0^{(3)}$ and $V_0^{(\infty)}$, in the form of heatmaps, for all four approaches. The heatmaps represent densities that integrate to 1, with the same colour representing the same quantile level and darker shades representing higher density values.

If the valuation actuary could foresee the future, the observations would all fall on the 45° line. The scatter plots show how for 3-year time horizon results, using a constant long-term bond yield as the valuation discount rate almost always under-estimates $V_0$. This is a matter of concern because such under-estimation of $V_0$ could lead to a misleading perception of insufficient funding levels for an otherwise healthy scheme on a run-off basis.

The estimates improve when the backing assets are used to obtain the valuation discount rate, and they improve further if the rate is adjusted over 15 years to converge to the long-run mean.

### 5.2 SENSITIVITY TO HIGHER BOND ALLOCATION STRATEGY

The middle panel of Figure 2 shows the density of $V_0^{(\infty)}$ (as a percentage of $A_0$), when the bond allocation of the UK pension scheme is increased from 30% to 70%. For ease of reference, the density of the base case result is also included as a grey curve in the background. We make the following observations:

1. For increased bond investment, the distribution of $V_0^{(\infty)}$ has moved to the left and has greater dispersion.

2. The leftward shift of the distribution indicates a greater probability of larger deficits. This is reflected in the median of $V_0^{(\infty)}$, which becomes a deficit of 23% as compared to a surplus of 24% for the base case. The $10^{th}$ and $0.5^{th}$ percentiles have moved by even greater magnitudes (see Table 5).

3. The sensitivity patterns can be explained by the fact that the expected returns from bonds are lower in the long term compared to equities. So, a higher allocation to bonds can lead to potentially lower levels of cumulative assets, which is reflected in the leftward shift and greater dispersion in the distribution.

4. Moreover, fixed interest bonds are a poor match for real liabilities (the UK scheme benefits are inflation-protected). Hence, an increased allocation to nominal bonds has exacerbated the risk, producing a fatter-tailed distribution.

Figure 5 shows the sensitivity of the results to a higher bond allocation strategy based on a 3-year time horizon. The most important observation here is that as the bond allocation of the scheme increases, a valuation basis based on bonds will produce results where the distributions of $V_0^{(3)}$ starts getting closer to that of $V_0^{(\infty)}$, as expected.

We summarise the main takeaways from the analysis so far as follows:

- It is natural for both pension regulators and scheme members to expect periodic assessments of the financial status of a pension scheme. For a triennial valuation, where the valuation discount rate is based only on long-term bond yields (plus an arbitrary fixed spread to allow for excess equity returns), the UK scheme appears severely underfunded. Looking at the base case results for $V_0^{(3)}$ given in Table 5, the scheme appears to have a staggering 92% chance of insufficient assets at the end of 3 years, with an average deficit of 94% of the current value of assets.

- However, on a run-off approach, the chance of a shortfall is only 27%, and there is an average *surplus* of 24% of current assets. Note that the huge apparent deficit over a three year horizon is an artefact of using a valuation basis that is not consistent with the actual underlying asset allocation strategy. The true financial status is actually unaffected by how the valuation actuary is required to value future liabilities.

- Now, if in response to a requirement to use long-term bond yield for liability valuation, the UK scheme tries to address the mismatch by changing its actual asset allocation strategy to involve greater bond investment, it is materially detrimental to the pension scheme over the long term. Note that the distribution of $V_0^{(3)}$ is broadly unaltered by the change in asset allocation as the valuation discount rate is unaffected by the change. However, as shown in Table 5, increasing the bond allocation from 30% to 70% now creates a *true* deficit for the scheme on a run-off basis. By increasing the bond allocation, the scheme has gone from 24% excess assets to 23% underfunded, on average. Moreover, now the chance of assets not being able to meet all future liabilities has increased from 27% to 68%.

- The impact of a 3-year assessment is somewhat less severe if the valuation discount rate is based on the returns on backing assets converging to the long-term returns over an appropriate duration.

### 5.3 SENSITIVITY TO CONTRIBUTION RATES

The bottom panel of Figure 2 shows the distribution of $V_0^{(\infty)}$ (as a percentage of $A_0$) when the contribution rate is increased or decreased by 7.5% of salary; and also if there were no future contributions. For ease of reference, the density of the base case result with a contribution rate of 22.5% is also included as a grey curve in the background.

The main observation here is that increasing or decreasing contribution rates by ±7.5% has the effect of shifting the median of the distribution by ±9% (of $A_0$). The relative shifts in the left tail increase slightly as we move further into the tails of the distributions. However the impacts are small compared to the impact of a higher bond allocation. In fact, the impact at the median of moving the bond allocation from 30% to 70% is more severe than eliminating all future contributions as shown in Figure 2.

Figure 6 shows the sensitivity of the results towards a higher or lower contribution rate when using a 3-year time horizon. Firstly, the directions of the shifts in the distributions are as expected, but the magnitudes of the shifts are much smaller than for the run-off case. This is an artefact of the valuation methodology adopted, i.e. the projected unit method, to calculate $L_3$, in which the actuarial liability is the discounted value of *accrued benefits* to date (i.e., ignoring future accruals based on future service and future contributions). Hence the liability calculations do not take into account future contributions. As a result, any contribution increase or decrease beyond year 3 has no impact on $L_3$. So the shifts in the distributions in the 3-year time horizon results are solely due to the higher or lower contributions only over the time-period $[0, 3]$.

The main conclusions that can be drawn from these results are as follows:

- The impact of changes in contribution rates is smaller compared to changes in asset allocation strategy. In particular, if a short time horizon is employed, the contribution rate change needs to be extremely large in order to have any meaningful impact on pension scheme risk.
- If instead of changing asset allocation strategy, a pension scheme wants to address any apparent deficit in $V_0^{(3)}$ using higher contribution rates, the scheme will have to raise the contribution rates by enormous amounts to have a material impact on the distribution of $V_0^{(3)}$.

## 6. STYLISED US PENSION SCHEME RESULTS

As with the UK scheme, for the stylised US scheme, we consider the following cases separately for the run-off and 3-year time horizon cases:

- the base case, in order to establish a baseline;
- the impact of changing the asset allocation to high bond content (30/70 in equity/bond) from the baseline allocation of high equity content (70/30 in equity/bond); and
- the impact of changing the contribution rate to 10% (high) and 3% (low), relative to the baseline contribution rate of 6.5% of salary.

### 6.1 BASE CASE

The top panel of Figure 7 shows the distribution, $V_0^{(\infty)}$ (as a percentage of $A_0$) for the stylised US scheme. The second and third panels show the sensitivity of this distribution

Table 6: Summary of results for the stylised US scheme using the graphical ESG.

| | | $50^{th}$ percentile | | $10^{th}$ percentile | | $0.5^{th}$ percentile | |
|---|---|---|---|---|---|---|---|
| Scenarios | $P[V_0 \leq 0]$ | VaR | ES | VaR | ES | VaR | ES |
| $V_0^{(\infty)}$ | | | | | | | |
| Base case | 0.28 | 24 | -17 | -40 | -86 | -194 | -267 |
| High bond allocation | 0.53 | -3 | -46 | -69 | -111 | -205 | -280 |
| High contribution | 0.22 | 31 | -8 | -28 | -71 | -170 | -237 |
| Low contribution | 0.33 | 18 | -25 | -48 | -95 | -201 | -270 |
| No contribution | 0.38 | 13 | -33 | -58 | -108 | -219 | -298 |
| $V_0^{(3)}$ **(Valuation rate based only on long-term bond yields)** | | | | | | | |
| Base case | 0.62 | -17 | -66 | -95 | -134 | -209 | -284 |
| High bond allocation | 0.66 | -20 | -58 | -79 | -108 | -166 | -240 |
| High contribution | 0.60 | -14 | -63 | -91 | -130 | -202 | -275 |
| Low contribution | 0.63 | -19 | -69 | -97 | -137 | -209 | -283 |
| $V_0^{(3)}$ **(Valuation rate based on returns on backing assets)** | | | | | | | |
| Base case | 0.34 | 34 | -58 | -120 | -200 | -330 | -416 |
| High bond allocation | 0.43 | 10 | -56 | -99 | -160 | -275 | -337 |
| High contribution | 0.33 | 36 | -55 | -116 | -196 | -326 | -406 |
| Low contribution | 0.35 | 31 | -60 | -122 | -202 | -335 | -416 |
| $V_0^{(3)}$ **(Valuation rate converging to long-term returns after 50 years)** | | | | | | | |
| Base case | 0.28 | 38 | -26 | -67 | -124 | -224 | -300 |
| High bond allocation | 0.39 | 15 | -38 | -71 | -116 | -205 | -270 |
| High contribution | 0.27 | 40 | -23 | -63 | -120 | -219 | -290 |
| Low contribution | 0.29 | 36 | -29 | -69 | -126 | -227 | -299 |
| $V_0^{(3)}$ **(Valuation rate converging to long-term returns after 15 years)** | | | | | | | |
| Base case | 0.21 | 41 | -5 | -33 | -76 | -156 | -230 |
| High bond allocation | 0.34 | 19 | -23 | -47 | -84 | -153 | -226 |
| High contribution | 0.20 | 43 | -2 | -30 | -72 | -150 | -220 |
| Low contribution | 0.22 | 39 | -8 | -36 | -78 | -158 | -229 |

to changes in the asset allocation and the contribution rate, which we discuss in detail in Sections 6.2 and 6.3, respectively. Table 6 provides all the summary statistics.

Focusing on the base case run-off result shown in the top panel of Figure 7 and the corresponding summary statistics in Table 6, we first observe its similarity with the corresponding result for the UK scheme as can be seen in Figure 2 and Table 5. The medians of the two distributions are the same, the probability of shortfall and the $10^{th}$ percentile are also broadly similar. However, the $0.5^{th}$ percentile for the stylised US scheme is farther to the left than that for the UK scheme, indicating that the distribution for the stylised US scheme has a fatter left tail.

The similarities between the base case run-off results for the UK and the stylised US schemes are partly by design to ensure a certain level of comparability. Recall that the membership profiles of the two schemes are the same. Also the starting asset value of the stylised US scheme is chosen specifically to produce the same median value of $V_0^{(\infty)}$. Although the benefit structures of the two schemes are different, with the stylised US scheme being less generous with no inflation indexation of benefits, this is compensated by a lower contribution rate.

However the fatter left tail of the distribution for the stylised US scheme is striking. This is the result of higher fixed guarantees involved in non-indexation of benefits. This feature plays a crucial role in the rest of the analysis for the stylised US scheme.

