# INQUIRY-BASED INSTRUCTION:

## THE ROLE OF MINDSET FOR STUDENTS WITH MATHEMATICS DIFFICULTIES

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## Declaration

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the Preface and specified in the text.

It is not substantially the same as any that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text. I further state that no substantial part of my dissertation has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other University of similar institution except as declared in the Preface and specified in the text.

It does not exceed the prescribed word limit for the relevant Degree Committee.

## Preface

I acknowledge that some of the argument present in this thesis has appeared in the following publication: (Rice, 2018).

# Abstract

The effectiveness of inquiry-based instruction (IBI) is well documented, however its use with students with mathematics difficulties (MD) has been limited, since it is often thought such methods are unsuitable for low attaining students. However, this belief is not entirely evidence based, and little research has explored how students with MD perceive IBI. In addition, previous research has demonstrated that students' beliefs can have an impact on mathematics performance. This thesis therefore explores two research questions: (1) How do students with mathematics difficulties perceive inquiry-based instruction? and (2) Are students' beliefs (e.g. mindset) associated with the effectiveness of inquiry-based instruction for students with mathematics difficulties? In a multiple case study involving two secondary schools from the United Kingdom, students with MD were taught using an inquiry approach to learning mathematics. Data were collected through a combination of questionnaires, lesson observations, student interviews, and pre-test/post-test assessments. Cross-case analysis suggested that students with mathematics difficulties perceived inquiry-based instruction according to four themes: IBI as a form of empowerment, IBI as a form of neglect, importance of the teacher, and importance of peers. The expression of the first two themes seemed to differ depending on the students' mindsets. In addition, the students' beliefs (including mindset) were analysed according to McLeod's (1992) framework of beliefs in mathematics. Beliefs about mathematics, mathematics teaching, the self (including mindset), and the social context appeared to be associated with the effectiveness of IBI. Students with fixed mindsets showed poorer engagement and persistence compared to students with growth mindsets, but surprisingly, this effect was not reflected in test scores. Future research should seek to further explore the contextual factors that contribute to the effectiveness of inquiry instruction for students with MD.


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*In loving memory of my mother, Donna Jeanne Rice.*

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# 1 Introduction

Reform efforts to improve the teaching of mathematics have been ongoing for some time (Permuth & Dalzell, 2013). Central to these efforts is a migration away from traditional instruction, such as tell-and-practice, towards teaching that places greater emphasis on student inquiry. In general terms, such inquiry-based instruction (IBI) in mathematics describes a pedagogic approach in which the teacher provides the students with problems in a domain of mathematics, however, offers minimal or limited guidance. Students are expected to explore the problem space prior to receiving instruction. The exact amount of guidance given during these inquiry exercises is highly variable, and in many ways, this leads to the lack of clear delimitations between inquiry and non-inquiry approaches. Pure discovery approaches, in which students receive zero instruction, have been shown to be ineffective (Hmelo-Silver et al., 2007; Kirschner et al., 2006; Mayer, 2004). However, studies suggest that mixing inquiry approaches with some form of direct instruction can be effective in teaching mathematics. Most studies suggest that IBI's biggest benefit is increased conceptual understanding, which 'consists of abstract or generic ideas generalized from particular instances, including knowledge of problem structures' (DeCaro & Rittle-Johnson, 2012, p. 555). On the other hand, IBI seems to have little advantage over direct instruction when assessing procedural knowledge, meaning 'the ability to execute action sequences to solve problems' (Rittle-Johnson et al., 2016, p. 577). This is an important feature of IBI given conceptual understanding is often espoused as the superior goal of mathematics instruction. Despite this, there is a growing consensus that both conceptual understanding and procedural knowledge are equally important since the two are intimately connected (Rittle-Johnson et al., 2015).

The factors that make IBI effective are unclear. Subjecting students to the cognitively demanding process of exploring the problem space during IBI would seem to increase their cognitive load, thereby reducing the capacity to encode new information into the long term memory (Kirschner et al., 2006; Sweller, 1988). Therefore, IBI would seem to be incompatible with Cognitive Load Theory. Various metacognitive mechanisms have been put forward to explain why IBI would seem to be effective despite its incompatibility with current understandings of cognitive architecture. By allowing students to explore the problems it is proposed they become conscious of gaps in their knowledge and that this awareness facilitates the assimilation of the missing pieces (Loibl & Rummel, 2015; Schwartz & Martin, 2004). Studies also suggest inquiry-based tasks help activate deeper awareness of the learning processes and prepare students for subsequent direct instruction (Kapur, 2010, 2011, 2014; Schwartz et al., 2011; Schwartz & Martin, 2004).

Despite the popularity of IBI, evidence for its effectiveness for students with mathematics difficulties has been mixed, and teachers have demonstrated a reluctance to use these techniques with this population of students (Darragh & Valoyes-Chávez, 2019; Lambert, 2018; Louie, 2017). Approximately 45 percent of teachers believe that the sort of higher order thinking needed for IBI is not appropriate for low achieving students (Zohar et al., 2001). A yearlong study of 104 third-grade mathematics students in the U.S. looked into the effectiveness of inquiry teaching approaches on mathematics performance (Woodward & Baxter, 1997). They found students with learning disabilities, and similar low attaining peers, made marginal gains when given IBI, whereas the control group that followed a traditional curriculum made dramatic gains. In a meta-analysis of 58 intervention studies, focused on elementary-level mathematics between the years 1985 and 2000, Kroesbergen and van Luit (2003) concluded direct and explicit instruction was the most effective instructional methodology for students with learning disabilities or identified as having mathematics difficulties. This would all seem to support the view that IBI should not be used for students with mathematics difficulties.

But why should IBI be effective for some students and not others? One area that has not been extensively researched is the extent affective factors, such as mindset, influence the utility of inquiry-based approaches. It is known that affective factors such as emotions, attitudes, and beliefs can influence a student's overall performance. The role of mindset is particularly interesting given its recent popularity (Boaler, 2013; Dweck, 2017b). According to Dweck, mindset can be categorised in two ways. Individuals with 'growth mindset' believe that intelligence is not fixed but malleable. They view learning as a process governed by effort, as opposed to some ingrained ability (Boaler, 2013). By framing learning within this context individuals are able to perform at a higher level (Dweck, 2017b). Alternatively, individuals with a 'fixed mindset' believe intelligence cannot be altered and ability or 'smartness' is something a person either has or does not. These individuals tend to focus on performance and objectives around demonstrating strong ability in the areas they believe they are superior. As such, they avoid challenges that might compromise this view (Dweck, 2017b; Yorke &

Knight, 2004). Students with growth mindsets, however, see challenges as learning opportunities and implement flexible learning goals. Thus, students with growth mindsets typically respond positively to failure and see it as an opportunity for increased learning and effort (Dweck, 2017b; Yorke & Knight, 2004). Given that IBI inherently relies on student failure, is it possible that such techniques are unsuitable for students with fixed mindsets?

In the U.K. and the U.S. the preponderance of ability grouping in mathematics (Boaler et al., 2000) means students with mathematics difficulties (MD) are typically grouped in 'low ability' classrooms. This 'ability grouping' has been shown to propagate fixed mindset and the stereotype that abilities are somehow genetic and fixed (Plomin et al., 2007). Research suggests that teachers adopt fixed mindsets in the way they instruct these ability groupings, even if they believe they are adopting mixed-ability mindsets (Marks, 2013). The prevalence of fixed mindset tendencies is greater in students with low prior academic achievements (Snipes & Tran, 2017). Before concluding that IBI is ineffective for students with MD, researchers need a greater understanding of how a student's mindset could be associated with its effectiveness.

This thesis seeks to address two research questions. Firstly, how do students with mathematics difficulties perceive IBI? And secondly, are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties? By using a multiple case study design, I develop detailed descriptions of two classrooms being taught using IBI. The choice of this method is appropriate, as a case study is used to develop a rich picture using various data collection methods designed to capture the perceptions, experiences and ideas of the cases' components (Yin, 2017). Over the course of two 10-week long case studies in two different classes, I collected interview and observation data to understand how students with MD and differing mindsets experienced IBI. In addition, pre- and post-test analysis of the students' conceptual and procedural understanding shed light on the effectiveness of IBI for these students.

# 2 Literature Review

Advances in the fields of science, technology, engineering and mathematics (STEM) are the engine of innovation and economic expansion. Today's students emerging from centres of learning are joining a world in which the rate of STEM related roles is growing at approximately one and a half times that of non-STEM roles (U.S. Department of Commerce, 2017). To contrast this, the U.S. Department of Education (2010) indicates that only 16 percent of American high school seniors are proficient in mathematics and interested in STEM careers. A similar situation can be seen in the U.K., where the Confederation of British Industry (2016) has indicated that 42 percent of businesses struggle to recruit for STEM roles. Furthermore, recent PISA results ranked the U.S. and the U.K. only 41$^{st}$ and 29$^{th}$ respectively in mathematics among 71 other industrialised nations (OECD, 2018).

This trend has persisted for some time and has given rise to numerous reform efforts in both countries, including substantial investment in STEM research initiatives. The most obvious efforts were the reform movements of the late 1990s in the U.S., which began to incorporate ideas from cognitive psychology and constructivism that originally rose in popularity two decades earlier (Permuth & Dalzell, 2013). The embodiment of reform mathematics in the U.S. are the Principles and Standards for School Mathematics by the National Council of Teachers of Mathematics (2000). These widely adopted standards are grounded in constructivist ideology and promote the idea of idiosyncratic learning, in which inquiry-based pedagogy features heavily. This contrasts with behaviourist theories of learning which advocate for more traditional, direct, and explicit instruction.

Conceptualising inquiry-based pedagogy is problematic, as various theoretical frameworks have emerged over the years. The nature of inquiry-based pedagogy can therefore vary depending upon which theoretical framework one considers. Taken as a whole, one might broadly say that inquiry-based pedagogy is the study of a subject in line with the practices of professionals within the subject (Artigue & Blomhøj, 2013). Many studies have demonstrated the effectiveness of various inquiry-based approaches. Other studies have gone further and begun to explore factors that are associated with the effectiveness of such approaches, for instance prior attainment (Laursen et al., 2011), amount and type of guidance (Lazonder & Harmsen, 2016), gender (Cooper et al., 2015), and task set-up (Jackson et al., 2013). Despite its apparent popularity there is concern in some circles, particularly in the field of special education, that such inquiry-based interventions are inappropriate for students with mathematics difficulties (MD), where more direct, explicit instruction is frequently favoured (Kroesbergen & van Luit, 2003). It is in this arena that this thesis focuses.

In this chapter, I begin by discussing the historical and theoretical origins of inquiry as a pedagogical practice. I then describe some of the reported benefits of inquiry and present a framework of inquiry-based instruction for use in this study. Following this, I describe the mediating factors that have been previously explored in inquiry research, with a focus on student prior attainment and student beliefs[1]. I argue that research to date has not focused on how students with low prior attainment (those with MD) perceive inquiry-based instruction or whether their beliefs are associated with its effectiveness. I conclude this chapter with two research questions.

## 2.1 Origins of inquiry as a pedagogical practice

When reviewing the literature on the origins of inquiry as a pedagogical practice, the general consensus is that credit belongs to John Dewey (1859-1952). Prior to Dewey, numerous educational philosophers had contributed to a general shift in epistemological thinking away from 'knowledge given as faith to knowledge based on thinking, reflection, experimentation and science' (Artigue & Blomhøj, 2013, p. 798). Dewey's point of departure was to turn this epistemological thinking into a pedagogical practice. Dewey had several names for this practice, such as the 'experimental practice of knowing' (Dewey, 1929), before settling on the term 'reflective inquiry' (Dewey, 1933). This notion captured his idea that 'the origin of thinking is some perplexity, confusion, or doubt. Thinking is not a case of spontaneous combustion' (Dewey, 1910, p. 12).

Dewey placed the problem–and the adaptive process that is experienced during the exploration and solving of this problem–at the centre of learning, stating that 'all reflective

[1] There are numerous other factors that may play an important role in how a student responds to IBI (e.g. student socioeconomic status, teacher beliefs, stereotype threat, etc.), however I do not address these in depth as part of this thesis.

inquiry starts from a problematic situation' (Dewey, 1929, p. 189). His approach is captured well by Hiebert et al. (1996):

> Dewey placed great faith in scientific (and ordinary) methods of solving problems…. He believed reflective inquiry was the key to moving beyond the distinction between knowing and doing, thereby providing a new way of viewing human behaviour. (p. 14)

The major features of reflective inquiry are problem identification, exploration, and resolution. Student exploration was what distinguished Dewey's approach from others. As Dewey (1929, p. 36) put it, 'The experimental procedure is one that installs *doing* as the heart of learning'. During student exploration the learner follows similar methods to that of professionals within the discipline. This requires certain habits of mind on the part of the learner, as exploration 'involves willingness to endure a condition of mental unrest and disturbance' (Dewey, 1910, p. 13).

Dewey was a pragmatic philosopher and believed that knowledge should be useful for understanding and solving real-world problems. For this reason, he was an advocate for building school experiences that reflected real-life situations and linking students' in-school experiences with their out-of-school lives. Through reflective inquiry, Dewey believed students would learn domain-specific knowledge as well as general habits for inquiry.

Dewey's work, along with that of Piaget (1973), contributed to the constructivist philosophy that says learning is idiosyncratic and must be constructed from the experiences of the learner. Reflective inquiry, and indeed all subsequent inquiry-based pedagogical approaches, can be said to fall under the constructivist philosophy. This contrasts with more traditional, direct instruction which can be said to fall under the *instructivist* philosophy that says teachers are the primary agents of learning (Kirschner et al., 2006; Skinner, 1965). Direct instruction typically includes an intervention in which the teacher provides the students with substantial guidance and structure such that errors and opportunities to develop misconceptions are reduced. Such teacher-led guidance can involve targeted instruction on a concept with the use of well-designed examples to facilitate learning of the canonical solution. This is typically followed by some form of independent practice. This form of instruction is often referred to as 'tell-and-practice' or 'traditional' instruction (Schwartz et al., 2011).

## 2.2 Frameworks of inquiry in mathematics education

Dewey was a former science teacher, and therefore much of his early work on inquiry-based instruction was focused on science, a subject in which he felt there was an excessive focus on facts and insufficient emphasis on thinking (Dewey, 1910). In this section, I discuss how inquiry-based instruction developed in science education and then migrated to mathematics education, where it met several existing ideas.

In the U.S., the national science curriculum was already under review when the Sputnik Crisis of 1957 incited the nation to challenge the quality of its science education. The result was a wave of changes to science education that provided for the development of teaching practices that encouraged students to 'think like a scientist' (DeBoer, 1991, p. 192). Ultimately, this led to the National Science Education Standards in 1996 with subsequent revision in 2000 (National Research Council [NRC], 1996, 2000). These standards outline five essential features of inquiry, which Barrow (2006, p. 268) describes as: '(1) scientifically oriented questions that will engage the students; (2) evidence collected by students that allows them to develop and evaluate their explanations to the scientifically oriented questions; (3) explanations developed by students from their evidence to address the scientifically oriented questions; (4) evaluation of their explanations, which can include alternative explanations that reflect scientific understanding; and (5) communication and justification of their proposed explanations'.

Around the same time in Europe a project called PRIMAS (which stands for 'Promoting inquiry-based learning (IBL) in mathematics and science across Europe') began incorporating similar features into its curriculum and teacher development materials (see Figure 2.1).

The NRC (1996, 2000) and PRIMAS (Abril et al., 2013) are well aligned on the features of inquiry. Both also come from the position that habits of inquiry are an independent, valuable, and developable skill *distinct* from the domain-specific knowledge being taught, as well as the idea that bigger concepts (those concepts that apply to a wide range of ideas or phenomena) emerge from smaller ones (ones that apply to a specific idea or observation).

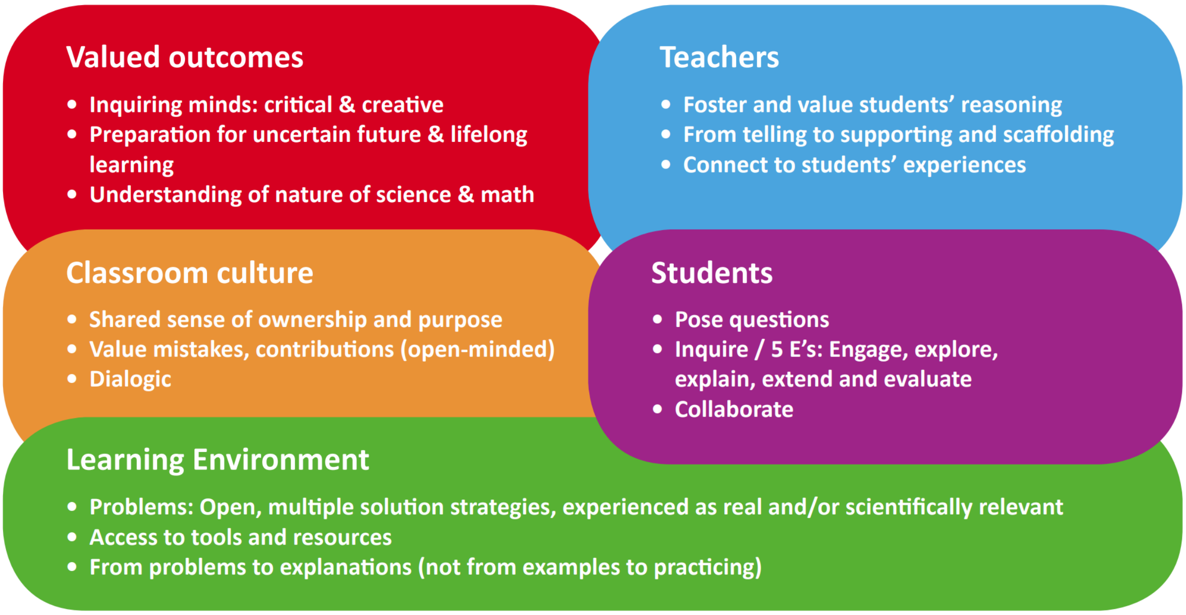


Figure 2.1: PRIMAS vision of inquiry-based instruction.

From 'Inquiry-based learning in maths and science classes,' by A. M. Abril et al., 2013, *PRIMAS Project*, p. 8. CC-BY-SA.

Inquiry-based pedagogy moved from science education into mathematics education as calls for economic competitiveness in the U.S. and the U.K. intensified (Laursen & Rasmussen, 2019), and both researchers as well as teachers began to see overlap between the two disciplines. Numerous theories of inquiry instruction in mathematics have emerged over the years. The proliferation of these, their associated terms, and the nuanced differences between them can lead to confusion and a lack of clear delimitations between different inquiry and non-inquiry approaches. To help conceptualise inquiry-based instruction it is helpful to review some of the significant theories and terms.

Laursen and Rasmussen (2019) describe two strands of inquiry-based mathematics education. The first strand is research-driven and known as Inquiry-Oriented Instruction (IOI). This form of inquiry instruction draws heavily from the work of Paul Cobb and Erna Yackel in the 1990s, whose work helped to solidify the term inquiry in the field of mathematics education (e.g. Cobb et al., 1991; Yackel & Cobb, 1996). Their observations of IOI in primary mathematics classrooms led them to define a series of sociomathematical norms (Yackel & Cobb, 1996). Two such norms were: (1) an understanding of what counts as an acceptable mathematical explanation, and (2) an understanding of what constitutes a mathematical difference.

The second strand of inquiry-based mathematics education identified by Laursen and Rasmussen (2019) is called Inquiry-Based Learning (IBL), and it is principally practitioner-led. The rise of this approach began as a movement to emulate the distinctive teaching practices of Robert Lee Moore, a mathematics professor at the University of Texas at Austin. This became known as the 'Moore Method' and then later the 'Modified Moore Method' and then finally 'IBL', partly in an effort to unite with the terminology being used in science education. IBL is not described as research-based practice but rather as 'consistent with and supported by education research' (Laursen & Rasmussen, 2019, p. 136).

Both strands of inquiry-based instruction persist and are characterised by challenging problems that require students to explore, explain, and evaluate new ways of working in mathematics. Central to both researcher-led and practitioner-led inquiry instruction is that students have the opportunity to explore new ideas before being taught directly.

Other conceptual theories of inquiry-based instruction include Problem-Based Learning (e.g. H.-C. Li & Stylianides, 2018), Realistic Mathematics Education (Freudenthal, 1973, 1991), the Theory of Didactical Situations (Brousseau, 1997), Active Learning (e.g. Freeman et al., 2014), Productive Failure (e.g. Kapur, 2014), Ambitious Mathematics (e.g. Jackson et al., 2013), and Complex Instruction (e.g. Boaler, 2006). This literature review does not undertake an exhaustive review of all of these theories. However, there are several which warrant further discussion. These are Problem-Based Learning, Realistic Mathematics Education, Theory of Didactical Situations, and Productive Failure.

*Problem Based Learning* (PBL) is a learning and teaching theory in which students are presented with a problem and a learning objective. As students attempt to solve the problem the teacher provides feedback and guidance. The objective is frequently to help guide the students towards the correct solution. Hence, this approach is sometimes labelled guided-discovery (Hmelo-Silver, Duncan, & Chinn, 2007). PBL is often conducted in collaborative settings, and student learning is scaffolded through the use of a domain knowledge expert (the teacher) as well as other materials. By providing question prompts, feedback on proposed solutions, clarifications, and drawing attention to critical aspects of the problem, teachers are able to scaffold their students toward the correct solution. As a teaching approach, however, PBL is principally focused on teaching students the habits of problem solving and hence is perhaps best considered a metacognitive strategy (Schoenfeld, 1992).

*Realistic Mathematics Education* (RME) traces its origins to Hans Freudenthal (1973). Realistic Mathematics is essentially the mathematisation of real events (but not necessarily everyday events). Freudenthal believed that even though not all students will go on to become mathematicians they will inevitably use mathematics in their lives, and therefore, the way they learn mathematics in school should reflect this. Whereas PBL might be described as the epistemology of problem-solving, RME might be said to be the epistemology of problematising. RME starts from two central principles: (1) mathematics is a human activity, and (2) meaningful mathematics is constructed from rich contexts (Freudenthal, 1991). As a result, teaching practices should focus on presenting problems and creating environments that allow students to 'construct the meaning of the abstract concepts and methods gradually through mathematization of meaningful real-life situations' (Artigue & Blomhøj, 2013, p. 804). Realistic problems appeal to the learner's common sense, present situations that are useful beyond the problem context, and allow a broad range of approaches and solution pathways (Wang et al., 2018). In general, RME leads to the philosophy of teaching through *reinvention*, a concept that lays at the heart of many inquiry-based teaching approaches. During RME students take real-world problems into the world of mathematical notation through a process called *horizontal mathematisation* (Gravemeijer & Terwel, 2000). As students then reflect upon and challenge these mathematical models, they begin to form more abstract mathematical models through a process called *vertical mathematisation* (Gravemeijer & Terwel, 2000).

The *Theory of Didactical Situations* (TDS) was put forward by Guy Brousseau (1997). Central to this theory is the notion of a 'situation', which is broadly understood to be a system of interactions between a student, the student's peers, the teacher, the mathematical concept being taught, and a 'milieu'. According to Brousseau, teachers seek to create a 'didactical situation' with their student, the goal of which is to create or modify the knowledge or understanding of the student. Through a process called 'devolution' the teacher passes authority of the problem to the student, who in taking ownership has entered into an 'a-didactical situation'. Whilst in an a-didactical situation the student moves through three further situations: (1) the Situation of Actions–the exploration of the problem as problem-solvers; (2) the Situation of Formulation–the development and discussion of the problem and solutions; and (3) the Situation of Validation–the evaluation of their thinking and proposed explanation of the phenomena. Finally, the teacher moves the student through a process of 'institutionalisation' whereby the student's understanding is aligned to the established mathematical norms being taught. TDS mirrors other inquiry-based theories in that it frames learning as a process in which students take ownership of problems, explore problems individually and collaboratively, evaluate their solutions during classroom discussions, and move toward a canonical solution.

Turning to another theory in this space, Kapur (2016) argues that existing inquiry approaches provide too much guidance, and therefore he advocates for a teaching strategy called *Productive Failure.* Under this approach students undertake a pure discovery exercise, in which students are permitted to repeatedly fail without any guidance or feedback, followed by a period of direct, explicit instruction of the mathematical concepts. Kapur argues that studies have shown that even when students initially solve problems in sub-optimal or even invalid ways, they ultimately demonstrate deeper conceptual learning (Kapur, 2014). Despite repeated incorrect attempts at solving problems, the students in Kapur's studies were better prepared for the direct instruction that followed and thus were better able to appreciate the superiority of the canonical method over their own.

Students who are allowed to solve problems before direct instruction are also able to more accurately gauge their understanding in both the short and long term (DeCaro & Rittle-Johnson, 2012). It seems failure plays an important role in the learning process (Tawfik et al., 2015). It is precisely when students reach an impasse during instruction that they are presented with a valuable learning opportunity (VanLehn et al., 2003). Kapur argues that even guided discovery tasks such as PBL are too restrictive and constrict the creative landscape for students. It has long been known that adult instruction restricts children's creativity, as they believe the adult has provided them with all the necessary information (Bonawitz et al., 2011).

In this section, I have presented several theories of inquiry instruction. Many of the theories in this space share more similarities than they do differences. In some respects, they all theorise on the amount and nature of guidance to provide to students when they reach an impasse during an inquiry problem. Therefore, before leaving this section, it is worth touching on one of the more extreme form of inquiry learning in which students receive *zero* teacher guidance, called 'pure discovery learning'. There appears to be agreement that pure discovery learning is a poor method of instruction (Hmelo-Silver et al., 2007; Kirschner et al., 2006; Mayer, 2004). Numerous experimental and quasi-experimental studies have demonstrated that there is little benefit to permitting students to undertake a cognitively demanding search of the problem space with little hope of discovering the correct solution (Sweller, 2009). In a meta-analysis of 164 publications looking at different inquiry implementation Alfieri, Brooks, Aldrich, and Tenenbaum (2011, p. 13) found that 'the effects of unassisted-discovery tasks seem limited, whereas enhanced-discovery tasks requiring learners to be actively engaged and constructive seem optimal'.

For this reason, all of the previously discussed theories have a clear role for the teacher in providing some form of guidance and instruction. In practice, no inquiry-based curricula advocate for a position as extreme as pure discovery learning, but rather they argue for a balanced approach in which inquiry-based activities precede or are interwoven with some form of more direct instruction or guidance. A study by Aulls (2002) found that few teachers implement inquiry-based instruction in a pure form, but rather tend to move towards the hybrid options such as PBL or RME.

Many critics portray a dichotomy between inquiry-based instruction and direct instruction, with direct instruction contrasted against pure discovery learning. Accepting the finding that pure discovery learning is ineffective does not necessarily indicate pure direct instruction is the optimal method by which learning can occur, nor that all forms of IBI are ineffective in mathematics classrooms. I, therefore, see this as a false dichotomy. I prefer to think of inquiry instruction in line with the view put forward by Bruder and Prescott (2013, p. 811) who argue 'the situation is less black-and-white so it is more practical to see these terms as being at either end of a continuum'. This continuum generally moves from teacher-centred approaches (such as direct instruction) at one end to student-centred approaches (such as IBI) at the other. Between these two poles lay a continuum of possibilities in which optimal learning for different circumstances may occur. Terms such as inquiry-based instruction, realistic mathematics education, discovery learning, exploratory learning, problem-based learning, minimally guided instruction, experiential learning, constructivist learning, and more recently productive failure all have homes on this continuum.

In an effort to advance a unified vision of Inquiry-Based Mathematics Education (IBME), Laursen and Rasmussen (2019, p. 138) suggest four pillars: '(1) students engage deeply with coherent and meaningful mathematical tasks; (2) students collaboratively process mathematical ideas; (3) instructors inquire into student thinking; and (4) instructors foster equity in their design and facilitation choices'. I believe these pillars are generally a helpful framework in conceptualising inquiry-based instruction in mathematics. However, in presenting this framework, I put forward a small amendment to the first pillar so that it reads: 'students engage deeply with coherent and meaningful mathematical tasks *prior to receiving instruction*.'

## 2.3 Benefits of inquiry-based instruction

There are significant benefits to inquiry-based instruction that have been evidenced in the literature. The primary benefit of IBI is thought to be the development of students' mathematical proficiency. Mathematical proficiency is defined by Kilpatrick, Swafford, and Findell (2001, p. 5) as the composition of five interwoven strands. The five strands are:

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's efficacy.

In this section, I focus on the impact of inquiry instruction on students' conceptual understanding and procedural fluency.

As a complement to the definition given by Kilpatrick *et al*. (2001), procedural fluency can be seen as an extension of procedural knowledge, meaning 'the ability to execute action sequences to solve problems' (Rittle-Johnson et al., 2016, p. 577). Conceptual understanding can refer to conceptual knowledge, meaning 'one's mental representation of the principles that govern a domain' (Rittle-Johnson et al., 2016, p. 577). Whereas procedural knowledge is intimately connected to the specific problem, conceptual knowledge 'consists of abstract or generic ideas generalised from particular instances, including knowledge of problem structures' (DeCaro & Rittle-Johnson, 2012, p. 555).

Historically, mathematics educators and researchers have advocated for the superiority of conceptual understanding over procedural knowledge (Llewellyn, 2014). However, there is a growing consensus that both conceptual understanding and procedural fluency are equally important since the two are intimately connected and increasing one has a direct impact on the other (Rittle-Johnson et al., 2015). It has been argued that inquiry-based instruction is an efficient teaching approach since it has been shown to improve both conceptual and procedural knowledge (Rittle-Johnson et al., 2016).

There have been numerous studies that have argued that inquiry-based approaches lead to improved conceptual understanding without sacrificing procedural fluency. One of the early studies demonstrating this effect was conducted by Cobb et al. (1991). Cobb et al. compared ten classes who each undertook a yearlong inquiry-based mathematics project with eight classes who did not. The results were that those students who were exposed to the inquiry learning approach scored similarly on procedural mathematics questions but demonstrated superior performance on questions designed to assess conceptual mathematics understanding. Also, the inquiry students 'held beliefs about the importance of understanding and collaborating; and attributed less importance to conforming to the solution methods of others, competitiveness, and task-extrinsic reasons for success' (Cobb et al., 1991, p. 3). In 102 classroom observations of inquiry-based lessons (60 in science and 42 in mathematics) in an urban middle school district in the U.S., Marshall and Horton (2011) noted a positive correlation between the amount of time dedicated to exploration of a novel problem and the cognitive level at which students operated.

In a multiple case study of U.K. secondary schools, Boaler (1998) observed two schools with differing mathematics instructional philosophies over three years. One school implemented an open, problem-based curriculum while the other implemented traditional instruction. The conclusion was that students who received problem-based instruction 'were able to achieve more in test and applied situations than the [traditionally instructed] students; they also developed more positive views about the nature of mathematics' (p. 60). Boaler's study, like many case studies, is open to the criticism that case-specific contextual factors make comparison problematic. However, similar observations were seen when reviewing the implementation of a problem-based mathematics curriculum in three California high schools (Clarke et al., 2004). In this study, students exposed to the problem-based curriculum scored equivalent to or higher than their peers on the mathematics portion of the Scholastic Achievement Test (SAT).

There have also been numerous studies that have found similar results in various fields such as secondary school mathematics (e.g. Kandil & Işıksal-Bostan, 2019; Kapur, 2011; Salim & Tiawa, 2015), undergraduate mathematics (e.g. Kwon et al., 2005; Rasmussen et al., 2006; Rasmussen & Kwon, 2007), college pre-service mathematics education (e.g. Laursen et al., 2016), secondary school economics (e.g. Mergendoller et al., 2006), secondary school physics (e.g. Schwartz et al., 2011), and medical school (e.g. Schmidt et al., 2009; Vernon & Blake, 1993).

It is worth noting that not all of the literature supports these findings and some studies have concluded that direct instruction is superior at developing both procedural and conceptual knowledge (e.g. A. L. Brown & Campione, 1994; Klahr & Nigam, 2004; Moreno, 2004; Tarmizi & Sweller, 1988). A review of 30 years of research into such minimal guidance approaches by Mayer (2004) found there was 'sufficient research evidence to make any reasonable person skeptical about the benefits of discovery learning' (p. 14). Most of these studies, however, were contrasting direct instruction with *pure discovery* rather than the more common inquiry theories which include a supportive role for the teacher.

Beyond the benefits of mathematical proficiency, studies have also suggested that inquiry-based instruction can help to moderate the gender gap. Work by Laursen et al. (2014) demonstrated a reduction in the gender gap for mathematics students at the undergraduate level. In addition, after a 4-year study into the effect of inquiry-orientated instruction in secondary schools, Boaler (2006) found 'the work of students and teachers … was equitable partly because students achieved more equitable outcomes on tests, but also because students learned to act in more equitable ways in their classrooms' (p. 45). In one instance, IBI actually created a gender gap in favour of females (Cooper et al., 2015).

IBI has also been shown to enhance a number of dimensions of student affect. For example, confidence (Kogan & Laursen, 2014), attitudes towards mathematics (McGregor, 2014), and agency (Hassi & Laursen, 2015).

Various mechanisms have been put forward to explain why IBI seems to be effective. By allowing students to explore the problems it is proposed they become conscious of gaps in their knowledge and that this awareness facilitates the assimilation of the missing pieces (Loibl & Rummel, 2015; Schwartz & Martin, 2004). Studies also highlight that inquiry-based tasks help activate deeper awareness of the learning processes and prepare students for subsequent direct instruction (Kapur, 2010, 2011, 2014; Schwartz et al., 2011; Schwartz & Martin, 2004). In addition, IBI has been shown to increase student motivation (Glogger et al., 2013; Hmelo-Silver, 2004) and increased motivation has a positive impact on learning and transfer (Belenky & Nokes-Malach, 2012). Piaget (1970b, p. 715) said, 'Each time one prematurely teaches a child something he could have discovered for himself, that child is kept from inventing it and consequently from understanding it completely'.

## 2.4 Criticisms of inquiry-based instruction

There are several axes upon which the literature criticises inquiry-based approaches. While this literature review does not make an exhaustive list of these criticisms, it does highlight several. Herein I address issues regarding: (1) coverage, (2) implementation, and (3) cognitive architecture.

**'You cannot cover all the material'**

Coverage issues relate to the challenges that teachers face when attempting to meet every learning standard set for their students within a school year. Teachers are under immense pressure to cover these learning standards in part because their students, their schools, and their teaching abilities are assessed against them using standardised tests (e.g. the National Curriculum Assessments in the U.K.). Coverage is also thought to be necessary so that students are adequately prepared for their next mathematics course in the sequence. Critics of inquiry-based curricula complain that inquiry approaches to instruction take too much time, and this wasted time results in less coverage of the topics students are required to learn. A typical teacher worried about coverage might say, 'I need to cover all these topics, so I don't have enough time to do student-centred activities' (Yoshinobu & Jones, 2012, p. 304). But what is the price of coverage? In a yearlong study of a tenth-grade geometry class in the U.S., Schoenfeld (1988) found that using traditional, direct teaching approaches ensured coverage but not understanding. In this study, the teacher successfully taught all the material on the curriculum, and the students scored well on standardised tests. Nevertheless, Schoenfeld found that the students significantly lacked conceptual understanding of geometry.

**'It is too difficult to implement'**

Turning to the implementation criticism of IBI, Hiebert and Stigler (2004) found that despite teachers advocating for inquiry-based instruction, most struggle to implement it successfully and instead stick to a traditional teaching approach. It appears the main challenges that teachers face include difficulty embracing the facilitator role (H.-C. Li & Stylianides, 2018) and struggles with student engagement (Stylianides & Stylianides, 2014). Dorier and García (2013) argue, as have others, that greater emphasis must be placed on the development of high-quality inquiry materials to help practitioners with implementation. Today there are more inquiry resources available than ever. For example, Bowland Maths[2], PRIMAS[3], and NRICH[4] have all developed useful professional development resources for IBI.

**'It is not compatible with cognitive architecture'**

One final criticism of IBI is its poor compatibility with Cognitive Load Theory (CLT; Sweller, 1988). CLT is built upon the standard cognitive model proposed by Atkinson and Shiffrin (1968) in which human cognitive architecture is composed of sensory memory, long term memory, and working memory. The latter two are most relevant for learning. According to Sweller (2011), working memory is placed under cognitive load during learning, and this load can be differentiated into intrinsic, extraneous, and germane (see Table 2.1).

The extraneous component of cognitive load is thought to be directly affected by the instructional approach. The oft-cited article by Kirschner, Sweller, and Clark (2006) points to excess extraneous load as the fundamental reason why inquiry approaches to instruction do

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[2] https://www.bowlandmaths.org.uk/

[3] https://primas-project.eu/

[4] https://nrich.maths.org/

not work. However, the authors seem to conceptualise all inquiry approaches as pure discovery learning, which as discussed in Section 2.2, has long been rejected. The authors, therefore, overlook ways in which CLT may be relevant to modern forms of inquiry instruction which incorporate different levels of teacher guidance. Furthermore, Kirchner and colleagues do not attend to the possibility that IBI can result in increased germane load (the type of load required to achieve long term learning) or the potential metacognitive benefits of such minimal instruction (Foster, 2014; Loibl & Rummel, 2015; Schwartz & Martin, 2004).