Continuing with the base case, but now focusing on a shorter 3-year time horizon, Figure 8 shows the same alternative approaches as for the UK scheme that a valuation actuary might take to value the liabilities, $L_3$, at the end of year 3.

1. Firstly, we look at valuation discount rate based solely on the prevalent bond yield at the end of 3 years. Unlike for the UK scheme, where we added a spread to the long-term bond yield for consistency with the USS 2014 valuation, for the US scheme we do not introduce this complexity of adding a fixed spread.

   In the top-left panel of Figure 8, we show $V_0^{(3)}$ (as a percentage of $A_0$), when $L_3$ is calculated using the prevalent long-term bond yield at the end of 3 years. Clearly, this approach produces a very different distribution of $V_0^{(3)}$, as compared to $V_0^{(\infty)}$, with a large median loss and a large negative $10^{th}$ percentile.

   However, it is instructive to observe that the difference between the distributions of $V_0^{(3)}$ and $V_0^{(\infty)}$ for the stylised US scheme is not as drastic as that for the UK scheme. This is due to the fact that for the UK scheme, fixed interest bonds are a poor match for real liabilities. However for the stylised US scheme with non-indexed benefits, liabilities calculated using long-term bond yield are still poor, but less so.

2. The top-right panel of Figure 8 shows the result if the third year's returns (weighted by asset allocation) are used as the discount rate for all future years. The bottom-left and bottom-right panels of Figure 8 show the result if the time period of convergence

to long-term mean is 50 years and 15 years, respectively. Commentary on these panels is the same as for the UK scheme.

Similar to the UK scheme results, we show the scatter plots, or joint densities, of $V_0^{(3)}$ and $V_0^{(\infty)}$ for the stylised US scheme in Figure 9. Although using a constant long-term bond yield as the valuation discount rate underestimates $V_0$, it is less prominent for the stylised US scheme than that for the UK scheme. However, the estimates do improve when returns based on backing assets, in conjunction with convergence to the long-run mean, are used to obtain the valuation discount rate.

### 6.2 Sensitivity to higher bond allocation strategy

The middle panel of Figure 7 shows the distribution of $V_0^{(\infty)}$ (as a percentage of $A_0$), when the bond allocation of the stylised US scheme is increased from 30% to 70%. For ease of reference, the density of the base case result is also included as a grey curve in the background. We make the following observations:

1. Similar to the UK scheme with increased bond investment, the distribution of $V_0^{(\infty)}$ has moved to the left and has greater dispersion.
2. The leftward shift of the distribution indicates a greater probability of larger deficits. This is reflected in the median of $V_0^{(\infty)}$, which shows a deficit of 3% as compared to a surplus of 24% for the base case results.
3. The leftward shift can be explained by the fact that the expected returns from bonds are lower in the long term compared to equities, leading to potentially larger losses.
4. However, the leftward shift of the distribution for the stylised US scheme is less severe than that for the UK scheme. As the UK scheme benefits are fully inflation-protected, high fixed interest investment is far more detrimental for the UK scheme.

Figure 10 shows the sensitivity of the results to a higher bond allocation strategy when using a 3-year time horizon. The most important observation here is that as the bond allocation of the scheme increases, a valuation basis based on bonds will produce results where the distributions of $V_0^{(3)}$ starts getting closer to that of $V_0^{(\infty)}$, as expected.

As for the UK scheme, we provide a summary of the main takeaways from the analysis so far for the stylised US scheme:

- If we consider a triennial valuation, where the valuation discount rate is based only on long-term bond yields, the scheme appears severely underfunded. Looking at the results for $V_0^{(3)}$ given in Table 6, under the base case assumption, the scheme appears to have a 62% chance of insufficient assets at the end of 3 years, with an apparent median deficit of 17%. On the other hand, using the run-off basis, the chance of a deficit is only 28%, and the scheme has a true average surplus of 24% of current assets.

- As with the UK scheme, the apparent deficit is an artefact of using a valuation basis that is not consistent with the actual underlying asset allocation strategy. The true financial status is actually unaffected by how the valuation actuary is required to value future liabilities.

- However, if this valuation method is a prescribed requirement, the stylised US scheme might be obliged to address this mismatch by changing its asset allocation strategy to involve greater bond investment. This course of action is actually materially detrimental to the long-term financial health of the pension scheme.

- As shown in Table 6, increasing the bond allocation from 30% to 70%, now creates a *true* deficit for the scheme, even on a run-off basis. By increasing its bond allocation, the fund has gone from 24% excess assets to 3% underfunded. Moreover, now the chance of assets not being able to meet all future liabilities has increased from 28% to 53%.

- We also note that the severity of the impact of the change in the asset allocation strategy is less severe for the stylised US scheme than that for the UK scheme.

### 6.3 SENSITIVITY TO CONTRIBUTION RATES

The bottom panel of Figure 7 shows the distribution of $V_0^{(\infty)}$ (as a percentage of $A_0$), when the contribution rate is increased or decreased by 3.5%; and also if there were no future contributions. For ease of reference, the density of the base case result with contribution rate of 6.5% is also included as a grey curve in the background.

Consistent with the results for the UK scheme, the main observation is that increasing or decreasing contribution rates by ±3.5% has the effect of shifting the median of the distribution by roughly ±6% (of $A_0$). The relative shifts in the left tail increase slightly as we move further into the tails of the distributions. However the impacts are small compared to the impact of a higher bond allocation. In fact, the impact at the median of moving the bond allocation from 30% to 70% has over twice the impact of eliminating all future contributions.

Figure 11 shows the sensitivity of the results towards a higher or lower contribution rate when using a 3-year time horizon. As with the UK scheme, the direction of the shifts in the distributions are as expected, and the magnitudes of the shifts are much smaller than for the run-off case.

As for the UK scheme, the main conclusion here is that the impact of changes in contribution rates is smaller compared to changes in asset allocation strategy. In particular, if a short time horizon is employed, the contribution rate change needs to be extremely large in order to have any meaningful impact on pension scheme risk. If instead of changing asset allocation strategy, a pension scheme wants to address any apparent deficit in

$V_0^{(3)}$ using higher contribution rates, the scheme will have to raise the contribution rates by enormous amounts to have a material impact on the distribution of $V_0^{(3)}$.

## 7. SUMMARY AND CONCLUSIONS

In this article we examine the impact of employing different time horizons to evaluate the risk of two pension schemes. The aspiration of any pension scheme is to be able to provide for all promised benefits. This suggests a very long-term planning horizon. However, members of the scheme and pension regulators have an interest in interim assessments of the financial status of a pension scheme, which suggests a short-term planning horizon.

We show that the risk assessment horizon has an impact on the quantification of the risk profile of pension schemes. This suggests that the assumptions used to determine the financial status of a scheme in the short term should minimise, to the extent possible, the differences between the risk profiles over the two risk assessment horizons.

In general, we find that the approach to setting the discount rate that generates the least distortion, relative to the discount rate that would be set by a fully prescient actuary, is one that uses a time-varying discount rate. That discount rate increases (or decreases) from the investment return actually realised in the third year to its long-term mean over an intermediate period of time, say 15 years.

Regardless of the approach taken to setting the discount rate, the difference in time horizon generates different conclusions regarding the best approach to manage risk through changes in asset allocation. Over the long term, increasing a scheme's allocation to long-term bonds worsens the risk profile – both reducing the median level of surplus (increasing the median level of deficit) and increasing the spread between the median and the $10^{th}$ percentile level of deficit. When examined over the shorter time horizon, increasing a scheme's allocation to long bonds reduces the median to $10^{th}$ percentile spread. However, this perceived short-term risk-reduction by shifting to bond investment might create true deficits in an otherwise financially healthy pension scheme on a run-off basis.

We also find that the choice of risk assessment time horizon can have a differential impact on the quantified risk of the UK scheme and the stylised US scheme. The UK scheme with inflation-protected benefits is more adversely affected by a shift from equities to bonds.

Overall, we believe that it would be in the interests of all relevant parties to minimise any distortion created by taking a short-term view of a scheme's financial status. We also find that the impact on a scheme's financial status of changing contribution rates is much smaller than the impact of changes to the scheme's asset allocation.

All of our results remain qualitatively the same when using a different ESG.

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# APPENDIX

## A. UK PENSION SCHEME RESULTS USING THE WILKIE MODEL

### A.1 THE WILKIE MODEL

One of the earliest published ESGs for actuarial use in the UK was Wilkie (1986). It has subsequently been updated and extended in a series of papers (Wilkie, 1995; Wilkie et al., 2011; Wilkie and Şahin, 2016, 2017a,b,c, 2018). In this article, we will refer to this ESG generally as the Wilkie model.

The Wilkie model is a multivariate autoregressive time series model, where chosen economic variables are modelled using a cascade structure, as depicted in Figure 12. The model carefully calibrates the cascading structure that builds up future scenarios in an explicit sequence, taking into account the dynamic properties of the individual variables being modelled. For brevity, we do not include the Wilkie model parameterisation here; these can be found in Wilkie et al. (2011).

The Wilkie model contains all the key economic variables required for analysing a UK DB scheme. Although, there has been a recent attempt by Zhang et al. (2018) to calibrate the Wilkie model for the US economy, one key variable required for analysing pension schemes, namely salary inflation, has been left out in Zhang et al. (2018). So the Wilkie model is not used for analysing a US DB scheme in this article.

In this appendix, we examine the results for the UK scheme using the Wilkie model ESG. As with the case of the graphical ESG, we consider three different cases separately for the run-off and 3-year time horizon:

- the base case;
- the impact of changing the asset allocation to high bond content (30/70 in equity/bond) from the baseline allocation of high equity content (70/30 in equity/bond); and
- the impact of changing the contribution rate to 30% (high) and 15% (low), relative to the baseline contribution rate of 22.5% of salary.

### A.2 BASE CASE

The top panel of Figure 13 shows the distribution, $V_0^{(\infty)}$ (as a percentage of $A_0$) for the UK pension scheme using the Wilkie model ESG. Table 7 provides all the summary statistics.

We make the following observations:

1. As with the graphical ESG, the density of $V_0^{(\infty)}$ based on the Wilkie model ESG has a left skew, but the distribution is more concentrated in the centre. The median of the distribution is a surplus of 15% of $A_0$ which is smaller than the corresponding value of 24% for the graphical ESG; while the $10^{th}$ and $0.5^{th}$ percentiles are closer to the median compared to the ones based on the graphical ESG. This points to a smaller dispersion along with less fat in both tails of the distribution.