Table 2.1: Three components of cognitive load

| Component | Explanation |
|---|---|
| Intrinsic | Relates to the inherent complexity of the subject matter |
| Extraneous | Relates to the demands created through the manner in which information is presented |
| Germane | Relates to the processing and construction of schemas into the long-term memory |

## 2.5 Students with mathematics difficulties

The purpose of this study is to explore the efficacy of inquiry-based instruction for students with mathematics difficulties (MD). Having spent some time discussing the nature of inquiry-based instruction, I shortly review how the literature has explored the effectiveness of this approach for students with MD. To effectively do so however, it is worth discussing what 'mathematics difficulties' means and how educators might identify such students.

The idea that a student can be deficient in a domain of knowledge is well understood and evidenced by the widespread acceptance of reading difficulties such as dyslexia. However, the application of this notion to mathematics has been emergent over the last decade. Terms such as mathematics disability, mathematics difficulties, and dyscalculia are used, somewhat interchangeably, to describe poor mathematics performance.

The literature on MD has been criticised for using a fluid definition of 'mathematics difficulties'. More generally MD is recognised as a learning disability and as such is considered by many to be a brain disorder with neurological, cognitive or congenital roots in the same way as dyslexia (Shalev et al., 2000). Some studies have supported this idea through identification of familial lineages (Shalev et al., 2001). The specific mode of action within the brain that is impacted in MD is unclear. Studies which have looked at the brain have identified certain regions in the parietal lobes that are associated with arithmetic performance that when damaged through injury or underdeveloped due to premature birth result in reduced arithmetic abilities (Butterworth, 1999; Isaacs et al., 2001). Ansari (2010) argues that we are unsure the exact regions of the brain that relate to mathematical cognition, however they are numerous and developmental deficits in any of these could manifest in phenotypically poor mathematic performers.

Whilst thinking about MD within this cognitive context is useful there is evidence within the literature that this purely cognitive view is insufficient. A longitudinal study of three thousand U.K. pre-schools demonstrated that students from low socio-economic backgrounds were more likely to underperform in arithmetic tasks and be classified as having MD (Sammons et al., 2002). Further evidence supports the idea that lack of exposure to mathematics within the environment at a young age, as is often the case in students from low socio-economic backgrounds, impacts student performance in mathematics (Baroody & Dowker, 2003; Geary, 2004). This would seem to suggest that MD could result from simply not learning the required skills early enough. The publishing of successful intervention techniques that seek to teach fundamental concepts to children with MD would seem to support this view. Kaufmann, Handl, and Thöny (2003) found that ‘children with developmental dyscalculia benefit from a numeracy intervention program that focuses on basic numerical knowledge and conceptual knowledge’ (p. 564).

As such, the view of MD as a learning disability, limited to congenital and cognitive factors, is the subject of active debate as the literature explores a more complex set of contributors including social and environmental factors. The study of mathematics difficulties ‘involves various disciplines, such as cognitive psychology, child development, and curriculum based assessment’ (Gersten et al., 2005, p. 293) in addition to cognitive and neurological factors (Mazzocco, 2005). Despite its growing recognition as a distinct phenomenon, no definition of MD has been accepted within the literature. In fact most of the literature is mixed on how it refers to what may be the same construct, with studies using terms such as mathematics disabilities (e.g. Geary, 1993, 2004; Murphy et al., 2007), mathematics difficulties (e.g.

Gersten et al., 2005; Isaacs et al., 2001; Lucangeli & Cabrele, 2006; Raghubar et al., 2009), and dyscalculia (e.g. Kosc, 1974; Shalev et al., 2000, 2001). In a separate review of forty years of research in this area, K. E. Lewis and Fisher (2016) argue that attempts to combine these different terms into a single construct are at the root of the confusion. They conclude that mathematics learning *disability* is a cognitive construct and distinct from non-cognitive ideas such as mathematics *difficulties*.

> To understand MLD [mathematics learning disability] it is necessary to differentiate between cognitive and non-cognitive sources of mathematics difficulties. Conflating students with an MLD and students with low achievement due to another cause makes it difficult to draw conclusions about either group of students. (K. E. Lewis & Fisher, 2016, p. 340)

Therefore, the extent to which these are the same or different constructs is still being explored, and for my purposes, I use the term mathematics difficulties (MD) which is in line with the recommendation discussed by K. E. Lewis and Fisher (2016).

> Those studies that do not report demographic differences between groups might more accurately classify students as having 'mathematics learning *difficulties*' or as being '*at risk* for mathematics learning disabilities' rather than students with an MLD. The label of *disability* connotes a cognitive difference not warranted in studies that are not attending to the existence of potential confounding environmental factors. In keeping with this recommendation, some researchers tend to use the term difficulties rather than disabilities to refer to students classified with achievement measures alone. (K. E. Lewis & Fisher, 2016, p. 364)

Assuming these different terms refer to the same construct, there is further debate into the actual processes that are deficient in MD. Suggested characteristics include poor number comprehension (Butterworth, 2005), executive functioning (Bull & Scerif, 2001), arithmetic fact retrieval (Jordan et al., 2003; Raghubar et al., 2009), and number sense (Landerl et al., 2004). These latter characteristics of arithmetic fact retrieval and number sense are repeating themes in the literature. This lack of consistent characteristics has been commented upon by some researchers as questioning whether MD is merely a loose term representing the lower end of the performance continuum rather than a truly distinct construct (Baroody & Dowker, 2003; Macaruso & Sokol, 1998). In her review into dyscalculia, Gifford (2006) asserts 'there seems to be no differences between children with a specific difficulty, those who are generally low achievers and those merely poor at arithmetic' (p. 38).

Some studies estimate that the prevalence of MD is as low as three to six percent, similar to that of reading disabilities and attention deficit disorder. They also report that MD is persistent (meaning students demonstrate the characteristics consistently over time) for around half of affected students (Shalev et al., 2000). However, these studies have tended to limit the definition of MD to the more cognitive focused 'mathematics disability', rather than the broader definition which includes non-cognitive sources of mathematics difficulties. Crucially, MD has been shown to be distinct from other learning disabilities such as dyslexia, and attention deficit disorder. However, co-morbidity between MD and dyslexia is around 40 percent, although this number varies between studies (Landerl & Moll, 2010; C. Lewis et al., 1994). Further, co-morbidity between MD and attention deficit disorder has also been explored within the literature and studies show similar co-morbidity rates to reading disabilities, although the general body of evidence is less conclusive (Lucangeli & Cabrele, 2006; R. M. Marshall et al., 1997).

Having no clear definition for MD has hindered the development of diagnostic tools. Even limiting the discussion to neuro-genetic and cognitive factors does not help, since these hardly provide practical means for diagnosis in the field. An often-used method for detection is the use of students' standardised test scores (Berch & Mazzocco, 2007). The validity of using standardised test scores as a differential diagnosis tool for MD has been explored in several studies (D. H. Bailey et al., 2012; Murphy et al., 2007). These studies, along with others, have supported the use of standardised tests scores, with the $10^{th}$ to $25^{th}$ percentile emerging as common cut-offs. However, their sole reliance is not universally accepted or without criticism.

The first challenge with using cut-offs on standardised tests is the implicit assumption that MD represents the low end of a distribution, as is typically agreed upon for dyslexia (Shaywitz et al., 1999). This assumption is supported somewhat in the literature (Girelli et al., 2000; Mazzocco & Myers, 2003). Secondly, cut-offs fail to recognise the volatility within test results by which students can move in and out of the cut-off criteria from one school year to the next. Mazzocco (2005) reported that ‘of 24 kindergartners meeting strict criteria for MD, 13 also met criteria in first grade, and 11 did not. An additional 6 children who had not met criteria for MD in kindergarten did meet these criteria in first grade’ (p. 320).

Thirdly, usage of single point cut-off criteria ignores other diagnosed or undiagnosed learning difficulties. For example, students with low IQ are possibly exhibiting poor mathematics performance for reasons that should not be characterised as mathematics difficulties. Fourthly, students can often compensate for difficulties with other strategies and therefore may not be identified as having MD based on standardised tests (Jordan et al., 2003). Finally, the content of typical standardised tests is not designed to provide a diagnosis of mathematics difficulties (Mazzocco, 2005).

Despite these limitations, the use of standardised tests is practical in the field due to their ubiquitous use in modern schooling. Therefore, to increase their effectiveness as a diagnostic tool, researchers supplement the standardised scores with other factors such as multiple years of test scores (to identify students persistently underperforming in mathematics), or eliminating students with pre-diagnosed learning disabilities or IQ below some threshold (Fuchs & Fuchs, 2002; Geary et al., 2000; Mazzocco & Myers, 2003). Incorporating teacher input is also used (Fuchs & Fuchs, 2002). In critique of using standardised test scores, K. E. Lewis and Fisher (2016) argue that MD is a not a single construct and that mathematics disability is distinct from low achievement due to non-cognitive causes (such as MD). The authors state that ‘commonly used cut-offs (10th percentile and 25th percentile) identified groups of students with different cognitive profiles’ and ‘students classified as having an MLD in one study might not be similarly classified in another, which limits our ability to meaningfully compare and synthesize findings across studies’ (p. 340).

In summary, there is a growing body of research into mathematics difficulties, however there is still much to learn in this developing field. As is typical in an emerging area of research the broad array of definitions and inclusion criteria makes comparisons between studies troublesome (Gifford, 2006). It is generally accepted that (1) MD is a construct distinct from other learning difficulties; (2) cognitive and non-cognitive factors may play a role, (3) poor arithmetic fact retrieval and number sense are common characteristics; and (4) existing standardised test scores should be supplemented with other screening criteria in identification.

## 2.6 Inquiry for all? A vision left unrealised

Despite the popularity of IBI, evidence for its effectiveness for students with mathematics difficulties has been mixed, and teachers have demonstrated a reluctance to use these techniques with this group of students (Darragh & Valoyes-Chávez, 2019; Lambert, 2018; Louie, 2017). In particular, teachers of students with MD are less familiar with and feel less equipped to teach this type of mathematics (Maccini & Gagnon, 2002). In addition, 45 percent of teachers believe that higher order thinking is inappropriate for low achieving students (Zohar et al., 2001). Teachers are likely to engage in unhelpful discourses of 'ability' when referring to students with MD (Alderton & Gifford, 2018) and often characterise them as 'dependent' learners who require more direct models of instruction (Mazenod et al., 2019). Even in the event inquiry tasks are selected by the teacher, these tasks often get reduced to simpler tasks focused on skill development. However, as Foster (2013b) argues, 'When reduction takes place *for* the student, rather than *by* the student, it may be experienced as dangerously disempowering' (p. 564).

A yearlong study of 104 primary students in the U.S. looked into the effectiveness of inquiry-based teaching approaches on mathematics performance (Woodward & Baxter, 1997). They found that students with learning disabilities and similar academically low achieving peers made only marginal gains when given IBI. The authors conclude that inclusive environments, in which children with learning disabilities as well those at risk for special education are mixed into general education classrooms, required teachers to provide considerable assistance and guidance to the low achieving learners. This is not compatible with the inquiry-based models embodied in reform mathematics.

In a meta-analysis of 58 intervention studies focused on primary mathematics between the years 1985 and 2000, Kroesbergen and van Luit (2003) concluded direct, explicit instruction was the most effective instructional approach for students with learning disabilities or identified as having MD. Most of the studies included in the meta-analysis used cut-off points on students' mathematics achievement scores as inclusion criteria. Similar results were replicated in other meta-analyses (Dennis et al., 2016; Gersten et al., 2009).

The specific group of students that have MD is heterogeneous and therefore researchers must be careful in drawing broad inferences. Students with MD often demonstrate poor generalisation and transfer skills and suffer from problems in applying both cognitive and metacognitive skills (Kroesbergen & van Luit, 2003). Therefore, it is not clear from the literature which factors contribute to the supposed poor response to the inquiry interventions.

The above data would seem to give support to the recommendation that students with mathematics difficulties are best instructed in an explicit manner, and instructional methods captured in reform curricula are not suitable for the needs of students with MD because they are too discovery-oriented (Carnine et al., 1994). However, before making this conclusion, I believe an important area of research has been neglected. Namely, how do student beliefs interact with the way students with MD perceive and perform in an inquiry-based classroom? The next section of this literature review explores the area of epistemological beliefs.

## 2.7 Students' personal epistemologies

Epistemology is the term used within philosophy to describe the nature of knowledge and justification of beliefs. It concerns questions that relate to the source and limits of human knowledge. These questions have become of central interest to educational psychologists under the term 'personal epistemology', wherein psychologists seek to explore 'how individuals acquire knowledge, the theories and beliefs they hold about knowing, and how these beliefs are a part of and influence cognitive processes, especially thinking and reasoning' (Muis, 2004, pp. 317–318).

The origins of personal epistemology within educational psychology draw roots from the work of Piaget (1970a) and Perry (1970). Piaget's work focused on whether an individual's epistemology can be viewed separately from the process of intellectual development. By approaching these questions from the view of developmental biology, Piaget (1970a) explored the relationship between knowledge and the knower. He sought to answer such philosophical questions as, 'Does knowledge depend on the individual (idealism), is it completely independent of the individual (realism), or is truth somewhere between these two extremes (constructivism)?' (Muis, 2004, p. 319).

Perry (1970) explored the epistemological question through a two-decade-long study of university students as they progressed through their studies. Perry argued that students move through nine different epistemological beliefs, organised into four phases. Students begin in the phase he called ‘dualism’, which is the belief that all problems have a correct solution, and this solution is only known to authority figures who represent the source of knowledge. Secondly, students move into the phase called ‘multiplicity’, wherein they believe that there are two kinds of problems, those for which we have solutions and those for which we do not. Students in this phase believe that people may differ in their views and for a student to gain knowledge they must develop their own position. Next, students progress to the phase of ‘relativism’, in which knowledge of solutions must be contextualised and supported by reasoning. The final progression is to the phase of ‘commitment’, in which students accept the ambiguity of knowledge and integrate what they learn from others with their own experiences.

Others have argued that these unidimensional models which focus on cognitive development are too simplistic and a student’s personal epistemology is complex and multidimensional. Schommer (1990, p. 498) said, ‘a more plausible conception is that personal epistemology is a belief system that is composed of several more or less independent dimensions.’

Schommer puts forward five dimensions of personal epistemology: (1) structure of knowledge–beliefs of knowledge as simple or complex; (2) certainty of knowledge–beliefs of knowledge as certain or tenuous; (3) source of knowledge–beliefs that knowledge is derived from an authority figure or from one’s own critical reasoning; (4) control of knowledge–beliefs about one’s ability to learn; and (5) speed of learning–beliefs about the speed at which one can learn and the role of effort in that speed. While Schommer may argue that her multidimensional model differs from the Perryan view, it is clear to see common ground between the two. The first three dimensions of Schommer’s (1990) personal epistemology align well with Perry’s (1970) cognitive development view of epistemology. The final two dimensions align well with the work on implicit theories of intelligence (Dweck, 1986; Dweck & Leggett, 1988). I discuss implicit theories of intelligence in more detail in Section 2.9, as it is central to this study.

Along each of Schommer’s dimensions, she identified beliefs as being either ‘naïve’ or ‘sophisticated’ (1990). Students’ beliefs about sources of knowledge can range from the ‘naïve’ belief that knowledge can only come from an authority figure to the more ‘sophisticated’ belief that knowledge is constructed through critical assimilation of information into one’s own experiences and contexts. The use of labels in this area is not universally agreed upon, with some terms being more emotive than others. Terms such as naïve or sophisticated incorrectly suggest that educational psychologists agree which end of these epistemological views is best. Muis (2004) argues for the more neutral terms ‘availing’ and ‘non-availing’.

Much of the early research into personal epistemology defended a global view, wherein students’ beliefs were general across all domains. However, based on the idea that academic domains can fundamentally differ in the nature of problem-solving and knowledge, as well as the idea that schools mirror these domain differences in their curriculum designs, it became clear that epistemic dimensions of students’ beliefs might themselves be domain-specific (Op ’t Eynde et al., 2006). There have been several studies which have supported this idea (Buehl & Alexander, 2002; Hofer, 2000; Schommer & Walker, 1995). Based on this research, it seems that elements of students’ epistemic beliefs exist at a domain-specific level. However, this does not mean that global beliefs are not also present, but rather the explanatory power of domain specificity is higher in certain contexts. Accepting this view, Buehl and Alexander (2001) argued for a *multidimensional* and a *multi-layered* view of epistemic beliefs. They put forward a hierarchical view containing three levels ‘nested within each other: (1) domain-specific beliefs, (2) academic epistemological beliefs; (3) general epistemological beliefs’ (Op ’t Eynde et al., 2006, p. 60).

## 2.8 Beliefs in mathematics

As discussed above, the idea that a student may have a mathematics domain-specific personal epistemology is supported by the literature. Within the field of mathematics education, the idea of personal epistemology has been studied more typically under the general term ‘mathematics beliefs’ (Muis, 2004). In his seminal paper, McLeod (1992) clearly placed mathematics beliefs within the broader construct of student affect. McLeod’s conceptualisation of affect put forward three constructs–beliefs, attitudes, and emotions. These are summarised with examples in Table 2.2. McLeod’s framework suggests that emotions are the most intense, least stable, and least cognitive of the three constructs, while beliefs are the least intense, most stable, and most cognitive. Attitudes sit in the middle.

Goldin (2002, p. 61) added a fourth dimension, values, and summarised the McLeod states of affect as:

- emotions (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context)
- attitudes (moderately stable predispositions toward ways of feeling in classes or situations, involving a balance of affect and cognition)
- beliefs (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured)
- values, ethics, and morals (deeply-held preferences, possibly characterized as 'personal truths', stable, highly affective as well as cognitive, may also be highly structured).

Other researchers have built upon these ideas. For example, Hannula (2011) describes the temporal dimension of the affective constructs. Under Hannula's framework, a distinction is made between trait-type affect constructs describing stable dispositions and state-type constructs describing more dynamic and rapidly changing dispositions. Different research methodologies suit the measurement of different temporal aspects. Quantitative methods can reveal stable trait-type constructs, whereas qualitative observational methods may allow the researcher to study more volatile state-type affect constructs (Di Martino, 2019).

Table 2.2: McLeod's framework for affect

| **Category** | **Example** |
|---|---|
| Beliefs about mathematics | 'Mathematics is based on rules' |
| about self | 'I am able to solve problems' |
| about mathematics teaching | 'Teaching is telling' |
| about the social context | 'Learning is competitive' |
| Attitudes | 'I dislike geometric proof' |
| Emotions | 'I feel frustrated with this problem' |

Another addition to the affect construct was the introduction of motivation (Hannula et al., 2007). Within the McLeod (1992) framework motivation was addressed as part of the beliefs construct (motivational beliefs). However, the literature has since established motivation as a core concept within the field of mathematical affect.

The purpose of my study is to evaluate whether students' beliefs are associated with the effectiveness of inquiry-based instruction. As such this literature review focuses on the beliefs construct within the broader affective domain. In his pioneering work on mathematical beliefs, Schoenfeld (1985) states that 'belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks' (p. 45). Before addressing beliefs further, it is worth briefly highlighting the evidence that links beliefs to academic performance.

**Beliefs and academic performance**

Many studies have explored the effects of beliefs on mathematical performance. Through a series of case studies Schoenfeld (1985, 1988) illustrated how students' mathematics worldviews (their beliefs about mathematics) impacted how they performed on mathematical problems. For example, students' performance was negatively impacted if the students held what Schoenfeld called inappropriate beliefs about mathematics. Such inappropriate beliefs include the belief that mathematical problems can be solved quickly, the belief that only geniuses can thrive in mathematics, and the belief that proofs in mathematics only confirm that which is obvious (Schoenfeld, 1985). In her work on epistemic beliefs (discussed in Section 2.7), Schommer (1990) categorised beliefs into those which are 'naïve' or 'sophisticated'. She found that holding 'naïve' beliefs was associated with poor academic performance (Schommer, 1990; Schommer et al., 1992).

> It is clear from work using either research methodology that the idea of sophisticated beliefs embraces evolving knowledge, multiple approaches to the justification of knowledge, integration of knowledge, and for those willing to entertain a broader conception of epistemological beliefs, gradual learning, and ever growing ability to learn. (Schommer-Aikins, 2002, pp. 112–113)

The belief that the source of mathematical knowledge is an authority figure is also linked to poor performance on problem solving tasks (Schoenfeld, 1985). Others have called this phenomenon learned helplessness, 'the perceived inability to surmount failure' (Diener & Dweck, 1978, p. 451).

**A focus on beliefs**

A focus of this thesis is on understanding how students' beliefs interact with their perceptions as well as the effectiveness of inquiry-based instruction. As such, this section centres upon the belief component of the affect construct. The research on beliefs is perhaps the most extensive amongst the dimensions of affect (Di Martino, 2019). Despite this, however, a universally accepted definition of beliefs has yet to come forward. For the purposes of this study, I adopt a broad definition of beliefs in line with that of Philipp (2007) in which beliefs are viewed to be 'psychologically held understandings, premises or propositions about the world that are thought to be true' (p. 259).

McLeod's framework (see Table 2.2) focuses heavily on beliefs and highlights that the role of beliefs is central to the attitudes and emotions experienced by students when learning mathematics. Within this framework, McLeod puts forward four subdomains of beliefs: beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the social context. I discuss each of these below.

**Beliefs about mathematics**

Research suggests that students hold many beliefs about mathematics. Examples include the belief that mathematics is important, difficult, and based on rules (C. A. Brown et al., 1988), mathematics is the domain of excellence (Op 't Eynde et al., 2006), mathematical problems can be solved in ten minutes or less (Schoenfeld, 1985), mathematics involves following rules and memorisation (Dossey et al., 1988), mathematics is skill-orientated (Greenwood, 1984), mathematics is about computation and all problems can be solved in just a few steps (Frank, 1988), the goal of mathematics is to get the correct answers (Frank, 1988), and mathematics cannot be easy (Kouba & McDonald, 1987). The body of research has pointed out that holding inappropriate beliefs impacts upon students' abilities to solve non-routine problems (Schoenfeld, 1988). When contrasting beliefs about mathematics to other subdomains of affect, McLeod states that 'these beliefs about mathematics, although not emotional themselves, certainly would tend to generate more intense reactions to mathematical tasks than beliefs that mathematics is unimportant, easy, and based on logical reasoning' (1992, p. 579).

**Beliefs about self**

Students hold various beliefs about themselves as mathematics learners. Examples of self-beliefs include one's belief that they are good at mathematics (Dossey et al., 1988), self-efficacy (Bandura, 1997), confidence (Reyes, 1984), intrinsic motivation (Schoenfeld, 1987), and Attribution Theory (Jones et al., 1987). Also within this subdomain I include the beliefs that students hold about intelligence, that is, their 'implicit theories of intelligence' (Dweck, 1986; Dweck & Leggett, 1988) otherwise known as 'mindset' (Dweck, 2017b). Both of these are discussed in Section 2.9.

**Beliefs about mathematics teaching**

McLeod argues that students hold many beliefs about how mathematics should be taught, which they develop through exposure to mathematics teaching in school (McLeod, 1992). While the research in this area is not as developed as some of the other beliefs, examples are seen within the literature, such as the belief that teaching mathematics should involve a range of tasks (Yackel & Rasmussen, 2003), the teacher should help students learn mathematics (Kloosterman et al., 1996), the role of the teacher is to transmit knowledge whilst that of the student is to receive knowledge (Frank, 1988), mathematics learning is done individually (Kloosterman et al., 1996), and mathematics involves mainly seatwork (Stodolsky, 1985).

**Beliefs about the social context**

McLeod's framework suggests that the beliefs students hold about the social context of mathematics impacts the way students tackle mathematical problems. Examples here include the belief that mathematics is a socially valuable skill, mathematics is competitive, peers are able to help with mathematics, and mathematics is part of one's out-of-school life (Cobb et al., 1989; Grouws & Cramer, 1989; McLeod, 1992).

The breadth of the McLeod framework is helpful in that it covers a wide range of possible beliefs that could interact with how students learn mathematics. However, the four subdomains are not mutually exclusive and can overlap. For example, the belief that mathematics is an individual activity could relate to one's beliefs about the social context as well as beliefs about mathematics teaching.

Another influential voice within this area is that of Schoenfeld (1983), who argues that cognitive behaviours that underpin learning rest upon an individual's beliefs about the task at hand, the social context in which that task occurs, and themselves as individuals performing the task in the current social context. It is easy to see overlap with McLeod's subdomains of beliefs.

## 2.9 Mindset as an epistemological belief

An aim of this study is to explore whether the effectiveness of inquiry-based methods for students with mathematical difficulties is influenced by students' implicit theories of intelligence (called mindset). Within the McLeod framework, discussed in Section 2.8, these beliefs fall under the category of beliefs about self. An important contribution to this discussion was made by Dweck (2017b) in her review of several decades of research into the effect of differing implicit theories of intelligence on achievement. As a result of these studies, Dweck proposes the idea that mindset can be categorised in two ways. Individuals with 'growth mindset' believe that intelligence is not fixed but malleable. They view learning as a process governed by effort, as opposed to ingrained ability (Boaler, 2013). By framing their learning within this context individuals are able to perform at a higher level (Dweck, 2017b). Alternatively, individuals that have a 'fixed mindset' believe that intelligence cannot be altered, and that ability or 'smartness' is something a person is born with. Individuals with fixed mindsets tend to focus on performance and set objectives around demonstrating strong ability in the areas they believe they are superior. As such, they avoid challenges that might compromise this view (Dweck, 2017b; Yorke & Knight, 2004). Students with growth mindsets, however, see challenges as learning opportunities and implement flexible learning goals. As such, students with growth mindsets typically respond positively to failure and see it as an opportunity for increased learning and effort (Dweck, 2017b; Yorke & Knight, 2004). Studies have shown that approximately 40 percent of students in the U.S. have growth mindsets and 40 percent have fixed mindsets. The residual population demonstrate mixed mindsets (Dweck, 2017b).

### 2.9.1 Criticisms of mindset

Emerging in literature as a result of a series of studies co-authored by Carol Dweck (e.g. Dweck, 1986; Dweck et al., 1995; Dweck & Leggett, 1988; Elliott & Dweck, 1988; Mueller & Dweck, 1998), mindset has reached an almost cult status, with a large number of schools regularly incorporating growth mindset material into their lessons. However, there are some notable objections put forward, which I discuss below.

**'Lack of replicating research'**

Critics argue that much of mindset research is based upon a few isolated studies, and most attempts to replicate these have failed to generate the same results (e.g. Y. Li & Bates, 2019). Whilst it is true that one might expect more supporting literature, especially given the popularity of the construct, I disagree with this criticism. Many studies have been able to replicate the positive correlation between mindset and academic performance (e.g. Aditomo, 2015; Aronson et al., 2002; Blackwell et al., 2007; Claro et al., 2016; Rattan et al., 2012; Sahlberg, 2011). More recently, a few larger-scale studies have added to the support in favour of this correlation. Notable examples of this include a study of 160,000 tenth-grade students in Chile (Claro et al., 2016).

Part of the challenge of replicating previous mindset results is the magnitude of the observed effect. In Mueller and Dweck's (1998, p. 36) study, 67 percent of students who received praise for their intelligence (thought to induce a fixed mindset) chose to work on 'easy' problems that could show how 'smart' they are, compared to only 8 percent of students who received praise for their effort (thought to induce a growth mindset). These are exceptional results and have led many to question the rigour of the analysis. Several recent meta-analyses of mindset research have generally supported the claim that incremental theories (growth mindset) are positively associated with academic goal setting and performance, albeit to a lesser extent than Dweck suggested (e.g. Burnette et al., 2013; Sisk et al., 2018).

Several studies, however, have failed to generate the above effect. Furnham, Chamorro-Premuzic, and McDougall (2003) conducted a study of U.K. university students and found personality traits *were* a predictor of academic performance, and whilst mindset was correlated with personality traits, the relationship between mindset and academic performance was *not* significant. Two separate studies of 222 and 211 Chinese pupils aged 9 to 13 years old also failed to find any effect of mindset. The authors found that 'the predicted association of growth mindset with improved grades was not supported' (Y. Li & Bates, 2019, p. 1640). Furthermore, a mixed methods study of 12 high school students with reading difficulties reported no effect of mindset on self-reported motivation and only a small effect on learning, although the author acknowledges the small scale of the study (Baldridge, 2010).

It is clear that further research into the construct of mindset is needed, particularly when it comes to the idea of teaching mindset. In response to criticisms of her work, Dweck (2017a, para. 1) said, ‘Growth mindset is on a firm foundation, but we’re still building the house’.

**‘Mindset studies lack context’**

One of the criticisms of mindset research is that it fails to properly account for contextual factors, such as socioeconomic status. For example, the prevalence of fixed mindset has been shown to be higher in students with ‘lower prior academic achievement, English learner students, and Black students’ (Snipes & Tran, 2017, p. i). Other studies have shown the almost opposite result. Hwang, Reyes, and Eccles (2019) found that students from more advantaged backgrounds (e.g. White or higher SES) tended to have fixed mindsets as they preferred to be recognised as ‘naturally intelligent’ as opposed to ‘hard workers’ (p. 262).

Recent evidence has further brought to light the importance of contextual factors such as SES in mindset research. A study by Claro et al., (2016) indicated that the positive effect of mindset was present across all socioeconomic levels, however the prevalence of fixed mindset was correlated with poverty.

> Students from lower-income families were less likely to hold a growth mindset than their wealthier peers, but those who did hold a growth mindset were appreciably buffered against the deleterious effects of poverty on achievement: students in the lowest 10th percentile of family income who exhibited a growth mindset showed academic performance as high as that of fixed mindset students from the 80th income percentile. (Claro et al., 2016, p. 8664)

There is evidence a growth mindset intervention can help minority students guard against stereotype threat and materially improve their academic performance (Aronson et al., 2002). Similarly, when considering students with ‘lower prior academic achievement’ a double study by Blackwell, Trzesniewski, and Dweck (2007) demonstrated that student belief that intelligence was malleable was an accurate predictor of subsequent academic performance over the following two years. In a second study, Blackwell and colleagues (2007) also demonstrated that providing an intervention targeted at teaching a growth mindset resulted in increased student achievement in mathematics over the subsequent 18 months, as compared to a control group. Perhaps most interestingly, this second study focused on students with MD (as determined by falling below the 35th percentile on the national test; see Section 2.5 for a discussion of MD), indicating that changes in beliefs about intelligence are possible in these students. The fact that students have been shown to respond favourably to growth mindset interventions has garnered much interest from teachers and parents. However, Yeager, Paunesku, Walton, and Dweck (2013) argue that our understanding of how to teach mindset is still in its infancy and more study is needed before attempting to scale such interventions.

**'It is too simplistic'**

Many studies have shown that a short intervention which focuses on students' thoughts and beliefs rather than academic knowledge can yield improvement in academic performance. In the previously mentioned study by Blackwell et al. (2007) the intervention was only eight sessions yet yielded stark improvements in academic performance for the entire school year. How can something so simple, that did not even focus on academic content, generate such large results? As of yet there is no clear answer to this question. In a review by Yeager and Walton (2011) the authors argue that huge effect-size of seemingly 'small' interventions have been demonstrated in other fields, such as the introduction of a one-page checklist in operating rooms that reduced surgical deaths by 47 percent. The authors argue that a similar phenomenon in education is beginning, with numerous studies of small interventions showing substantial improvement in student outcomes. However, the comparability of the checklist example is questionable as the impact of a tangible tool used for every procedure is hard to compare to an intervention which is supposed to have effects that persist long after.

The three categories of mindset (fixed, mixed, and growth) are certainly a simple concept, but are there really such hard delimitations between them? Proponents of mindset are rarely advocating for such hard delimitations. 'All of us have elements of both—we're *all* a mixture of fixed and growth mindsets' (Dweck, 2017b). Moreover, mindset can vary by subject:

> People can also have different mindsets in different areas. I might think that my artistic skills are fixed but that my intelligence can be developed. Or that my personality is fixed, but my creativity can be developed. We've found that whatever mindset people have in a particular area will guide them in that area. (Dweck, 2017b, p. 47)

The idea of subject-specific mindsets is not surprising given the general acceptance of domain-specific epistemic beliefs, discussed in Section 2.7.

Whilst Dweck may be credited for coining the term growth mindset, her work is very much grounded in a broader field of study that has its origins in Attribution Theory (Jones et al., 1987), which discusses the ways in which individuals explain the causes of outcomes, and Personal Construct Theory (Kelly, 1955) which argues that people see the world as a 'scientist' who builds constructs to explain certain events. Despite these deeper roots, the literature around mindset is still emerging. A search of literature discussing mindset in the decade before Mueller and Dweck's 1998 study found c18,000 articles. In the two decades since there have been c240,000 articles. Despite this, the number of large-scale studies conducted by independent research teams has been thin and further such studies are needed.

## 2.10 Fixed messages in mathematics

Inquiry-based instruction relies upon students attempting, often *unsuccessfully*, to solve problems. How a student responds to this failure might determine the effectiveness of the IBI. Given that students with fixed mindsets respond poorly to failure, and often see it as further evidence that they lack mathematics ability (Boaler, 2016; Dweck, 2017b) it would seem plausible that students with fixed mindsets would respond poorly to IBI.

Teacher attitudes towards low achieving students has been shown to propagate a fixed mindset (Boaler, 2013; Marks, 2013). The strong preference for mathematics ability grouping in the U.S. and the U.K. further drives the stereotype that abilities are somehow genetic and hence fixed (Plomin et al., 2007). It is estimated that 83 percent of pupils in England are grouped by ability for maths by the time they reach Key Stage 3 (Kutnick et al., 2005). It has been suggested in the literature that teachers adopt fixed mindsets in the way they instruct these groupings, even if they believe they are adopting mixed-ability mindsets (Marks, 2013). The effect of ability grouping on students' self-esteem, engagement, and perceived ability leave many students feeling like their academic performance is something out of their control (Braddock & Slavin, 1992). Boaler (2010) suggests that mathematics is the worst offender when it comes to perpetuating a fixed mindset. A comprehensive meta-analysis of research from 1970 to 2000 found that labelling a student with a 'learning disability' significantly changed teachers' attitudes and behaviours towards the student and resulted in lower expectations and fixed mindsets (Osterholm et al., 2007). The consequence is clear; students who demonstrate MD are more likely to encounter teaching practices that propagate fixed mindset through a combination of teacher beliefs and institutional ability grouping.

### 2.11 Summary of literature review

A review of the literature suggests that IBI can be an effective instructional approach for enhanced mathematical proficiency. The benefits of IBI are possibly metacognitive, in that the learner becomes aware of gaps in his or her knowledge and is therefore primed to learn better from subsequent instruction. However, the results of studies into the effectiveness of these inquiry-based approaches for students with MD have been mixed (see Section 2.6). Studies into mindset show that students with MD can fall into an educational system that propagates fixed mindsets, the result being these students are less likely to respond to failure as positively as students with growth mindsets. Given the prevalence of failure within an inquiry-based curriculum it might be possible that holding a fixed mindset reduces its effectiveness. Could it be that mindset is associated with the effectiveness of IBI for students with MD?

Little literature has looked into the ways in which mindset and other beliefs may influence students with MD in inquiry-based classroom environments. As part of this dissertation, I explore two research questions:

RQ1. How do students with mathematics difficulties perceive IBI?

RQ2. Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties?

# 3 Methodology

The principal objective of research is concerned with learning and understanding new information. However, in undertaking this goal, researchers make several key assumptions. Such assumptions include fundamental questions regarding *what* the researcher will learn new things about. This is not the same as the research question(s), but rather the researcher's fundamental philosophy about the nature of reality and to what extent research output is generalisable. A second assumption concerns *how* the researcher believes knowledge of reality comes to be known. These two groups of assumptions are respectively called a researcher's ontological and epistemological beliefs. A good methodology will do more than state the researcher's assumptions but will also provide an analysis of these assumptions and acknowledge that they are likely to influence methodology choices, from study design to data collection and analysis (Taber, 2013).

## 3.1 Epistemology and ontology

It is important to note that there is no 'right' ontological or epistemological view to adopt. Therefore, it is crucial for researchers to maintain an open mind in order to appreciate the value of peer research. Regardless of the choice of theoretical framework it is important that a researcher's views flow logically throughout their study from the focus of their research questions through to the design of their methods.

From an ontological perspective, the two dominant schools of thought are realism and relativism. Classic natural sciences, with the belief that there exists a single objective reality, align with the views of realism. Under this viewpoint knowledge can be generalised around this single reality or truth. The difficulties applying such views to social animals led to the ideas of relativism, which rejects the central tenets of realism and proposes the view that no single version of the truth or reality exists. Absent a single reality, any attempts to generalise are hampered. The consequences of these different views can drive different methodological approaches.

More recently mixed approaches, and subsequently mixed methods, have risen in popularity. Researchers who do not adopt one of these two dominant schools have sought refuge in ideas of pragmatism and critical realism. It is under the latter view that this study is conducted. Critical realism attempts to 'reconnect social theory with research practice' (Lipscomb, 2011, p. 4) and stems from the belief that a single version of reality exists but the complexity of social structures mean knowledge of it is unattainable. Under this philosophy, research methods should be selected that seek to triangulate around reality by measuring a phenomenon from multiple angles, collecting data at multiple levels, and mixing quantitative and qualitative tools to address research questions. One of the goals of research should be to reduce the possibility of incorrect conclusions, whilst acknowledging that such errors cannot be entirely eliminated. Therefore, critical realism seeks to deploy a variety of data collection techniques with the aim of reducing this error. In reference to critical realism, Scott (2005) explained, 'holding a belief that an independent reality exists does not entail the assumption that absolute knowledge of the way it works is possible' (p. 635). The 'critical' dimension of critical realism could be argued to provide an uncontentious way to hold a constructivist epistemological and a realist ontological view. Scott further summarises critical realism as:

> … realist, because it is accepted that there are objects in the world, including social objects, whether the observer or researcher can know them or not. Critical … because any attempts at describing and explaining the world are bound to be fallible, and also because those ways of ordering the world, its categorisations and the relationships between them, cannot be justified in any absolute sense, and are always open to critique and their replacement by a different set of categories and relationships. (Scott, 2005, p. 635)

One's epistemology and choice of methods ought to be connected. Epistemology addresses the *philosophy* of how we come to know truth while methods address the *practicalities*. Some researchers argue that one's ontological assumptions and epistemological choices can be connected (Gee, 2011). A realist might design methods to discern between different hypotheses and build samples large enough to approximate the 'true' population. Such a realist's methods may focus on quantitative studies with statistical tests designed to differentiate what can be said to be true from what cannot. If the phenomena of interest cannot be measured, it cannot be known, and therefore should not be studied. Such views are popular amongst behaviourists, and the epistemological approach of these realists is called positivism.

In contrast, by rejecting that 'truth' (in a social setting) can ever be known, relativists often adopt constructivist and interpretivist epistemologies. Methods under these schools of thought seek to contextualise the research question and may prefer more idiosyncratic, qualitative approaches. Observations, interviews, and open ended questions feature heavily, as constructivists are concerned with 'interpretation, multiplicity, context, depth, and local knowledge' (Ramey & Grubb, 2009, p. 80). Such studies allow the researcher to gain detailed insights in a tight context. One challenge with these approaches is their usefulness in designing applications. In the case of educational research this can make translating constructivist research into teaching frameworks more difficult. It should be noted that some versions of the epistemology of constructivism are not incompatible with ontological realism. Constructivists challenge the empiricists' monopoly on the ability to describe social reality, not necessarily the existence of a real world (Sayer, 1997).

A critical realist might adopt a post-positivist epistemology which rejects the positivist view that only measurable things can lead to knowledge and instead recognises that studies are fallible, and data have limitations. By believing that researchers should deploy a broader range of tools to triangulate, a post-positivist might use a mixture of both quantitative and qualitative methods where appropriate. Such a study might be said to be using 'mixed methods'. By using several different fallible methods researchers seek to triangulate around reality, accepting that it can never be truly known. While this thesis has been conducted under a post-positivist framework, it does not go so far as to propose a mixed methods approach and rather focuses on qualitative methods, supported and supplemented by quantitative tools.

Some researchers seek to move views of post-positivism closer to those of constructivism by defining post-positivism to align with a relativist ontological view. O'Leary (2004) explains, 'What might be "truth" for one person or cultural group may not be "truth" for another' (p. 6). This paper adopts Creswell and Poth's interpretation of post-positivism. 'In practice, postpositivist researchers view inquiry as a series of logically related steps, believe in multiple perspectives from participants rather than a single reality, and espouse rigorous methods of qualitative data collection and analysis' (2017, p. 23).

## 3.2 Multiple case study design

Before laying out a suitable set of methods, it is important to establish the methodological approach that has informed the research design. As discussed earlier, my interpretation of a post-positivist epistemology permits a broad use of methodologies collecting qualitative and quantitative data. Methodologies that might be considered suitable would include phenomenology, grounded theory, and case study. There are elements of a phenomenological methodology which are relevant to my research question and epistemological view, however as Creswell and Poth (2017) suggest, the best criteria to determine the use of phenomenology is when the research problem requires a profound understanding of human *experiences* common to a group of people. Hence, whilst a phenomenological approach does have merit in helping understand how a student experiences IBI, it is not useful in assessing student *performance* and therefore a pure phenomenological methodology does not feel appropriate. Although, it is acknowledged there is some truth to the statement that 'all qualitative research has a phenomenological aspect to it, but the phenomenological approach cannot be applied to all qualitative researchers' (Padilla-Díaz, 2015, p. 103). Grounded theory as a methodology requires the researcher to permit the design to emerge from the study itself and avoid preconceived ideas (Glaser & Strauss, 1967). This approach feels inappropriate given this study has developed a detailed scope as a result of a literature review and my personal background and experiences as a mathematics teacher.

Another methodology would be the use of case studies. Yin (2017) defines a case study as 'an empirical method that investigates a contemporary phenomenon (the "case") in depth and within its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident' (p. 15). As such, a case study is often used to develop a rich picture, using various data collection methods to capture the perceptions, experiences, and ideas of the cases' components. A case study design is appropriate for classroom research since 'teaching and learning present the kind of complex phenomena that are most suitable for case studies' (Taber, 2013, p. 145). This methodology aligns with my epistemological view and is appropriate for the research questions proposed.

Accepting that a case study is the most appropriate methodology leads to the question, *what is used as a case*? In the context of this study the most obvious options were a school, a class (a group of students), or a student. The focus of this study is targeted at the classroom context, specifically how classroom-based IBI is perceived by students with mathematics difficulties, how it contrasts to their everyday experiences in the mathematics classroom, how mindset features, and how much learning occurs. Choosing a whole school as representing the case would have prevented the development of a deep understanding of the contextual factors that are most relevant to the students' experiences of inquiry mathematics. My focus is not at the program or curriculum level, which would have made the choice of school a more appropriate one. At the other extreme, selecting a single student as representing a case would have seemed to be a suitable choice, however this would have restricted the data collection. The goal of this research is to understand how students with mathematics difficulties perceive IBI and the role of beliefs (e.g. mindset). This is best served by collecting data on numerous students, and therefore I chose to use the class as the case.

A common criticism of case studies is that researchers cannot generalise from small sample sizes in a reliable way (Gomm et al., 2009). This challenge to reliability is hard to avoid. However, the use of a case study is not intended to establish a generalisation (Stake, 1995) but rather to put forward findings that may form the basis of a future framework, which through future research may emerge into a reliable generalisation (Yin, 2017). Case studies do not seek absolutism but recognise the contextual limitations.

To further respond to challenges of reliability the use of additional cases is a commonly used technique to add to the weight of arguments. 'Multiple-case sampling adds *confidence* to findings. By looking at a range of similar and contrasting cases, we can understand a single-case finding, grounding it by specifying *how* and *where* and, if possible, *why* it behaves as it does' (Miles et al., 2019, p. 29).

In choosing additional cases a researcher may either select similar cases ('replication cases') or different cases ('contrasting cases'). For the purposes of this study I have selected two 'replication cases.' Yin (2017) defines 'replications' as any two or more cases that 'predict similar results' (p. 55). However, the use of the word 'replications' is problematic since no two classes of students are truly alike. Here the term 'replications' is used to refer to two classes who share relevant features to the study, e.g. from a secondary school, located in the U.K., taught by a mathematics teacher, and characterised as having mathematics difficulties (please see Section 3.3). While this approach increased the complexity of the study, the increased strength of the design allowed for triangulation and a more robust understanding of the phenomena.

### 3.3 Identifying students with MD

Having established that a class represents the case, this section discusses how students with mathematics difficulties (MD) were identified in the study. Within the literature, no consensus has emerged concerning whether chronic difficulty in mathematics can be explained entirely through cognitive factors, contextual factors (e.g. SES, race, gender), or some combination of both (see Section 2.5). One diagnostic approach emerging within literature is to identify students with MD by reviewing scores on a relevant standardised test (such as the National Curriculum Assessments in the U.K.) and using the 25$^{th}$ percentile as an appropriate cut-off (D. H. Bailey et al., 2012; Murphy et al., 2007). Using this approach, students who score below the 25$^{th}$ percentile could be identified as having MD. The merits of this approach are discussed in Section 2.5.

Drawing such a cut-off from a large-scale standardised test, as opposed to a school-specific test, does have some advantages. Whilst a standardised test is still a relative measure its principal advantage is the avoidance of idiosyncratic, school-specific testing conditions. These standardised scores are often an essential consideration used by schools to categorise students into attainment based 'sets' in mathematics (Wiliam & Bartholomew, 2004). As a result, selecting the lowest set within a school could be a helpful starting point in finding a valid case. However, doing so does not guarantee students within that set would have scored below the 25$^{th}$ percentile on the relevant standardised test.

I, therefore, considered two options. Option one was to select only students who scored below the 25$^{th}$ percentile from a year group and form a separate class for the purposes of the study. This has the obvious drawback that the case environment would differ from the normal environment of the case components (the students and teacher). Alternatively, a second option considered was to select a lower set classroom and acknowledge any students within the class who did not score below the 25$^{th}$ percentile, if this information is available. This has the advantage of providing a familiar class environment but the disadvantage that a portion of the students may be illegible for full inclusion in my analyses (e.g. lesson observations and student interviews). To keep the case studies as naturalistic as possible, I decided to implement this second option.

An important consideration is the presence of non-mathematics disabilities, such as dyslexia or ADHD, as these may confound any observations. Such confounding disabilities are acknowledged in each case description and taken into consideration in all analyses according to the recommendation of Ansari (2019).

# 4 Methods

## 4.1 Case study

This thesis includes two case studies, with each case representing a class of students with mathematics difficulties (MD). These cases were selected from two U.K. comprehensive secondary schools. The advantage of a case study approach is the breadth of data that can be collected, such as observations, interviews, and pre- and post-tests. These data serve to triangulate the findings (Yin, 2017). Please see Section 3.2 for a discussion of the advantages of the case study approach.

Within each case study I: (1) surveyed the students using the Attitudes Towards Mathematics Inventory (ATMI), the Implicit Theories of Intelligence Scale (ITIS), and the modified Implicit Theories of Intelligence Scale (m-ITIS); (2) assessed procedural and conceptual understanding using a pre- and post-test; (3) observed each lesson and rated it against the Electronic Quality of Inquiry Protocol (EQUIP); and (4) interviewed the students about their perceptions. Figure 4.1 represents how each data source contributed to an understanding of the cases and was used to address the research questions.

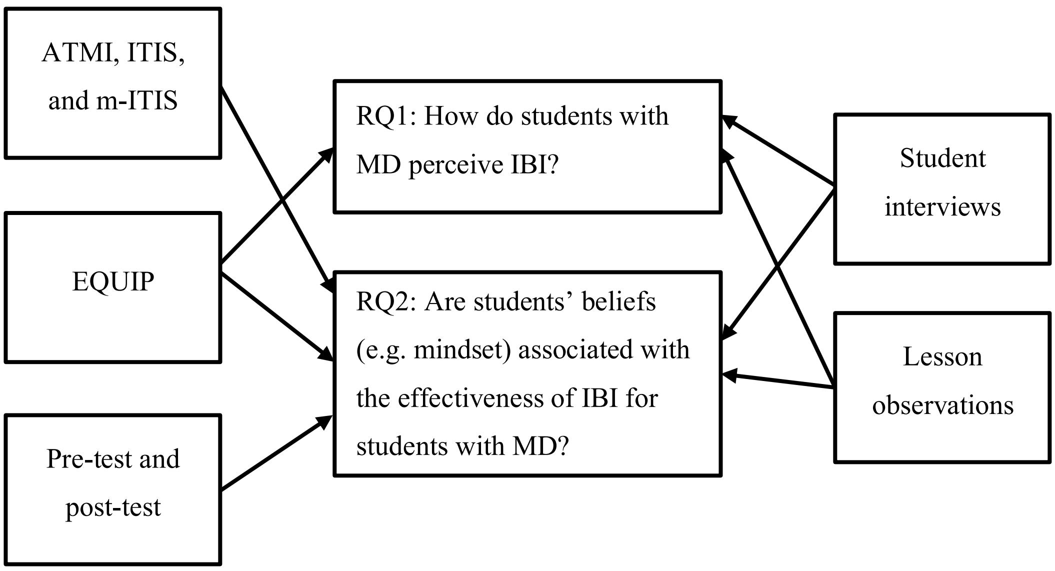


Figure 4.1: Mapping methods and research questions

Each case study took place over approximately 10 weeks, from initial contact with the teacher to completion of the post-test. The pre-test, ATMI, ITIS, and m-ITIS were administered one week before the IBI intervention. Student interviews began about halfway through the unit and continued until the last lesson. The post-test was administered one week after the completion of the unit.

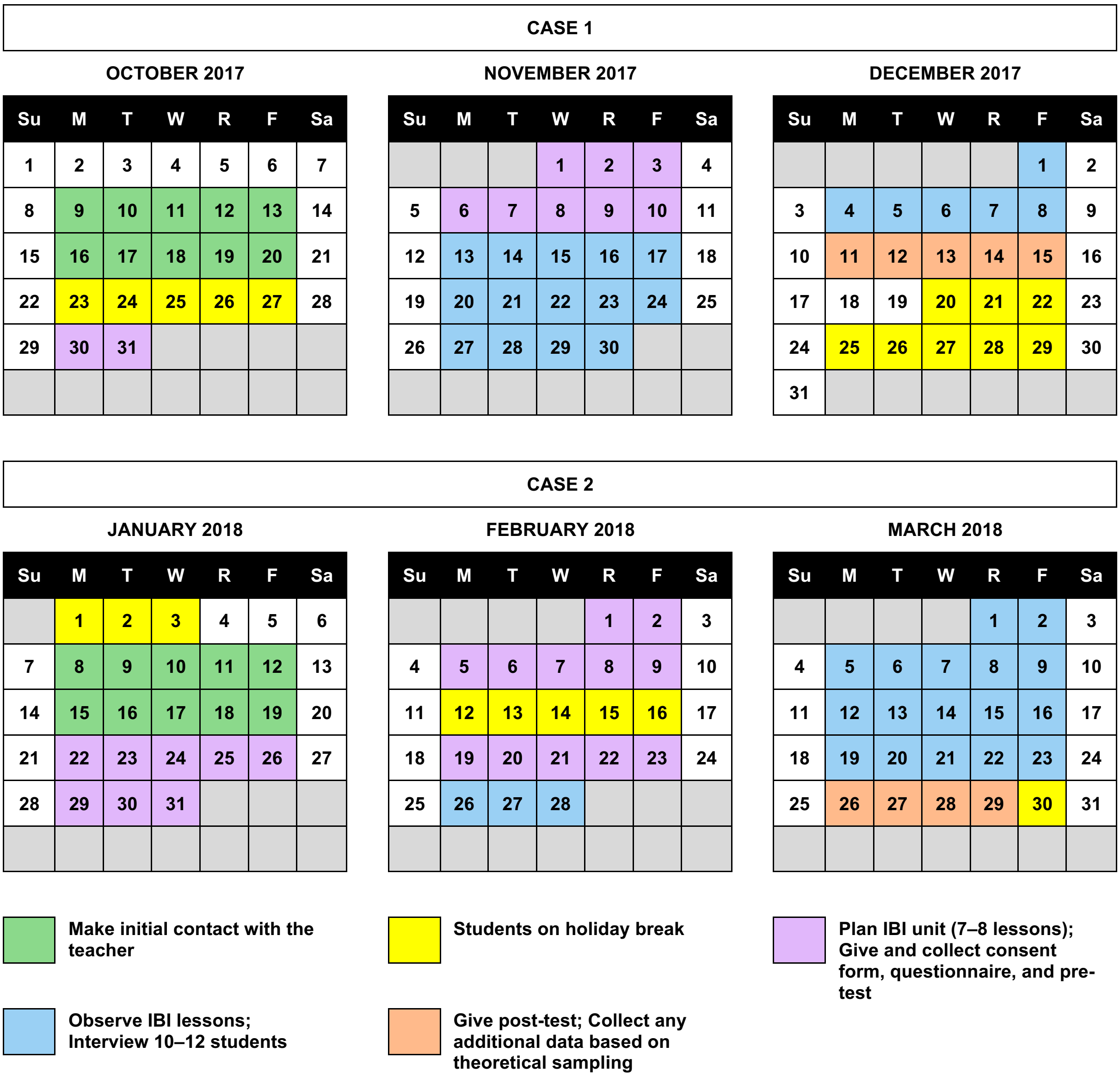


Figure 4.2: Study timeline for both cases studies

A timeline for both case studies, with the IBI unit and data collection methods indicated, is presented in Figure 4.2. The detail for each of the methods, along with their common critiques, is discussed below. However, before expanding on these data collection methods, it is worth reviewing how the cases were selected as well as the IBI unit itself.

## 4.2 Selection of the cases

In May 2017 I contacted 31 comprehensive secondary schools in the United Kingdom. I emailed each school's Head of Mathematics (or equivalent) using the contact information I found on each school's website. Teachers at six of the schools expressed an interest in taking part. I visited each of these schools to meet with the teachers and explain more about what their participation in my study would involve. After the introductory meeting, four teachers (one teacher from each of four schools) wished to take part. I chose two of these teachers from two different schools to take part in the case studies. I chose these teachers primarily based upon ease of access to the school.

## 4.3 IBI unit

In each case, the classroom teacher chose the topic for the IBI unit, ensuring it was suitable for the selected class, given their position in the curriculum at the time of the study. Neither teacher had access to suitable inquiry lesson materials for the selected topics, so we developed them together according to the design principles set out by previous research (see Section 2.2). The duration of the IBI unit was determined alongside the teacher and influenced by the topic chosen. In the first case study there were seven inquiry lessons, and in the second case there were eight.

I observed and rated each lesson using the EQUIP (see Appendix A; J. C. Marshall, Smart, & Horton, 2010). The EQUIP is an inquiry-based observation protocol that contains four rubrics designed to assess a lesson's instructional, discourse, assessment, and curriculum factors. For example, the item 'student role' (considered an instructional factor) could be given a score of one to indicate 'students were consistently passive as learners (taking notes, practising on their own)', or alternatively, a score of four to indicate 'students were consistently and effectively active as learners (highly engaged at multiple points during lesson and clearly focused on the task)'. The four levels are described as pre-inquiry (level 1), developing inquiry (level 2), proficient inquiry (level 3), and exemplary inquiry (level 4). The teacher's questioning level is assessed using the revised Bloom's Taxonomy (Krathwohl, 2002).

I chose to use the EQUIP over other observation protocols (e.g. the Reformed Teacher Observation Protocol [RTOP] or the Scholastic Inquiry Observation [SIO] instrument) because it is one of the only protocols designed and field-tested for use in mathematics classrooms. The EQUIP is also superior for its use of a descriptive rubric in addition to a Likert scale, making it an ideal choice for use with teachers. While it is true that most of the literature cited in the validation paper for the EQUIP (J. C. Marshall et al., 2010) comes from science education, the constructs used in the protocol are equally supported by literature in mathematics education (see Table 4.1).

Table 4.1: EQUIP constructs supported by research in mathematics education

| **EQUIP Factor** | **EQUIP Construct** | **Selected supporting evidence from mathematics education literature** |
|---|---|---|
| Instructional | Instructional strategies | (Rittle-Johnson et al., 2016) |
| | Order of instruction | (DeCaro & Rittle-Johnson, 2012) |
| | Teacher role | (Lampert, 1990)* |
| | Student role | (Cobb et al., 1990)* |
| | Knowledge acquisition | (Stein et al., 2009) |
| Discourse | Questioning level | (Yackel & Cobb, 1996) |
| | Complexity of questions | (Stein et al., 2008) |
| | Questioning ecology | (Stein et al., 2008) |
| | Communication pattern | (Ruthven et al., 2011) |
| | Classroom interaction | (Lampert, 1990)* |
| Assessment | Prior knowledge | (Fyfe et al., 2012) |
| | Conceptual development | (Hiebert & Grouws, 2007) |
| | Student reflection | (Labuhn et al., 2010) |
| | Assessment type(s) | (Heritage & Wylie, 2018) |
| | Role of assessing | (Mason, 2000) |
| Curriculum | Content depth | (Makar, 2012) |
| | Learner centrality | (Hassi & Laursen, 2015) |
| | Integration of content and investigation | (Jessen et al., 2017) |
| | Organising and recording information | (Swan et al., 2013) |

* indicates original evidence provided by Marshall, Smart, and Horton (2010)

The lessons in both case studies were designed to a proficient level of inquiry, or higher, as per the EQUIP. However, the quality of the overall IBI units ultimately relied on the teachers' implementation. For this reason, the teacher in both cases taught a practice IBI lesson before commencing the study, followed by a lesson debrief. In order to ensure the teacher maintained a high level of inquiry throughout the IBI unit, the teacher and I developed an atmosphere of ongoing feedback between lessons. In both case studies, the IBI unit as a whole achieved the level of 'proficient.' Please see Section 5.4.1 for the EQUIP ratings of Mr Scott's case and Section 6.4.1 for the EQUIP ratings of Ms Silver's case.

## 4.4 Questionnaires

All students within each case were asked to complete three questionnaires one week before the start of the IBI units. The students' attitudes towards mathematics were assessed using the Attitudes Towards Mathematics Inventory (ATMI; Appendix C; Tapia, 1996; Tapia & Marsh II, 2004), and their mindsets were assessed using both the Implicit Theories of Intelligence Scale (ITIS; Hong, Chiu, & Dweck, 1995) and a modified Implicit Theories of Intelligence Scale (m-ITIS). Both mindset instruments are presented in Appendix B. The ATMI, ITIS, and m-ITIS are discussed further below.

Evaluation instruments measuring attitudes must overcome several challenges, not least of which is the fact that attitude is a psychological construct and, therefore, difficult to measure. '[Society] has not agreed upon what constitutes psychological constructs such as anxiety or interest. As such, quantifying them is maximally problematic, but not impossible' (Chamberlin, 2010, p. 169).

Such evaluation instruments often require specialist analysis and skills to administer, and the training hurdle pushes the tools out of reach for most teachers. Researchers have developed several instruments which, with some success, address these concerns. In a review of six instruments, Chamberlin (2010) identifies the ATMI as an instrument demonstrating excellent reliability and ease of use. According to Chamberlin, '…the mathematics education community has not engaged this instrument to the extent that it has other similar instruments and its long-lasting effect may yet be realized' (p. 174). The instrument comprises four subscales and forty questions.

Table 4.2: Summary of the subscales of ATMI

| ATMI subscale | Measurement |
|---|---|
| Enjoyment | How much a student likes or dislikes mathematics |
| Motivation | How likely a student is to engage or disengage with mathematics |
| Self-confidence | How good or bad a student believes he/she is in mathematics |
| Value | How useful or useless a student thinks mathematics is |

Students' mindsets were assessed using the ITIS. This is a three-question instrument validated through numerous studies (Dweck et al., 1995; Hong et al., 1995). The instrument has demonstrated good reliability ranging from 0.94 to 0.98 and is not correlated with self-esteem, self-preservation, optimism, political view, religion, or cognitive or motivation needs and styles (Dweck et al., 1995).

The ITIS asks participants to rate how much they agree with each of the following statements on a six-point Likert scale: (1) You have a certain amount of intelligence, and you really can't do much to change it; (2) Your intelligence is something about you that you can't change very much, and; (3) You can learn new things, but you can't really change your basic intelligence. Given the existence of domain-specific mindsets, the m-ITIS was developed to include three mathematics specific statements: (1) You have a certain amount of MATHS intelligence, and you really can't do much to change it; (2) Your MATHS intelligence is something about you that you can't change very much, and; (3) You can learn new things, but you can't really change your basic MATHS intelligence. This wording is in line with recent studies of mathematics-specific mindset (e.g. Adhitya & Prabawanto, 2019; Bostwick, Martin, Collie, & Durksen, 2019; Sun, 2018). The study included both the ITIS and the m-ITIS in the event a student's general mindset differed from his or her mathematics-specific mindset.

As a data collection method, questionnaires are limited by their inability to cover: (1) the breadth of topics which could be relevant, nor (2) the depth to explore beyond a few questions. Students' attitudes and beliefs about mathematics and their response to an IBI intervention are likely to be multidimensional and unlikely to be knowable a priori in a manner permitting the design of a questionnaire. However, questionnaires do have clear advantages in permitting the researcher to collect substantial amounts of data quickly and simultaneously (Groves et al., 2009). Furthermore, the resulting data is readily analysable and amenable to making comparisons across groups. Moreover, questionnaires are useful when the scope and context are limited, such as diagnostic instruments. Finally, questionnaires, when used in conjunction with observations and interviews, can be a significant source of triangulation, as was the case in the present study.

The students in each case completed the questionnaires one week before the first IBI lesson. This helped establish a baseline for the students' attitudes and mindsets towards mathematics and avoided the possibility that the IBI unit itself or subsequent data collection methods, unduly influenced the questionnaire results. The students in both cases completed the questionnaires on a computer using SurveyMonkey[5].

## 4.5 Pre-test and post-test

All students were given an assessment, designed in collaboration with their classroom teacher, to assess conceptual and procedural knowledge before and after the IBI unit. A problem was considered to be conceptual if it tested 'abstract or generic ideas generalised from particular instances' or 'knowledge of problem structures' (DeCaro & Rittle-Johnson, 2012, p. 555). For example, the following test item was selected to assess students' conceptual understanding of volume (see Figure 4.3). For this question students were asked to determine whether the volume of the object on the right is greater than, less than, or equal to the volume of the object on the left. The students had not seen any question like it during the IBI unit.

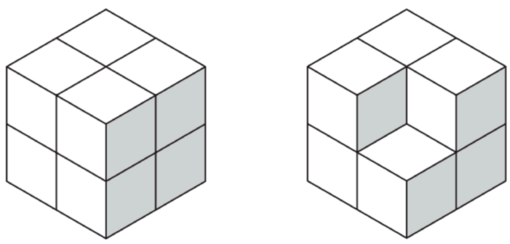

Figure 4.3: Test item for conceptual understanding of volume

[5] SurveyMonkey is an online survey platform that allows participants to complete questionnaires electronically.

A problem was considered procedural if it tested 'the ability to execute action sequences to solve problems' (Rittle-Johnson et al., 2016, p. 577). For this type, I selected questions for which the students had learned a specific procedure in order to solve. For example, the following test item was selected to assess students' procedural knowledge for calculating volume (see Figure 4.4).

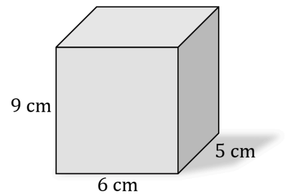


Figure 4.4: Test item for procedural knowledge of volume

It was anticipated that this assessment would take the students 30 to 40 minutes to complete. Please see Appendix F for the pre- and post-test used in the first case and Appendix G for the pre- and post-test used in the second case.

In each case, the pre-test and post-test were given to the entire class under the normal testing conditions of each school. The goal of the pre-test was to establish a baseline level of knowledge from which to assess the effectiveness of the IBI unit. Following the IBI unit, the students were given a post-test to assess their procedural and conceptual understanding of the topic. The results of these tests provided insight into the amount and nature of learning that took place over the course of the IBI unit.

The pre- and post-tests were constructed using identical problems. It is therefore possible that a practice effect might be observed (Lezak et al., 2012). However, given the length of time between the pre-test and the post-test in each case (approximately 6 weeks) this was felt unlikely.

Please see Section 5.4.4 for the results of the pre- and post-test in the first case, and Section 6.4.4 for the results of the pre- and post-test in the second case.

## 4.6 Observations

A crucial part of the data collection phase was to observe the students throughout the IBI unit. These observations were open-ended with attention paid to behaviours and reactions to the task (e.g. putting head down on the desk). Observations were instrumental in helping to anchor and shape one-on-one interviews with students.

During observations, I followed the note-taking procedure put forward by Creswell and Poth (2017) who recommend the observer take both descriptive (objective) and reflective (subjective) notes. Please see Appendix D for an example of the notes I took during the Case of Mr Scott's Class. During observations, I mostly moved around the room eavesdropping, taking notes, and identifying instances to follow up on in student interviews. I interacted directly with the students (much like the teacher) in order to obtain a better sense of the students' progress and conversations.

The lessons throughout each IBI unit were video recorded to supplement my observation notes and act as an additional data collection source. Replaying the video during data analysis helped to highlight salient points that were missed within my field notes. Video recordings were also instrumental in assessing each unit's overall EQUIP scores (see Section 5.4.1 for Mr Scott's case and Section 6.4.1 for Ms Silver's case).

## 4.7 Interviews

I interviewed ten students in the first case study and 12 students in the second case study. All students interviewed were identified as having MD (see Section 3.3). Each student was interviewed once for up to 30 minutes in a one-on-one format. These students were chosen for interview because (1) they had indicated their willingness to be interviewed on their returned consent forms, and (2) their schedules allowed for the interview to take place. Interviews began about halfway through the IBI unit and continued until just before the students took the post-test.

The interview process was crucial as it allowed me to investigate further the behaviours I observed throughout the lessons (e.g. 'I saw you put your head down on your desk today. What was going through your mind?'). The interviews also gave students a chance to share their thoughts and perceptions about the IBI unit.

The interviews followed a semi-structured format, meaning the discussions were conversational in style but addressed important ideas related to the research questions. Each interview was audio recorded to facilitate later transcription. Example interview questions included:

- How do you feel about your maths class?
- Did you find anything in maths class difficult today? Why or why not?
- Is there anything you would change about your maths class?
- During class, I noted [insert behaviour]. How were you feeling at this moment?

Facilitating students' recall of previous lessons can be a challenge. There are advantages to using video footage from the lessons (called 'video-stimulated recall') to help refresh students' memories. Showing relevant clips of key moments may have encouraged the student to share more. However, because the audio was not always discernible from the video recordings, I opted to have the students review their relevant notes and worksheets instead. Reviewing their notes from previous lessons facilitated good recall of the lessons and resulted in lengthier and more vivid student reflections than could have been achieved otherwise.

In conducting an effective interview it is essential to acknowledge the inherent power imbalance between the interviewer and interviewee (Kvale & Brinkmann, 2015). For the most part this inequity is unavoidable, as the interviewer controls the context and content of the interview. However, reducing this imbalance and creating a comfortable atmosphere is often desirable in collecting authentic data (Seale et al., 2004). Establishing a rapport and trust-based relationship can help, such as showing an interest in the student and sharing previous experiences that do not compromise the study or interview (J. Johnson & Rowlands, 2012). I sought to build relationships with the students in each case by establishing a friendly presence in the classroom throughout the IBI unit. Taking opportunities to join in on informal discussions helped me build trust with the students. In addition, I made sure to casually talk to the student before the interview as we walked to the room and got settled in our seats to put them at ease. Once in the room, I re-explained the purpose of the interview and ensured they were aware of their right to withdraw at any time.

One of the challenges in conducting good interviews is developing a high level of rapport with students quickly, and I found the above methods to work well. At the end of each interview I made sure to thank the student for their time and to also allow them to ask me questions about myself and my research.

## 4.8 Theoretical sampling

Real-world contexts make prescriptive methods challenging, especially as it relates to qualitative data collection methods such as observations and interviews. Overprescribing such collection methods upfront may lead to the researcher missing valuable insights that fall outside of tightly predefined collection parameters or failing to capture contexts that were not originally envisaged. Several methods can be used to minimise this. The first is to collect as much data as possible, while at the same time bearing in mind the ethical imperative not to collect data that is unlikely to be analysed. The second is to use a theoretical sampling approach. This study sought to achieve both. By using theoretical sampling, I continuously reflected upon the nature of the collection methods being used as well as the data being collected and modified the methods as needed. For example, within interviews, I altered my initial interview structure to allow for more open discussions with each student. Allowing such flexibility within the research design ensured useful data was not lost.

## 4.9 Threats to trustworthiness

A common criticism of case studies is the propensity for biases. The reliance on the researcher for large portions of the qualitative elements of the studies is where bias is likely to manifest. Within interviews and observations conducted under a post-positivist framework, researchers must seek to remain impartial and objective (to the extent that it is possible) while also retaining the flexibility to ensure sufficient data is collected. An overly structured interview would limit the ability of the researcher to engage with the subject and hear the 'story'.

Conversely an unstructured interview can lead the interviewer to interpret data and change the substance of the interview on the fly, thus increasing the opportunity for bias to taint the data (C. R. Bailey, 2006). For this study, I used semi-structured interviews, in which I defined key objectives, pace, and language without pre-scripting. Within observations 'the qualitative case study researcher keeps a good record of events to provide a relatively incontestable description for further analysis and ultimate reporting' (Stake, 1995, p. 62). The goal is 'not to interpret relationships along the way, wary that moving to that level of thinking might alter the objectivity' (p. 62). During the proposed study, I implemented such approaches to mitigate the risk of bias.

Finally, as discussed above, the use of multiple data sources and different levels allowed me to triangulate findings from each case study, thereby increasing the trustworthiness of outcomes (Lincoln & Guba, 1985).

### 4.10 Analytical approach

As discussed above, case studies represent a broad class of research designs and, as in the present study, can provide a blend of quantitative and qualitative collection techniques. This inherent complexity in design also presents challenges in the analysis.

In Yin's (2017) comprehensive review of case study design he outlines the importance for researchers using case studies to approach data analysis with an 'analytical strategy' and an awareness of the 'analytical techniques' that may follow. Yin proposes four analytical strategies and five analytical techniques. I conducted the analysis using what Yin calls 'Relying on Theoretical Propositions'. Under this approach the analytical means are determined through the theoretical propositions that generated the research study.

The theoretical framework used within this study calls upon ideas presented by Dweck (2017b) in which students' implicit theories of intelligence can influence their academic performance and responses to certain types of mathematical problems. Also, I draw upon ideas by McLeod (1989, 1992, 1994) which describe the different beliefs that students may hold about mathematics. McLeod provides four main categories of beliefs: (1) beliefs about mathematics, (2) beliefs about self (under which I include mindset), (3) beliefs about mathematics teaching, and (4) beliefs about the social context. These four components provide the framework under which I explore whether the students' beliefs were associated with their perceptions of the IBI unit or its effectiveness. The resulting framework for analysis is shown in Figure 4.5.

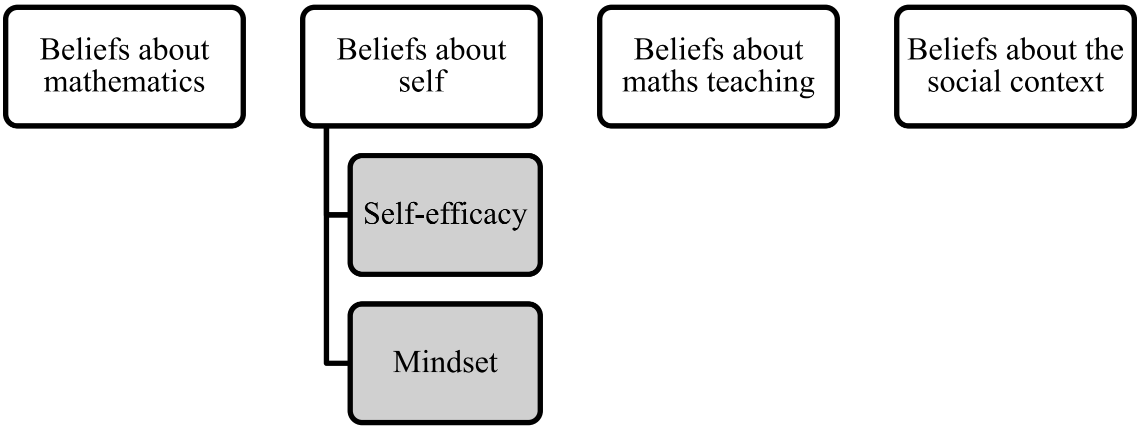


Figure 4.5: Categories used to describe student beliefs

When analysing the data within each of the four categories, I sought to explore, where appropriate, whether the students' beliefs were associated positively or negatively with the effectiveness of the IBI unit. This was done by evaluating the students' attention on the task at hand, the depth of discussion (including level of questioning) within their groups or during whole-class discussions, responses to questions or challenges given by the teacher or myself, and the workings in their notebooks or worksheets.

The McLeod framework guided the analysis by focusing my attention primarily on the ways students' attitudes and beliefs affected their response to the IBI unit. An alternative approach to 'Relying on Theoretical Propositions' includes a 'ground-up analysis' (Corbin & Strauss, 2015) in which no theoretical propositions are put forward, and data is analysed freely with the aim of noticing patterns. Given the extent to which the literature review has influenced my methods this approach feels inappropriate. Researchers using analytical strategies that rely on theoretical propositions must guard against the introduction of bias within their analysis. Such safeguards might include clear documentation of all data collected and rigorous use of rival explanations and propositions.

In addition to Yin's analytical strategy of 'Relying on Theoretical Propositions' I also make use of Yin's analytical technique of 'Explanation Building' (2017, p. 141) in which 'the goal is to analyze the case study data by building an explanation about the case'… 'to "explain" a phenomenon is to stipulate a presumed set of causal links about it, or "how" or "why" something happened' (see Figure 4.6).