Table 7: Summary of results for UK scheme using the Wilkie model ESG.

| | | $50^{th}$ percentile | | $10^{th}$ percentile | | $0.5^{th}$ percentile | |
|---|---|---|---|---|---|---|---|
| Scenarios | $P[V_0 \leq 0]$ | VaR | ES | VaR | ES | VaR | ES |
| $V_0^{(\infty)}$ | | | | | | | |
| Base case | 0.30 | 15 | -12 | -27 | -51 | -92 | -111 |
| High bond allocation | 0.91 | -52 | -93 | -119 | -148 | -205 | -225 |
| High contribution | 0.17 | 27 | 2 | -13 | -35 | -72 | -92 |
| Low contribution | 0.46 | 3 | -26 | -42 | -67 | -110 | -130 |
| No contribution | 0.77 | -22 | -54 | -73 | -100 | -148 | -168 |
| $V_0^{(3)}$ **(Valuation rate based only on long-term bond yields)** | | | | | | | |
| Base case | 0.96 | -97 | -181 | -232 | -309 | -461 | -532 |
| High bond allocation | 0.98 | -118 | -202 | -253 | -324 | -465 | -532 |
| High contribution | 0.95 | -92 | -176 | -227 | -304 | -454 | -522 |
| Low contribution | 0.96 | -101 | -186 | -237 | -314 | -466 | -537 |
| $V_0^{(3)}$ **(Valuation rate based on returns on backing assets)** | | | | | | | |
| Base case | 0.45 | 15 | -164 | -282 | -479 | -880 | -1076 |
| High bond allocation | 0.68 | -33 | -149 | -214 | -354 | -647 | -836 |
| High contribution | 0.43 | 19 | -159 | -276 | -472 | -875 | -1070 |
| Low contribution | 0.46 | 10 | -169 | -287 | -484 | -887 | -1082 |
| $V_0^{(3)}$ **(Valuation rate converging to long-term returns after 50 years)** | | | | | | | |
| Base case | 0.40 | 20 | -70 | -128 | -210 | -375 | -454 |
| High bond allocation | 0.65 | -22 | -91 | -132 | -200 | -339 | -412 |
| High contribution | 0.38 | 25 | -65 | -123 | -205 | -370 | -446 |
| Low contribution | 0.42 | 15 | -75 | -133 | -216 | -380 | -460 |
| $V_0^{(3)}$ **(Valuation rate converging to long-term returns after 15 years)** | | | | | | | |
| Base case | 0.30 | 27 | -21 | -50 | -91 | -178 | -216 |
| High bond allocation | 0.59 | -10 | -53 | -79 | -117 | -187 | -228 |
| High contribution | 0.27 | 32 | -16 | -45 | -86 | -172 | -209 |
| Low contribution | 0.33 | 22 | -26 | -55 | -97 | -184 | -222 |

2. However, the main conclusions for the base case result are still the same, i.e. on average the scheme has a surplus, while significant deficits are possible, but with smaller probabilities.

For the case of 3-year time horizon, Figure 14 shows the same alternative approaches as for the graphical ESG that a valuation actuary might take to value the liabilities, $L_3$, at the end of year 3.

1. Firstly, we look at valuation discount rate based solely on the prevalent bond yield at the end of 3 years. In the top-left panel of Figure 14, we show $V_0^{(3)}$ (as a percentage of $A_0$), when $L_3$ is calculated using the prevalent long-term yield at the end of 3 years plus a spread of 1.8%. As observed for the graphical ESG, a characteristic that stands out is that the left tail of the distribution is much fatter than the one for the full run-off.

2. The top-right panel of Figure 14 shows the result if the third year's returns (weighted by asset allocation) are used as the discount rate for all future years. The bottom-left and bottom-right panels of Figure 14 show the result if the time period of convergence to long-term mean is 50 years and 15 years, respectively. Commentary on these panels is the same as for the graphical ESG.

Figure 15 shows the scatter plots, or joint densities, of $V_0^{(3)}$ and $V_0^{(\infty)}$ for the UK scheme. The conclusions are the same as that for the graphical ESG, i.e. using a constant long-term bond yield as the valuation discount rate almost always underestimates $V_0$, which could lead to a misleading perception of insufficient funding levels for an otherwise healthy scheme on a run-off basis. The estimates improve when the backing assets are used to obtain the valuation discount rate, and they improve further if the rate is adjusted over 15 years to converge to the long-run mean.

### A.3 SENSITIVITY TO HIGHER BOND ALLOCATION STRATEGY

The middle panel of Figure 13 shows the distribution of $V_0^{(\infty)}$ (as a percentage of $A_0$), when the bond allocation of the UK scheme is increased from 30% to 70%. For ease of reference, the density of the base case result is also included as a grey curve in the background. We make the following observations, which are similar to those for the graphical ESG:

- For increased bond investment, the distribution of $V_0^{(\infty)}$ has moved to the left and has greater dispersion.
- The leftward shift of the distribution indicates a greater probability of larger deficits. This is reflected in the median of $V_0^{(\infty)}$, which shows a deficit of 52% as compared to a surplus of 15% for the base case results.
- The severity of the impact of higher bond investment on the financial health of the scheme is greater under the Wilkie model ESG.

Figure 14 shows the sensitivity of the results to a higher bond allocation strategy when using a 3-year time horizon. The most important observation here is again that as the bond allocation of the scheme increases, a valuation basis based on bonds will produce results where the distributions of $V_0^{(3)}$ start getting closer to that of $V_0^{(\infty)}$, as expected.

### A.4 SENSITIVITY TO CONTRIBUTION RATES

The bottom panel of Figure 13 shows the distribution of $V_0^{(\infty)}$ (as a percentage of $A_0$), when the contribution rate is increased or decreased by 7.5%. Figure 17 shows the sensitivity of the results towards a higher or lower contribution rate when using a 3-year time horizon. The observations regarding the scheme's sensitivity to changes in contribution rates are the same as when we use the graphical ESG.

Qualitatively, both the results and the implications from using the Wilkie model are the same as when we use the graphical ESG, though some of the details are a little different.

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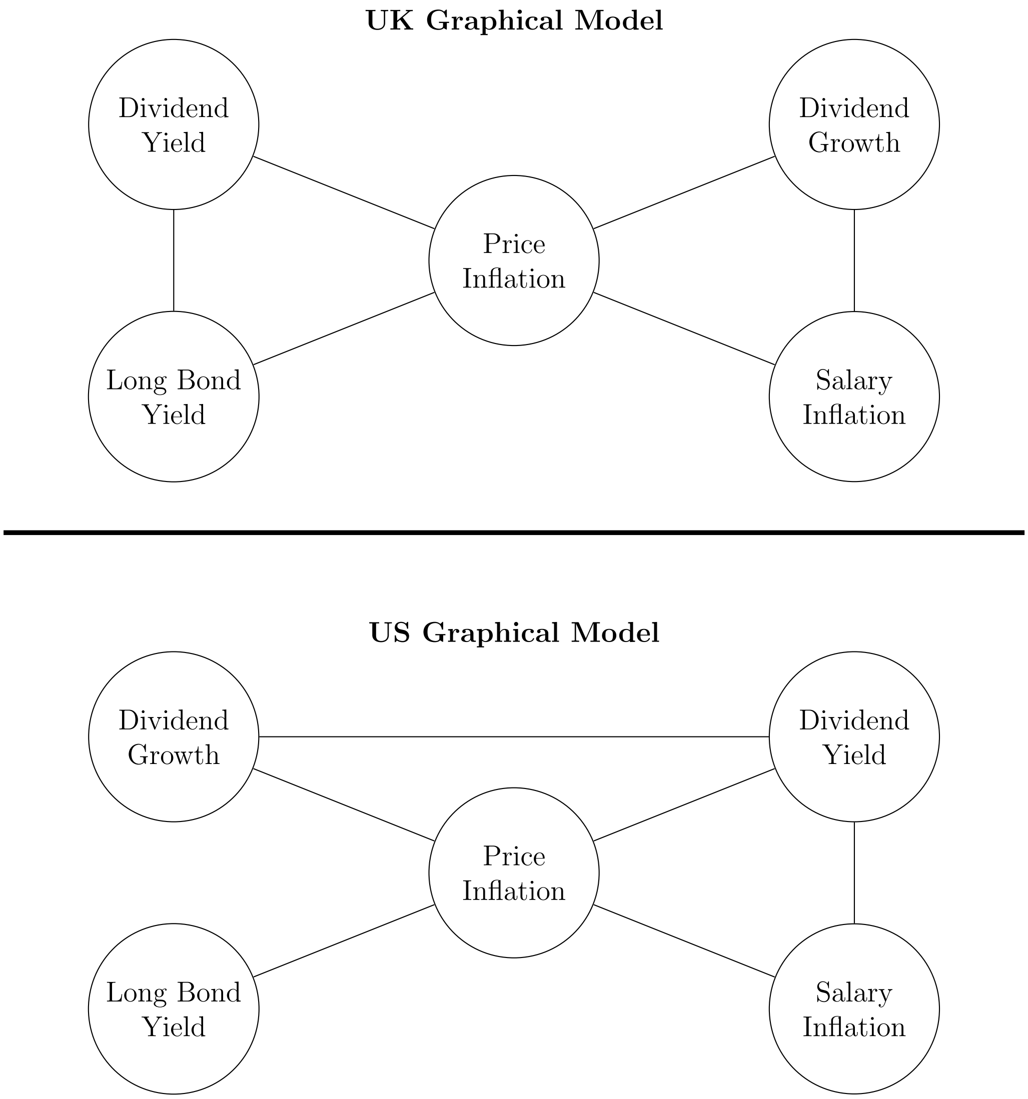
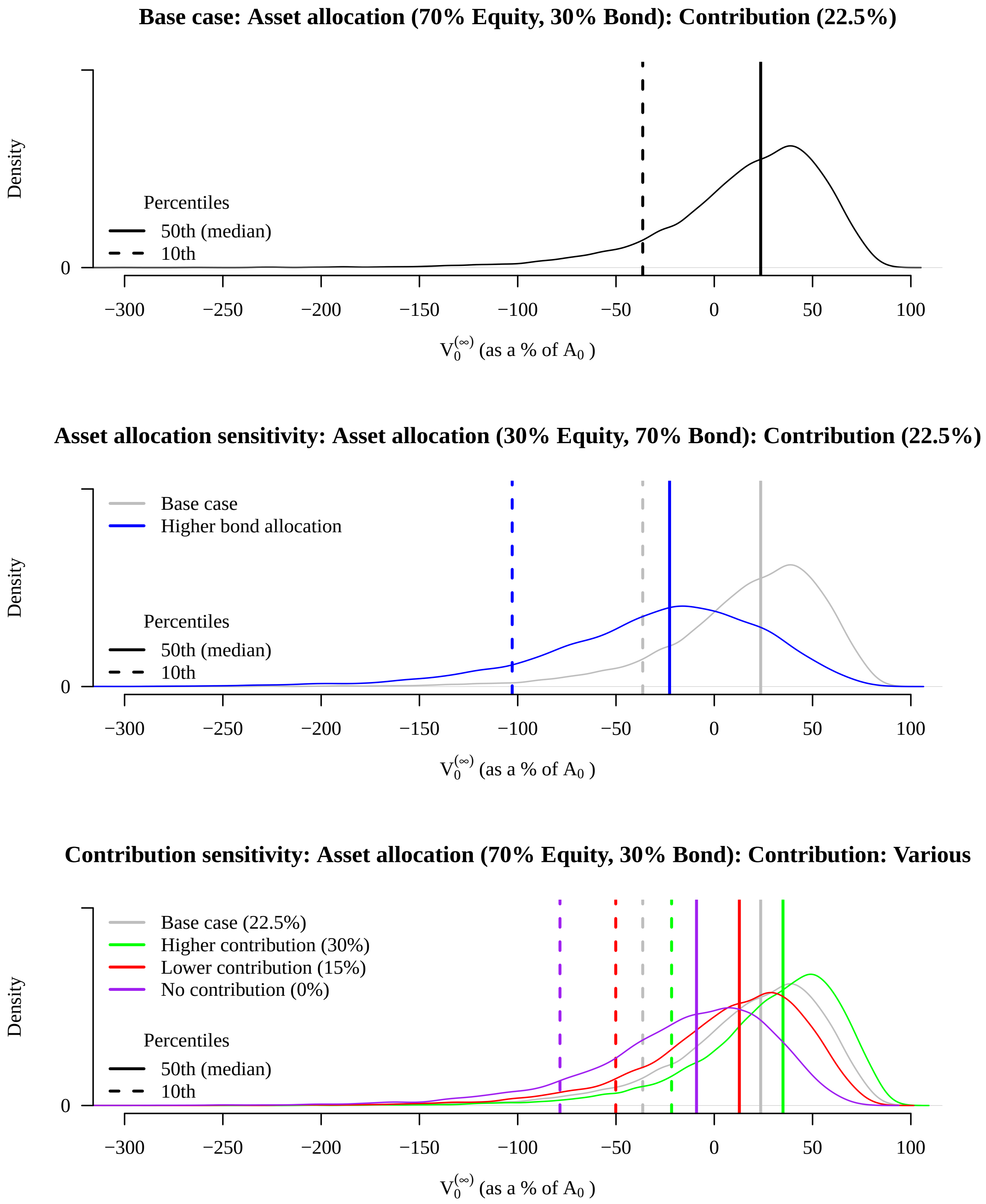