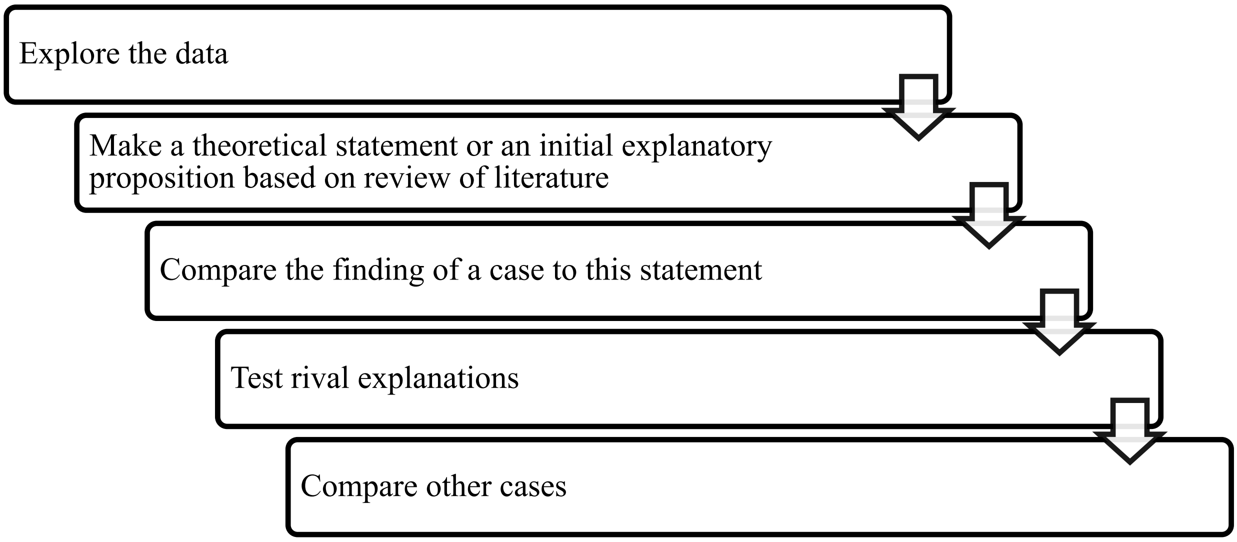


Figure 4.6: An outline of the steps undertaken in analysing the data obtained

**Explore the data**

The rich data that lies at the heart of the case study design necessitates a substantial period of exploration and familiarisation. Table 4.3 outlines the various types of data that were collected in each case study, split between that which is natively amenable to quantitative analysis and that which is more qualitative.

The collection of quantitative data took place at a whole class level, meaning all students within each case took the pre-test, post-test, ATMI, ITIS, and m-ITIS. Descriptive statistics of these tests and questionnaires were inspected to shed light on the association between student beliefs and the effectiveness of IBI. The results of these analyses supplemented the overall picture and helped to form propositions that shaped the exploration of the qualitative data.

Table 4.3: Examples of data types collected

| Qualitative data | Quantitative data |
|---|---|
| Observation transcripts | Scores on the latest standardised test |
| Interview transcripts | Pre-test data |
| Discussions with teacher | Post-test data |
| Historic student records | ATMI |
| Environmental notes | ITIS |

All interviews and lesson observations were transcribed using the software ExpressScribe. Linguistic fillers, such as ‘err’ and ‘uhm’, were not included in the transcripts, however other paralanguage elements were documented. For the lesson observations, only talk that was clear and identifiable was transcribed (e.g. the teacher giving directions, a student explaining his or her solution to the rest of the class). This means most of the talk which took place among the students during the explore portions of each lesson could not be transcribed. Once the first round of transcribing was complete, I reviewed the written transcripts and listened to the audio recordings several more times. From this, I was able to ensure the quality of the transcription, as well as better internalise the tone the students used when speaking.

I then applied an initial coding structure (Merriam, 2016). Under Yin’s ‘Theoretical Propositions’ strategy (2017) the initial coding structure was influenced by the research questions and the literature review as well as the data collection itself, and then gradually refined. The coding structure was first developed by interpreting each transcript line by line (within the context) and applying a code to each. However, subsequent steps and reworking of explanatory propositions (as well as rival propositions) necessitated multiple revisits and refinement to this coding structure. As Merriam (2016) points out, during the initial exploration the goal is to gain familiarity with the dataset, reduce the dataset, and begin to identify themes. I did this by looking for patterns across all the transcripts, consolidating and reorganising the codes as necessary. This process was repeated until several primary themes emerged. Please see Section 5.4.3 for the themes resulting from the interviews in the first case and Section 6.4.3 for the themes resulting from the interviews in the second case.

In both cases, the same theme appeared to be expressed by multiple students but to different extents. In order to distinguish the relative strength of a particular theme expressed by multiple students, I employed intensity scoring (Boyatzis, 1998). Intensity scoring, in its simplest form, requires a researcher to simply count the number of times a code for a particular theme occurs in a person’s interview transcript. The greater the number of codes then the greater the intensity of the theme. However, this approach to intensity scoring feels inappropriate since some students spoke more than others and, moreover, segments of text coded to a particular theme varied. For these reasons, I chose to employ intensity scoring as the percentage of transcribed words in a given transcript coded to a particular theme. For example, if a student spoke about mathematics using 400 words and 100 of these were coded to the theme 'disaffection' then this student would be assigned an intensity score of 25 for the theme 'disaffection'. Please see Table 5.3 and Table 6.3 for the intensity scores presented in case 1 and case 2 respectively.

**Make a theoretical statement or an initial explanatory proposition**

The purpose of this stage is to formally acknowledge that through my literature review, I have formed some theoretical propositions. By stating these, they can be assessed as part of the analysis and modified or abandoned as needed. The goal of the proposition is to offer up an explanation as to how or why a phenomenon occurred, e.g. students with fixed mindsets give up on challenging problems sooner than those with growth mindsets.

**Compare the finding of a case to this statement**

The initial propositions were derived from an understanding of the literature and then compared to the data obtained in each case. In cases in which the data failed to support the proposition, it was necessary to consider a different proposition.

This approach to analysing the data is not without risk, and it is possible that the researcher, through numerous iterations, may devise new theoretical statements which are incompatible with the purpose of the study and the research questions (Vaughan, 1992). Alternatively, the researcher may be accused of allowing the proposition to distort the interpretation of the data. To guard against this critique, I have kept comprehensive documentation of the data and have made thorough use of rival explanations.

**Test rival explanations**

Rival explanations are alternative hypotheses that explain the observed data. For example, when exploring the proposition that growth mindset is associated with positive student perceptions of IBI (see Section 5.4.3 and Section 6.4.3) several rival hypotheses were tested. For example, the four factors measured using the ATMI (enjoyment, motivation, self-confidence, and value) were inspected to see if they were also associated with positive student perceptions of IBI. It turned out that these rival hypotheses did not fit the data (Section 5.2.2 and Section 6.2.2).

Being mindful of rival explanations during the data-gathering phase helped to ensure data was collected that could address those rivals in the analytical phase. Rival explanations were sought from the literature or emerged from the data itself. During this stage of the analysis rivals were considered, and the data were analysed to reject or not reject these rivals. In some cases, rival explanations could not be rejected, and these are put forward as areas for future study (see Section 7.5).

**Compare other cases**

The goal of a successful case study should be to advance an explanation or theory that fits each of the cases. In this regard, each case is powerful, and a single contradictory case can disprove a theory. Therefore, any propositions emerging from the data analysis should be supported or at least not contradicted, by both cases. Finding a theory is supported by multiple cases adds weight to its trustworthiness (see Section 4.8).

Before moving on, it is worth highlighting an important consideration when looking at case study data, namely that a researcher should address all the data. If an explanation is to withstand critique it must avoid the accusation of selectively ignoring data that was not in support of it. The use of well-crafted rival propositions is one safeguard for this.

### 4.10.1 Contextual factors

There is an inexhaustible array of contextual factors that bear upon the external validity of this study. For example, both case studies were conducted in English-speaking schools in the United Kingdom. The proximity to influential centres of research (e.g. nearby universities) may have afforded a culture within these schools that was supportive of research studies within classroom ecologies. This affordance may constrain the applicability to other parts of the world. In addition, all classroom environments are unique and students' previous exposure to IBI varied within each case. Furthermore, each case focused on secondary school mathematics and therefore its relevancy for other contexts is subject to future study.

As a further contextual consideration this research required me to interact with the students during lesson observations and during interviews. The Hawthorne effect says that when human beings are aware they are being observed their behaviour is modified (Landsberger, 1958).

Overall, the unique factors present within these case studies limit the extent to which conclusions may be applied to wider contexts. Furthermore, I cannot claim an unbiased interpretation of my data despite efforts to remain objective. As described previously (Section 4.5), I interacted with students in both case studies much like their classroom teacher. I therefore adopt an attitude of reflexivity, in which I acknowledge my active role throughout the research process and the direct effect my presence and ongoing interpretations had on the results of both case studies. My choice of research questions, methods, and what should count as a finding have all been directly influenced by my past experiences as a student and teacher of mathematics and should, therefore, be viewed as a limitation of this thesis.

## 4.11 Ethical considerations

This study followed the BERA ethical guidelines (2018) and conformed to the ethics procedures laid out by the University of Cambridge. Central to the BERA guidelines is the idea of 'voluntary informed consent' (p. 9). According to BERA, participants 'should be told why their participation is necessary, what they will be asked to do, what will happen to the information they provide, how that information will be used and how and to whom it will be reported' (p. 9). The teacher in both cases determined the content of the mathematics instruction, and therefore, consent was not required for attendance throughout the IBI unit. However, participation in the questionnaires and interviews required consent. I, therefore, informed students and teachers of the research aims and obtained necessary consent (see Appendix E for consent documents). The only exception was that students were not informed of their attitude and mindset results as this was thought to unduly alter their behaviours. Students were made aware of their right to withdraw at any time. Any participant looking to exit the study would not have been influenced or coerced into remaining, though this situation never arose. The above information was written in simple language and reviewed with participants in advance (see Appendix E).

The BERA guideline state that 'all social science should aim to maximise benefit and minimise harm' (2018, p. 4). As described in Chapter 2, it is thought that IBI approaches are unsuitable for students with mathematics difficulties. Therefore, the design of this research study could be accused of being unethical since it required students with MD to be instructed under an inquiry approach. Similarly, BERA states 'researchers should take steps to minimise the effects of research designs that advantage or are perceived to advantage one group of participants over others' (p. 20). At the outset of this study, it was preconceived that students with fixed mindsets would underperform during IBI. Therefore, one could argue that the present study was unethical since it subjected students to an intervention that was predicted to be more advantageous to some students than others. However, these drawbacks are unavoidable as they hit at the central purpose of the study. As I outlined in Section 2.5, the definition of MD is poorly defined, yet teachers are regularly using such terms and student groupings to decide who gets exposed to IBI and who does not. Therefore, I argue that this research is necessary to ensure potentially inequitable practices do not take place in schools. However, in undertaking this study it is important to put in place methods to address any lingering concerns. Ensuring the intervention took place over a short period of time minimised these potentially harmful effects (see Section 7.4). Moreover, I shared growth mindset materials with the cooperating teacher at the end of the unit, which each teacher then shared with their students.

## 4.12 Pilot study

Before implementing the proposed methods in a full-scale study, it is beneficial to test if critical elements of the design are viable by running a pilot study (Yin, 2017). This is also an opportunity to trial some of the materials and methods to identify learning opportunities. Hence a small-scale pilot study was conducted in June 2017 (Rice, 2018).

This pilot study was conducted over two IBI lessons in two U.K. secondary schools (one lesson per school). Students were given the ITIS, m-ITIS, and ATMI before the IBI lesson. Each lesson was observed by me. Following the lesson, several students were interviewed to determine their perceptions of the IBI. Overall, the pilot study was limited in scope but helped to address the following four key questions.

**Can the questionnaires be effectively administered and yield analysable data?**

The distribution of the questionnaire was effective in both paper and electronic form via SurveyMonkey. Outputs were analysable and yielded insightful content. Interestingly, the proportion of students identified as growth, fixed, and mixed mindset did not align with findings within the literature (Dweck, 2017b). There was a greater representation of mixed mindset than expected based on data collected during the pilot study.

**Am I able to effectively observe a class experiencing IBI?**

The pilot study flagged some concerns regarding the ability to video a small group within an active classroom. While the video was not without useful material, at best it served as a supplement to extensive field notes. Also, the pilot led me to identify opportunities to increase the level of inquiry within the interventions and improve the EQUIP scores. This experience highlighted the need to engage with the teachers ahead of time to ensure a common understanding of the main features of an inquiry lesson. I incorporated these learnings into the main study.

**Does interviewing students yield insightful information into their beliefs about mathematics and IBI?**

Having conducted only four interviews, the insight from the pilot was limited; however, the material collected suggested that students were able to effectively reflect on their beliefs about mathematics and articulate their perceptions of IBI.

**Does the pilot study confirm the effectiveness of the proposed methods as a means to answer the research questions?**

Overall, I was encouraged to see several themes emerge from the discussions with students. It was clear that some students felt that IBI was empowering and fostered persistence and engagement. Other students felt the opposite and saw the lack of direction as a form of neglect by the teacher. Moreover, it appeared, from this limited study, that students with different mindsets expressed views of IBI as empowering to different extents (Rice, 2018). The fact that I was able to conduct the proposed methods on this small pilot and obtain results which appeared relevant to my research questions led me to conclude that the proposed methods would be an effective way to address the research questions as part of this more extensive thesis.

### 4.12.1 Lessons learned and modifications to methods

Following the pilot study, I modified several aspects of the methods. The use of video as a recall stimulant was ineffective. In light of the pilot study, the video footage was chiefly used to assess the EQUIP scores for the lesson and to supplement fieldnotes. In the place of the video footage, recall was stimulated by reviewing the student materials used during the lesson.

It was clear that the selected teachers had historically avoided inquiry-based teaching for this student group. As a consequence, the implementation of IBI was challenging, and EQUIP scores left space for improvement. For the full study, I was sure to discuss with the teacher how he or she might effectively administer IBI and what strategies might increase the EQUIP scores. The teacher also gave a practice IBI lesson with substantial feedback from myself based on the EQUIP. Finally, during the full study, the teacher and I established feedback discussions following each lesson to ensure opportunities to improve the inquiry score were implemented.

Under the instruction of the headmaster, the interviews at one of the pilot schools were conducted under supervision from a teacher's aide. This likely created a dynamic that impacted the data gathered. In the full study I engaged with the headmaster to address any concerns they had and ultimately ensured all interviews were conducted in private in a place suitable to the headmaster.

### 4.12.2 Limitations of the pilot study

Several elements of the full study were not tested in the pilot. The leading example of this is the use of a pre- and post-test to measure the students' procedural and conceptual understanding. The pilot study included just one IBI lesson, so it was thought to be too short to measure learning effectively. The full study took place over a more extended period and consisted of seven lessons in Mr Scott's case and eight lessons in Ms Silver's case.

In addition, the use of the bottom set as a proxy for students scoring below the $25^{th}$ percentile on the standardised test, and therefore being within the criteria I have defined for MD, is problematic. Access to historical student test scores was not provided in the pilot study. It is *possible* that the lower sets I observed during the pilot were above average on a national scale and no students within either case fell below the $25^{th}$ percentile. Of course, the requirement for access to student test scores must be balanced against the school's consent. A justification for the class chosen in the first case is provided in Section 5.1.2 and a justification for the class chosen in the second case is provided in Section 6.1.2.

# 5 The Case of Mr Scott's Class

In this chapter I examine the results of a case study in which a class of secondary school students with mathematics difficulties were taught seven inquiry-based lessons (from now on referred to as the IBI unit). All students were observed throughout the unit and ten students were interviewed. Students completed a pre-test and post-test as well as three questionnaires designed to evaluate their attitudes towards mathematics, implicit theories of intelligence (also known as mindset), and mathematics-specific implicit theories of intelligence. For more details about the methods used in this study, please refer to Chapter 4.

The study was designed to explore two research questions: (RQ1) How do students with mathematics difficulties perceive IBI? and (RQ2) Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties?

## 5.1 The setting

The study took place at Harrison School[6], a U.K. comprehensive secondary school, in the autumn term of 2017. Six months prior to the start of the case study, I sent an email sent to the Head of Mathematics[7] at Harrison School, and several teachers expressed interest in learning more about the case study. After meeting with these teachers in the summer term of 2017 and explaining what the case study would entail, Carl Scott volunteered to take part. He explained he was eager to learn more about IBI and was confident he would have the time to take on the additional responsibility.

A recent Ofsted report rated Harrison School as a 'good school'. The report described the school as having a positive culture in which pupils respect each other's differences and exhibit good behaviour. Ofsted commended the school for the quality of its teaching and learning.

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[6] All names, including school names, teacher names, and student names, have been changed to protect the identity of those involved in the study.

[7] The email address for each Head of Mathematics was found on their school's online staff directory.

### 5.1.1 The teacher

Mr Scott served as a mathematics teacher at Harrison School throughout the duration of the case study. Before joining the mathematics department at Harrison School, Mr Scott received his undergraduate degree in a mathematics related field. The case study was conducted during Mr Scott's second year at Harrison School and fifth year teaching overall.

Mr Scott described himself as a passionate teacher invested in his students. He explained he thinks it is important to get to know his students well before reading any historical reports. He incorporated light-hearted games into each of his lessons in order to get to know the students better. From a pedagogical point of view, Mr Scott described himself as a hands-on teacher who enjoys planning lessons that are interactive and incorporate plenty of student choice.

### 5.1.2 The class

Harrison School sets students into six sets for mathematics based on a combination of factors including their Key Stage 2 results, yearly Cognitive Abilities Test (CAT) results, and mathematics teacher recommendation. At the time of the case study, Mr Scott taught eight classes which spanned all sets and ranged from year 7 to year 13. In selecting the appropriate class, Mr Scott and I reviewed his lowest three sets (set 4, set 5, and set 6 classes). Please see Section 3.3 for more on how students with mathematics difficulties were identified in this study.

Mr Scott and I considered the class size and instance of disability in each group. We also considered Mr Scott's overall schedule and when during the week would be most manageable for him to meet with me. Mr Scott's year 9 set 5 class was selected for the case study. There were 18 students in the class ranging in age from 12 to 13.

Of the 18 students, ten were female and eight were male. Mr Scott shared with me that four students qualified for free school meals and seven for the pupil premium. On reviewing the school records for this class, it was identified that three students had Special Educational Needs (SEN) indicators, one with a speech and language disability, one with an emotional disability, and one with dyslexia. Thirteen students in the class had received literacy support in the past. All students in the class met my criteria for MD (discussed in Section 3.3), meaning they were placed in a lower set and scored below the 25th percentile on the most recent standardised test.

Mr Scott taught the selected class five times in every two weeks. He saw them every Wednesday morning, every other Thursday morning, and every Friday afternoon. Each lesson lasted for one hour. Following each Wednesday session Mr Scott had a one-hour planning period, therefore we agreed this would be used as an opportunity to discuss and reflect on the IBI lessons and student progress.

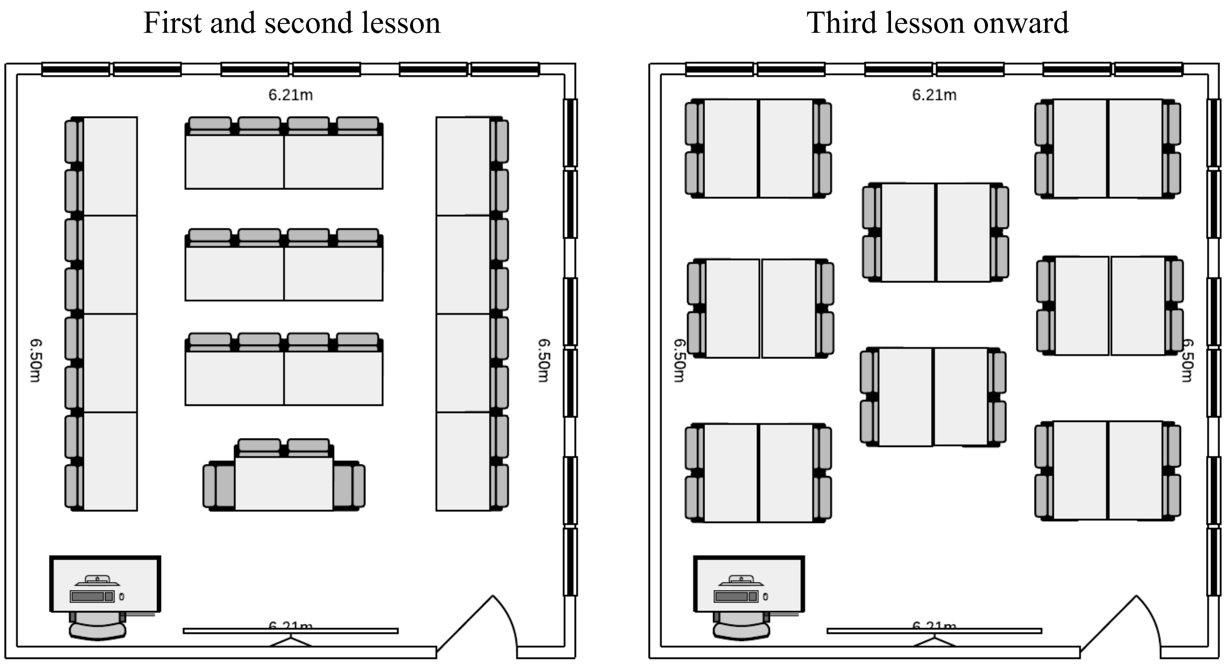


Figure 5.1: Mr Scott's classroom arrangement during the IBI unit

The classroom contained a large white board, table-desks, and student work hung around the room. The room benefited from ample sunlight with two of its four walls lined with large windows. A SmartBoard[8] was mounted to the whiteboard at the front of the classroom, although Mr Scott said he did not use it for much more than displaying PowerPoint presentations and other media.

[8] A SmartBoard is an interactive display designed for use in classrooms (www.smarttech.com).

Student desks were typically arranged in rows, consisting of between four and eight students per row. The total student capacity of the room was 32. Whilst this was the typical configuration, the classroom was rearranged after the second IBI lesson to include table-groups of four students each (see Figure 5.1). Mr Scott thought this would facilitate greater collaboration and discussion.

### 5.1.3 Voluntary informed consent

I introduced myself to the class in October 2017, explaining that I was a PhD researcher from the University of Cambridge and that I had previously been a mathematics teacher in the United States. I distributed and read the information sheet to the class which outlined the purpose of the study as well as what would be involved (see Appendix E for a copy of the consent materials). Students were asked to review the information sheet with a parent or guardian and then return the signed consent form indicating their willingness to participate in the study and interview by the following week. Having introduced myself, I stayed for the remainder of the lesson to answer any questions and make some preliminary observations. I noted the flow of the lesson led by Mr Scott and the good rapport between him and the pupils.

Before the start of the study, all students returned the consent forms signed and indicated their wish to be included in the study. Ten students wished to be included in the interview process, with the remaining eight wishing to be excluded.

## 5.2 Lesson development

Mr Scott and I met several times before the start of the case study to plan lessons and assessments. All teachers within the school used subject specific ‘schemes of work’ which provided a timeline of the specific topics and learning standards the students were to cover. Accordingly, Mr Scott’s scheme of work indicated the pupils were scheduled to cover the topic measurement during the month in which the case study was to take place. Therefore, Mr Scott and I discussed the major learning objectives for the study’s IBI lesson sequence to cover the topic of measurement. These were aligned to the Pearson KS3 Maths Progress and

Edexcel GCSE (9-1) Mathematics curriculum[9] being used at Harrison. This curriculum is aligned to Key Stage 3 of the National Curriculum. We agreed that, by the end of the unit, students should be able to: (1) recognise perimeter as a 'distance' (in cm for instance); (2) recognise area as a 'space' (in $cm^2$ for instance); (3) calculate perimeters and areas of rectangles; and (4) calculate volumes and surface areas of cuboids.

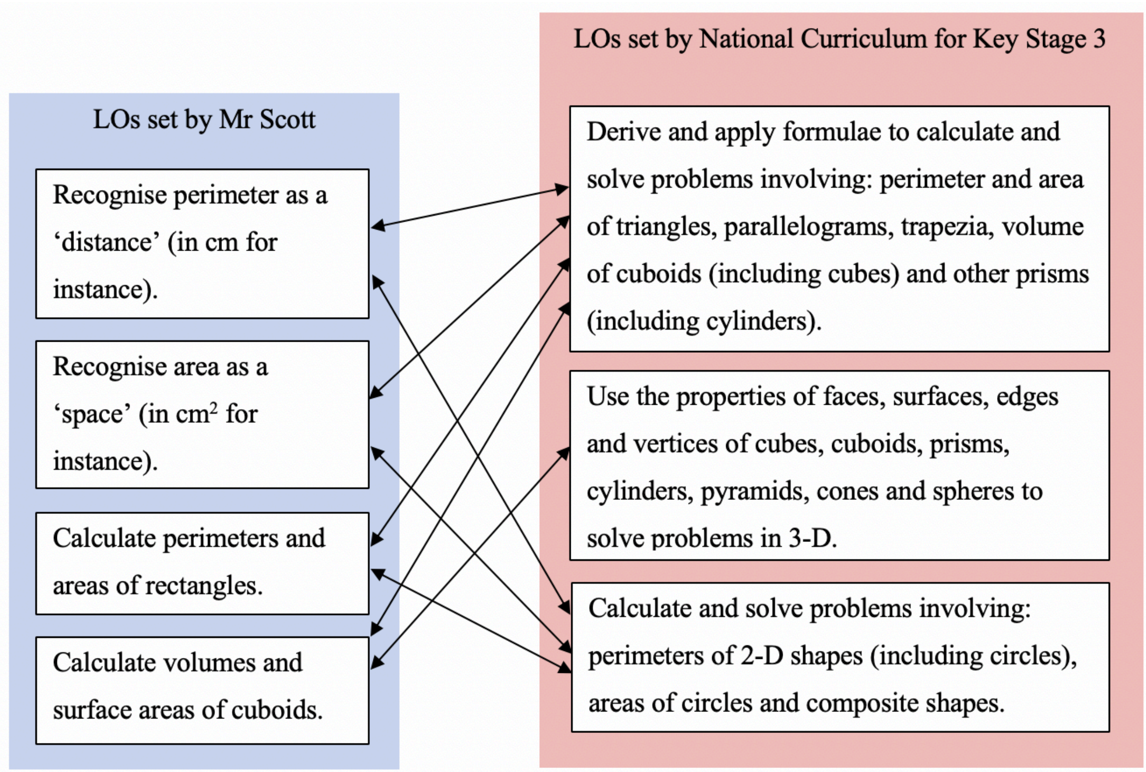


Figure 5.2: Mapping Mr Scott's LO's to the National Curriculum

It was anticipated that it would take at least six full lessons (60 minutes each) to achieve these learning goals. Mr Scott developed each lesson plan, typically one week in advance. Together, we reviewed the lesson plans to ensure they complied with a high level of inquiry. This was done with reference to the Electronic Quality of Inquiry Protocol (EQUIP; Section 4.3), which gives guidance on the instructional, discourse, assessment, and curriculum factors related to quality inquiry teaching.

Mr Scott's teaching style might be best described as experiential. He preferred leading lessons that required students to be active. As such, many of the inquiry activities we

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[9]https://www.pearsonschoolsandfecolleges.co.uk/AssetsLibrary/SECTORS/Secondary/SUBJECT/Mathematics/11-16-maths-story/W38411-16MathsGuideA424pgWEB.pdf

designed together required students to get out of their seats and engage in an activity. However, to provide a more balanced IBI experience across the case study we agreed to incorporate at least two lessons in which students engaged with purely desk-based inquiry. The purpose of this was to avoid students conflating IBI with simply getting out of their seats.

### 5.2.1 Pre-test and post-test development

Having selected the topic of measurement for the IBI unit, Mr Scott and I then constructed the pre-test and post-test (see Section 4.5). In all, the pre-test and post-test were composed of six questions of procedural knowledge and six questions of conceptual understanding. Mr Scott and I selected all six conceptual questions from previous National Curriculum assessments (known as 'SATs') which had been aligned to the Key Stage 3 programme of study. We selected two procedural questions from previous National Curriculum assessments and wrote four additional procedural questions. We did this by mimicking the form of procedural problems we knew the students would complete during the unit. Please see Section 4.5 for the definitions of procedural and conceptual knowledge used in this paper.

The assessment duration was designed to take approximately 30 minutes (half of one lesson). Both the pre-test and post-test were administered under exam conditions within the students' normal mathematics classroom. The pre-test was administered one week before the first IBI lesson. The post-test was administered one week following the last IBI lesson.

### 5.2.2 ATMI, ITIS, and m-ITIS

The Attitudes Towards Mathematics Inventory (ATMI), the Implicit Theories of Intelligence Scale (ITIS), and a modified Implicit Theories of Intelligence Scale (m-ITIS) were given one week before the start of the IBI unit, however on a different day to the pre-test (see Section 4.4 for more details on the ATMI, ITIS, and m-ITIS). Students completed these questionnaires individually in the computer lab using the online platform SurveyMonkey. Mr Scott supervised the students while they completed the questionnaires.

### 5.2.3 Observation protocol

To ensure detailed observations, I took handwritten notes in addition to audio recording and video recording each lesson. As described in Section 4.6, my notes were both descriptive and reflective (Creswell & Poth, 2017; see Appendix D for a sample of my observation notes). The main purpose of my written notes was to capture observations that would facilitate subsequent interviews as well provide a record of the lesson's flow for future documentation and analysis. I also audio recorded the lesson. An audio recorder was placed on the teacher's desk since this was located towards the front of the room where the teacher normally stood to give whole class directions (see Figure 5.1). As such it provided a valuable record of the instruction the students received as well as any verbal feedback and questions directed towards the teacher. Finally, I used a video camera set up at the back of the classroom (furthest from the white board) to record the lesson. This video aided my post-lesson reflections as well as development of tailored interview questions and later analysis.

### 5.2.4 EQUIP rubric

The purpose of the EQUIP rubric was to assess the level of inquiry in each lesson on a scale of 1 to 4: (1) 'pre-enquiry', (2) 'developing enquiry', (3) 'proficient enquiry', and (4) 'exemplary enquiry' (please see Appendix A for copy of the rubric and Section 4.3 for a discussion of its use). One of my goals was to ensure that the majority of the lessons of the IBI unit met or exceeded the criteria for 'proficient'. To this end, Mr Scott and I reviewed the rubric before each lesson was developed and discussed ways in which to achieve a high level of inquiry. In addition, I observed Mr Scott teach an IBI lesson before the intervention began. Following this lesson, we debriefed to grade the lesson as per the rubric and discuss improvement areas. The principal feedback from this practice lesson was to increase the amount of classroom discussion following the inquiry activity.

### 5.2.5 Interview protocol

All interviews were conducted at Harrison during the school day. A small meeting room near the reception desk was reserved for the interviews. Since only a few interviews were able to take place during Mr Scott's lesson, most interviews took place during the student's alternative maths lesson, or occasionally during their Art or Physical Education lesson. In every instance, I was given permission from the class teacher for the student to miss approximately 30 minutes of the lesson that day in order to be interviewed. Student consent to miss class was also obtained. Scheduling conflicts meant it was not possible to avoid students missing classes.

All students who consented to an interview were interviewed. I began each interview by reminding the student of the purpose of the study as well as their right to skip questions or end the interview at any time at their request. I also checked with the student that it was still okay for me to audio record the interview. The interview followed a semi-structured approach, with the five main topics to cover being: (1) feelings about mathematics, (2) perceptions of the IBI lessons, (3) impressions of teaching in IBI, (4) self-reported effectiveness of IBI on learning, and (5) handling impasses.

For all interviews, I began with the question, ‘In general, what do you think about the subject of mathematics?’ Based on the student’s response to this initial question, I allowed the interview to flow naturally, being sure to keep the discussion broadly on track and cover the five main topics.

#### 5.2.6 Other data collection

In addition to the questionnaires, pre-test, post-test, observations, and interviews, I also collected the students’ worksheets to supplement my observation data and analysis. These worksheets were also helpful in facilitating recall of the IBI lessons during student interviews.

### 5.3 Overview of the IBI lessons

Several IBI problems were chosen that aligned to the learning objectives (see Table 5.1). The problems were selected from a variety of sources including curricular websites, textbooks, and Mr Scott’s prior teaching experience. Each problem is presented in the table below alongside the lesson number in which the task appeared (L1 stands for Lesson 1, L2 stands for Lesson 2, and so on). Each lesson’s primary learning objective is also indicated.

Table 5.1: Overview of the seven IBI lessons at Harrison School

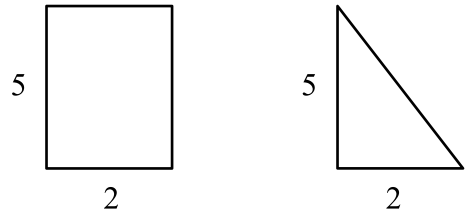
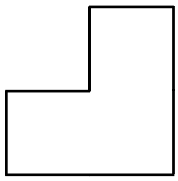
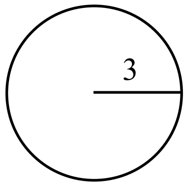
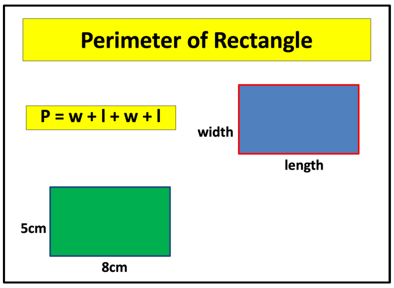
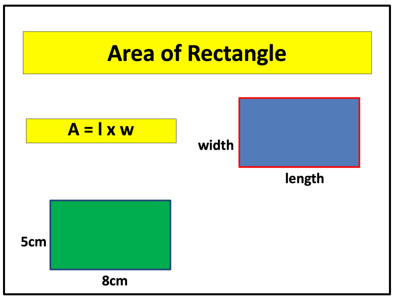
| | Learning Objective | IBI Task |
|---|---|---|
| L1 | Assess prior knowledge of perimeter, area, and volume | **Stations problem**<br>1) Find the area of the following shapes.<br><br>2) Find the perimeter of the following shape.<br><br>3) Find the circumference and area of the circle.<br><br>4) Find the surface area and volume of the given cuboid.<br>(*Students given a manipulative like the one shown below.)*<br> |
| L2 | Recognise perimeter as a 'distance' (in cm for instance) | **Basketball problem (part 1)**<br>Measure your school's basketball court using any method.<br>Describe how your group measured the basketball court. |
| L3 | Recognise area as a 'space' (in $cm^2$ for instance) | **Basketball problem (part 2)**<br>What is the court's perimeter? How did you determine this?<br>What is the court's area? How did you determine this? |

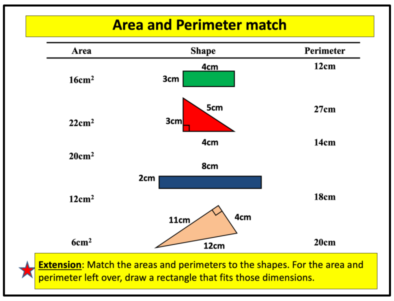
Table 5.1 (continued)

L4 — Calculate perimeters and areas of rectangles

**Area and Perimeter PowerPoint**

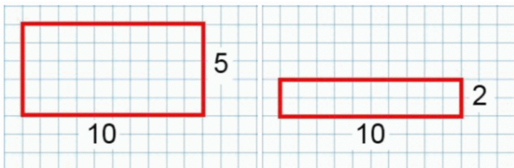




L5 — Calculate perimeters and areas of rectangles

**Equable problem**

Charlie has been drawing rectangles:




The first rectangle has a perimeter of ___ units and an area of ___ square units.

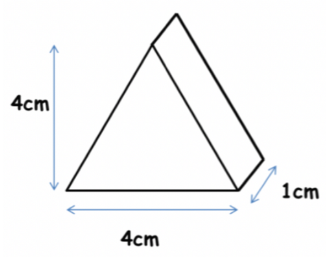
The second rectangle has a perimeter of ___ units and an area of ___ square units.

Charlie wondered if he could find a rectangle, with a side of length 10 units, whose perimeter and area have the same numerical value.

Is it possible to construct a shape in which the numerical values of its area and perimeter are the same?

(NRICH, n.d.)

Table 5.1 (continued)

| | | |
|---|---|---|
| L6 | Calculate surface areas of cuboids | **Toblerone problem**<br>Toblerone are making a new family sized bar. Each piece will have the following dimensions.<br><br><br>The bar will consist of 12 equal pieces.<br>What will the total surface area of the wrapper need to be? |
| L7 | Calculate volumes of cuboids | **Classroom volume problem**<br>How many of these water bottles do you think it would take to fill up our classroom? (*Students given a 500 ml water bottle)* |

## 5.4 Analysis

This case study seeks to address the following research questions: (RQ1) How do students with mathematics difficulties perceive IBI? and (RQ2) Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties? The following aspects of the data collected were analysed to address the above two research questions. Firstly, the lessons as implemented by the teacher and enacted by the students are analysed using the four rubrics of the EQUIP (Appendix L and Section 5.4.1). Then, evidence of students' beliefs throughout the IBI lesson observations are analysed using McLeod's (1992) framework (RQ2; Section 5.4.2). Next, students' perceptions of IBI are analysed using Merriam's (2016) approach to coding (RQ1; Section 5.4.3). Finally, students' learning outcomes as measured by the pre- and post-test are analysed using descriptive and bivariate statistics (RQ2; Section 5.4.4). Please see Section 4.10 for a complete description of my analytical approach.

### 5.4.1 An analysis of the quality of inquiry instruction

With a view towards understanding students' perceptions of inquiry instruction in mathematics, it is important to first establish whether the observed teaching unit could be characterised as inquiry (see Section 4.3). In Appendix L, I present a detailed assessment of the quality of the overall IBI unit, which consisted of seven lessons. I do so according to the four factors of EQUIP: Instructional factors, Discourse factors, Assessment factors, and Curriculum factors. I present a summary of this analysis in Table 5.2. As previously discussed in Section 4.3, the EQUIP is an instrument that has been validated for use in mathematics classrooms. For a copy of the EQUIP, please see Appendix A.