Figure 1: UK and US graphical models.



Base case: Asset allocation (70% Equity, 30% Bond): Contribution (22.5%)

Asset allocation sensitivity: Asset allocation (30% Equity, 70% Bond): Contribution (22.5%)

Contribution sensitivity: Asset allocation (70% Equity, 30% Bond): Contribution: Various

Figure 2: Run-off results for the UK scheme using the graphical ESG. Top panel shows base case. Middle panel shows sensitivity to asset allocation strategy with higher bond allocation. Bottom panel shows sensitivity to changing contribution rates.

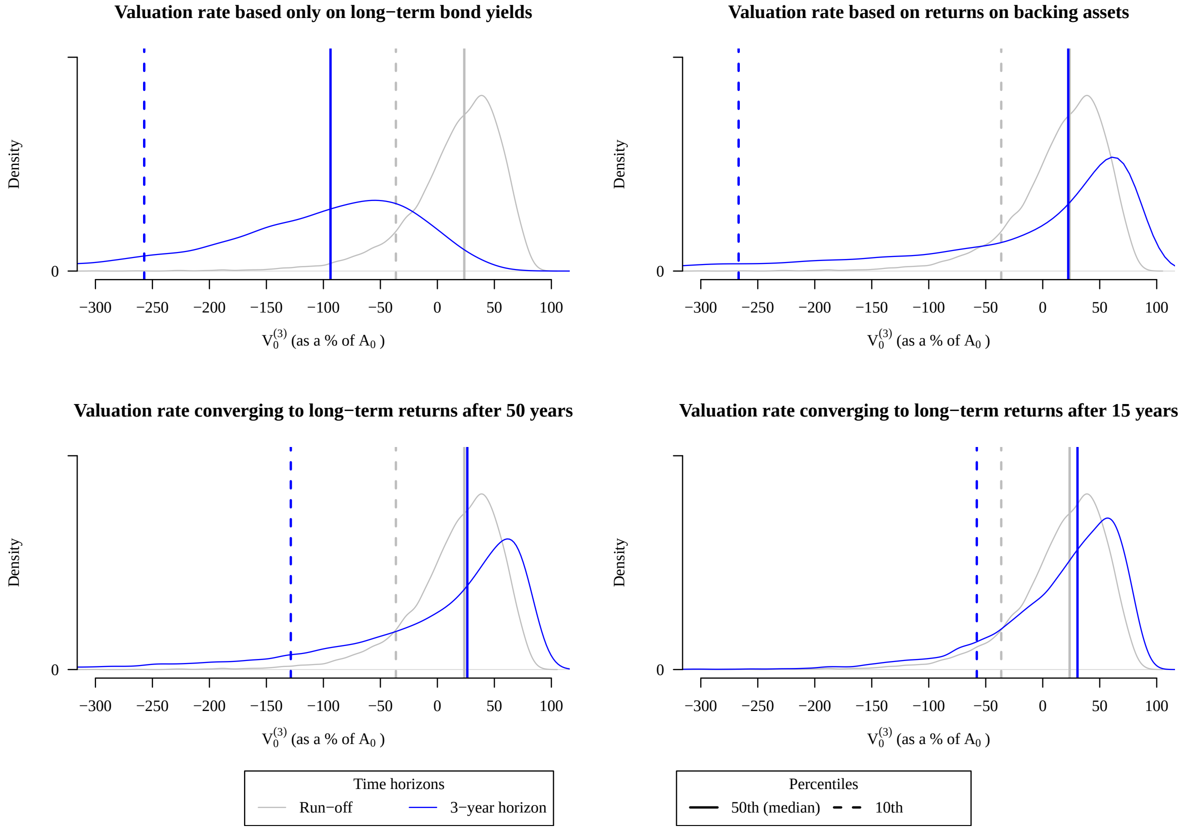


Figure 3: 3-year time horizon results for the UK scheme using base case assumptions and the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

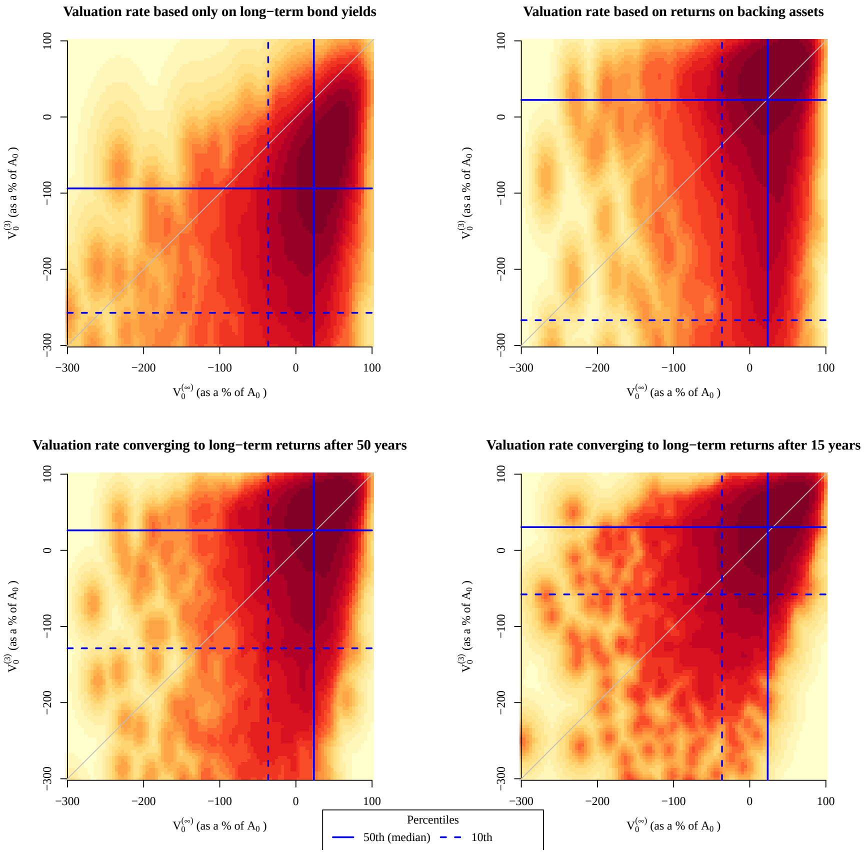
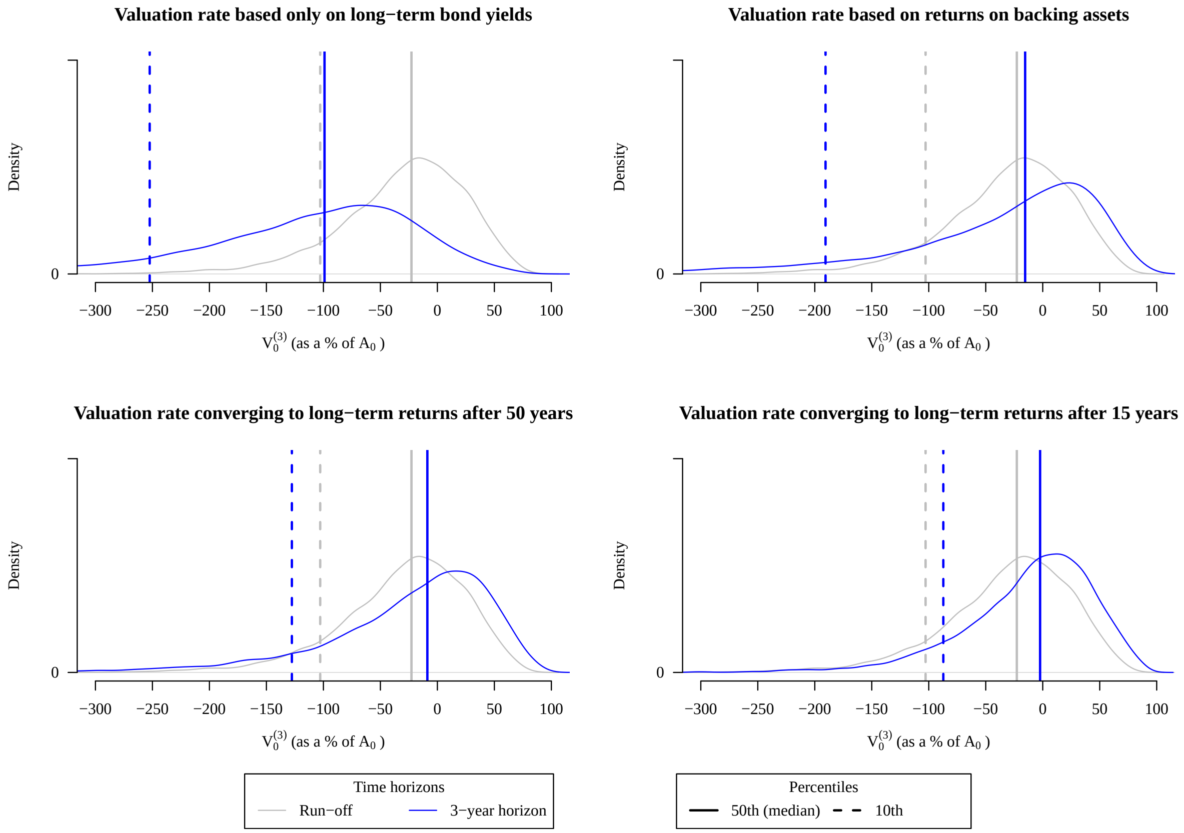


Figure 4: 3-year time horizon results for the UK scheme using base case assumptions and the graphical ESG. Each panel shows joint density of $V_0^{(\infty)}$ and $V_0^{(3)}$, as heatmaps, based on different discount rate approaches used in the valuation basis.



Valuation rate based only on long−term bond yields

Valuation rate based on returns on backing assets

Valuation rate converging to long−term returns after 50 years

Valuation rate converging to long−term returns after 15 years

Figure 5: 3-year time horizon results showing sensitivity to higher bond allocation strategy for the UK scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

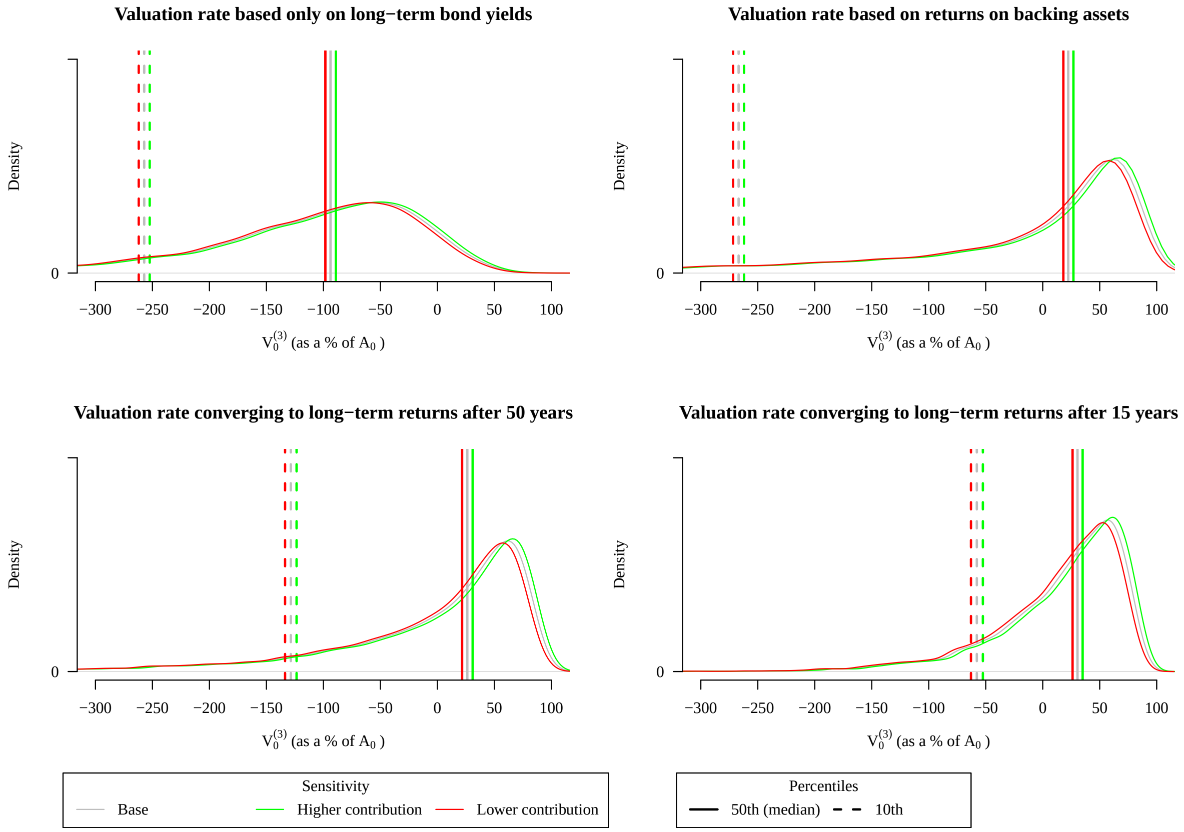


Figure 6: 3-year time horizon results showing sensitivity to higher and lower contribution rates for the UK scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the base case, $V_0^{(3)}$, is also shown, as a grey curve, for reference.

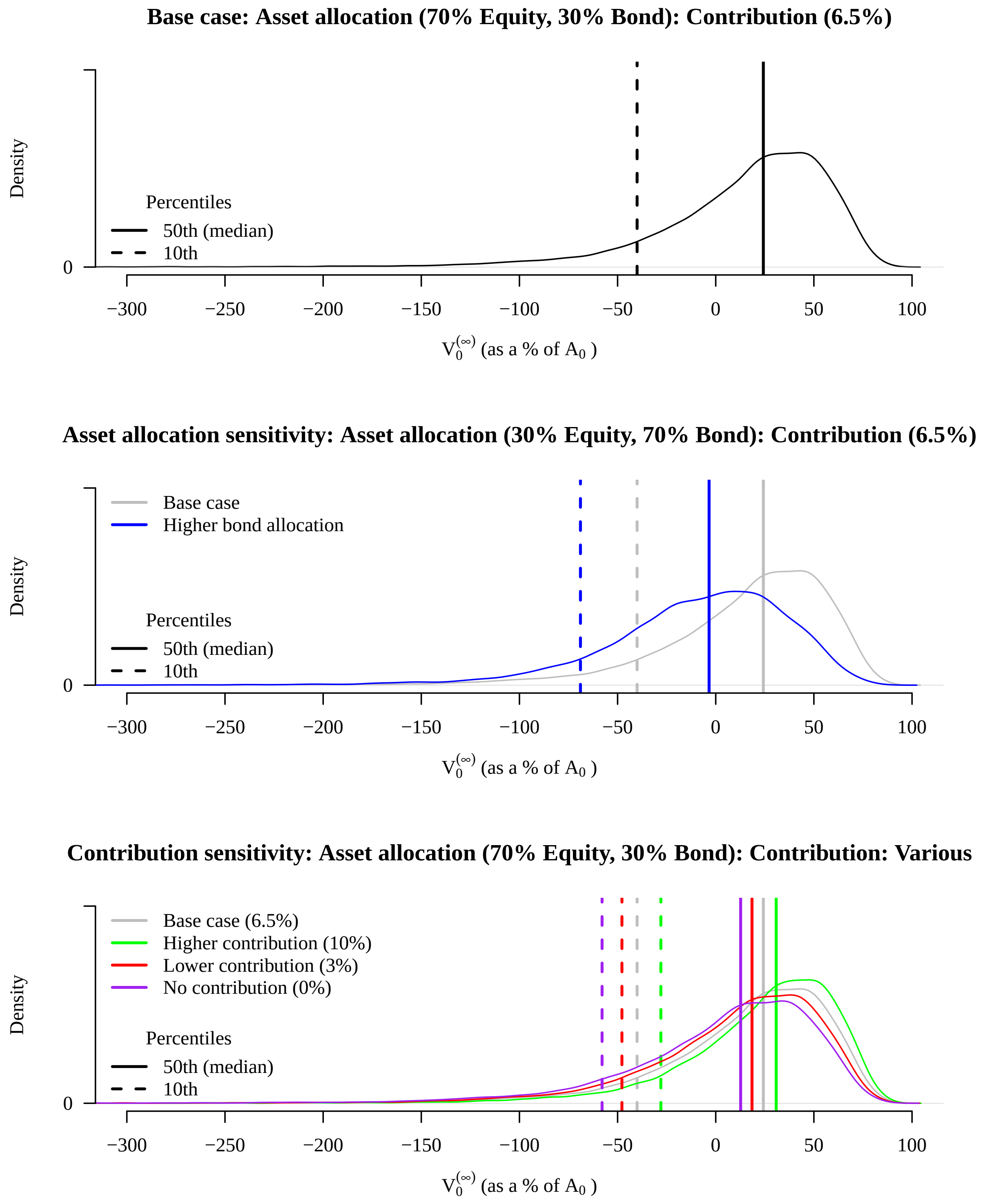


Figure 7: Run-off results for the stylised US scheme using the graphical ESG. Top panel shows base case. Middle panel shows sensitivity to asset allocation strategy with higher bond allocation. Bottom panel shows sensitivity to changing contribution rates.