Table 5.2 Assessment of the quality of the inquiry instruction in Mr Scott's case

| Factor | Sub-factor | Level assessed | Section |
|---|---|---|---|
| Instructional Factors | Instructional strategies | Proficient | Appendix L.1.1 |
| | Order of instruction | Proficient | Appendix L.1.2 |
| | Teacher role | Exemplary | Appendix L.1.3 |
| | Student role | Proficient | Appendix L.1.4 |
| | Knowledge acquisition | Proficient | Appendix L.1.5 |
| Discourse Factors | Questioning level | Proficient | Appendix L.2.1 |
| | Complexity of questions | Proficient | Appendix L.2.2 |
| | Questioning ecology | Proficient | Appendix L.2.3 |
| | Communication pattern | Pre-inquiry | Appendix L.2.4 |
| | Classroom interactions | Developing | Appendix L.2.5 |
| Assessment Factors | Prior knowledge | Proficient | Appendix L.3.1 |
| | Conceptual development | Proficient | Appendix L.3.2 |
| | Student reflection | Developing | Appendix L.3.3 |
| | Assessment type | Proficient | Appendix L.3.4 |
| | Role of assessing | Proficient | Appendix L.3.5 |
| Curriculum Factors | Content depth | Developing | Appendix L.4.1 |
| | Learner centrality | Proficient | Appendix L.4.2 |
| | Integration of content and investigation | Proficient | Appendix L.4.3 |
| | Organising and recording information | Proficient | Appendix L.4.4 |

As is shown in Table 5.2, the most common score assigned to the different components of the EQUIP was that of proficient inquiry. Therefore, the unit as a whole could be best described as meeting the requirements of proficient inquiry.

Mr Scott was successful in leading instruction, discourse, assessment, and curriculum that met many of the goals of inquiry. Students were routinely asked to explore problems before instruction and discuss their ideas with peers. The classroom was highly student-centred, and Mr Scott assessed students' understanding frequently. To have achieved a higher level of inquiry, Mr Scott could have provided more opportunities for students to interact directly with one another during whole-class discussions (without these interactions needing to be mediated by the teacher). In addition, students could have had more opportunities to reflect on their learning.

One of the most important aspects of an inquiry lesson is the amount of time the students spend exploring the problem. EQUIP states that during proficient inquiry teachers 'only occasionally lecture'. During the seven lessons observed, Mr Scott allowed students to explore the problems (either individually or with a group) for a substantial part of the lesson time, with the exception of L4 since this lesson was intended to serve as the 'explanation phase' of the two previous lessons.

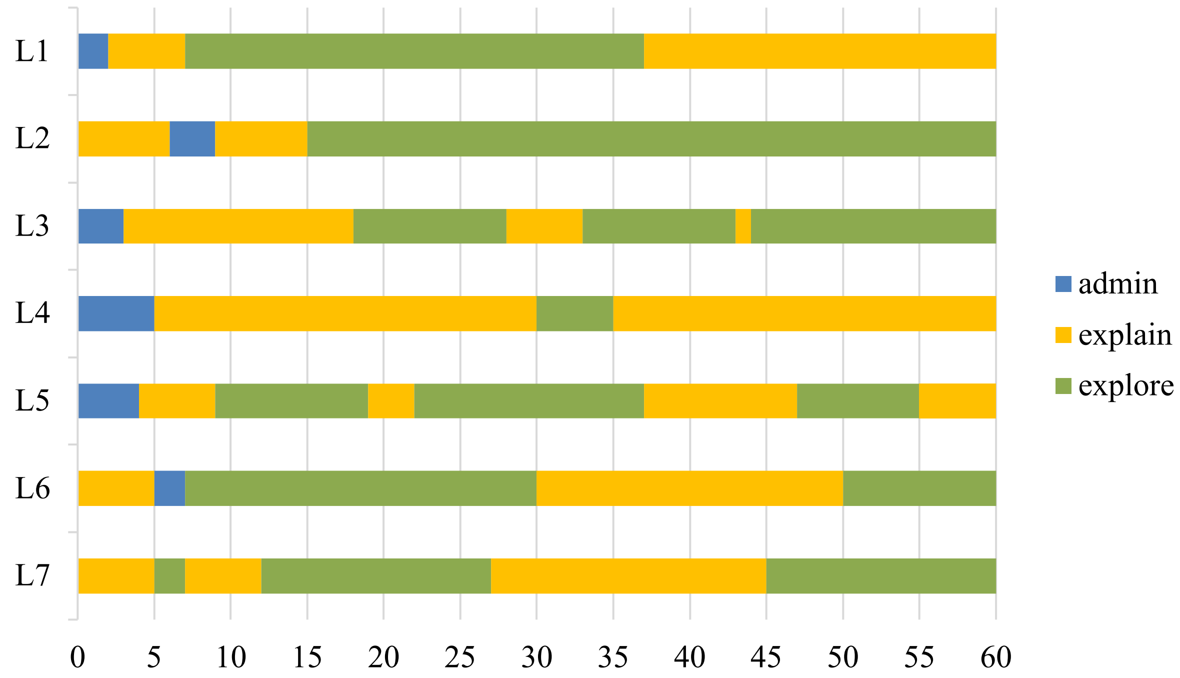



Figure 5.3: Time dedicated to administration, explanation, and exploration at Harrison

Figure 5.3 shows how the lesson time throughout the IBI unit was divided among administration, explanation, and exploration. Administration was considered to be tasks that the teacher and students completed in order to prepare to begin or progress the lesson, for example, taking the register or passing out papers. Explanation was considered any time a person (the teacher or a student) spoke to the entire class. This included explanations of the task or solution methods. Exploration was considered any time the students freely explored the IBI task. However, distinctions between explanation and exploration in a classroom context are not clear cut. It is possible that elements of exploration took place during the phases I coded as 'explain' and elements of explanation took place during the phases I coded as 'explore'.

### 5.4.2 An analysis of student beliefs throughout the unit

In Section 5.2 I provide an overview of the seven one-hour lessons organised around the topic of measurement. These lessons were observed, video recorded, and detailed field notes were kept. In addition, student work (e.g. worksheets) was collected and reviewed. These data were used to analyse how students' beliefs were evidenced and whether these beliefs were associated with their perceptions of the IBI unit as well as its effectiveness. In addition, the results of students' responses to the questionnaires are used within this analysis to help consider the extent to which mindset impacted the effectiveness of the inquiry-based approach. The results of this analysis are presented in Section 5.4.3.

Before discussing the beliefs that students expressed throughout the unit, it is useful to briefly present the results of the questionnaires the students completed prior to the commencement of the IBI unit: Attitudes Towards Mathematics Inventory (ATMI), Implicit Theory of Intelligence Scale (ITIS), and modified Implicit Theory of Intelligence Scale (m-ITIS). Please see Section 4.4 for a discussion of these instruments.

#### 5.4.2.1 Results of the ATMI, ITIS, and m-ITIS

Eighteen students completed the two versions of the ITIS, one for general mindset and another for mathematics-specific mindset. The results of these are shown in Figure 5.4.

Just over half of the students reported as having a fixed general mindset, which is higher than the 40 percent suggested in the literature (please see Section 2.9), but somewhat unsurprising given it has been demonstrated that low attaining students tend to hold more fixed views (Snipes & Tran, 2017). The number of students with growth general mindsets is also different than literature would suggest, at only 11 percent. Alternatively, looking at the students' reported mathematics-specific mindsets, the distribution is more aligned to the 40-20-40 (fixed-mixed-growth) split expected from the literature. Somewhat surprisingly, students held more growth orientations according to the mathematics-specific instrument than they did according to the general instrument.

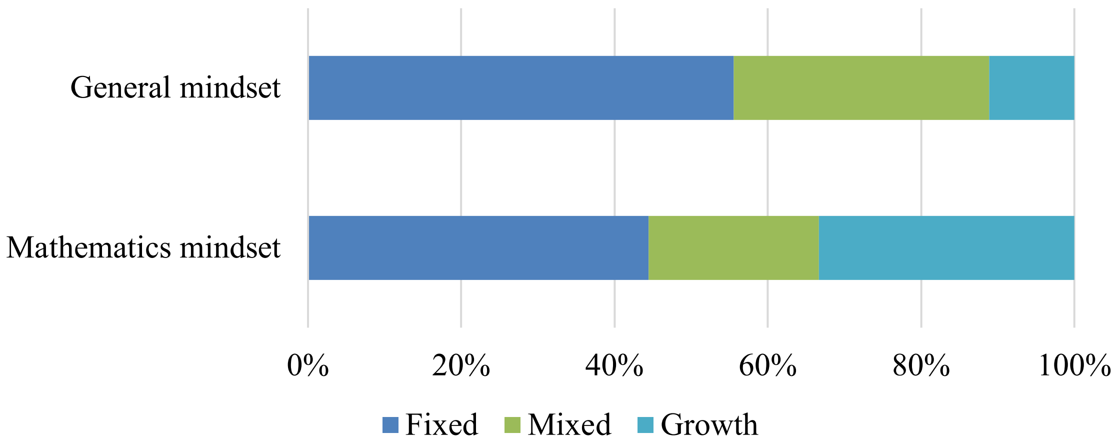



Figure 5.4: Harrison School ITIS results

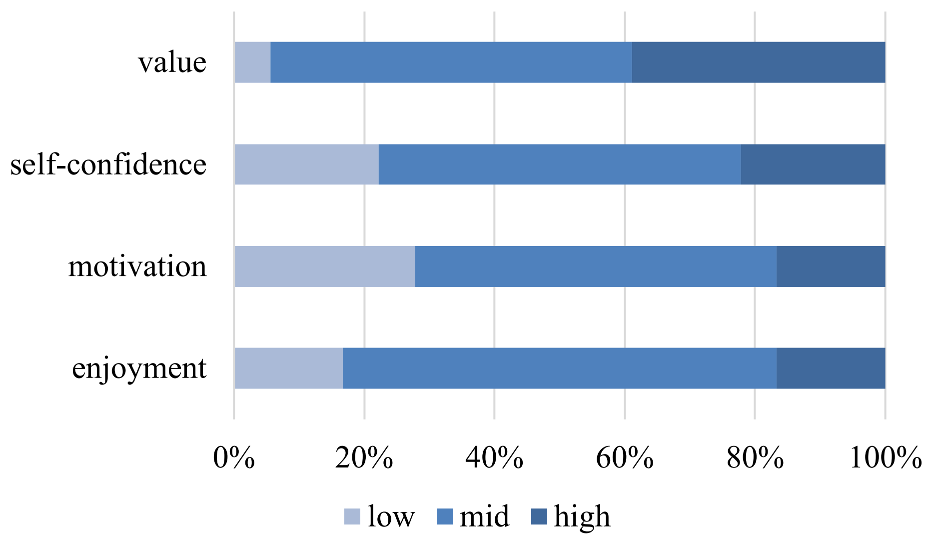



Figure 5.5: Harrison School ATMI results

The ATMI assessed students along four sub-scales: enjoyment of maths, motivation in maths, self-confidence in maths, and value of maths (please see Table 4.2 for a description of each term). Figure 5.5 shows the students' responses to the ATMI categorised into low, mid, and high by sectioning the possible scores for each construct into thirds. For example, students' responses to the motivation construct could range from five to 25. Therefore, any score less than 12 was labelled 'low' and any score greater than 18 was labelled 'high'. The remaining scores were labelled 'mid'. This somewhat crude segregation of the data into thirds, while not based upon any established authority, allows for simple visualisation of the students' views towards the ATMI categories. The students' responses on each construct were mainly neutral ('mid'), however a relatively large proportion of students expressed high value of maths and relatively low motivation in maths.

#### 5.4.2.2 Student beliefs emerging from observations

As previously discussed in Section 4.10, I use McLeod's framework of student beliefs to analyse how students beliefs were expressed throughout the IBI unit and to what degree these beliefs may or may not have been associated with the effectiveness of the IBI unit. The McLeod framework breaks students' beliefs into four constructs: (1) beliefs about mathematics, (2) beliefs about self, (3) beliefs about mathematics teaching, and (4) beliefs about the social context.

##### 5.4.2.2.1 Beliefs about mathematics

In this section I explore how students' beliefs about mathematics were observed during the seven IBI lessons. These beliefs tended to fall within two sub-categories, the idea that mathematics is unrelated to reality and the idea that mathematics is something one 'does'.

***Mathematics is unrelated to reality***

Across the seven lessons students were exposed to questions that varied from abstract mathematical problems (e.g. find the perimeter of a rectangle with given side lengths) to real world challenges (e.g. determine the amount of fencing needed to enclose a basketball court). By analysing the observations across these different problems, I was able to identify instances in which students' beliefs about the nature of mathematics may have been implied. For example, during L1 a student questioned the usefulness of determining the area of several shapes.

> This is pretty useless. When are we ever going to need this? (Irene, L1)

The above quote by Irene, the sentiment of which was common amongst the students, suggests that Irene believed that the question bore no connection to her real life. Her engagement in the activity was low, despite several of her group members appearing to take a more active role. It is possible that Irene's inability to connect the problem to something useful contributed to her apparent poor engagement.

Conversely, there were several points across the unit in which students made connections between the nature of the mathematical problem they were facing and situations they had encountered in the real-world. Others have noted that this close connection between real-life situations and mathematics problems seems to provide an opportunity for greater engagement (Wang et al., 2018). For example, L2 and L3 required the students to tackle a real-world problem, namely how much fencing would be required to surround the perimeter of the school basketball court.

> T: We have a basketball hoop in our garden, but I can only use it when dad is at work.
>
> R: Why is that?
>
> T: When I miss a shot the ball often hits dad's car.
>
> R: So maybe you could work out how much fence you'd need at home too?
>
> T: We already have a fence around the bottom bit, just not up where my dad's car is.
>
> (Tyler and Researcher, L2)

The above exchange suggests Tyler has made a connection between the abstractness of the mathematical concept of perimeter with the realities of his own home life. He continued to talk excitedly about his garden and even drew out an imaginary line with his feet to demonstrate to his other classmates how big his home court was. Tyler stood in the middle of the basketball court and held out his arms, as if to pretend he was a human corner fence post.

> T: This would be about the size of my basketball garden.
>
> D: That's a lot smaller. You'd definitely need less fence than this one.
>
> (Tyler and Daniel, L2)

This exchange highlighted not only that Tyler was beginning to apply the concept of perimeter to his real-life situations, but that this was having an effect on his classmate Daniel, who was able to relate to the discussion and join in.

Making connections between the mathematics and the real world outside of the classroom might be one way in which students can challenge a belief that mathematics is unrelated to reality. Another approach might be to make the mathematics problem itself suitably authentic. For example, during L7 students were asked to estimate how many water bottles would be needed to fill their classroom. Whilst this task is more artificial than the basketball court task used in L2, it has the advantage of connecting the problem with the real-world space in which the students spend their time. When asking students for ways to measure the volume of the room in water bottles, the students were highly engaged in exploring the problem and making suggestions which connected to real life activities (e.g. swimming, going to the store).

> We could fill the room like a swimming pool! (David, L7)
>
> We could buy as many bottles as possible and just count how many will fit. (Michelle, L7)

The exploration phase of this lesson was productive, and students applied a variety of approaches to solve the question of volume by measuring the dimensions of the room. In this case it appeared the real-world aspect to the problem resulted in greater exploration of the problem. The subsequent explanation phase led by the teacher suggested the students had developed an improved concept of volume, describing it as the amount of 'stuff' inside a space. Disappointingly, however, a review of the students' handwritten notes suggest that most were not able to use the data they collected to develop a meaningful estimate.

Despite these fleeting episodes of seeing maths as connected to real life, some students' belief that maths is unrelated to reality persisted. Students often communicated ideas that maths, or at least the particular question under investigation, was not relevant to their real lives. The following exchange took place in L7 when students were attempting to find the volume of the classroom.

M: This is dumb.

R: Why's that?

M: Because when are we ever going to fill a room with water?

I: And with water bottles? It's pretty pointless really.

R: I agree it's not likely, but one day you will want to fill something up. Like maybe your car for example?

I: My mum lets me fill up the car every week, and I just keep pulling until it stops.

R: Just the other day I used the volume of a box to figure how much packing peanuts I should buy to protect my friend's present. And besides, the point might be to just investigate the problem for the fun of it.

M: [laughing] So swotty.[10]

I: [laughing] Yeah, that's not for us.

R: What does 'swotty' mean?

M: Like kids in the top set.

(Maddie, Researcher, and Irene, L7)

This exchange continued for a few more moments before I left the students, who appeared to start working on the problem but contributed little to the follow up class discussion and whose written notes suggested little exploration had taken place. This exchange suggests that the students had poor motivation to understand the task in L7. In this case, their lack of motivation might have stemmed from seeing little connection between the task and their real lives.

***Mathematics is something one 'does'***

The second subcategory of beliefs about mathematics relates to the belief that mathematics is a series of procedures that are 'done', rather than concepts to be understood. As an example, during L1 I approached a group of six girls who were trying to solve the area of a rectangle.

[10] The term 'swotty' is an informal word sometimes used to describe someone who is overly concerned with academics, often to the exclusion of other activities such as socialising.

L: What does area mean?

E: Pretty sure that's when you add up all the sides.

(Linda and Erica, L1)

The above exchange typifies the belief that mathematics means *applying* a procedure rather than a concept to which procedures can be *applied*. In this case Erica demonstrated her belief that 'area' *is* a procedure, rather than applying the more conceptual definition of area as a measure of space within a two-dimensional shape. This belief could impact the effectiveness of IBI, as students see the inquiry activity as a means of determining prescribed steps to solve the problem, rather than understanding the deeper concepts underlying the problem.
This recurring belief emerged in other ways, such as students frequently asking for the answer or necessary procedure to apply. For example, when exploring the volume of the classroom the following exchange took place:

E: We measured the sides of the room and this one here [pointing to a diagram of the room].

L: So, do we add them?

E: I don't know, I don't know what it means.

L: When we did stuff like this the other day, we added them.

E: Let's ask Sir.

(Erica and Linda, L7)

The students were unable to get the teacher's attention and subsequently disengaged from the activity to discuss off-task topics. Their belief that mathematics was a procedural 'doing' task meant that, absent the procedure, they were unable to engage further and therefore missed potential further learning opportunities.

Later in the same lesson (L7), Mr Scott began to lead a discussion about how to find the volume of the room now that they had measured its length, width and height. One student called out during classroom discussion.

Do we multiply them together? (David, L7)

Again, students were eager to know the required procedure. The curious tone under which David asked his question suggests he may have simply guessed at what the procedure could be without an underlying understanding of why.

Another example occurred during L1 when a group of boys were exploring the concept of perimeter for a compound shape. One student was overheard saying:

> We haven't been taught this. This looks different to that other one where we just added. (Timothy, L1)

Whilst I was unable to continue observing the group's conversation, a subsequent review of their worksheets indicated no attempt, and subsequently no observable learning, had occurred. Like previous students, Timothy seemed to believe that mathematics is primarily about 'doing' something with the numbers. In addition, he seemed to have the expectation that mathematics needs to be taught, with the teacher being the primary agent of knowledge. This may have contributed to the student's failure to explore the problem further. Beliefs about mathematics teaching is discussed in Section 5.4.2.2.3.

Another aspect to the belief that mathematics is something one 'does' is the idea that mathematics cannot emerge from collaboration. It was clear that some students felt that the procedural aspects to mathematics precluded the need for collaborative exploration. Student collaboration was often stunted when students realised no one in their group knew the answer or could recall a procedure. For example, when working on problems relating to area in L5 a group of boys appeared off-task. I approached to discuss their work.

> R: Do you know what you need to do?
> T: Not really.
> R: Does anyone in your group know?
> T: Nope. [The group laughs]
> R: Have you tried discussing it together?
> T: Not really, no. [Looks around at group members]
> R: Well what does everyone here think? What are we trying to do for this problem?

D: We need to find a rectangle where the perimeter and the area are the same number.

T: But Sir hasn't told us the trick yet.

(Researcher, Timothy, and Daniel, L5)

This exchange suggests these students felt mathematics was something transmitted from someone who knew (in this case the teacher or possibly a peer) to someone who did not know. It would appear the students felt there was a correct way to approach each problem and it only remained for them to be taught the way, i.e. the 'trick'. They failed to view the importance of exploration and as such missed out on the opportunity to develop an understanding of area. Again, this is closely linked with students' beliefs about mathematics teaching which is discussed in Section 5.4.2.2.3.

It is worth noting, however, that the belief that mathematics is not something that can result from collaboration was not universally adopted. There were times during the IBI unit when students acknowledged that their collective group was able to explore the problem more completely than if they were left to work on their own. They seemed to appreciate collaborative exploration was able to generate more ideas. When solving the perimeter of a compound shape in L1 a group had successfully arrived at the solution after an active discussion.

R: Tell me about your group's approach.

E: Well I didn't get this bit at first [pointing to upper left region of the compound shape (see Figure 5.6)] but then Linda had the idea, she said–

L: I thought you had to make sure you didn't count this bit twice [pointing to the top side of the shape].

E: That got us the answer, but then Michelle did it another way. I mean we checked it a different way too.

R: What did you do, Michelle?

M: Well the whole thing looks a bit like a square with the corner folded down. So if you just do the long side times four it's the same.

R: Wow, great work. Sounds like you guys really worked well together.

E: Yeah, we figured that if we all looked at something a bit different then we'd eventually figure it out.

(Researcher, Erica, Linda, and Michelle, L1)

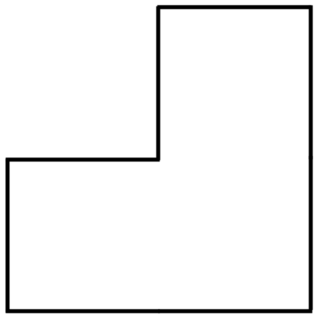


Figure 5.6: Perimeter problem from L1

It would appear that these students were able to see that, through collaboration, they could explore the problem more effectively and construct new approaches that led to greater understanding. In addition, these students were demonstrating an understanding that there was more than one way to approach the problem and that a single 'correct' procedure was not present. In this instance the IBI approach had allowed the students to step away from the idea of mathematics as 'doing' towards mathematics as constructing understanding through collaboration. Students' views of their peers throughout the IBI unit would later emerge as a central theme from the interviews (please see Section 5.4.3).

A belief that is also closely linked to mathematics as 'doing' is the belief that to be excellent in mathematics requires extensive practice (e.g. 'skill and drill'). This view of mathematics implies that memorization of seemingly unconnected procedures is required to be highly successful. As others have observed, students often nuanced their beliefs about their skills in mathematics across the multidimensionality of the subject (Leder, 1987). For example, they might say they were 'good at fractions' but 'bad at algebra'.

*5.4.2.2.1.1 Summary of beliefs about mathematics*

The above section highlights instances when students' beliefs about mathematics, as outlined by McLeod (1992), were observed and how some of these beliefs may have interacted with the perceptions and effectiveness of the IBI unit. Students initially struggled to connect the relevancy of mathematical problems to their real lives, and this behaviour was often associated with low motivation and engagement levels. However, the IBI lessons appeared to create opportunities for students to make connections between the mathematics and real-world problems, thereby breaking down some of the abstract barriers that students may have held. These connections appeared to stimulate the exploration phase of the problem solving. However, it was not abundantly clear this led to increased learning.

Students frequently felt that mathematics was something one ‘does’, in the form of a series of procedures, rather than something one understands on a deeper conceptual basis. Although this was not universally true, and some students were able to use the collaborative group exploration part of IBI to recognise that mathematics need not be limited to carrying out procedures but rather could include developing new ideas with their peers.

**5.4.2.2.2 Beliefs about self**

In the following section, I present observations of students’ behaviours indicative of beliefs about self in two primary categories: student self-efficacy toward mathematics (Bandura, 1997) and student mindset (Dweck, 2017b).

*5.4.2.2.2.1 Self-efficacy*

IBI requires students to explore novel problems without having a clear understanding of the mathematics concepts or procedures needed, but for which they have some prior knowledge (e.g. the properties of rectangles), as discussed in Section 2.2. Success with this type of instruction calls upon students’ self-efficacy towards mathematics, sometimes referred to as confidence. Students with low self-efficacy in mathematics might find the exploration aspect of IBI challenging. In observing the IBI unit, there were several times when students’ beliefs about their ability manifested, typically with the result that learning or engagement in the task was reduced. For example, during L4:

T: Do you know what you are supposed to be doing?
K: [shrugs]
T: Which question are you on?
K: I don’t know…
T: It looks like you’re on this one.
K: Yeah, but I don’t know what to do.
T: This is just like the one we did last lesson. Do you remember?

K:	I didn't get it last time either. I'm not good at these.
(Teacher and Kevin, L4)

This interaction, which typifies several similar interactions with different students, suggests that Kevin's self-efficacy for this problem type is low. The result of this belief appears to be a diminished willingness to engage in the task. Following the above exchange, Kevin placed his head on the table for several minutes, after which he distracted others around him by making faces. An alternative explanation is that Kevin's inability to engage fully with the task is due to his having a fixed mindset (as measured by the mathematics-specific ITIS). Students' mindset beliefs are discussed further in Section 5.4.2.2.2.2.

In a separate exchange during L1, I observed a group of three boys and one girl handling a rectangular prism manipulative. The boys were off task while the girl, Heather, was diligently working on her worksheet.

R:	Are you all finished?
H:	Heather knows how to do it.
T:	That's great, but it's important that you understand how to do it too.
Make sure you are talking and checking with each other, please.
(Researcher, Henry, and Teacher, L1)

In this example, Henry does not seem to believe he knows how to complete the problem so defers to Heather. Perhaps he and the other group members (James and Steven) doubted their abilities to add value to the activity (i.e. low self-efficacy) and allowed their more competent peer to shoulder the work. Mr Scott's interjection was timely and helpful in reminding these students a key element of IBI is that of exploration (in this case through collaboration with peers). The effect of the teacher's prompting was favourable, and the group appeared to be discussing things together after proceeding to the next problem. An inspection of Henry's notes suggested increased working during the problem immediately after the rectangular prism problem.

An alternative explanation for Henry's actions is that he believes the only thing that matters in mathematics is getting the correct answer. Henry might have high self-efficacy for the problem they are working on, but what does it matter when the correct answer is already in hand thanks to Heather? With this view, allowing Heather to work on the problem while Henry and the other group members do something else is perfectly acceptable. It might even be viewed as a more economical use of their energies.

Often the students exhibited high self-efficacy in 'doing' mathematics as it pertains to procedural knowledge, but low self-efficacy in understanding mathematics on a deeper level. They were uncomfortable exploring the problems and were frequently seeking the formula (the procedure). In the below exchange John and Alex discuss how to use the measurements for width, height, and length of the classroom to determine the volume.

> J: I don't get what we do with this now.
> S: Add them together?
> J: But for the area we timesed them together, like this one and this one times together [pointing to the length and width].
> S: But then what about this? [pointing to the height and the width].
> J: Multiply again?
> S: Or maybe you add them?
> J: Sir, we aren't sure what to do.
> T: Alright, tell me what you're thinking so far.
> S: We were thinking maybe you add them together … or--
> T: Maybe think about just the height for a moment. How does just the height help you here? Think about that and you'll be closer to finding the formula.
> (James, Steven, and Teacher, L7)

In this exchange the boys exhibited some positive behaviours, by exploring and discussing a few different ways in which volume may be calculated. However, the discussion quickly focused on the procedure and guessing what to do with the numbers rather than trying to conceptualise what the numbers meant. As such, the boys' exploration was shallow and terminated with the incorrect idea that adding the product of length times width with the product of height times width. Mr Scott's response could have had an unhelpful effect by reinforcing the boys' beliefs. By telling the boys they will get 'closer to finding the formula' he implied that the formula was the ultimate goal. This might serve to deepen the boys' beliefs that success in this problem, and perhaps maths more broadly, is to find the correct procedure.

A similar event was observed during L1 when students were exploring the perimeter of a compound shape.

> T: What has this group found about the problem?
> D: Nothing really.
> T: What sort of things have you tried?
> D: None of us gets any of it.
> (Teacher and David, L1)

Mr Scott proceeded to instruct the boys by redrawing the shape on their worksheet. He labelled each side and worked through the process of solving. David, assuming the role of spokesperson for the group, responded.

> D: Oh yeah, I get it. It makes loads more sense when you explain it like that.
> T: Okay, that was one way to solve it. Can you think of another slightly different way?
> (David and Teacher, L1)

Under the IBI philosophy, the decision of Mr Scott to show the students the solution, as in the above exchange, may have taken away an important learning opportunity. IBI relies on these impasses as valuable learning opportunities (Doerr & Tripp, 1999; Goldin, 2014; also see Section 2.3). By overcoming these impasses students develop an awareness of their knowledge gaps, thereby facilitating subsequent explanation. Therefore, the decision of the teacher, whilst appropriate under a direct instruction teaching model, was arguably a poor choice in the case of IBI since students had not had sufficient opportunity to explore the problem and gain awareness of gaps in their knowledge.

The teacher's willingness to solve the problem for the group may propagate learned helplessness amongst the students (Dweck, 1975; Yates, 2009). Learned helplessness is the idea that a student's self-efficacy can be reduced through teaching practices that promote the over reliance upon others as the source of knowledge, and the under reliance upon their own abilities. However, Mr Scott did attempt to re-inject a sense of self-efficacy and inquiry with his parting comment, encouraging students to try and find ‘another slightly different way’ of solving the problem. Sadly however, this was insufficient to stimulate the group, who neglected to attempt any further approaches having felt satisfied that they had the answer.

There were also observed instances where students appeared to hold high self-efficacy for mathematics, and these were typically associated with a more positive reaction to the IBI problems.

S: Miss, is this right?
R: Well, do you think it’s right?
S: I think it’s right because it’s similar to the one on the board but this one is just bigger.
R: Well I think it looks right too. How did you feel about the problems?
S: They are okay, I find them quite easy. We did harder stuff last year.
R: You did harder topics with Mr Scott last year?
S: No, not Mr Scott. I was dropped down a set last year because my tests weren’t good. It’s not that I can’t do maths, I just don’t do good on the tests and miss my homework.
R: How do you feel about that?
S: It’s alright, I miss my old mates a bit. I’m pretty good at maths, so hopefully I can get back up to middle set next year.
(Susan and Researcher, L4)

In this example Susan clearly expressed a favourable view towards her mathematics ability, and this manifested as an expressed desire to do well. Her efforts on the problem set as well as her expressed desire to move up a set are possible sources of motivation. As discussed in Section 2.3, there is a positive correlation between motivation and performance, and in this case, Susan stood out as being one of the most motivated across the entire IBI unit.

*5.4.2.2.2.2 Mindset*

The above discussion explores how students' self-efficacy beliefs may have been associated with the way they perceived and performed during the IBI unit. Here, I discuss observations that suggest students' mindsets and the ways these mindsets may have been associated with their perceptions and performance during the IBI unit (see Section 2.9 for a discussion of mindset). In addition, I bring together student observations with the results from the ITIS and m-ITIS where relevant. One of the most common ways in which students expressed mindset is typified in the below exchange, which took place during L5.

R: Have you made any progress?
D: No.
R: Okay, where are you struggling?
D: I don't know. I think I'm supposed to work out the area and the perimeter?
R: Yes, for different rectangles.
D: Yeah, but I don't know what numbers to put in.
R: You are trying to find a shape where the area and perimeter are the same number. You did something similar earlier [pointing to earlier workings].
D: Yeah, I was good at that bit. I don't work like this though.
(Researcher and Daniel, L5)

Here Daniel is expressing a view that suggests, at this point in time, he is holding a fixed mindset about this task. His final comment, 'I don't work like this,' was a common sentiment that others expressed and suggested that Daniel believed he had some *inherent way* of working, and that this problem conflicted with that inherent way. It is worth noting that Daniels's m-ITIS results suggested that he held a fixed mindset, so the above comment was consistent with that result. The above interaction with Daniel might also suggest yet another example of a student's belief that mathematics is something one 'does', by which Daniel viewed the problem as searching for formula inputs, rather than searching for meaning or understanding.

In a separate example, during L6, the following exchange took place.

L: Is it done like that?

S: I don't think so. You have to add them all up.

L: How do you remember this stuff?

S: [shrugs]

L: You're dead good a maths. It's not my thing, but you're dead good.

(Linda and Susan, L7)

Here Linda, who obtained a fixed general mindset and a mixed mathematics mindset, expressed what could be viewed as fixed mindset views towards the subject of mathematics. Linda's suggestion that maths is not her 'thing' might be indicative of a belief that people have inherent, immutable 'things' they are good at, rather than the view that people can become good at things they put time and effort into. In this case, Linda is commenting holistically on mathematics as a subject ('…maths. It's not my thing') rather than limiting her statement to the unit at hand. The suggestion that Linda might be expressing a fixed mathematics-specific mindset, despite having appeared as mixed from her responses to the m-ITIS, is consistent with the idea that students' mindsets can oscillate over time (see Section 2.9). In this collaboration, Linda frequently deferred to Susan when reaching an impasse, contributed few ideas to the group's discussions, and tended to limit her contribution to praising others or off topic discussions.

Another observation which seemed to suggest student mindset appeared in the following exchange during L4.

T: How are we getting on here?

J: I don't know, not great.

T: Do you know what you are supposed to be doing?

J: We have to put numbers in here.

T: What numbers do you think the question is asking for?

J: I don't know … my brain's not very good at this.

(Teacher and Jackie, L4)

Mr Scott then helped to explain the major vocabulary terms in the question and went on to discuss some of the work done in the prior lessons. This had a favourable effect, and the student demonstrated improved effort. The exchange might suggest that Jackie holds a fixed mindset. Jackie outsourced the blame to her 'brain' which suggests she felt the struggle she was facing was inherent. It is possible that this mindset led Jackie to not fully engage with the task, which reduced her opportunity to learn.

The exchange in L1 between Erica, Linda, and Michelle (Page 91) might also suggest how mindset drives behaviour. Erica's comment ('…we figured that if we all looked at something a bit different then we'd eventually figure it out.') is indicative of a student who believes that successful outcomes can result from time and effort. This is a key feature of growth mindset. Erica recognised that with enough trying this group would eventually succeed or 'figure it out'. This apparent observation of Erica's mindset, however, does not fully align with her m-ITIS result which indicated a mixed mindset in mathematics. It might be possible that Erica's beliefs about her own intelligence, coupled with her perceptions of the IBI challenge, allowed her to see the benefit of the group's initial struggle and effort. Her fellow group members, Linda and Michelle, obtained m-ITIS scores which indicated a mixed and fixed maths mindset respectively. From my observation records it is unclear how the group interacted prior to this discussion as the audio and video quality at this time were poor. It would be interesting to explore how small groups with participants of varied mindsets interact. Could cooperative groups develop collective mindsets? Areas for further study are discussed in Chapter 7.

The teacher can play an important role in priming students' mindsets (Burns & Isbell, 2007). There were occasions when Mr Scott said things during the lesson which might have encouraged fixed beliefs.

> Excellent, Heather. Did everyone hear that? [pause] You should all be listening. Heather is our top student. (Teacher, L6)

Unknowingly the teacher's reference to Heather as the 'top student' could perpetuate a belief that mathematics is something that some people can do well, and others cannot. Without knowing the wider context of this classroom beyond this study I am unable to be sure that the use of the phrase 'top student' was not a regular, equitably distributed, title given out by Mr Scott. However, over the course of my observations, this was the only time this specific term was used.

Another example of this came following L1 when students gave a thumbs-up or thumbs-down as part of the whole class reflection. Several students held their thumbs down, indicating they did not feel the lesson went well.

> Some of you probably found that difficult, and that's okay. It's probably just not your thing. You will find other stuff which is your thing. (Teacher, L1)

Clearly this comment does not help students build a belief that mathematics is a learnable discipline.

*5.4.2.2.2.3 Summary of beliefs about self*

In the above sections, I have discussed several observations which suggest students' beliefs about self. Students often demonstrated low self-efficacy. This seemed particularly true when it came to the exploration of novel problems and developing deep understanding of mathematical concepts. Students frequently attempted to jump to the formula as a means of answering the problem. The fact that IBI shies away from providing the formula upfront meant students' low self-efficacy became particularly evident. This overlaps with the above discussion about students' beliefs about mathematics.

Furthermore, the role of the teacher in propagating these beliefs was observed during several lessons. On more than one occasion the teacher may have stunted opportunities for deeper thought by intervening with direct instruction to a struggling group or individual. Finally, students' fixed mindset views were often expressed through comments such as 'my brain doesn't work that way' or 'maths is not my thing'. These fixed views were frequently associated with underperformance on the IBI tasks. Growth mindset comments were also observed, with some students suggesting a recognition of the link between effort and outcomes. These instances were frequently associated with students or groups that had persisted through difficulties on a given task.

**5.4.2.2.3 Beliefs about mathematics teaching**

Through many years of schooling students begin to develop a series of beliefs about the way in which mathematics should be taught. These beliefs typically revolve around the role of the teacher (Boaler, 1998; Makar & Fielding-Wells, 2018; Op 't Eynde et al., 2006). In direct instruction, the role of the teacher is seen as the excerpt in the room and the source of knowledge. This type of instruction tends to focus on skill transmission and procedural understanding, without necessarily a heavy focus on conceptual understanding, as might be found in inquiry based environments (J. C. Marshall & Horton, 2011). At various points during the IBI unit students expressed views that demonstrated their beliefs about the way in which maths should be taught.

Mr Scott had explained to me before commencing the unit that this group of students had little previous exposure to IBI, particularly within a mathematics context. He echoed this sentiment to the students when he introduced the unit during L1, emphasising they were about to experience a 'different' type of learning. In some cases, I was able to observe students expressing favourable views towards the new working style. For example, below is an excerpt from a discussion between a small group of students whilst waiting to rotate to the next station during the L1 exercise.

> L: These are hard.
> M: Yeah, kind of. This one wasn't too bad.
> L: I'm not sure I get it. I'll get it better when he [the teacher] explains it.
> M: It's better than bookwork.
> L: Yeah.
> (Lisa and Maddie, L1)

The above discussion suggests several examples of the students' beliefs about the way in which maths should be taught. Firstly, Lisa, a student who is unsure of the relative value of the exercise, made the comment, 'I'll get it better when he [the teacher] explains it', suggesting the student felt she would develop a better understanding for the subject from the direct instruction of the teacher, and not from the shared knowledge of her peers or from her own exploration. Perhaps this student felt that the role of the teacher is that of explaining and the role of the student is that of understanding, and that by learning via inquiry they were not really learning (at least not until the explanation phase). Others have noted this behaviour, whereby students seem reticent to learn via IBI (Makar & Fielding-Wells, 2018).

In addition, the excerpt suggests that students found the IBI approach different and, in the case of Maddie, more enjoyable than their normal classes which were more focused on completing exercises from the textbook. These students' comments suggest they enjoyed the new structure, despite apparent difficulty with the task. This is a theme I explore further within the analysis of the interview data (Section 5.4.3).

During L2 and L3 the students spent most of the time exploring the question, 'What is the perimeter and area of the basketball court'? Towards the end of L3 one student called out, 'Do these always take so long? We've only done one problem'. Similar sentiments were repeated by other students and highlight the different nature of IBI activities as compared to the maths lessons they were accustomed to. Students may have been familiar with teachers breaking down problems and providing focused instruction followed by shorter practice exercises. Perhaps for this reason, students seemed to believe this was how all mathematics teaching should be done. The students did not appreciate the role of exploration in learning and perhaps felt that by taking so long over a single problem they were not learning as much as they would if they covered more problems. This could also be linked to the belief that all mathematics problems can be solved in less than ten minutes (Schoenfeld, 1985).

During a discussion between the teacher and a group of girls, the teacher probed the students' feelings about the lesson.

T: How are you guys finding this today?

J: It's a bit different really.

T: What is different exactly?

J: Well we aren't just sitting down doing questions, we are up and about.

T: And is that good or bad?

J: Well it's mostly just different. I suppose it's good because I like moving about in class and you get to really understand the question by talking it out. It's tricky but it's good, and it's a bit different.

E: I like the moving about bit, but don't like when you don't know what to do or when others don't know either. It's hard because you said we can't ask you and normally we'd ask you. I mean it's good and all, but if we're stuck– Like in a normal lesson we'd ask you, but today we can't really, and we just have to try figure it out.

(Teacher, Jackie, and Erica, L2)

This exchange suggested a series of beliefs students held. In the first instance students repeated the view that the lesson was different to their normal classes. Through probing, the teacher was able to explore the nature of these differences. Jackie suggested a view that the new ways of working were enjoyable, partly due to the collaboration, movement-based activities, and partly because the style of working allowed for a deeper understanding of the material. This student had made a connection between the investigation and being able to 'really understand'. Erica, however, raised some beliefs that were common during the IBI unit. It seems Erica was accustomed to using the teacher as the agent of knowledge when she got stuck. It appears she expected the teacher to facilitate her overcoming any impasses, and therefore felt ill equipped to tackle these impasses alone during the IBI. Clearly this belief would have an impact upon how the students perceived the IBI lessons.

In some cases, it appeared that students felt the lesson so strongly violated their understanding of the role of the teacher that the result was a total refusal to engage in the task. For example, during L5, whilst the students were exploring rectangles with perimeters and areas that were numerically the same, the following exchange took place.

> R: Have you had any success with this one?
> T: No.
> R: What have you tried so far?
> T: Dunno.
> R: Do you understand the question?
> T: Not really.
> R: We already saw an example where the area was bigger than the perimeter. That one is on the board, and we have seen rectangles where the perimeter is bigger than the area. We are now trying to find an example where the perimeter and area are the same number.
> T: He hasn't told us how though…
> R: Perhaps you could try it out anyway.
> T: If he's not gonna bother explaining it then why should we bother trying to work it out?
> (Researcher and Timothy, L5)

Following the above exchange Timothy placed his head on the table and refused to reengage with work. Sensing that further persistence might be counterproductive I left Timothy, but observations from across the room indicated he remained disengaged until the teacher began discussing possible solutions. Perhaps the IBI approach so conflicted with Timothy's understanding of his role within the classroom (as the recipient of knowledge), and that of the teacher (as the transmitter of knowledge), that he was unable to adapt to the new instructional approach.

*5.4.2.2.3.1 Summary of beliefs about mathematics teaching*

This section discussed whether students' beliefs about mathematics teaching were associated with their perceptions and the effectiveness of IBI. Students appeared to hold a number of beliefs, the first of which was that learning mathematics is best done by solving many short problems rather than one long one. Students seemed surprised that a single problem, such as finding the area and perimeter of the basketball court, could take two whole lessons.

The second belief centred around the role of the teacher as the transmitter of knowledge and the role of the students as the recipients. IBI seemed to challenge this belief, and as such, some students struggled with the shift in responsibility. Several students felt that the new approach allowed them to develop deeper knowledge, or 'really understand [the problem]', which aligns with the views of IBI proponents.

**5.4.2.2.4 Beliefs about the social context**

The social context includes the way students interact with each other, as well as the way they see mathematics as part of the broader society (McLeod, 1992). IBI requires students to challenge their beliefs and the beliefs of others. Challenge was frequently observed between group members, such as the below exchange whilst finding the area of a triangle in L6.

> S: Why do you have 8? Isn't it 16?
> J: It's a triangle, so you have to halve it.
> S: But I thought for area you have to multiply.
> J: Yeah, but it's not a rectangle. The ends are triangles.
> S: I still can't see how you got 8. I'm going to stick with 16.
> (Steven and James, L6)

The students in this group appeared to feel comfortable with challenging each other when they disagreed. Steven's final comment would seem to suggest that he was happy to 'agree to disagree' and stick with his answer. However, despite his apparent resignation Steven actually revisited this problem quietly and eventually proclaimed, 'I see how you got 8 now. I was counting this bit two times.' The feeling of being challenged, and the social context, in which Steven had clearly and publicly disagreed with his peer, led him to re-work his approach. This sort of interaction may help reinforce the sense that mathematics is something that can be shared and collaborated on in a social way, with the result being an improved final answer.

The above exchange highlights how IBI may have allowed students to move towards a whole group consensus through discussion. There were occasions, however, when the outcome was less consensus and more socially democratic. For example, when exploring how to approach the volume question in L3, Michelle and her group were debating how to best measure area.

> M: We could walk all over the court and count footsteps again?
> J: Maybe, but how would you keep track of where we stepped already?
> K: Can we choose something that won't take forever!
> M: But Sir said to think of measuring area like we did with perimeter…
> J: We could use something big like bedsheets maybe…?
> M: I like that!
> K: Brilliant, done.
> (Michelle, Jasmine, and Kevin, L7)

The above exchange shows students taking a more democratic approach to a problem. This type of debate may suggest students believe mathematics to be less black and white and more socially derived. Such opportunities are common with IBI and allow students to develop a more nuanced view of the mathematical world, one in which people's opinions can differ and this difference can influence the solution.

A similar example was seen during L2 when a group of girls were attempting to measure one side of the basketball court. The girls initially attempted to measure by estimating the length of a metre. Through a combination of arm stretches and leg paces they were debating just how long a metre was. However, the group observed a separate group of students measuring the length in feet (heel-to-toe). Upon contrasting their approach with that of the other group, the girls decided to abandon their original idea and instead adopt that of their peers. Whilst neither group were outright challenging each other, the girls were comfortable with seeing a contrasting method which conflicted with their own and adapting their approach in response. Perhaps the IBI lesson created opportunities for these students to realise that aspects of their social context, namely their peers, might be able to help them when tackling mathematical problems.

During L7 the students were estimating how many water bottles it would take to fill the classroom.

> D: Let's measure using my height again.
>
> J: But how does that help? We should measure in bottles. When would we ever want to fill a room with *you*? [laughing]
>
> D: When would we ever fill this room with water!
>
> T: Can you think of a time when you might want to know how much water it would take to fill something?
>
> D: I have a fish tank at home.
>
> T: And how do you fill it?
>
> D: Well you don't fill it really. You empty half out and then use a bucket to fill it back up again.
>
> J: We do the same with my sister's fish tank, but she uses the hose. It's a big tank.
>
> D: Yeah, it takes a while. I usually need to fill like four buckets.
>
> (David, Jess, and Teacher, L7)

On the one hand the above exchange is an example of students making connections between the task and their real worlds, which could align with the previous discussion regarding the belief that mathematics is unrelated to reality. However, this example is also suggestive that the students can take advantage of opportunities during IBI to have conversations about the mathematics being done, and to do so socially by interweaving their own lives and the role of mathematics in society. This creates opportunities for students to see how mathematics is important for society by thinking of ways people use mathematics in the world. It also brings up the notion of practical approaches to solving maths problems. David's suggestion was that the best way to know how much water the fish tank needed was to simply start filling the tank and counting. These sorts of conversations align with Jaworski's (2006) view that inquiry mathematics creates opportunities for deeper mathematics conversations than might not be possible in direct instruction.

The inaccuracies of the real-world mathematics came up several times during the unit, particularly as students made connections to sampling error work which they were encountering in science lessons (according to Mr Scott). When evaluating why two groups obtained different perimeters for the basketball court, as measured in toe-to-heel feet, the class was quick to identify different people having different shoes sizes being the culprit. Furthermore, when critiquing their estimate for the number of bottles that would fit into the classroom, they were able to identify measurement errors such as the omission of windows and the lack of right angles in the room.

Another example of the social context for these students comes from an awareness of the attainment-based groups the school deploys. Other researchers have highlighted the challenges that setting can have upon students' outlooks (Francis, Connolly, et al., 2017). Within this case, the students were aware that they were placed amongst the lowest groups. For example, whilst completing the end of unit post-test, one student (Irene) called out, 'Are we doing all this because we are bottom set?' This awareness of their social position seemed to impact their motivation and beliefs. Also, as previously discussed, Maddie identified with being bottom set and not 'swotty' like 'the top set'.

> M: [laughing] So swotty.
> I: [laughing] Yeah, that's not for us.
> R: What does 'swotty' mean?
> M: Like kids in the top set.
> (Maddie, Irene, and Researcher, L7)

The students seemed aware of their social positioning within the pool of mathematics students at the school. In the exchange the students appear to identify with what they believe to be the normal behaviour of their peers in this lower set, namely that mathematics is not enjoyable. The choice of the word 'swotty' in this social context is meant negatively, much like ‘geek’ or ‘square’. This point of view may have impacted the students’ levels of engagement throughout the IBI unit.

However, there were examples of setting having the opposite effect, for example by providing motivation to work hard. Typically, this would be expressed as students wanting to move up a set, as voiced by Susan in L4 (Page 96). Further examples are discussed in the analysis of the interviews (Section 5.4.3).

*5.4.2.2.4.1 Summary of beliefs about the social context*

This section has explored how students’ beliefs about the social context interacted with the IBI unit. IBI seemed to encourage social interactions, with students debating and challenging each other as well as deploying democratic approaches to working together. These sorts of rich conversations can help students develop a deeper understanding of mathematics (Jaworski, 2006). Students were also aware of their position within the attainment-based groups of the school. This awareness may have driven students to behave in ways that were consistent with their understanding of the social norms for this group, e.g. students in lower sets of mathematics do not enjoy mathematics. On some occasions these beliefs meant students disengaged from the IBI; on others it was a source of motivation.

### 5.4.3 An analysis of student perceptions of inquiry instruction

This section provides an analysis of ten student interviews following the IBI lessons. These interviews were designed to explore the students’ perceptions of the IBI unit as well as mathematics more generally. For a detailed discussion of the analysis methods please refer to Section 4.10. Several themes emerged: (1) IBI as a form of empowerment, (2) IBI as a form of neglect, (3) Importance of teacher, (4) Importance of peers, and (5) Mathematics disaffection. Table 5.3 presents the intensity of these themes for each student, organised by mathematics mindset. For a discussion of intensity scoring, please see Section 4.10.

Table 5.3: Intensity scores for interview themes at Harrison

| | Student | IBI empower | IBI neglect | Teacher importance | Peer importance | Maths disaffection |
|---|---|---|---|---|---|---|
| Fixed | Kevin | 12 | 25 | 15 | 10 | 9 |
| | Jackie | 8 | 30 | 9 | 9 | 7 |
| | Michelle | 14 | 29 | 5 | 7 | 3 |
| | Lisa | 5 | 19 | 14 | 3 | 4 |
| | Daniel | 7 | 23 | 16 | 3 | 5 |
| | Timothy | 13 | 32 | 10 | 7 | 4 |
| Mixed | Erica | 18 | 11 | 27 | 11 | 1 |
| | Linda | 28 | 5 | 22 | 3 | 4 |
| Growth | Henry | 31 | 12 | 5 | 8 | 8 |
| | James | 29 | 10 | 13 | 2 | 1 |

#### 5.4.3.1 IBI as a form of empowerment

Across all the interviews a consistent theme emerged around the view that IBI was a form of empowerment. These views align with comments made within the literature that IBI gives students a chance to 'have a go' and 'engage their brain'. For example, when discussing his approach to inquiry-based problems, Daniel made the following comment:

> Well I try and work it out myself, and see if I understand it, and if I don't understand I'll ask Sir. But if I do, I'll just carry on with the work and see how far I get into it. If I get something wrong, I just keep trying again. (Daniel)

Here Daniel expressed a view that mathematics requires him to demonstrate persistence. Whilst Daniel later expressed a slight preference for direct instruction (Page 118) he appeared to appreciate the exploration component of IBI. A similar example was noted during the interview with Lisa. When asked how she felt about the exploration portion of the lessons she said:

> It was hard [L5], but I kept trying and I found ways of doing it. (Lisa)

It is interesting to note that the idea of persistence, hinted at by Lisa in the above quote, did not necessarily imply the student had high self-efficacy. Some students expressed views that they understood their role as a student was to try, but at the same time believed this effort would ultimately lead to failure.

> I was just like, 'I'll have a go' [L1]. But I wasn't sure of myself. I didn't think I'd be able to. I kind of had doubt in myself, because I've never done it before and most likely was probably going to fail at some things or not have the right idea. But I was alright. I didn't really understand it, but there were some things that I got more than I thought I would. I was a step closer to actually getting it right. I enjoyed that lesson. That was a good lesson. (Erica)

On the one hand, Erica expressed a view that she understands her role is to 'have a go', but at the same time she appeared to exhibit low self-efficacy. Perhaps her initial willingness to try was a kind of 'resigned acceptance' as described by Nardi and Steward (2003, p. 346) rather than an intrinsic motivation to solve the problem. Her comments also suggest that, despite her low confidence, she enjoyed the lesson and was able to recognise where she was improving and where her knowledge gaps were. This aligns with the view from the literature in which IBI helps students become aware of their knowledge gaps and readies them for the direct instruction that follows (Chi et al., 1994; Schwartz & Martin, 2004).

In the above quote, Erica seems aware of her low self-efficacy. Other students seemed to be similarly aware of their beliefs about themselves. For example, when asking James how he felt about the IBI unit he said:

> I think it [the IBI unit] made me feel more confident in the subject and the topic we're doing. (James)

It appears the students may have felt an increased confidence as a result of the IBI lessons.

In addition to expressing views of increased confidence, students also felt that IBI afforded them greater independence. James's comment during his interview is one example of this:

> You're learning better, because you're more independent I'd say. (James)

It is possible that as students explored the IBI problems in an unconstrained and unguided way this fostered greater ownership of their learning. It is interesting to note that when introducing each IBI problem, Mr Scott never suggested that inquiry learning was ‘better’. Furthermore, the students had little experience with IBI in the mathematics classroom. Despite this, however, James was able to express a dual view that IBI prompted more independence and that independent learning for mathematics was better.

During several of the interviews, students drew a contrast between the direct approach (often described as ‘PowerPoint and practice’) and the IBI approach (often described as ‘try and discuss’). In the below extract, I asked Kevin if he had a preference for one type of instruction over the other. Kevin expressed a preference for the IBI approach since he sees it as an opportunity to ‘have a challenge’.

> R: Which of those two do you like more? Between the PowerPoint and practice or the try it on your own and then we’ll discuss.
> K: Try it on your own and then discuss about it.
> R: Why do you like that more?
> K: Because I like to be able to have a challenge and not just know what it is. Because if he’s going to discuss after, like what has happened and all that, then it’s a lot easier. And then he can go through it with us after to explain how to do it. And then we could try it again after that to get it right.
> (Researcher and Kevin)

Timothy expressed a similar preference for the challenging aspects of IBI during his interview.

> It was challenging [L1] because you normally think if someone doesn’t know that another person will, but then for one of them none of us knew what to do. And then we kept trying and trying, but we couldn’t figure out what to do. …It was fun, because it was challenging. (Timothy)

Here, Timothy expressed his surprise that none of his group members knew how to solve the given problems. But instead of feeling overwhelmed or frustrated by this, Timothy reported feeling enjoyment ('it was fun because it was challenging'). This is somewhat surprising given Timothy's fixed result on both the general and mathematics-specific mindset questionnaire.

As discussed previously in the analysis of the lesson observations (see Section 5.4.2.2.1), many students held the belief that mathematics is about 'doing' something with the numbers, usually applying the formula or procedure (Kouba & McDonald, 1987). However, during the interviews it was not uncommon to hear comments that suggested the IBI approach was able to challenge this belief. As an example:

> I like that because you got to try and do it yourself [L1]. Instead of getting told what to do you could try different ways to find out how you liked to do it. It's like there is not just one way to do it, not just the way teacher does it. You can give-- well like try different ways and maybe you find a different way that gets it right too. (Henry)

Henry's comments suggest that the IBI was able to challenge the idea that mathematics is a search for a single solution method. Furthermore, it might indicate that Henry was able to see mathematics as something which is not necessarily black and white, and something in which the teacher is not necessarily the sole source of knowledge.

Student engagement with a task is a key component to learning in mathematics (Fung et al., 2018). Proponents of IBI have argued that lessons which incorporate inquiry-based activities and exploration of novel problems can generate greater engagement. Feedback from the student interviews during this case would support this view, with many students expressing that they felt engaged by the IBI task and believed this engagement had a favourable impact upon their learning. Two excerpts from interviews with Jackie and Henry highlight this:

> Well, there was a lot more work going on, and I could engage more with the learning, and I could actually get into it [the IBI unit]. (Jackie)

> That was an alright lesson [L1], but it was like-- we learnt stuff we didn't really know. So, it was a bit more thinking, and we concentrated a bit more. (Henry)

As discussed in the analysis of the lesson observations (Section 5.4.2), students often held the belief that mathematics problems should be quick, and that speed and accuracy were good measures of doing well in mathematics. Clearly different students learn and operate at different speeds, and highly proficient IBI lessons can accommodate this pace by allowing students time to deeply explore a single problem, often from multiple angles. During the IBI unit students who seemed to complete the task quickly were encouraged to find alternative solutions or to discuss their solutions with peers. For example, during the interview, Timothy was asked to pretend he was the teacher and to decide whether to adopt an IBI or direct instructional approach:

> R: Why do you think that would be a better way of ordering your lesson, rather than just telling them [the students] upfront?
>
> T: Because they [the students] wouldn't really learn anything. Sometimes if you tell them straight away, they have to do some stuff on their own. Because it's challenging. So, they could get good stuff on it but then if I help them out first thing they will just say, 'Oh I get it,' but then no, they forget about it when the test comes. They should do it themselves. And do it at their own pace at times, because good people go at it really fast and they think, 'Oh I've got to do it really fast,' but then they get [it] horribly wrong, because they don't really know what to do. They just guessed everything which is- you can guess because sometimes you might be lucky and win or get the right answer, but sometimes you can guess and rush and won't get anything right. (Timothy and Researcher)

Timothy's comments seem to suggest an understanding that students operate at different paces and that an IBI approach could empower the students with greater control over this pace.

Of the ten students who were interviewed, all ten expressed the view that IBI was empowering in some way (see Table 5.3). This varied between IBI created greater engagement, improved understanding, improved control over the pace, and led to greater self-efficacy. Examples of these have been outlined in this section. However, when looking at the frequency with which students expressed these views there were notable differences between students of differing mindset groups. Students who, at the time of taking the m-ITIS, scored as growth mindset expressed views of IBI as empowering with greater intensity (Boyatzis, 1998) than those who scored as fixed mindset, as measured by the percentage of the interview that was coded to this theme (please see Section 4.10 for a discussion of intensity scoring).

#### 5.4.3.2 IBI as a form of neglect

At some point during the interview process all students expressed the view of IBI as a form of empowerment. However, every student also expressed the contrary impression of IBI as a form of neglect, albeit to different intensities. These views revolved around the lack of teacher support and explanation. Students perceived the absence of these explanations as neglectful and increasing the difficulty of the task. For example:

> If he tells you what to do then you have more of a chance of getting the question right, if you get what I mean. You have more understanding of what you have to do than if he just gave you a hint because maybe you didn't fully understand what you had to do. (James)

In the above excerpt, James expressed his view that the teacher's failure to explain the solution reduced his capacity to understand the task. As discussed earlier in Section 5.4.2.2.1, many students viewed mathematics as a 'doing' subject, often doing something procedural (Kouba & McDonald, 1987). James's comments might have been an expression of this belief and his view that without being told the procedure to 'do' he was not really understanding. A similar view arose during Daniel's interview.

> D: I enjoyed it [L5], but I would have done a different lesson.
>
> R: Why is that?
>
> D: I don't know. Because like say if you didn't understand it then you didn't know what to do.

(Daniel and Researcher)

This excerpt from Daniel could be said to reflect a belief about mathematics similar to James, namely that the purpose of mathematics is to ‘do’ problems. Daniel’s last sentence suggested he saw understanding as a prerequisite for ‘doing’, which is somewhat contrary to the IBI approach in which students explore problems as *means* to develop understanding. It seems Daniel did not believe that understanding of mathematics could arise from exploration.

The most common theme to emerge under the idea of IBI as neglectful was simply that the approach made mathematics ‘harder’. This is typified as:

> Because doing it without knowing what you are doing is hard, and when you know what you are doing it’s easier. (Lisa)

When asked to explain how the IBI lesson made them feel students provided a range of responses. For example, Jackie, who in the previous section on ‘IBI as a form empowerment’ (see Section 5.4.3.1) expressed views of positive engagement, found herself being annoyed.

> Yeah, I got annoyed because I couldn’t do it [L7], because I couldn’t find-- because it was like taking forever. (Jackie)

Jackie also expressed a common belief that mathematics problems can be solved quickly. Whilst Jackie’s choice of the word ‘annoyed’ was a strong expression of her feeling, other students expressed the same sentiment through the choice of phrases such as ‘frustrated’ or ‘confused’. For example:

> I was quite confused [L5]. And I was kind of listening to everyone else’s first and seeing if I could pull it together. But I didn’t really understand it very much, so I was kind of listening to them, and I still didn’t understand it. (Erica)

In Section 5.4.2, I explored how students frequently expressed the belief that the purpose of mathematics is to get the answer, typically by doing the procedure. I heard elements of this belief during some of the interviews. For example:

> Like just explains to you what to do like how to do it and how to get the answer. (Daniel)

Daniel seemed fixated on the procedural aspects of mathematics. His apparent focus on getting the answer quickly could bias him towards a search for the procedure rather than a search for understanding. Given the lack of up-front procedures in an IBI approach, it is possible this contributed to Daniel's negative feelings towards the IBI unit.

In a discussion with Erica about what mathematics lessons she prefers, instruction followed by practice versus exploration followed by instruction, she made the following comment:

> I do like to be told first because my mind processes it better instead of after where I'm like really confused. It takes longer for me to understand it. We normally do it that way with Mr Scott but not those ones. (Erica)

Here Erica suggested that the order of instruction in the IBI unit, with exploration preceding direct instruction was a source of confusion. It seemed her expectation of the teaching of mathematics was two-fold. Firstly, that it should minimise her confusion, and secondly that it should minimise the amount of time it takes for her to understand. Proponents of IBI would argue that confusion minimisation should not be an aim of optimal instruction and that being confused is part of the exploration process. Erica repeated her sentiments later with the comment:

> I don't know, it's just something about knowing what we are doing instead of like being introduced to something new but not learning about it first. (Erica)

Taken together Erica's comments suggest that her preferences are not compatible with IBI, which she described as confusing. Her perception of the IBI unit seems to conflict with how she believes she learns best. Likewise, Lisa also reported feeling neglected by the IBI unit.

> Some of it was confusing, and why would he not explain the topic first, and why would he make us do it first and not explain it to us? It wasn't helpful. (Lisa)

Lisa reported confusion with a tone almost to the point of bewilderment that Mr Scott would not explain the new topic before assigning problems to solve. When asked why she thought Mr Scott made those choices she responded that she did not know.

The desire for explanations (and the apparent lack of them during the IBI unit) was also a feature of several interviews. For example, when discussing how he felt about Mr Scott's IBI approach compared to how he would teach the lesson, Kevin said:

> He [Mr Scott] just didn't explain it enough. I'd teach them so that they understand, would understand, and wouldn't get frustrated, so that I would explain it. And if they didn't get it the first time, I would explain it again and again and again until they got it. (Kevin)

Kevin expressed a desire for Mr Scott to explain material more since he 'didn't explain it enough'. Moreover, Kevin felt the explanation should be persistent until all the students achieve understanding ('until they got it') and avoid feeling 'frustrated'. In this exchange, Kevin did not appoint responsibility for learning to the student. Instead, Kevin seemed to believe that failures to achieve understanding are due to a lack of persistence on the part of the teacher.

In the previous example, Kevin suggested that a lack of explanation during IBI could lead to frustration. This was echoed during Linda's interview, when she made the following point:

> I prefer when we got told what it is and then we have to answer the questions because then you won't be stuck and then people won't get frustrated because you won't be able to answer it. Like in that lesson with the worksheets in the corners of the room [L1], it would be better to know before so I could do them straight away. I prefer that I know how to answer it, so I can do it straight away. (Linda)

In the above comment, Linda suggested that failure to answer the questions can lead to frustration. In addition, Linda communicated that the goal of mathematics is to do it 'straight away'. But, because the IBI approach requires students to explore problems before instruction, Linda was not able to solve the problems 'straight away', which may have resulted in her feelings of frustration and perhaps neglect by the teacher.

Some students took a more balanced view, recognising that the IBI approach may have created opportunities for engagement but failing to make the connection that this engagement, or the lesson in general, might lead to improved understanding.

> It was fun [L7], but I don't really know what you're going to learn from that to be honest. (Kevin)

Many studies have demonstrated that engagement in mathematics improves learning (Fung et al., 2018) and therefore Kevin's recognition that the IBI approach was fun should indicate it created an opportunity for improved understanding. However, if learning did occur it was not at a conscious level for Kevin. A similar comment was made by Daniel during his interview:

> I enjoyed it [L1], but I would have liked for him [Mr Scott] to just have explained it a bit first. I felt a bit … stranded. (Daniel)

Much like I discussed in the previous section, 'IBI as a form empowerment' (see Section 5.4.3.1 and Table 5.3), all ten students interviewed expressed views of IBI as neglect at some point during their interview. Some students expressed this view numerous times, such as Kevin who made seven separate references to this theme, whereas others expressed this view only once, such as Henry. When considering the students' mindset scores, there were again differences between those students who scored as growth mindset when assessed with the m-ITIS as compared to those who scored as fixed mindset (Table 5.3). Students with fixed mindsets expressed views that were associated with 'IBI as a form of neglect' with greater intensity (Boyatzis, 1998) than those with growth mindsets. This is the opposite of what was seen in the theme 'IBI as a form of empowerment'. Please see Section 4.9 for a discussion of intensity scoring.

#### 5.4.3.3 Importance of the teacher

A clear and persistent theme to emerge was around the role of the teacher throughout the IBI unit. Two subthemes emerged around the importance of the teacher in (1) creating engagement and (2) providing support.

The teacher's role in creating engaging experiences was expressed by all students interviewed. For instance, Daniel stated:

> I quite liked the one where we measured the basketball court [L2], because it was something that we were—like it wasn't something I had no idea about. I play basketball with my mates all the time… It's nice when Mr Scott does problems like that with sport.

Daniel's comments appear to suggest he feels best engaged in a mathematics problem when the problem is something to which he can relate, and he attributes this good choice of problem to his teacher. This also aligns with the discussion in Section 5.4.2.2.1 in which students expressed beliefs that mathematics is unrelated to reality but were able to make connections to their real lives throughout the IBI unit. Here Daniel may have looked upon the IBI lesson as more enjoyable because his teacher selected a context that was familiar to him. This suggests Daniel sees the teacher as a key factor in creating engaging content. Another example of this same view came from James.

> Mr Scott is a really good teacher, and he made the lessons interactive. (James)

When discussing how he might run the classroom if he were the teacher Henry made the following comments.

> H: I'd want the students to really get into it. I'd do lessons that let them do that.
>
> R: What would you do to help achieve that?
>
> H: Like give them a chance to get stuck in, not just writing on the board, or doing stuff out of the textbooks. Sometimes going through it on the board is good, but I'd let students have a good go first and try figure it

> out and maybe, if some don't get it, I'd go through it on the board to make sure everyone gets it before moving on.
> (Henry and Researcher)

Henry suggested he understood the role of the teacher to make lessons engaging and to help students 'get into it'. When probed further Henry appeared to describe what sounded like an IBI approach, with students trying to 'figure it out' first with possible direct instruction to follow.

During Linda's interview, she also seemed to express the idea that the teacher made the IBI lessons engaging.

> Like Mr Scott he is good because he's supportive and wants everyone to understand, and he can make it fun at the same time. It's not all textbooks and boring worksheets. (Linda)

Judging by Linda's comment, she seemed to enjoy the IBI unit because of Mr Scott's decision to limit his use of textbooks and worksheets. In her view this made the lesson more fun. In this comment, Linda also brought up another common idea to emerge regarding the role of the teacher, namely the teacher's role to provide support.

Throughout the interviews, many students spoke positively of Mr Scott's role during the IBI unit to support them and how this support was frequently a source of motivation. For instance:

> That's what some teachers do, they just focus on one lesson and then move on. But, this time it's not like that. Mr Scott has focused on it for a week and then moved on, because everyone understood it. (Erica)

Erica explained that Mr Scott performed his role as a teacher well because he adjusted the pace of the lesson throughout the IBI unit and provided enough time for students to understand the topic. Erica saw Mr Scott recognising that each student needs to achieve understanding before the lesson should move forward.

Other students noted Mr Scott's support by having high expectations for his students.

> He [Mr Scott] doesn't give up until you have done it really well, because he doesn't want us to fail and have a really bad mark or anything. (Linda)

In the above comment, Linda recognised that Mr Scott cares about each of his students and he does this by not giving up on them or allowing them to hand in work which does not reflect their best. Linda may see the high expectations of Mr Scott as indicative of his dedication to his students' education and success in mathematics.

Other students noted Mr Scott's support by communicating clear directions.

> Yeah now, Mr Scott, he's much better because Ms Brown like she just gives the work out. She doesn't explain it well. But Mr Scott writes on the board and explains it [the directions] and then we have to do the work. (Daniel)

Here Daniel seemed to communicate a clear preference for Mr Scott's approach. He, like many of his peers, felt that teachers who clearly explain the material before asking the students to 'do the work' were superior. Initially, this seemed to suggest that Daniel preferred a direct approach over an IBI approach, but further questioning revealed Daniel was referring to Mr Scott's clear directions at the start of each class. Frequently Mr Scott clarified to the students the directions for the task without telling them how to solve the problem (e.g. 'Talk with your group', 'Try it out', 'Compare your answers').

Many of the students felt that teachers should provide support to students. In addition to this, some students seemed to suggest that teachers need not just provide support during IBI, but that this support should be equitable. For example, when discussing the IBI unit:

> Yeah, sometimes he'd explain to a certain group like what they had to do. But never really explained to the whole group. (Henry)

My observations of the class suggest that Mr Scott rarely entered into explanation-based discussion with individual groups. When he did engage with a group it was usually to provide prompts to remain on task or to clarify what the students were supposed to be doing.

Therefore, Henry's comments more likely reflect his perception that Mr Scott was helping others but not him. Possibly this, compounded with Henry's confusion regarding the IBI task, may have led to feelings of inequality.

#### 5.4.3.4 Importance of peers

One of the themes which students most frequently expressed during the interviews was the importance of peers throughout the IBI unit. The expression of this theme was not limited to using peer groups as a means to spend time with friends socially. Moreover, the discussions suggested the presence of two sub-themes: (1) peers as a knowledge sharing network and (2) peers as source of enjoyment.

The first sub-theme, peers as a knowledge-sharing network, often revolved around the idea that, if a student is struggling on a task, they can leverage the knowledge of the group.

> Usually asking someone on my table helped because sometimes teacher is seeing someone else. So, I'll ask my friends first what they think. (Erica)

Erica's comment shows how she saw her peers as a source of knowledge throughout the IBI unit, in much the same way as the teacher. When the teacher was busy attending to other students (which was often, given Mr Scott's role as facilitator), Erica looked to her peers for support. When asked whether this had always been the case, Erica explained in the past she was used to working by herself in maths.

Occasionally students shared that the IBI unit provided them with an opportunity to share knowledge when they believed their peers could benefit.

> I don't know, I find it better to work with someone so if someone gets it wrong you can try and help them, and if you get it wrong someone can try and help you. (Henry)

In this excerpt Henry expressed how he saw opportunities for a two-way sharing of knowledge when exploring the IBI problems. The IBI unit had a strong emphasis on small group and class discussion, which may have created opportunities for Henry to see the benefit of exploring problems with his peers. In a separate interview Jackie expressed a similar idea:

> It was good [L1] because other people were figuring it out like you were and you could help. And if you got stuck you could ask them, because they were doing the same thing as you and they might have been stuck as well. So, combining your ideas with someone is really good because you're sort of doubling the learning by your knowledge and their knowledge. (Jackie)

Here Jackie expressed a similar view to Henry regarding the bi-directional nature of learning when exploring problems in a collaborative way. This speaks to the social interdependence which collaborative learning researchers suggest is crucial to effective group work (D. Johnson & Johnson, 2016). In addition, Jackie seems to be making a tangible connection to her own learning being enhanced as a result ('doubling the learning'). Daniel expressed a similar view.

> Well I found you can work together and try and work it out instead of one by one, because it makes it harder for yourself. (Daniel)

In addition to peers providing additional avenues to developing understanding, there was also evidence that students felt a sense of reassurance when their peers were struggling with a problem in the same way they were. When discussing which aspect of the IBI unit she liked Michelle said:

> It's better that way, because if they don't know it either then it sort of makes you feel good. Well not good as in happy, but you don't feel as bad that you are struggling. And you can both sort of try stuff. Even though you both don't know what you are doing you can talk it out. (Michelle)

Here Michelle expressed that she felt reassured by the apparent failings of her peers. Perhaps by seeing her peers struggle with the problems this challenged any assumptions she may have held that mathematics is an inherent skill which some people have, and some people lack. This might be one reason to explain why Michelle felt she was able to work with an equally confused peer to explore the problems in L1 and discuss possible solutions ('talk it out'). When Michelle was questioned further, she said:

> Because we had to try and like work with a team [L1] but then if someone didn't really agree with your answer then it would just – you just have to try and battle to see who was really right and who was wrong. (Michelle)

Michelle appeared to suggest that when students within a collaborative IBI setting disagree a mathematical debate may ensue. This would seem to align with the views of D. Johnson and Johnson (2016) who suggest that during inquiry activities students tend to move towards a general consensus around a solution. These sorts of debates, or 'battles' are a strong opportunity for students to develop deeper conceptual understanding of the mathematical constructs.

Several students suggested that peers could be an important source of creativity. A good example of this sentiment was expressed by Jackie during her interview:

> If you're stuck or like don't know what to do next, then you can ask your mates and they might know a way. Or maybe you think your way is best, but by chatting with your mates they may look at it in a better maybe more creative way. So, you can have a better understanding. (Jackie)

Jackie expressed that she valued her peers as a source of alternative ways to tackle the IBI problems. From her perspective, these approaches were creative and by exploring them together her understanding improved. It is also possible that Jackie's statement reflects an element of 'free-riding' as discussed earlier, in which Jackie need not engage in an exhaustive search for alternate approaches, since her peer network can be relied upon to do this for her.

The second subtheme after peers as a knowledge sharing network was peers as source of enjoyment. Here students viewed IBI as more fun when working in groups. For example:

> Doing the volume one that was fun because you got to engage with other people who you wouldn't normally engage with, so you see it from different

> perspectives. And you got to do questions with them as well. So, you're still working but you're having fun whilst working. (Jackie)

Here Jackie expressed two views. Firstly, the IBI unit allowed her to interact with peers beyond those with whom she normally interacts, and perhaps this allowed her to access networks with more varied approaches ('see it from different perspectives'). Secondly, the peer interactions during the IBI exploration are enjoyable ('having fun whilst working'). It is possible this enjoyment created greater engagement and discussion in the mathematical concepts being investigated.