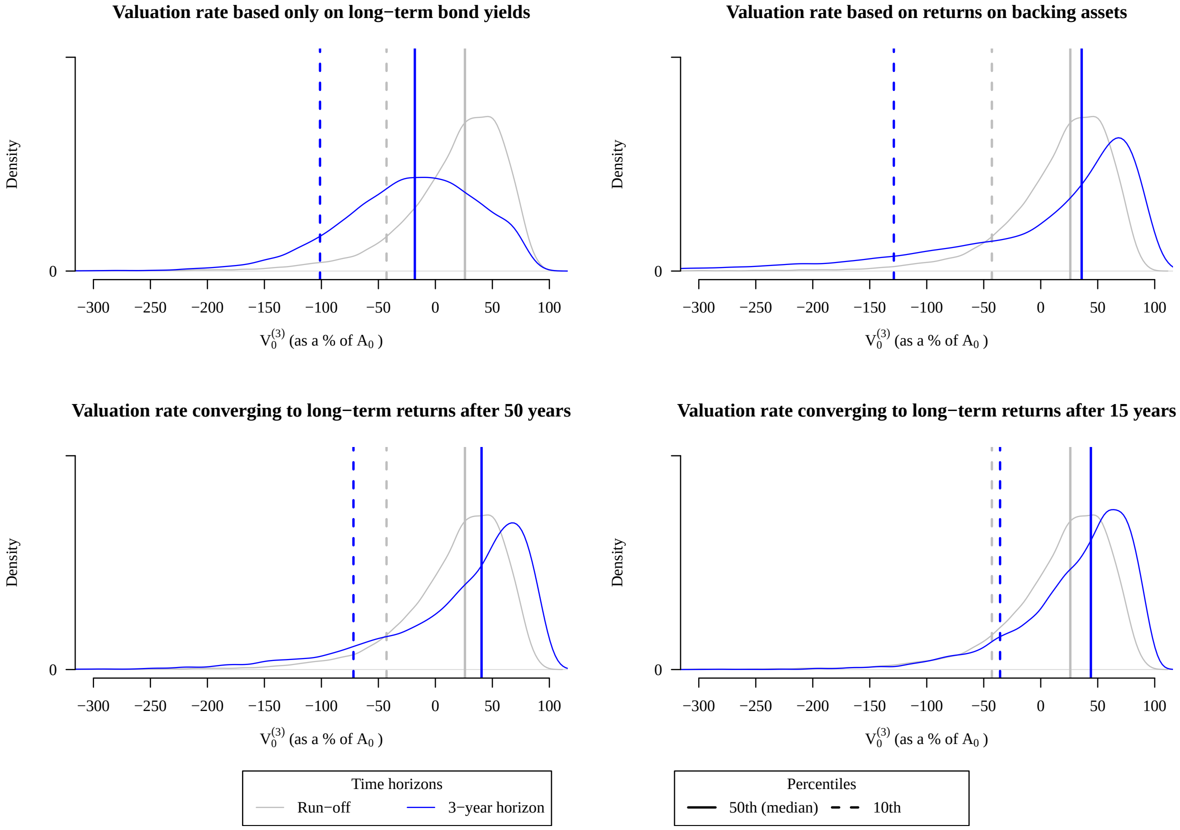


Figure 8: 3-year time horizon results for the stylised US scheme using base case assumptions and the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

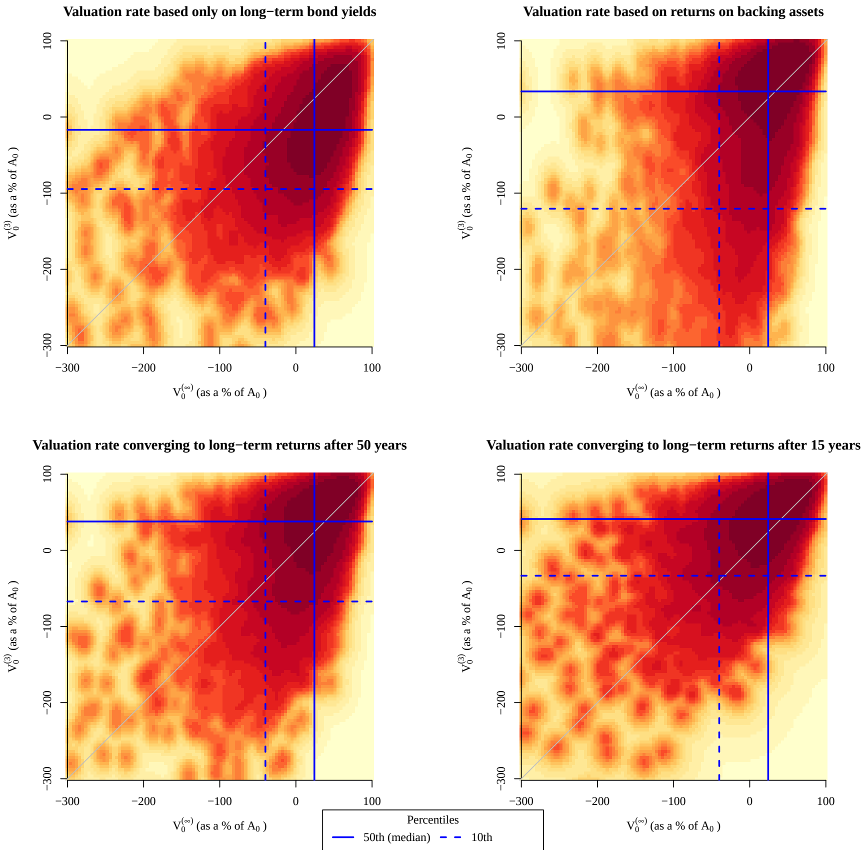


Figure 9: 3-year time horizon results for the stylised US scheme using base case assumptions and the graphical ESG. Each panel shows joint density of $V_0^{(\infty)}$ and $V_0^{(3)}$, as heatmaps, based on different discount rate approaches used in the valuation basis.

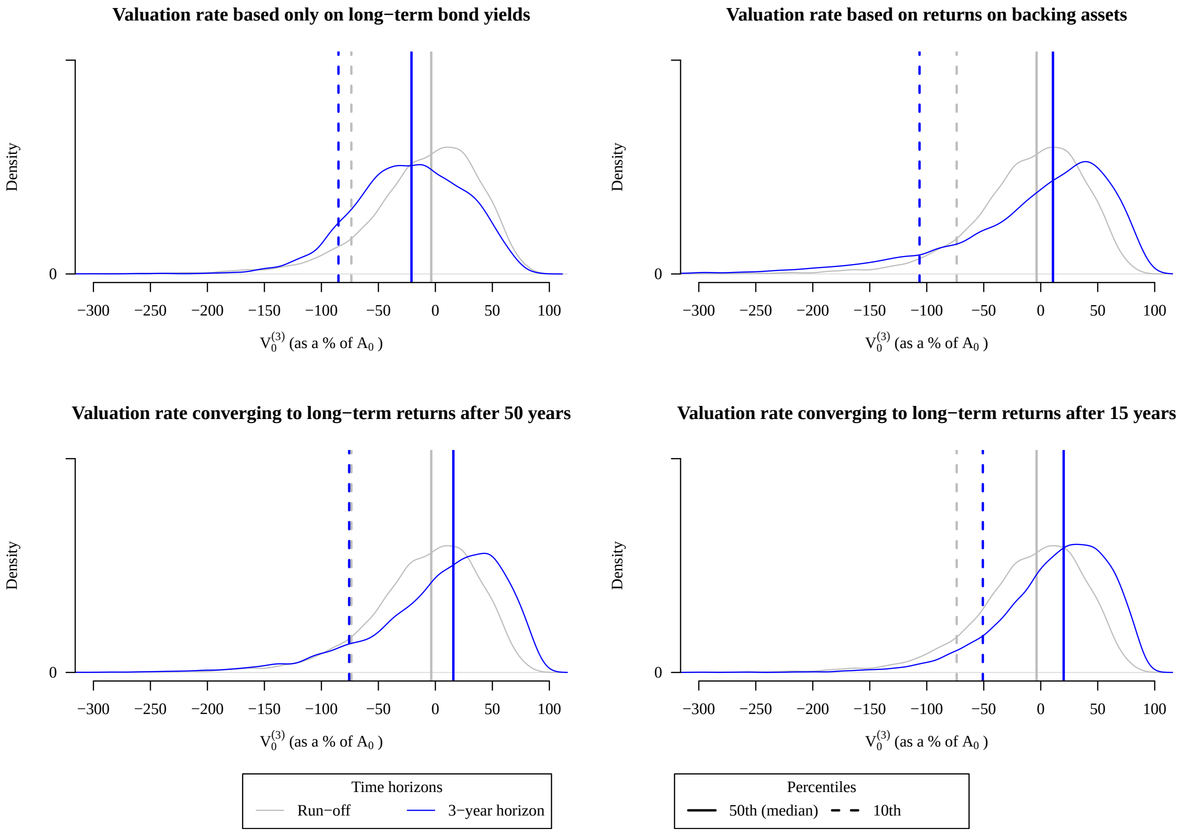


Figure 10: 3-year time horizon results showing sensitivity to higher bond allocation strategy for the stylised US scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

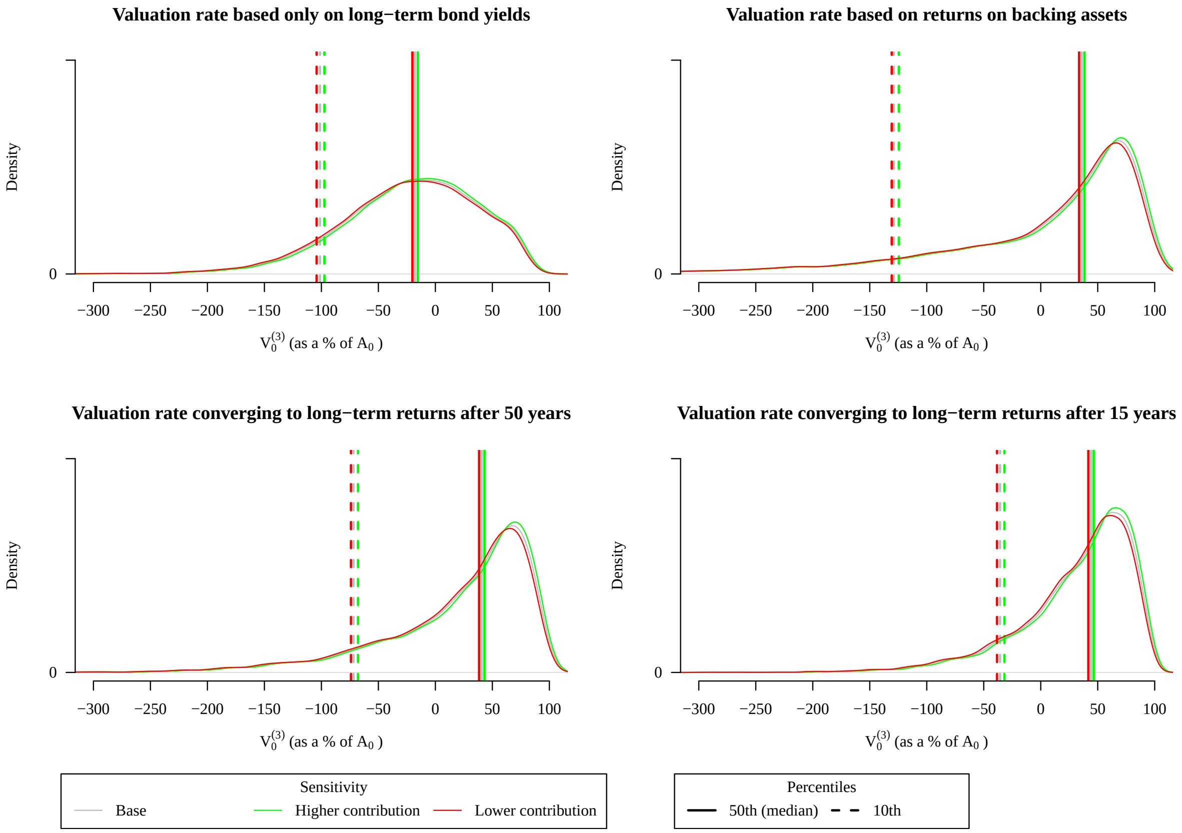


Figure 11: 3-year time horizon results showing sensitivity to higher and lower contribution rates for the stylised US scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the base case, $V_0^{(3)}$, is also shown, as a grey curve, for reference.

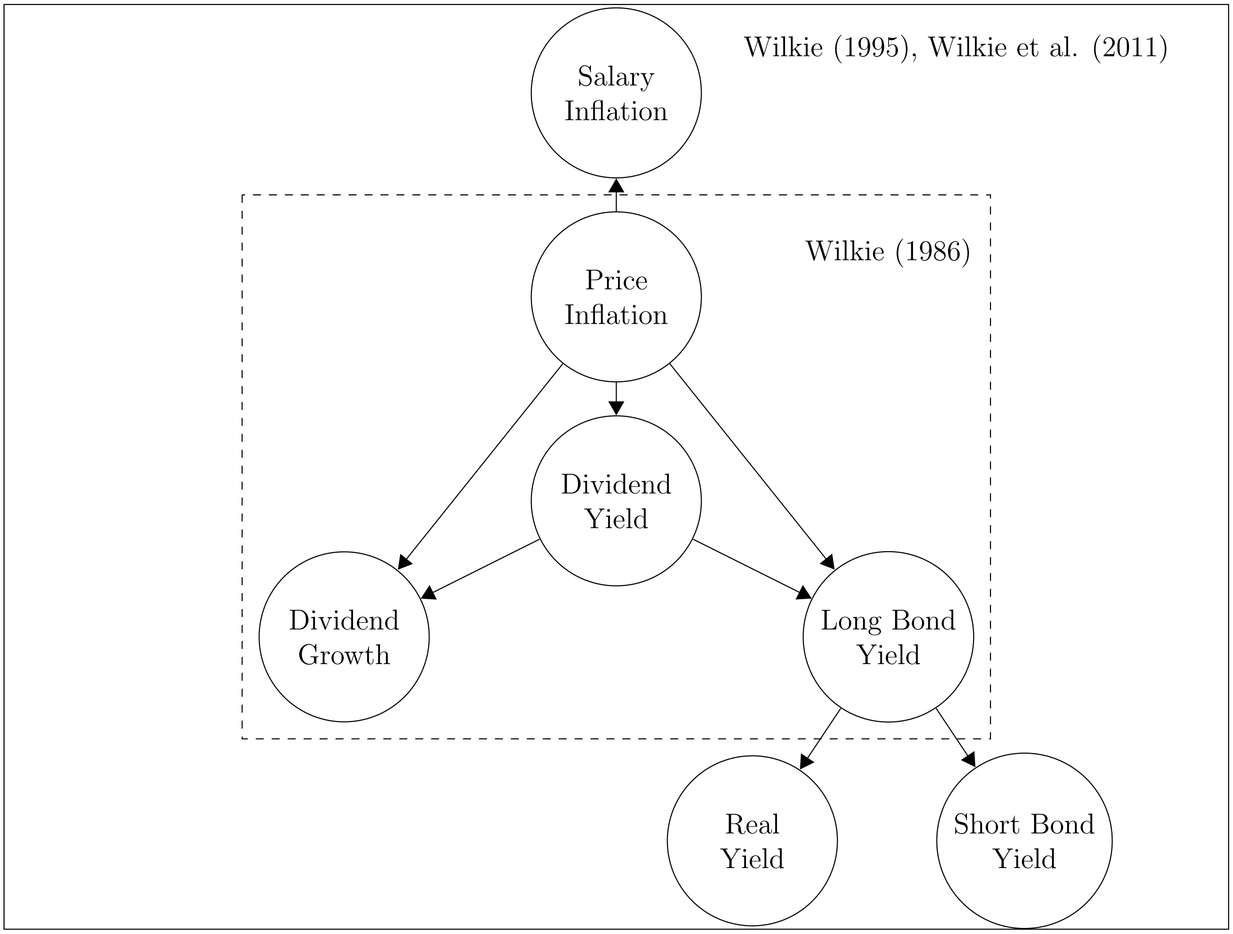


Figure 12: The Wilkie model.

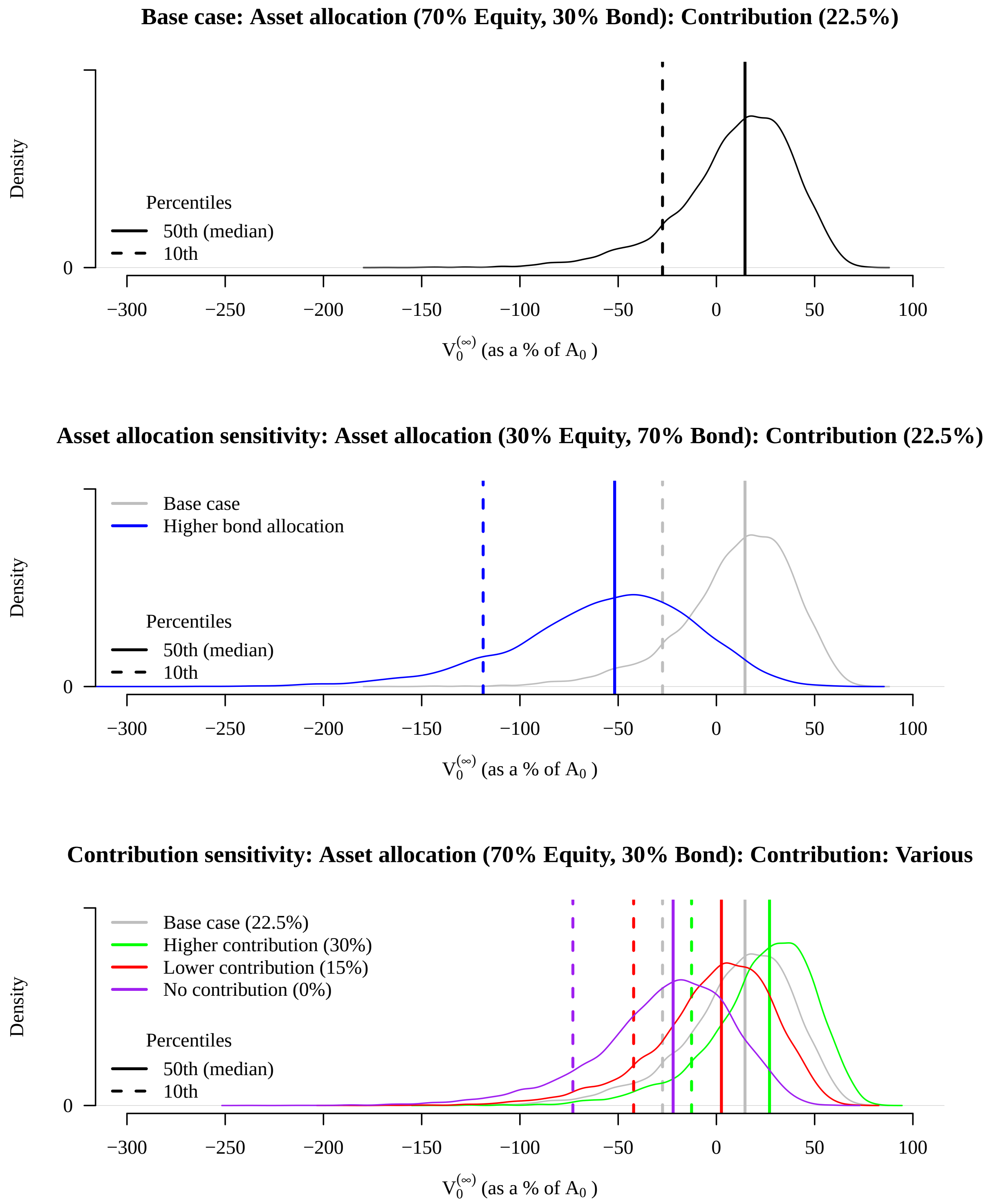


Figure 13: Run-off results for the UK scheme using the Wilkie model ESG. Top panel shows base case. Middle panel shows sensitivity to asset allocation strategy with higher bond allocation. Bottom panel shows sensitivity to changing contribution rates.

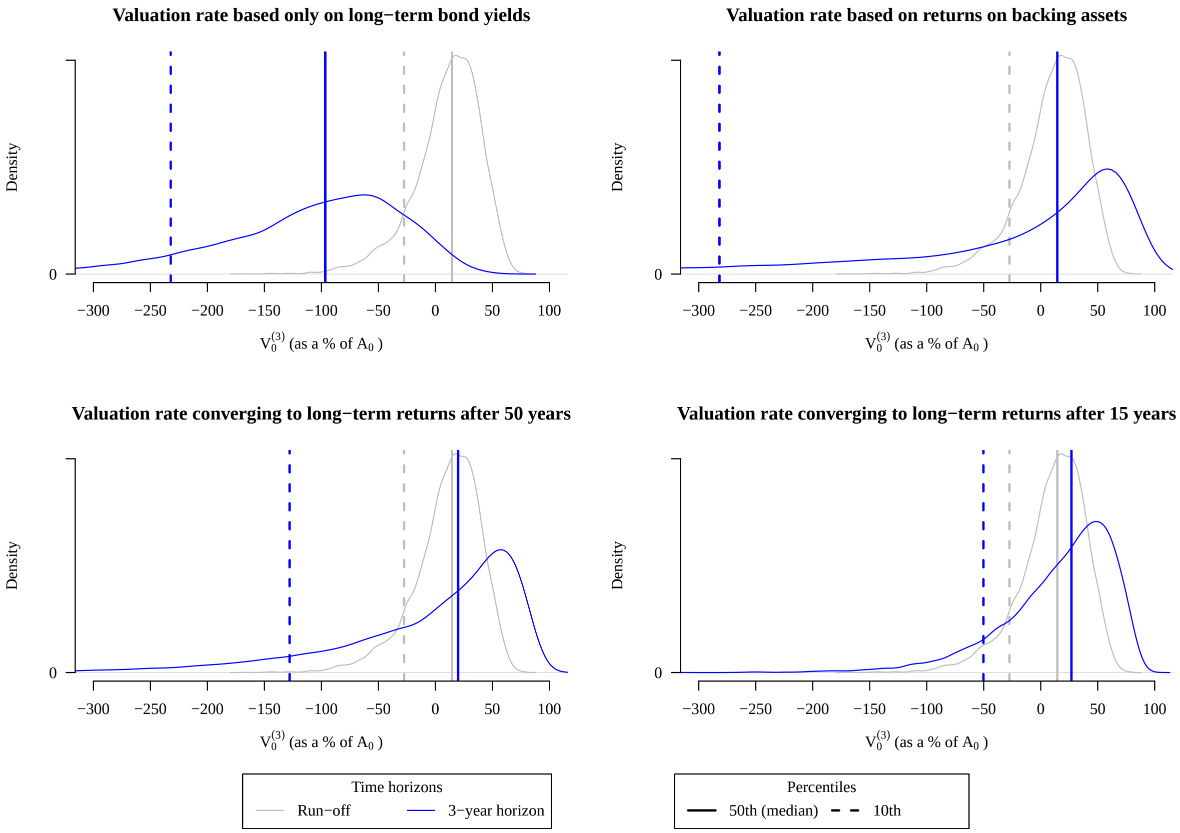


Figure 14: 3-year time horizon results for the UK scheme using base case assumptions and the Wilkie model ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