In explaining his preference for the IBI unit during his interview, Timothy expressed a view that he enjoyed the opportunity to work with peers. When pushed to give more clarity he made the following statement:

> Because when I'm on my own sometimes I don't know, so I just like sit there and like panic. Well not panic, but like guess what to do and normally I'll get it wrong or not even bother. But I like working with my friends because they know what to do and then it's just fun working with them. (Timothy)

There were frequent instances during the observations in which students were disengaged with the task, often placing their heads on the desk or engaging in off-topic conversations. When challenged these students would typically explain they were confused about the problem and didn't know what to do. It is therefore interesting to hear Timothy's response above which suggested that, during the IBI unit he viewed his peers as a source of motivation despite his apparent lack of understanding of the problem.

Across both subthemes references to working with peers in IBI were broadly positive and referenced the enhancing effect of peer interactions on the student's perceptions of the IBI lessons. Occasionally, however, students would indicate a preference to work alone or an issue with working in a group. When asked how she felt about exploring novel mathematics problems with peers Jackie said:

> It kind of depends on who you're with. So, like, not pointing the finger at anyone but James, he is quite annoying sometimes. And if you're in a group

> with him then he doesn't really shut up. And you're trying to work and that just gets annoying. But then if you're with someone like Linda then she'll get on with it, but she'll have a laugh at the same time. (Jackie)

Here Jackie shared how working with a group during the IBI unit could be a source of distraction (or even annoyance) at times. Students are individuals, and therefore, it is natural that some group compositions will work better than others. Groups with poor interaction (such as the one Jackie described) may hinder learning. Perhaps, James himself put it best when he said:

> I prefer sometimes to work on my own than with a group, and sometimes I prefer to work in a group than on my own. It's like 50-50. (James)

There appeared to be no difference between how students expressed the views of peer importance across the different mindsets.

#### 5.4.3.5 Mathematics disaffection

A final theme that emerged from the analysis of the interviews was the idea of mathematics disaffection. All ten students expressed views of mathematics disaffection. Although it is worth noting that these disaffection comments were sometimes directed towards mathematics in general rather than the IBI unit. Most comments referenced a general sense of disaffection with the way mathematics is taught.

> They [mathematics lessons] were just really boring. You just sit there in the back of the classroom writing down questions from the sheet and answering them. (Henry)

Henry's comment that mathematics is boring seemed to be aimed at two aspects of how he sees mathematics teaching. Firstly, that mathematics is mostly seatwork and that mathematics is mostly worksheet based. Prior to commencing the IBI unit, the students within Mr Scott's class had little exposure to IBI in mathematics. The relatively short duration of the IBI unit apparently did not change Henry' expressed views towards mathematics. Figure 5.7 graphically represents the words the students used when asked to describe mathematics in three words, with the size of the word representing its frequency of occurrence across the interviews. This figure shows the general trend toward disaffection. This view was similarly expressed during Timothy's interview.

> I don't like it. Honestly, I think it's boring. (Timothy)


Figure 5.7: 'What three words would you use to describe maths?' at Harrison School[11]

On the one hand, disaffection can result in a lack of engagement and missed opportunities to learn. In more extreme cases, disaffection may cause students to abstain from their mathematics lessons altogether.

> At my primary school I kept on skipping maths classes and I kept on walking out of them. (Jackie)

When addressing the question of how he felt about mathematics Kevin made the following comment.

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[11] Generated using https://www.wordclouds.com/

> It's alright. It's like when I have got maths next it's not like it's a bad thing. It's just normal. When it's with Ms Brown it's quite bad, but Mr Scott is alright. (Kevin)

This comment, which was similar to those made by others, suggests that disaffection towards mathematics might be amplified by disaffection towards certain mathematics teachers. The different teaching approaches of different teachers is known to play a meaningful role in students' overall attitudes towards mathematics (Boaler, 2002; Boaler & Greeno, 2000; De Corte et al., 2010). Jackie also touched on the idea of teachers having an influence over how enjoyable mathematics is.

> The teachers that I've had they've made the work fun, but you still learn. So that people do actually enjoy maths, because it's not a very enjoyed subject. (Jackie)

Here, Jackie expressed a societal view that mathematics is 'not a very enjoyed subject' but that it is possible to enjoy maths if the teacher makes the work fun.

When expressing views of mathematics, the predominant opinion was not *always* one of disaffection. In some interviews, students expressed the contrary opinion that mathematics could be enjoyable, indicating mathematics *affection*. However, unlike the comments around disaffection, those expressing affection tended to isolate their comments to small instances, such as a particular lesson or topic. This suggests that students may see pockets of enjoyment within the broader IBI unit, whilst still retaining a view of disaffection towards mathematics in general. Examples include:

> Like we did that lesson the other day with Mr Scott. I really enjoyed that. I just got that straight away. Some lessons, like fractions, I just struggle with. (Erica)

> In secondary school you get more into the aspect of algebra and the more complicated stuff. That's quite interesting. (Henry)

In the case of Erica her comments suggested a very binary point of view between affection for those topics which she can do, and disaffection for those in which she struggles. Henry, however, shows affection for areas that are complicated, suggesting his feelings of disaffection might arise from material that is not complicated enough.

Throughout the interviews, students also spoke about the value of mathematics. The below comments by Henry and Linda typify the way in which these sentiments were expressed.

> It's not the funnest subject, but it's a subject which you need to learn. Like to get on in life. (Henry)

> I think it's a very strong subject, like you need it in your life on an everyday basis. But it can be quite hard. (Linda)

Henry's comment suggested he does not find mathematics fun, and Linda's suggested she finds it hard. However, both students seem to acknowledge the value of mathematics in their future lives. This is reminiscent of the utilitarian view held by participants in Nardi and Steward's (2003) study of mathematics disaffection.

#### 5.4.3.6 Summary of student perceptions

Students' perceptions of the IBI unit were varied across five main themes: inquiry as a form of empowerment, inquiry as a form of neglect, importance of the teacher, importance of peers, and mathematics disaffection. Notably, students' views of inquiry were somewhat conflicted between feelings of empowerment and feelings of neglect. On the one hand, the inquiry approach was empowering. Without a prescribed method for solving problems students felt open to trying out different methods, or 'having a go', and this challenge was engaging. On the other hand, students felt neglected by their teacher's reluctance to present explanations up-front. Absent a prescribed method, students felt 'stranded' and at times 'frustrated'. For all students, though, an appreciation for their peers was evident. And despite Mr Scott's decentralised role as facilitator, students still expressed appreciation for his role in designing engaging activities and holding high expectations for them. For all students, however, mathematics disaffection was evident.

### 5.4.4 An analysis of student learning outcomes

The pre-test and post-test results were analysed to determine the extent to which students' knowledge had improved (see Table 5.4). Overall the students' scores improved from 5.8 to 6.3 out of a maximum possible score of 12. The procedural and conceptual scores improved by a similar amount (8.7 percent and 8.6 percent respectively).

Table 5.4: Pre- and post-test results for Mr Scott's class

| | Test component | Max | Pre-test *M* | Pre-test *SD* | Post-test *M* | Post-test *SD* |
|---|---|---|---|---|---|---|
| All students (*n* = 18) | Overall | 12 | **5.8** | 1.9 | **6.3** | 1.5 |
| | Procedural | 6 | **2.3** | 0.8 | **2.5** | 0.8 |
| | Conceptual | 6 | **3.5** | 1.7 | **3.8** | 1.2 |
| Fixed mindset (*n* = 8) | Overall | 12 | **5.5** | 2.2 | **5.8** | 1.2 |
| | Procedural | 6 | **2.3** | 1.0 | **2.5** | 0.9 |
| | Conceptual | 6 | **3.3** | 2.1 | **3.3** | 0.9 |
| Mixed mindset (*n* = 4) | Overall | 12 | **5.9** | 1.6 | **7.0** | 0.8 |
| | Procedural | 6 | **2.3** | 0.5 | **2.3** | 0.5 |
| | Conceptual | 6 | **3.6** | 1.3 | **4.8** | 1.0 |
| Growth mindset (*n* = 6) | Overall | 12 | **6** | 2.0 | **6.5** | 2.1 |
| | Procedural | 6 | **2.3** | 0.8 | **2.8** | 0.8 |
| | Conceptual | 6 | **3.7** | 1.6 | **3.8** | 1.3 |

Separating the results by maths mindset shows that students with growth, fixed, and mixed mindsets all made improvements during the intervention. Students with growth mindsets slightly outperformed their peers with fixed mindsets on the post-test (see Table 5.5).

Looking across test items from the pre-test to post-test suggests some patterns in how students' understanding of the unit topic changed (see Appendix F for a copy of the test items). Since the pre-test and the post-test were composed of identical questions it is possible that any observed changes are due to a practice effect (Lezak et al., 2012). However, this is thought to be unlikely since over a month elapsed between each assessment.

Table 5.5: Growth and fixed post-test results for Mr Scott's class

| | Test component | Max | Fixed $n = 8$ | | Growth $n = 6$ | |
|---|---|---|---|---|---|---|
| | | | *M* | *SD* | *M* | *SD* |
| Post-test | Overall | 12 | **5.75** | 1.2 | **6.5** | 2.1 |
| | Procedural | 6 | **2.5** | 0.9 | **2.8** | 0.8 |
| | Conceptual | 6 | **3.3** | 0.9 | **3.8** | 1.3 |

An increase from pre-test (*M*=0, *SD*=0) to post-test (*M*=.41, *SD*=.49) was observed on test item 2a, which measured procedural calculation of volume. This is noteworthy because students only spent about ten minutes during L7 specifically practicing this procedure. The majority of the class time was spent in the 'explore' phase of inquiry with students measuring and estimating how many water bottles it would take to fill up their classroom. When it came time to discuss how to calculate or estimate the volume of the classroom (i.e. the 'explain' phase) the discussion was cut short by a student's early venture that the correct method was 'to multiply'. Rather than probing this student's thinking further or opening up a class discussion about why this procedure might be effective, Mr Scott swiftly accepted the answer and passed out a worksheet for students to practice. The students then spent the remainder of the class multiplying the three dimensions of given cuboids to find their volumes. It is unusual that this alone could have resulted in such a boost in the students' scores. The 'explain' phase was limited, and the 'explore' phase provided few explicit connections to the big idea. The increase in score might simply be because L7 was the last lesson of the unit and therefore the freshest in the students' memories. Future research may wish to incorporate the use of a delayed post-test to address potential recall bias.

By contrast, the improved performance observed in calculating the volume of a rectangle on item 2b fell away on similar test item 4a, possibly due to the given rectangular prism being presented in a different way. Rather than a line drawing of a rectangular prism with a given length, width, and height, item 4a presented students with a drawing of a prism constructed using unit blocks with no labelled dimensions. It is possible the students' difficulties with this problem shows not a lack of procedural knowledge but rather a lack of knowledge transfer. This explanation seems more plausible when taken together with test item 4c, which showed no change in students' conceptual understanding of volume.

The most consistent improvement observed from pre- to post-test was students' conceptual understanding of area. Every test item of conceptual understanding of area saw an increase. This is compatible with observations made of the students throughout the unit. In the beginning, students described area as merely an operation (e.g. 'to multiply') however by the end of the unit their description of area was more conceptual and varied, e.g. area as the amount of space inside a shape, or area as the number of squares that cover a surface.

Test items which covered surface area saw either no change or a decrease in accuracy. This is unsurprising since students spent relatively little time on this concept. The most common error was for students to confuse calculating surface area with calculating volume, possibly because both measures pertain to rectangular prisms. Perhaps with more time students could have increased their scores in this content area as well.

Neither mindset nor any of the constructs measured by the ATMI (enjoyment, motivation, self-confidence, value) were strongly correlated with the students' post-test scores. Please see Table 5.6 for the results of this analysis.

Table 5.6: Spearman's rank correlation for post-test, mindset, and ATMI

| | Post-test $n = 18$ |
|---|---|
| | $r_s$ |
| Maths mindset | **.33** |
| Enjoyment | **.05** |
| Motivation | **.08** |
| Self-confidence | **-.04** |
| Value | **.20** |

## 5.5 Summary of the Case of Mr Scott's Class

Analysis of the implementation of the IBI unit was conducted using the EQUIP rubric. The scores assigned to each factor of the rubric (instructional, discourse, assessment and curriculum) indicate the teacher was successfully able to implement an IBI unit for students with MD. The unit as a whole met the level of proficient inquiry.

The students' beliefs towards mathematics (as evidenced by the lesson observations) were analysed using McLeod's (1992) framework. It was observed that many of the students held the belief that mathematics is a procedural activity. This was often expressed as mathematics is something one 'does'. As such, students unduly focused on obtaining the formula or the procedure needed rather than on developing their understandings of the problem. This belief conflicts with the principles of IBI which may have led to task disengagement.

Many students appeared to have low self-efficacy in their mathematics abilities. These students disengaged from student-led investigations claiming they were not good at the task or attempting to outsource the activity to a more knowledgeable peer. Often this low self-efficacy would be directed towards problems that challenged students' conceptual understanding of the problem, whereas problems of procedural application did not elicit similar self-efficacy views. Similarly, many students expressed fixed mindset beliefs such as mathematics is not their 'thing', or their brain does not 'work that way'. These students frequently disengaged from the investigations and as such struggled to gain much from the exploratory component of the IBI lessons.

Beliefs about how mathematics should be taught appeared to be two-fold. Firstly, students felt that learning mathematics is best done by solving many short problems rather than one long one. Secondly, students believed that the teacher was the primary transmitter of knowledge, with the students serving as recipients. IBI challenged both these beliefs and, as such, some students struggled with the shift in responsibility. Conversely, several students felt that the IBI approach allowed them to develop greater knowledge, or 'really understand [the problem]', which aligns with the views of IBI proponents.

Students' beliefs about the social context also appeared to interact with the IBI unit. Students appeared to hold the belief that social interactions could be a source of mathematical knowledge, and this may have led students to debate each other as well as deploy democratic approaches to working together. These sorts of interactions can help students develop a deeper understanding of mathematics (Jaworski, 2006). Students were also aware of their position within the attainment-based groups of the school. This awareness may have driven students to behave in ways that were consistent with their understanding of the social norms for this group, e.g. the belief that students in lower sets of mathematics do not enjoy mathematics.

Analysis of the interview transcripts suggested that the students in Mr Scott's class perceived the IBI unit according to five themes. These were:

1. IBI as a form of empowerment
   Students felt that IBI provided them with interesting challenges and forced them to think deeply. They felt the lessons were more engaging and encouraged students to 'have a go'.
2. IBI as a form of neglect
   Students felt the lessons were frustrating and confusing, and believed it was the teacher's responsibility to explain content before trying it on their own.
3. Importance of the teacher
   Students felt the teacher took responsibility for (1) creating engagement and (2) providing support.
4. Importance of peers
   Students felt that peers played an important and positive role in their learning during the IBI unit. These feelings tended to fall into two sub-themes: (1) peers as a knowledge sharing network; and (2) peers as a source of enjoyment.
5. Mathematics disaffection
   Students expressed a dislike for mathematics as 'boring' or 'hard'. Additional views of mathematics as a 'necessary evil' were common.

Interestingly, the intensity of the theme IBI as empowerment was substantially greater in those students with growth mindsets (see Section 4.10 for a discussion of intensity scoring and Table 5.3 for the results). Likewise, the intensity of the theme IBI as a form of neglect was greater in those students with fixed mindsets than those with growth mindsets.

Students' beliefs about their intelligence were measured on both a general as well as a maths-specific basis. The results of these measurements align with the theory of mindset in which students may hold domain specific beliefs (e.g. a student may be growth mindset generally but fixed mindset in mathematics), as discussed in Section 2.9.

Despite the differences in the perceptions of IBI for students with differing mindsets, the actual learning outcomes as measured by the pre- and post-test suggest little difference in learning between the two groups in this case. Students with growth mindsets scored only slightly higher than those with fixed mindsets on the post-test.

When separating learning between procedural and conceptual, as set out in Section 4.5, both measures increased by similar amounts. This would seem to contradict other studies, which suggested that conceptual understanding would increase to a greater extent than procedural knowledge. In addition, separating the students by mindset did not provide any evidence that students with growth mindsets outperformed those with fixed mindsets on either conceptual understanding or procedural knowledge.

# 6 The Case of Ms Silver's Class

In this chapter I report the results of a case study in which a group of secondary school students with mathematics difficulties (MD) were taught eight inquiry-based lessons (hereon referred to as the IBI unit). All students were observed throughout the unit and 12 students were interviewed. Students completed a pre-test and post-test designed to measure any change in their understanding of linear relationships as well as three questionnaires designed to evaluate their attitudes towards mathematics (ATMI) and their implicit theories of intelligence (ITIS and m-ITIS). For more details about the methods used in this study, please refer to Chapter 4.

The study was designed to explore two research questions: (RQ1) How do students with mathematics difficulties perceive IBI? and (RQ2) Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties?

## 6.1 The setting

The study took place at Stratham College[12], a U.K. secondary school, in the spring term of 2018. This school was chosen after I sent an email to its Head of Mathematics[13] and they expressed their interest in participating. After meeting with several interested teachers to explain what the case study would entail, Mary Silver volunteered to take part. She explained she had recently completed her master's thesis on the topic of mindset and was keen to further her studies in this area. Several months before the full case study commenced, Ms Silver participated in a pilot study which helped her to gain familiarity with the selected methods.

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[12] All names, including school names, teacher names, and student names, have been changed to protect the identity of those involved in the study.

[13] The email address for the Head of Mathematics was found on their school's online staff directory.

A recent Ofsted report described Stratham College as a ‘good’ comprehensive secondary school and commended the school’s staff for their strong planning, content knowledge, and behaviour management.

### 6.1.1 The teacher

Ms Silver served as a mathematics teacher at Stratham College throughout the duration of the case study. Before joining the faculty, Ms Silver received her master’s degree in Mathematics Education from a respected university. Some years prior to that she earned her undergraduate degree in Mathematics. The case study was conducted during Ms Silver’s first year teaching at Stratham College and second year teaching overall.

Ms Silver described herself as an enthusiastic teacher who holds high expectations for her students. Having completed her master’s dissertation on the topic of mindset, Ms Silver reported that she taught her students about the importance of a growth mindset at the beginning of the school year. She self-reported that she emphasised to them the importance of perseverance and ‘having a go’ in the face of a challenge. Ms Silver said that she continued giving the students positive mindset messages throughout the year.

### 6.1.2 The class

Stratham College sets students into three sets for mathematics based on a combination of factors including Cognitive Abilities Test (CAT) results, Key Stage 2 results, and mathematics teacher recommendation. The three sets were called top, middle, and bottom.

At the time of the case study, Ms Silver taught nine classes which spanned all sets and ranged from year 7 to year 11. In selecting the appropriate class, Ms Silver and I initially reviewed her middle and bottom sets, however upon learning that her bottom sets included a large majority of students with special educational needs (e.g. dyslexia, social emotional disorders) we decided to narrow our selection to only her parallel middle sets.

Middle sets at this school included students from a wide range of prior attainment. For each class, I was able to review their Fischer Family Trust Band (FFTB), which is a type of prior attainment measure based on the students’ Key Stage 2 exam results. A FFTB of ‘higher’ indicates a score in the top third of all students in the U.K. who took the exam the same year.

Likewise, ‘middle’ indicates a score in the middle third, and ‘lower’ indicates a score in the lower third. Upon reviewing the FFTB indicators in Ms Silver’s two middle sets, about half the students were indicated as ‘lower.’ In other words, half of the students in the middle sets had Key Stage 2 results that were among the lowest third of all students who took the exam that year in the United Kingdom. For this reason, about half of the students in the middle sets could be described as having mathematics difficulties (MD) according to the definition used in this study. As described in Section 3.3, previous studies of MD have used a similar approach, for example by selecting students at the 25th percentile or below on a national assessment.

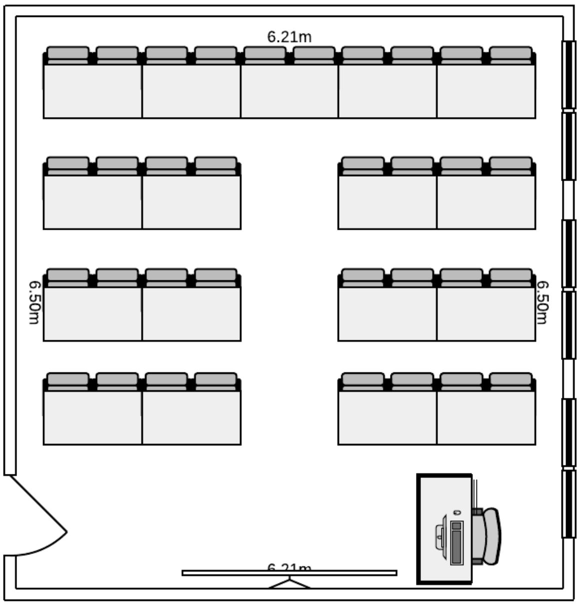


Figure 6.1: Ms Silver’s classroom arrangement

In addition to incidence of disability and FFTB, Ms Silver and I also considered her overall schedule. Ms Silver’s year 8 middle set class was selected for the case study. There were 30 students in the class ranging in age from 11 to 12. The class included 18 students in the ‘lower’ FFTB, nine students in the ‘middle’ FFTB, and three students in the ‘higher’ FFTB.

Of the 30 students, 21 were female and nine were male. No specific reason was given to explain why the majority of the class was female. Ms Silver shared with me that four students qualified for free school meals and six for the pupil premium (an indicator of SES). On reviewing the school records for this class, it was identified that two students had Special Educational Needs (SEN) indicators, one with a moderate learning difficulty and another labelled as social, emotional, and mental health.

Ms Silver taught this class twice a week on Wednesday mornings and Friday afternoons. Each lesson lasted for 50 minutes. Following each Wednesday and Friday session Ms Silver and I discussed and reflected on the IBI lessons and student progress. We also exchanged emails before and after sessions to do additional planning.

The classroom contained a Promethean ActivPanel[14] with small whiteboards on either side of it, table-desks, and several motivational posters. The room had one full wall of windows. Some parts of the room appeared in need of repair. For instance, when it rained water leaked from the ceiling onto the students' desks. Student desks were arranged in rows. The total student capacity of the room was 34 (see Figure 6.1).

### 6.1.3 Voluntary informed consent

I introduced myself to the class in February 2018, explaining that I was a PhD researcher from the University of Cambridge and that I had previously been a mathematics teacher in the United States. I distributed and read the information sheet to the class which outlined the purpose of the study as well as what would be involved (see Appendix E for a copy of the consent materials). Students were asked to review the information sheet with a parent or guardian and then return the signed consent form indicating their willingness to participate in the study and interview by the following week.

Having introduced myself, I stayed for the remainder of the lesson to answer any questions and make some preliminary observations. I noted the flow of the lesson led by Ms Silver and some apparent behaviour difficulties.

Before the start of the study, all students returned the consent forms signed and indicated their wish to be included in the study. Nineteen students wished to be included in the interview process, with the remaining 11 wishing to be excluded.

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[14] A Promethean ActivPanel is an interactive display designed for use in classrooms (www.prometheanworld.com).

## 6.2 Lesson development

Ms Silver and I met several times before the start of the case study to plan the IBI unit. At Stratham College the mathematics teachers had developed their own curriculum aligned to the National Curriculum. According to the school's year 8 scheme of work, the selected class Ms Silver and I chose were scheduled to learn about the topic of linear relationships during the month in which the case study was to take place.

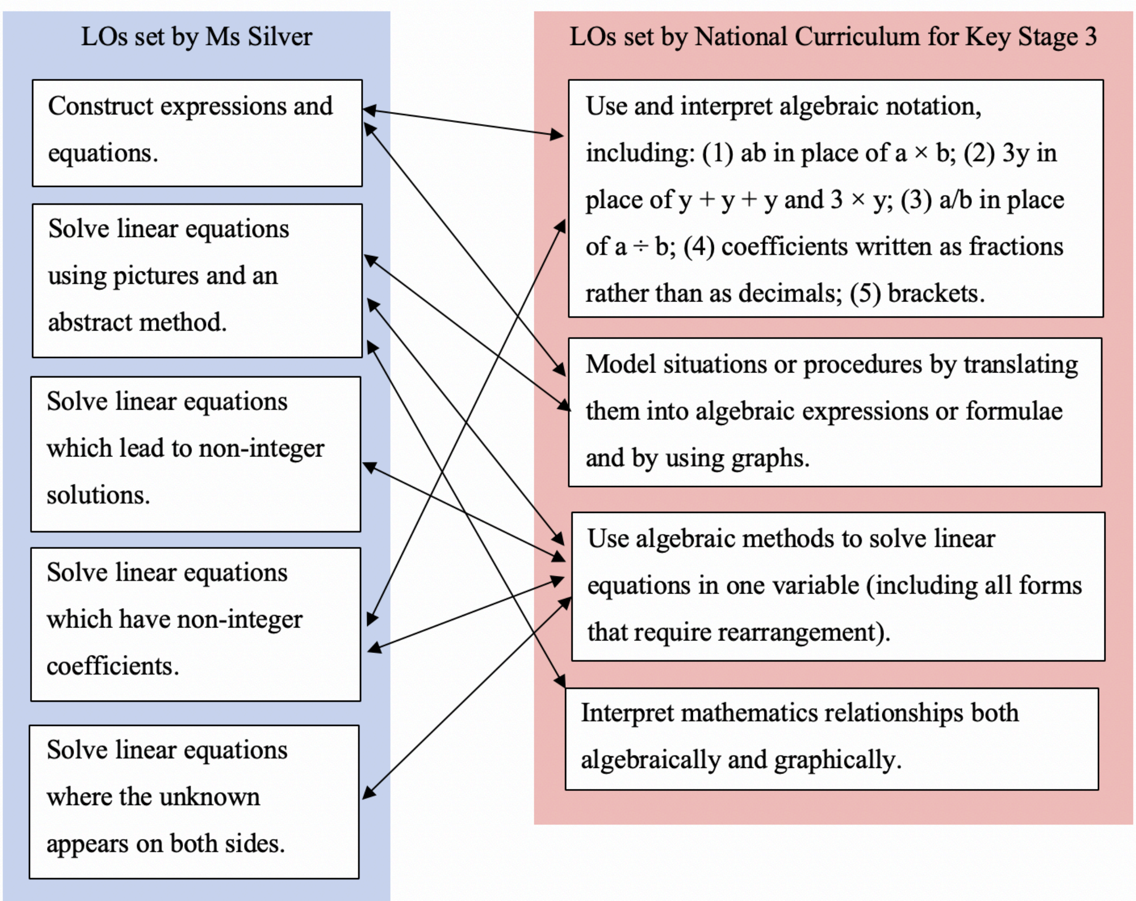


Figure 6.2: Mapping Ms Silver's LO's to the National Curriculum

To begin planning, Ms Silver selected a set of learning objectives (LOs) for her students. The learning objectives were that students should be able to: (1) construct expressions and equations; (2) solve linear equations using pictures and an abstract method; (3) solve linear equations which lead to non-integer solutions; (4) solve linear equations which have non-integer coefficients; and (5) solve linear equations where the unknown appears on both sides. How each of these learning objectives align to the National Curriculum is shown in Figure 6.2.

Ms Silver and I reviewed each lesson plan about one week in advance to ensure each complied with a high level of inquiry. This was done with reference to the Electronic Quality of Inquiry Protocol (EQUIP; Section 4.3). As mentioned above, Ms Silver had participated in a pilot study the previous school year, so she was already familiar with the EQUIP.

Ms Silver's teaching style might be best described as demonstrative. Based on previous observations, it seemed to me that she preferred leading lessons in which she had control and was the primary demonstrator in the classroom. As such, I anticipated that Ms Silver would need additional support in facilitating classroom discussions throughout the IBI unit.

### 6.2.1 Pre-test and post-test development

Having selected the topic of linear relationships for the IBI unit, Ms Silver and I then constructed the pre-test and post-test (see Section 4.5). Ms Silver and I selected test items from previous National Curriculum assessments (known as 'SATs') which had been aligned to the Key Stage 3 programme of study. To assess conceptual knowledge, 12 questions were identified, and to assess procedural knowledge, 21 questions were identified. Please see Appendix G for a copy of the pre- and post-test. As previously described (Section 4.5) the test items on the pre- and post-test were identical.

Both the pre-test and post-test were administered under exam conditions within the students' normal mathematics classroom. Each assessment took the students approximately forty minutes to complete. The pre-test was administered one week before the first IBI lesson. The post-test was administered one week following the last IBI lesson (i.e. six weeks later).

### 6.2.2 ATMI, ITIS, and m-ITIS

The Attitudes Towards Mathematics Inventory (ATMI), the Implicit Theories of Intelligence Scale (ITIS), and the modified Implicit Theories of Intelligence Scale (m-ITIS) were given one week before the start of the IBI unit, however on a different day to the pre-test (see Section 4.4 for more details on the ATMI, ITIS, and m-ITIS). Students completed these questionnaires individually in a computer lab using the online platform SurveyMonkey. Ms Silver supervised the students while they completed the questionnaires.

### 6.2.3 Observation protocol

To ensure detailed observations, I took handwritten notes in addition to audio recordings and video recordings of each lesson. As described in Section 4.6, my notes were both descriptive and reflective (Creswell & Poth, 2017; see Appendix D for an example of my observation notes). The main purpose of my written notes was to capture observations that would facilitate subsequent interview sessions as well provide a record of the lesson's flow for future documentation and analysis. Finally, I used a video camera set up at the back of the classroom (farthest from the white board) to record the lesson. This video aided my post-lesson reflections as well as development of tailored interview questions and later analysis.

### 6.2.4 EQUIP

The EQUIP was used to assess the level of inquiry in each lesson on a scale of 1 to 4: (1) 'pre-enquiry', (2) 'developing enquiry', (3) 'proficient enquiry', and (4) 'exemplary enquiry' (please see Appendix A for copy of the rubric and Section 4.3 for a discussion of its use). One of my goals was to ensure that the majority of the lessons of the IBI unit met or exceeded the criteria for 'proficient'. To this end, Ms Silver and I reviewed the rubric before each lesson was developed and discussed ways in which to achieve a high level of inquiry. In addition, I observed Ms Silver teach an IBI lesson before the intervention began. Following this lesson, we debriefed to rate the lesson as per the EQUIP and discuss improvement areas. The principal feedback from this practice lesson was to provide the students more time to explore the IBI problem and to allow for discussion of the problem afterward.

### 6.2.5 Interview protocol

All interviews were conducted at Stratham College during the school day. A small meeting room next to the Headmaster's office was reserved for the interviews. Since only a few interviews were able to take place during Ms Silver's lesson, most interviews took place during one of the students' alternative maths lessons. Occasionally it was not possible to schedule the interview during any of the students' mathematics lessons, so some students missed lessons outside of mathematics (e.g. art). In every instance I was given permission from the class teacher for the student to miss approximately 30 minutes of the lesson that day in order to be interviewed. Student consent to miss class was also obtained in order to partake in the interview. Scheduling conflicts made it impracticable for students to avoid missing some class time during the interviews.

Nineteen students consented for an interview. I excluded seven students since their FFTB was not 'lower' and therefore did not meet my criteria for MD. As a result, 12 students were interviewed. I began each interview by reminding the student of the purpose of the study as well as their right to skip questions or end the interview at any time at their request. I also confirmed that they still consented to having the interview recorded. The interview followed a semi-structured approach, with the five main topics to cover being: (1) feelings about mathematics, (2) perceptions of the IBI lessons, (3) impressions of teaching in IBI, (4) self-reported effectiveness of IBI on learning, and (5) handling impasses.

### 6.2.6 Other data collection

In addition to the questionnaire, pre-test, post-test, observations, and interviews, I also collected the students' worksheets to supplement my lesson observations as well as stimulate the students' recall of the IBI lessons when being interviewed.

## 6.3 Overview of the IBI lessons

Several IBI tasks were chosen that aligned to the learning objectives (see Table 6.1). The problems were selected from a variety of sources including curricular websites and textbooks. Each problem is presented in the table below alongside the lesson number in which the task appeared (L1 stands for Lesson 1, L2 stands for Lesson 2, and so on). Each lesson's primary learning objective is also indicated. Each problem took approximately one lesson to explore and then discuss, with the exception of the Henri and Emile problem which took two lessons (L5 and L6).

Table 6.1: Overview of the eight IBI lessons at Stratham College

| | Learning Objective | IBI Task |
|---|---|---|
| L1 | To solve linear equations using pictures and an abstract method. | **Laila and Julius problem**<br>Laila tells Julius to pick a number between one and ten. 'Add three to your number and multiply the sum by five,' she tells him. Next, she says, 'Now take that number and subtract seven from it and tell me the new number.' 'Twenty-three!' Julius exclaims. |

Table 6.1 (continued)

| | | |
|---|---|---|
| | | a. Write an expression that records the operations that Julius used.<br>b. What was Julius' original number?<br>c. In the next round Laila is supposed to pick a number between 1 and 10 and follow the same instructions. She gives her final result as 108. Julius immediately replies: 'Hey, you cheated!' What might he mean?<br>(Illustrative Mathematics, n.d.-a) |
| L2 | To construct expressions and equations. | **Expressions problem**<br>Write an expression for the sequence of operations.<br>a. Add 3 to *x*, subtract the result from 1, then double what you have.<br>b. Add 3 to *x*, double what you have, then subtract 1 from the result.<br>(Illustrative Mathematics, n.d.-b) |
| L3 | To solve linear equations which lead to non-integer solutions. | **Fibonacci problem**<br>A certain man proceeded to Lucca on business to make a profit, doubled his money, and he spent there 12 denari. He then left and went through Florence; he there doubled his money, and spent 12 denari. Then he returned to Pisa, doubled his money, spent 12 denari, and it is proposed that he had nothing left. It is sought how much he had at the beginning.<br>(Sigler, 2002) |
| L4 | To solve linear equations which have non-integer coefficients. | **Pizza problem**<br>Below are the prices for a medium 2-topping pizza and a medium 4-topping pizza from Domino's.<br>**Medium (12") Hand Tossed Pizza**<br>**Whole: Pepperoni, Green Peppers**<br>**£16.79**<br>**Medium (12") Hand Tossed Pizza**<br>**Whole: Ham, Chicken, Mushrooms, Green Peppers**<br>**£19.59** |

Table 6.1 (continued)

| | | |
|---|---|---|
| | | a. How much do you think Domino's is charging for each topping, and how much would you expect to pay for a plain cheese pizza with no toppings?<br>b. Write an equation you could use to determine the price of a pizza for a given number of toppings.<br>c. If you ordered your favourite medium pizza, how much would you expect to spend?<br>d. If you had £20 to spend on a medium pizza, how many toppings could you get?<br>(Mathalicious, n.d.) |
| L5 and L6 | To solve linear equations which have non-integer coefficients. | **Henri and Emile problem**<br>In Ms Chang's class, Emile found out that his walking rate is 2 meters per second. That is, Emile walks 2 meters every 1 second. When he gets home from school, he times his little brother Henri as Henri walks 100 meters. He figures out that Henri's walking rate is 1 meter per second. Henri walks 1 meter every second.<br><br>Henri challenges Emile to a walking race. Because Emile's walking rate is faster, Emile gives Henri a 45-meter head start. Emile knows his brother would enjoy winning the race, but he does not want to make the race so short that it is obvious his brother will win.<br><br>How long should the race be so that Henri will win in a close race?<br>(Pearson Connected Mathematics 3, n.d.) |
| L7 | To solve linear equations where the unknown appears on both sides. | **Ichiro problem**<br>It has been one month since Ichiro's mother entered the hospital. He has decided to give a prayer with his small brother at a local temple every morning so that she will be well soon. |

Table 6.1 (continued)

| | | |
|---|---|---|
| | | There are 18 ten-yen coins in Ichiro's wallet and just 22 five-yen coins in his smaller brother's wallet. They have decided every time to take one coin from each of them and put them in the offertory box and continue the prayer up until either wallet becomes empty. One day after they were done with their prayer, when they looked into each other's wallet the smaller brother's amount of money was bigger than Ichiro's. How many days has it been since they started praying? (The TIMSS Video Study, n.d.) |
| L8 | To solve linear equations where the unknown appears on both sides. | **Foster problem**<br>$\square x + \square = \square x + \square$<br>Can you construct an equation of the form above in which<br>a. the solution for $x$ is an integer?<br>b. the solution for $x$ is a non-integer?<br>c. there is no solution for $x$?<br>(Foster, 2013a) |

## 6.4 Analysis

This case study seeks to address the following research questions: (RQ1) How do students with mathematics difficulties perceive IBI? and (RQ2) Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties? The following aspects of the data collected were analysed to address the above two research questions. Firstly, the lessons as implemented by the teacher and enacted by the students are analysed using the EQUIP (Section 6.4.1). Then, evidence of students' beliefs throughout the IBI unit are analysed using McLeod's (1992) framework (RQ2; Section 6.4.2). Next, students' perceptions of IBI are analysed using Merriam's (2016) approach to coding (RQ1; Section 6.4.3). Finally, student's learning outcomes as measured by the pre- and post-test are analysed using descriptive and bivariate statistics (RQ2; Section 6.4.4). All analyses have been limited to the 18 students in the class identified as having MD (Section 6.1.2). Please see Section 4.10 for a more complete description of my analytical approach.