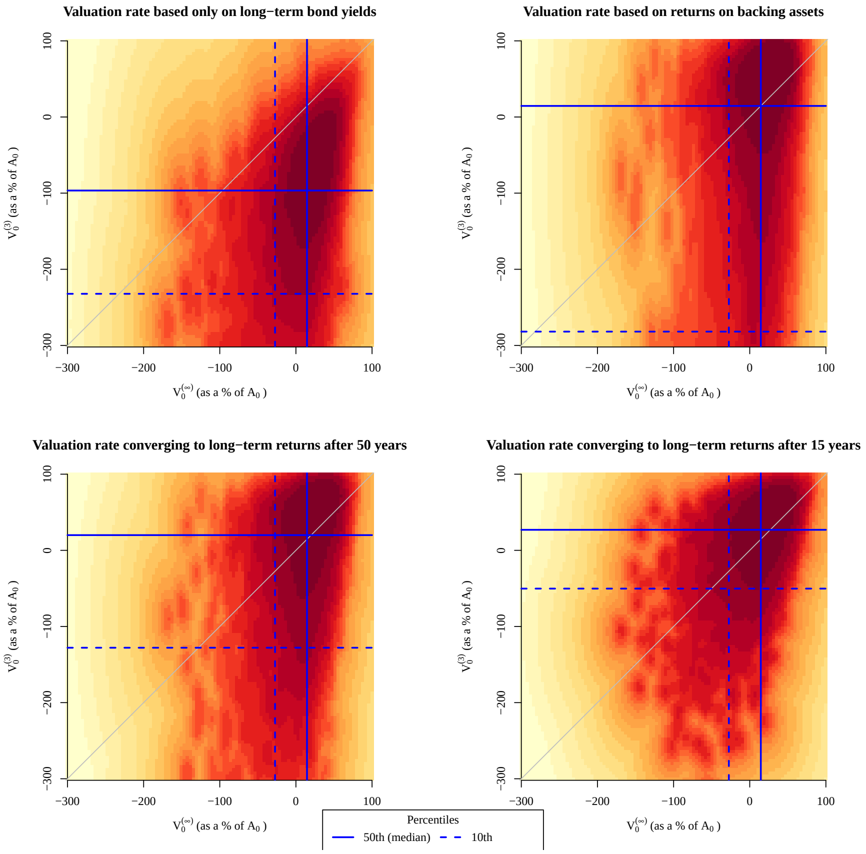


Figure 15: 3-year time horizon results for the UK scheme using base case assumptions and the Wilkie model ESG. Each panel shows joint density of $V_0^{(\infty)}$ and $V_0^{(3)}$, as heatmaps, based on different discount rate approaches used in the valuation basis.

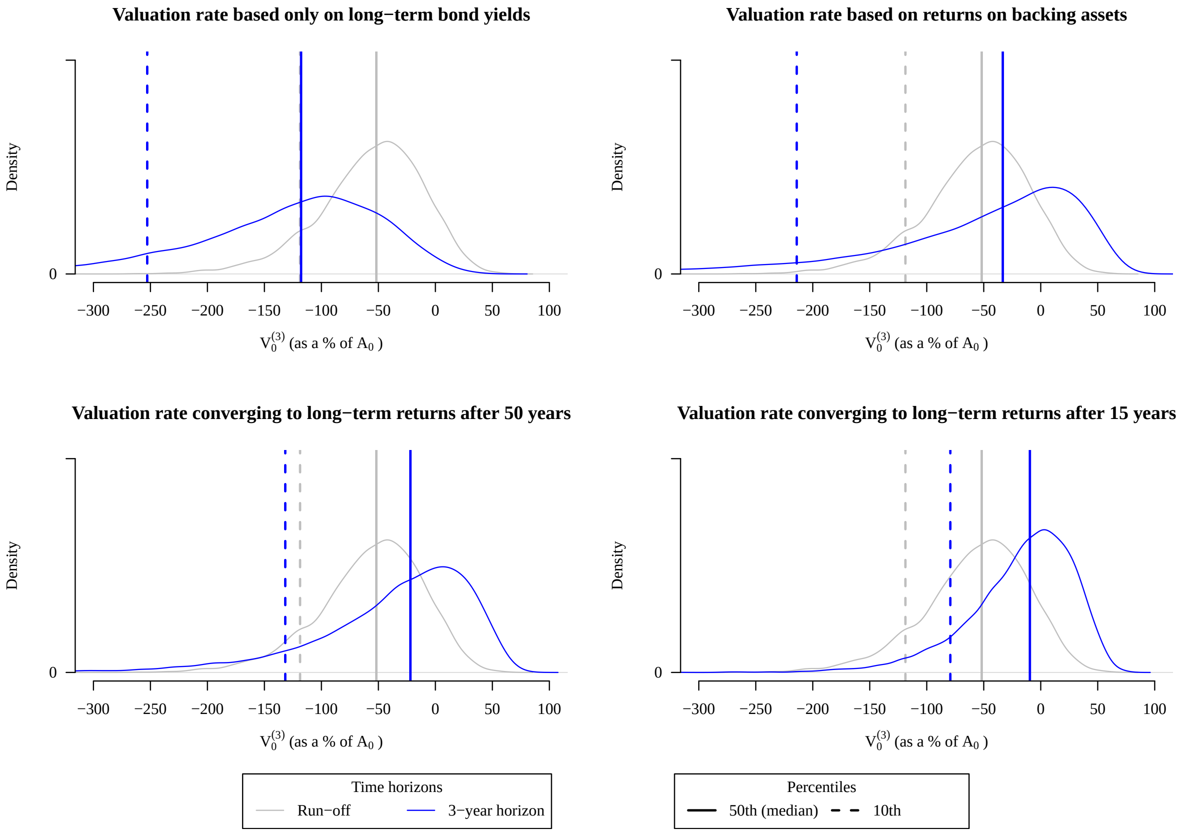
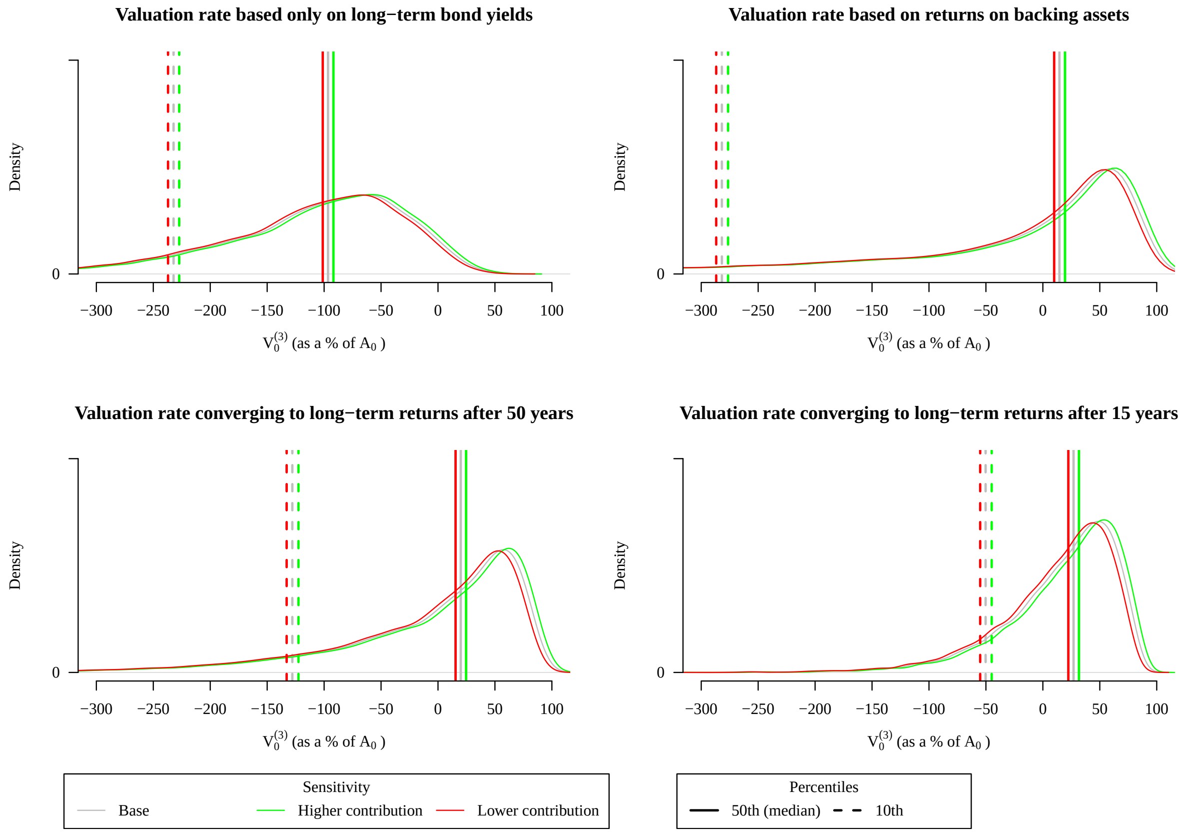


Figure 16: 3-year time horizon results showing sensitivity to higher bond allocation strategy for the UK scheme using the Wilkie model ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.


Valuation rate based only on long−term bond yields

Valuation rate based on returns on backing assets

Valuation rate converging to long−term returns after 50 years

Valuation rate converging to long−term returns after 15 years

Figure 17: 3-year time horizon results showing sensitivity to higher and lower contribution rates for the UK scheme using the Wilkie model ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the base case, $V_0^{(3)}$, is also shown, as a grey curve, for reference.