### 6.4.1 An analysis of the quality of inquiry instruction

To ensure the IBI unit met the criteria of proficient inquiry (discussed in Section 4.3) I evaluated the lessons within Ms Silver's case using the four factors of EQUIP: Instructional factors, Discourse factors, Assessment factors, and Curriculum factors. As previously discussed in Section 4.3, the EQUIP is an instrument that has been validated for use in mathematics classrooms.

Table 6.2 Assessment of the quality of the inquiry instruction in Ms Silver's case

| Factor | Sub-factor | Level assessed | Section |
|---|---|---|---|
| Instructional Factors | Instructional strategies | Proficient | Appendix M.1.1 |
| | Order of instruction | Proficient | Appendix M.1.2 |
| | Teacher role | Developing | Appendix M.1.3 |
| | Student role | Proficient | Appendix M.1.4 |
| | Knowledge acquisition | Proficient | Appendix M.1.5 |
| Discourse Factors | Questioning level | Proficient | Appendix M.2.1 |
| | Complexity of questions | Proficient | Appendix M.2.2 |
| | Questioning ecology | Proficient | Appendix M.2.3 |
| | Communication pattern | Pre-inquiry | Appendix M.2.4 |
| | Classroom interactions | Proficient | Appendix M.2.5 |
| Assessment Factors | Prior knowledge | Proficient | Appendix M.3.1 |
| | Conceptual development | Proficient | Appendix M.3.2 |
| | Student reflection | Proficient | Appendix M.3.3 |
| | Assessment type | Proficient | Appendix M.3.4 |
| | Role of assessing | Proficient | Appendix M.3.5 |
| Curriculum Factors | Content depth | Exemplary | Appendix M.4.1 |
| | Learner centrality | Proficient | Appendix M.4.2 |
| | Integration of content and investigation | Proficient | Appendix M.4.3 |
| | Organising and recording information | Proficient | Appendix M.4.4 |

A detailed discussion of this analysis is presented in Appendix M. However, in this section I present a summary of this evaluation (Table 6.2). As can be seen in Table 6.2 the most common score assigned to the different components of the EQUIP was that of proficient inquiry, therefore the unit as a whole could be best described as meeting the requirements of proficient inquiry. For more details, please see Appendix M.

Ms Silver was successful in leading instruction, discourse, assessment, and curriculum that met many of the goals of inquiry. Students were always given time to explore the IBI problems before receiving instruction. In addition, students were given many opportunities to explain and justify their ideas both with a partner and with the class. Ms Silver assessed students' understanding frequently, and she was especially skilled at providing depth of content.

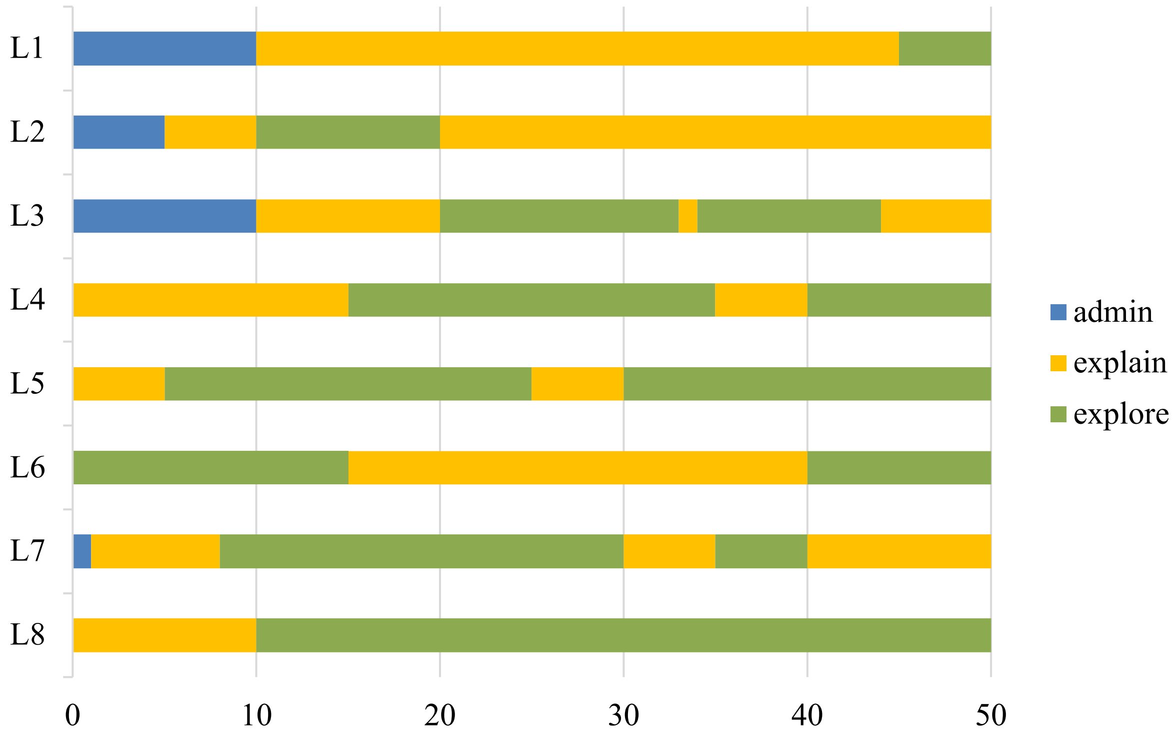


Figure 6.3: Time dedicated to administration, explanation, and exploration at Stratham

An important aspect of an inquiry lesson is the allocation of time between teacher-led instruction and student-led exploration, with the goal that teachers 'only occasionally lecture' (see Appendix A for a copy of the EQUIP). Figure 6.3 illustrates how the time in each lesson was divided between administration, explanation, and exploration for Ms Silver's IBI unit. Administration was considered to be tasks that the teacher and students completed in order to prepare to begin a lesson, for example, taking the register or passing out papers. Explanation was considered any time a person (the teacher or a student) spoke to the entire class. This included explanations of the task or solution methods. Exploration was considered any time the students freely explored the IBI task. However, distinctions between explanation and exploration in a classroom context are not clear cut. It is possible that elements of exploration took place during the phases I coded as 'explain' and elements of explanation took place during the phases I coded as 'explore'.

### 6.4.2 An analysis of student beliefs throughout the unit

In Section 6.3, I present an overview of eight 50-minute lessons organised around a single mathematics unit on the topic of linear relationships. These lessons were observed, video recorded, and detailed field notes were taken. In addition, student work (e.g. worksheets) was collected. These data were used to analyse whether students' beliefs were evidenced and how these beliefs may have been associated with their perceptions of the IBI unit as well as its effectiveness. In addition, the results of students' responses to the questionnaires were used within this analysis to help consider the extent to which beliefs (e.g. mindset) were associated with the effectiveness of the inquiry-based approach. An analysis of the students' perceptions is presented in Section 6.4.3.

Before discussing the beliefs that students expressed throughout the unit (Section 6.4.3), it is useful to briefly present the results of three questionnaires the students completed prior to the commencement of the IBI unit: Attitudes Towards Mathematics Inventory (ATMI), Implicit Theories of Intelligence Scale (ITIS), and modified Implicit Theories of Intelligence Scale (m-ITIS). Please see Section 4.4 for a discussion of these instruments.

#### 6.4.2.1 Results of the ATMI, ITIS, and m-ITIS

All 18 students with MD completed two versions of the ITIS, one for general mindset and another for mathematics-specific mindset (m-ITIS). The results of these are shown in Figure 6.4.

Five students reported as having a fixed general mindset while nine reported as having a growth general mindset. This is a different picture than the 40-20-40 split between fixed-mixed-growth mindsets that is suggested in the literature (please see Section 2.9). The distribution is somewhat different when looking at mathematics mindset with a smaller number showing as mixed and a greater number showing as growth. The relatively high number of students with growth mindsets might be due to Ms Silver's self-reported explicit teaching of growth mindset early in the year and daily growth mindset messages. Although, it is possible that students might hold a 'false growth mindset', meaning they report as having a growth mindset only because they believe this is socially desirable (Gross-Loh, 2016, para. 2).

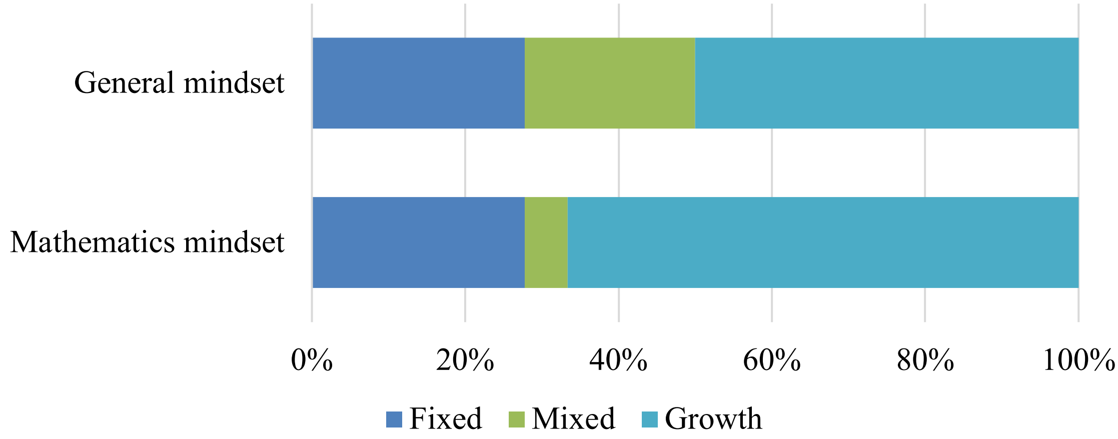


Figure 6.4: Stratham College ITIS results

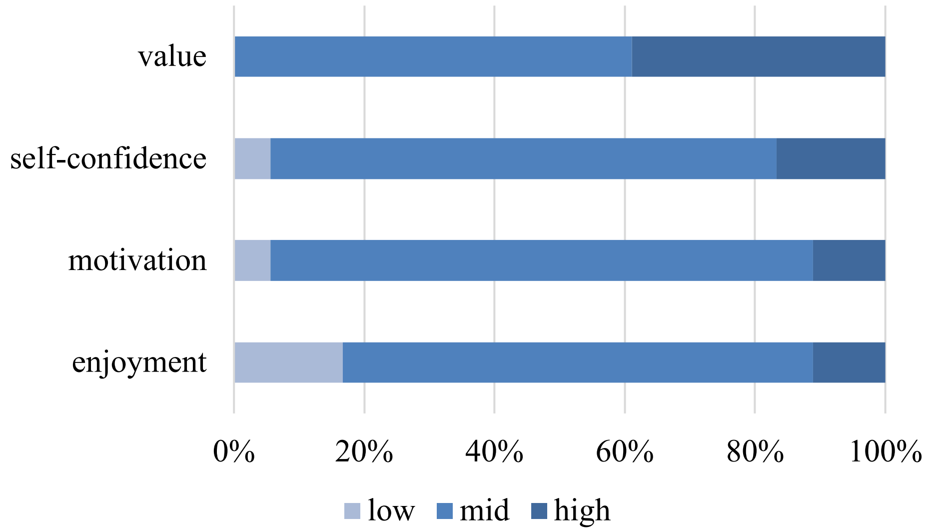


Figure 6.5: Stratham College ATMI results

The students' attitudes towards mathematics mainly clustered around neutral positions of enjoyment, motivation, and self-confidence. Value, on the other hand, was viewed favourably by the students with seven students rating the value of mathematics highly. Please see Table 4.2 for the definition of each attitudinal measure. Students' attitudes towards mathematics were categorised into low, mid, and high by sectioning the possible scores for each construct into thirds (Figure 6.5).

#### 6.4.2.2 Student beliefs emerging from observations

As previously discussed in Section 4.10, I use McLeod's framework of student beliefs to analyse how students' beliefs were expressed throughout the IBI unit and to what degree these beliefs may or may not have been associated with the effectiveness of the IBI unit. The McLeod framework breaks students' beliefs into four constructs: (1) beliefs about mathematics, (2) beliefs about self, (3) beliefs about mathematics teaching, and (4) beliefs about the social context.

##### 6.4.2.2.1 Beliefs about mathematics

In this section I explore the ways in which the students appeared to express beliefs about mathematics, and whether these beliefs were associated with the effectiveness of the IBI unit. Beliefs which emerged from the analysis include the idea that (1) mathematics is something one 'does'; (2) mathematics problems should not take long to solve; and (3) mathematics problems should have a single solution pathway.

Throughout much of the IBI unit the students appeared to feel that the correct approach to tackling the novel problems was to combine the salient information with the correct procedure, suggesting the belief that mathematics is something one 'does'. An example of this can be seen in the following extract.

> E: We're not sure if we have everything.
> R: What do you mean?
> E: We wanted to get all the numbers out first. Like, before we go on to the next bit.
> R: I see. Is that what this means? [pointing to numbers the student had underlined]
> E: Yeah, that's what we did in year 7 with these kinds of problems. Find the numbers first, then we can … add them or whatever.
> (Ekko and Researcher, L3)

In the above exchange, Ekko suggested that the process of solving the IBI problems relied on being able to clearly identify all the numbers and then apply a simple procedure. When presented with IBI problems, however, this approach may prove challenging since students are not provided with a procedure.

R: How are we getting on here?
C: We know we have to do something with this number and this one [pointing to circled numbers on the worksheet], and it has something to do with brackets, but we can't remember how to solve these.
(Researcher and Cayleigh, L5)

Like Ekko previously, Cayleigh seemed to believe the correct way to approach the problem was to firstly find the numbers (by circling) and then do something with them. Cayleigh and her peers struggled with the problem for several more minutes before getting off-task. This exchange was not uncommon as students frequently focused on finding a step-by-step approach to solving the IBI problems, namely: Step 1, identify the numbers, and Step 2, apply a known procedure. This approach is generally in conflict with IBI, which requires students to work on problems for which they have not been taught a procedure, a point that Ms Silver had emphasised to the students during each IBI lesson.

While exploring the Ichiro problem (L7) a group of students had correctly identified two expressions for the number of coins in each of the brothers' wallets. The students were discussing what to do next and were overheard saying the following:

K: But hold on, the smaller brother is supposed to have more.
D: What?
K: If this one here is Ichiro's and this one here is the smaller brother's then this [smaller brother's amount] must be bigger than this [Ichiro's amount].
D: Right, yeah.
K: Yeah. So, we can't put the equals then.
D: So that's not right. What do we do then?
K: I don't know. We let Ms explain it [laughs].
(Karson and Donald, L7)

At this point I attempted to encourage the group to continue exploring the problem, and if they did not feel they could solve the problem using their expressions, to try some alternative approaches. After some discussion, the students began to explore several other solution pathways, including working through the story day by day and keeping a record of the amounts in a table. However, prior to some coaxing from myself, the above exchange between Karson and Donald suggested they may have been about to give up. Perhaps the students were more familiar with a direct teaching approach and hence were tempted to rely on the teacher to explain ('let Ms explain it').

One interpretation of the above interaction might be that the students' unwillingness to engage in solving the problem further without explicit help from the teacher is an example of learned helplessness (Dweck, 1975; Yates, 2009). Some studies argue that IBI can help students overcome learned helplessness (Di Martino & Zan, 2009) and the ability of the group to re-engage with the problem after some coaxing from myself might be evidence of this. In this case, the students did demonstrate good effort and progress by finding two expressions for the amount of money each brother had. However, the students failed to ultimately make these expressions useful for solving the problem (for example, by writing an inequality or by interpreting their equation in a meaningful way).

Taken another way, perhaps the above extract is an example of these students simply reaching the limits of their cognitive stamina. Exploration of novel problems is a cognitively demanding task, utilising high levels of working memory and cognitive load. Cognitive Load Theory (CLT; Sweller, 1988) suggests that working memory is a limited resource and, when taxed, reduces performance.

Another belief about mathematics that these students expressed during the IBI unit related to the length of problems.

> D: Why are these always so wordy?
> T: Sometimes maths problems are not all tied up in a neat little bow. You have to go looking for them.
> D: But these ones take ages.
> (Dax and Teacher, L7)

Dax appeared to notice that the problems throughout the IBI unit took longer to solve than those he normally associated with mathematics. McLeod (1992) and Schoenfeld (1985) argue that many students believe that problems in mathematics can be solved in less than ten minutes and, moreover, students who held this view struggled to persevere when solving inquiry problems. The comments from Dax were not dissimilar to other comments made by students throughout the IBI unit. My observations were that these sentiments were often associated with signs of frustration and boredom, such as students placing their heads on their desks or defacing their paper (see Appendix K).

In addition to believing that problems should not take long to solve the students often expressed beliefs about how problems can be solved:

> R: That's a nicely drawn table. Can you explain to me what you've done?
> E: Yeah, we said we thought he would start out with 10.5 denari. Is that the answer?
> R: Let's put the answer to the side for a moment. Can you please explain what you've done here? [pointing to drawn table]
> E: But we're not sure if that's the answer though.
> R: Well, if you're not sure then maybe try a different approach and see if you get the same answer.
> (Researcher and Edith, L3)

Despite my prompting, Edith was hesitant to explain her group's approach. The group instead seemed fixated on getting the correct answer but at the same time were reluctant to try out other approaches to confirm the answer. A review of their written work suggested there was no evidence of further working. It is possible that the students held the naïve belief that mathematics problems only have one correct solution pathway. This may have resulted in this group of students feeling like they had completed the task and any further attempts were pointless. Other research has argued that questions which encourage students to explore multiple solution pathways allow students to be more innovative and to think about a wider range of alternatives (Makar, 2012).

*6.4.2.2.1.1 Summary of beliefs about mathematics*

In the above section I discuss my classroom observation which might evidence some of the beliefs the students held about mathematics. Students seemed to believe that mathematics is something that is done via a procedure, rather than by developing and applying an understanding of the concepts. As such, students sought guidance on 'the formula' or 'how to do it'. In addition, students felt the question length conflicted with their beliefs about maths problems, claiming that the IBI problems took too long.

Finally, prior experiences in school may have resulted in the students' belief that mathematics problems have a single solution pathway. Some students struggled with the idea that problems could be solved or confirmed using different approaches. The result was that students stopped exploring once they felt they had found a solution pathway.

**6.4.2.2.2 Beliefs about self**

There were several observations in which students appeared to express their self-beliefs during the IBI unit. In this section I discuss these under the groupings of student self-efficacy (Bandura, 1997) and student mindset (Dweck, 2017b).

*6.4.2.2.2.1 Self-efficacy*

Student self-efficacy for mathematics has been shown to impact how long students will persist at a task and how much efforts they will expend (Bandura & Cervone, 1986; Schunk, 1995). During the observations of the IBI unit, students expressed views that might have indicated their self-efficacy. Indications of low self-efficacy were closely associated with task disengagement or disruptive behaviour. For example:

T: I hear lots of noise coming from this group but most of it isn't about maths. Can someone show me where we are on the problem?

E: We don't know what to do Ms.

T: Were you listening before?

S: We were, but we just can't do them. I don't get any of it.

(Teacher, Elayne, and Sirena, L1)

Following this exchange, the teacher spent several minutes recapping some key terms and coaxing the students to try and reengage with the problem. This had little success and ultimately the teacher needed to move on to the next group. The exchange seemed to suggest that the students were exhibiting low self-efficacy. The students felt they could not do the problems, which possibly led to their poor engagement with the problem. A review of their worksheets suggested little progress before or after Ms Silver's coaching.

In a separate incident I observed two students working on the Ichiro problem. Upon closer inspection it appeared that one student, Elayne, was copying the work of her peer, Clay. When I questioned Elayne, the following exchange took place:

> R: Can you explain for me how you got this answer?
> E: [Shrugs shoulders and looks at Clay].
> (Researcher and Elayne, L7)

Student collaboration can be an important part of problem exploration. However, the above example shows how 'collaboration' can sometimes be one-sided. It appeared Elayne felt she lacked the ability to tackle the problem (suggesting low self-efficacy). However, an alternative explanation for Elayne's actions could be that she held the belief that the most important thing in mathematics is to get the correct answer, and therefore there is no harm in copying a peer's work (students' beliefs about mathematics are covered in Section 6.4.2.2.1). Elayne's actions might also be indicative of a fixed mindset, as she might have felt that her effort would not lead to improvement. Student mindset is discussed in more detail below.

Situations in which one student within a group appeared to do most of the work were frequent. Another example was observed during L4.

> R: It looks like you guys have finished over here.
> K: Not really. I don't get it, but Sirena has been helping me.
> S: We get pizza every Friday at my house, so I guess you could say I'm sort of an expert on this.
> (Researcher, Kent, and Sirena, L4)

These students worked together well, and while both failed to find a valid solution pathway, they approached the impasses they each encountered within the problem differently. Kent appeared to exhibit low self-efficacy, claiming he did not 'get it'. A review of his worksheets suggested that little written exploration had taken place, perhaps indicating that his low self-efficacy caused him to disengage with the task. Alternatively, it is possible Kent spent a majority of the class time talking with his peers in an attempt to make sense of the problem and did not have time to write down his thoughts. In contrast, Sirena appeared to make a personal connection with the subject matter of the problem explaining that she orders pizza every week. Prior studies have suggested that students who make connections to the problems (such as a family connection) exhibit greater ownership of their learning (Wang et al., 2018). A review of her worksheet, and my own observations, indicated that Sirena tried a variety of approaches to solving the problem, and her closing remark suggests that it was an overall positive experience.

When tackling the Henri and Emile problem (L5) I overheard Opal make the following comment.

> I felt like I was doing okay on these the other day but this one makes no sense at all. Help me. (Opal, L5)

It seems that Opal held a low belief in her ability to solve the IBI problem of L5 and believed the only way to overcome her inability was for someone to help her. Opal's view may be an example of learned helplessness (Dweck, 1975; Yates, 2009). Her peer's subsequent interjection of help may have inadvertently reinforced Opal's feeling of helplessness. The students' beliefs about teaching are addressed in Section 6.4.2.2.3.

*6.4.2.2.2.2 Mindset*

In the previous section I explored how students' beliefs about their self-efficacy appeared to manifest during the IBI observations, with some impact upon the students' performance. I categorise self-efficacy as a belief that students hold about themselves. Another belief that students can hold about themselves is mindset, which I explore in this section.

During the observations, students frequently made comments which suggested they held a fixed mindset. Perhaps one of the clearest examples of this was during a discussion between one student and me regarding the Ichiro problem (L7).

R: Are you having some trouble over here?
E: Yeah…
R: What have you tried so far?
E: Nothing really, I don't get any of it … These types of problems just really confuse me. My mum said she was the same way when she was in school.
(Researcher and Eleri, L7)

The above comment typifies similar observations I made during the IBI unit. Eleri appeared reluctant to try out the problem and seemed to almost give up on the problem as soon as she began. This behaviour might indicate a fixed mindset, in which students 'view their abilities as representing fixed traits over which they have little control' (Schunk, 2012, p. 257). It is worth noting that Eleri measured as mixed mindset on the general ITIS and yet growth mindset on the mathematics-specific ITIS. As I discussed in Section 6.4.2.1, Ms Silver undertook an explicit teaching of mindset at the start of the school year, and students were often exposed to mindset messages within the classroom. Therefore, one explanation of the conflict between Eleri's comment above (suggesting a fixed mindset) and her ITIS score (suggesting a growth mindset) is that Eleri had been trained to respond to the mindset questionnaire in a more growth orientated way. Dweck called this phenomenon 'false mindset' (Gross-Loh, 2016, para. 2). In addition, Eleri's unprompted raising of her mother's perceived abilities in mathematics might suggest Eleri believes her poor performance in mathematics might be inherited. This view also aligns with those of a fixed mindset. A potentially similar view was observed during a discussion with Zelda during L2.

R: How did you come up with this answer?
Z: I didn't really. Harper got the answer.
R: I see, did you try the problem for yourself?
Z: Not really … I'm rubbish at this stuff. Harper has got the brains [laughing].
(Researcher and Zelda, L2)

Zelda makes several references to performance in mathematics as a fixed trait (e.g. 'I'm rubbish at this' versus 'Harper has got the brains'). She suggests that Harper has some innate ability which makes her superior in mathematics. This aligns with the views of a fixed mindset in which students feel their abilities are largely outside of their control. This belief may have contributed to Zelda's low engagement in the IBI problem, as evidenced by my observations as well as her worksheet which, aside from the copied answer from Harper, showed no evidence of working. Zelda measured growth on the general ITIS and fixed on the m-ITIS. It is therefore possible that Zelda was expressing fixed messages in this instance because she was operating in the domain of mathematics.

Another potential manifestation of student mindset occurred during the pizza problem of L4. In a discussion with a pair of students the following exchange took place:

> R: You guys have a lot written down.
> S: Yeah! We think we got it. Is £1.40 correct?
> R: Yes, but how did you get there?
> C: We tried a few different bits, like if this one has two toppings and this one has four the difference is only two toppings. So, we divided by two and kind of went from there.
> R: Well that sounds like a very sensible approach. Nice work.
> C: Thanks, I liked this problem. I've always been quite good at maths.
> (Researcher, Simon, and Clay, L4)

This pair of students engaged well with the problem. My observations, as well as a review of their worksheets (see Appendix J), suggested they had attempted several different solution pathways, eventually finding one which led them to their final answer. Given Clay scored as fixed mindset on both the general and mathematics specific scales, his comment that he has 'always been quite good at maths' could relate to how he sees his perceived high abilities in mathematics as fixed. It is therefore possible that Clay's fixed mindset actually *helped* him succeed on the IBI problem, since he identified so strongly as someone who succeeds in maths. This phenomenon has been observed in other studies (Hwang et al., 2019). Despite this, it appeared that Clay had expended a lot of effort on this problem and yet did not connect his success with his effort. Perhaps, when faced with evidence which might conflict with students' beliefs about their intelligence, some seek out mitigating explanations (e.g. innate ability) rather than change their beliefs.

*6.4.2.2.2.3 Summary of beliefs about self*

In the previous section, I outlined several observations which might indicate the beliefs the students held during the case study. Students held a belief that they lacked the ability to perform well in mathematics, and this low self-efficacy may have contributed to poor engagement by some students. IBI problems that students were able to connect to the real-world seemed to counter this issue and result in greater effort.

On occasion, students seemed able to simultaneously hold two beliefs. The first was that mathematics is important (and the students need to have the correct answer). The second was the belief that they lacked the ability to explore the problems themselves. The result of students holding these two beliefs appeared to be cheating by copying answers.

Students' mindsets were suggested at several points throughout my observations, with several students suggesting that effort plays little role in their understanding. The idea that mathematics ability was innate and genetic was expressed by several students throughout the case and this was used as a reason for their disengagement. When students with fixed mindsets succeeded through effort, they were apt to seek mitigating explanations (e.g. innate ability, luck) rather than change their beliefs.

**6.4.2.2.3 Beliefs about mathematics teaching**

In preparing for the IBI unit Ms Silver informed me that it was unlikely that her students had received much exposure to IBI in the past. Furthermore, before commencing the IBI unit, Ms Silver said that she suspected the students would 'give up quite easily' when faced with the selected tasks. She acknowledged that her own teaching style is quite direct, and she tends to help students when they struggle. Through the consistent application of these teaching approaches students develop certain expectations and beliefs about how mathematics is taught (Boaler, 1998; Makar & Fielding-Wells, 2018; Op 't Eynde et al., 2006). I explore this interaction in this section.

During the pizza problem (L4) I had the following exchange with a group of students.

R:  How are you girls doing here?
I:  I think we are doing okay, thanks.
R:  How do you like these types of problems?
E:  I like them. It's good variety.
I:  Yeah.
R:  What do you mean?
E:  Like, everyone's different, so it's good they [the problems] are always different.
I:  Yeah, and I'm really visual, but some people can read these and just get them because they are more, like, 'wordy' learners. I like the pictures and diagrams.
(Researcher, Ida, and Edith, L4)

Ida's and Edith's references to 'variety' and 'different' suggest the students saw the IBI problems as different to their normal mathematics lessons. The comment, 'It's good they are always different', suggests Edith believed the IBI teaching approach was positive. Alternatively, Edith could have been referring to the variety *within* the IBI problems. Ida seemed to have some awareness of the concept of learning styles and the idea that different students can learn best in different ways. Perhaps this awareness is what allowed her to recognise the changed approach during the IBI unit.

Other students made comments throughout the IBI unit that suggested they saw the IBI approach as different. In addition to the previous example, the following exchange helps to highlight this.

W:  Is it like this because you're here, Miss?
R:  What do you mean?
W:  Like normally Ms Silver would explain everything at the board, but she doesn't do that anymore.
R:  And how do you feel about that?
W:  To be honest, I miss it. I felt like everything used to be a lot clearer.
(Winston and Researcher, L5)

Teacher feedback indicated that Winston had been performing well for much of the year despite his low prior attainment. However, Winston frequently expressed that he felt the IBI style of teaching was ineffective. Perhaps his historically strong performance using the direct teaching approach led him to believe this was the best teaching method. The idea that students might be reluctant to learn via IBI is discussed within the literature (Makar & Fielding-Wells, 2018). The extract suggests Winston feels the IBI approach is not as clear and he is not learning as much as before.

Favourable views of IBI were also observed within the IBI unit. During the Ichiro (L7) problem the following exchange occurred between me and a group of students.

> O: I wish Ms would just do an example with us first.
> E: Yeah, but I can sort of see how it might be better.
> R: What do you mean?
> E: Well you have to put more into it, and that makes it stick a little better.
> O: I don't know. I think it's a waste of time to keep trying stuff and then you never get it right.
> (Opal, Ekko, and Researcher, L7)

Both of these students appeared to agree that the IBI approach was more difficult. Interestingly, however, Ekko appeared to align with proponents of IBI who say that the increased attention to the problem in an IBI lesson may help develop deeper and more permanent understanding. It is worth noting that Ekko scored as growth mindset on the ITIS for both general and mathematics specific mindset. This would seem to align with her comments that effort, or putting 'more into it', can drive successful outcomes. Opal also scored as growth mindset on both ITIS assessments, although during this extract she appeared to demonstrate a more fixed view. Opal appeared to believe that the teaching of mathematics should follow the more traditional pattern in which the teacher uses direct instruction and then the students subsequently practice the material. To explore a novel problem without first being shown an example is therefore a 'waste of time'.

*6.4.2.2.3.1 Summary of beliefs about mathematics teaching*

In the previous section I explored how students' beliefs about the way mathematics should be taught were observed during the IBI unit. Students were sensitive to the change in mathematics teaching, with many able to articulate how they saw the new approach as different to their usual maths lessons. Students appeared to believe that mathematics problems should be short and the IBI problems were excessively long, thus reducing the pace at which they might be able to learn new material. This might suggest that the students believed mathematics teaching should be volume orientated.

Given the students' prior experiences of mathematics, it is unsurprising that some students believed teaching should follow a traditional transmit-receive relationship, in which the teacher, acting as the expert, transmits knowledge to the students. Recognition of the value of effort and exploration was mixed, with some students feeling it was a waste of time and others feeling it encouraged deeper understanding.

**6.4.2.2.4 Beliefs about the social context**

The social context relates to the beliefs the students hold about the interactions between mathematics and society (McLeod, 1989). This includes the role of mathematics in the wider world and the role of social interactions within mathematics. This section explores how these beliefs may have manifested themselves during my observations of the IBI unit.

During the second lesson of the IBI unit, a group of students were overheard discussing the utility of the task and mathematics more broadly.

> K1: I don't know but my dad always says no one ever uses the maths they learn at school.
>
> K2: Well, I wanted to be a programmer, but I got knocked down a set last year so that's unlikely now.
>
> K1: Why not?
>
> K2: Employers look for that, don't they, like if they compare two people and one was in lower set maths and one in higher set, they will choose the higher set.
>
> (Karson and Kent, L2)

The above exchange suggests several possible beliefs that these students held about the social context. Karson's initial comment highlights an important social setting, namely the family unit. It seemed within Karson's social context, mathematics was not a valued pursuit, and Karson's observed poor engagement with the task may have been informed by this social context. Separately, Kent appeared to recognise the real-world application of mathematics in his stated future career. Making connections to real-world applications can create engagement, and Kent's awareness that employers might expect to see evidence of mathematics achievement, could offer a source of engagement and motivation. However, in this exchange, Kent appeared to see the situation more negatively. He seemed to believe that within the school social context lower set students are not as proficient at mathematics, and therefore his expectation for his mathematics achievement might be lower (Francis, Archer, et al., 2017). He seemed to also believe that within the real-world social context future employers might have a negative view upon his recent demotion from a higher set. These factors might have combined to reduce his motivation.

Looking back at Section 6.4.2.2.2.2 in the excerpt with Eleri on Page 159, there was a similar instance of the family social context impacting a student's belief. In that instance the student felt that difficulties in mathematics were something she shared with her mother ('My mum said she was the same way when she was in school').

Another way in which students may have expressed their beliefs about the social context was in how they interacted with their peers while exploring the IBI problems. When tackling the Henri and Emile problem (L5) the following discussion between a pair of students was observed.

T: If he walks twice as fast, he will be done twice as quick.
E: Yeah but we need to know how far not how quick.
T: Well what if we try 100 meters like here.
E: No, but we were doing it with formulas and brackets before.
T: That sounds harder.
E: Yeah, but it would take forever to try all the lengths.
T: I'm gonna try 100 since it says that. You try your way.
E: Alright.

(Theresa and Ekko, L5)

In this exchange the students appeared to demonstrate a democratic style of interacting. The students seemed to understand that challenging each other, as well as themselves, was part of the process of exploration. The outcome of the exchange was that each student attempted the problem for several minutes before reconvening to discuss what they had done. The students appeared to understand that, within the classroom context, new knowledge could emerge through social interactions with peers. This sort of social collaboration and compromise while working on the problems could have had a positive impact on motivation (Brough & Calder, 2012).

Bringing real-world problems and experiences into the classroom has been shown to improve student performance (Lowrie & Clancy, 2003). There were observed instances within the IBI unit in which students seemed to make connections between the IBI problems and their out-of-school experiences. For example, when discussing the pizza problem (L4) I noted the following exchange.

D1: They don't really price it like that in real life.
D2: What do you mean?
D1: Well some toppings will cost them more so they would have to charge more. My uncle owns a takeaway and he's always saying he makes more money on some food than others.
(Dax and Dori, L4)

Discussions with the teacher prior to the commencement of the IBI unit had identified Dax as a student whose performance was towards the bottom of the class. My previous observations of Dax noted that he typically failed to engage with the problems, often producing no visible work and contributing little to class discussion. However, his engagement with the pizza problem was noticeably different and his worksheet suggested he had attempted several solution pathways. Problems that students can relate to their real-world social contexts can foster increased ownership (Lowrie & Clancy, 2003; Wang et al., 2018). The previous extract suggests that Dax was able to connect this problem with the real-world, out-of-school experiences of his uncle, and it is possible this connection contributed to his observed increase in engagement. It is also interesting that Dax shared this view with his peers.

Jaworski (2006) argues that IBI promotes greater social discussion around mathematics than traditional classroom contexts.

*6.4.2.2.4.1 Summary of beliefs about the social context*

In the previous section I discussed my observations of how students' beliefs about the social context for mathematics were evidenced during the IBI unit. Students appeared to be aware of multiple social contexts which influenced their beliefs. These included the family context (in which views of mathematics were expressed at home), the world of work and employment (relating to how employers might view mathematics achievement), the attainment-based setting of the school (relating to beliefs about the expectations their school holds for different sets), and the classroom context (wherein peers might democratically explore problems and develop knowledge). These social contexts provided a range of motivating and demotivating outcomes.

### 6.4.3 An analysis of student perceptions of inquiry instruction

This section provides an analysis of 12 student interviews following eight IBI lessons. These interviews were designed to explore the students' perceptions of the IBI unit as well as mathematics more generally. All 12 students were identified as having MD (Section 6.1.2).

Table 6.3: Intensity scores for interview themes at Stratham

| | Student | IBI empower | IBI neglect | Teacher importance | Peer importance | Pace and format |
|---|---|---|---|---|---|---|
| Fixed maths mindset | Cayleigh | 18 | 23 | 7 | 6 | 7 |
| | Clay | 12 | 12 | 5 | 4 | 8 |
| | Eleri | 10 | 22 | 11 | 12 | 5 |
| | Karson | 2 | 17 | 13 | 13 | 5 |
| | Zelda | 13 | 23 | 6 | 4 | 7 |
| Growth maths mindset | Adele | 19 | 7 | 10 | 10 | 7 |
| | Elayne | 16 | 12 | 13 | 19 | 4 |
| | Elva | 14 | 8 | 11 | 11 | 5 |
| | Ethel | 31 | 14 | 8 | 10 | 4 |
| | Harper | 32 | 1 | 6 | 4 | 5 |
| | Opal | 18 | 9 | 18 | 7 | 18 |
| | Sirena | 19 | 11 | 15 | 7 | 9 |

For a detailed discussion of the analysis methods, please refer to Section 4.10. Several themes emerged: (1) IBI as a form of empowerment, (2) IBI as a form of neglect, (3) Importance of teacher, (4) Importance of peers and (5) Lesson pace and format. Table 6.3 presents the intensity of these themes for each student, organised by mathematics mindset. For a discussion of intensity scoring, please see Section 4.10.

#### 6.4.3.1 IBI as a form of empowerment

Through one-to-one interviews I encountered students who expressed views that best aligned with a sense of empowerment. Students expressed ideas that the IBI unit allowed them to 'engage their brains' and 'get stuck in'.

> I mean, it was a bit frustrating because I guess I was sort of thinking, 'I'm just gonna have to guess it', because I haven't really done much on it. But once you get one idea, it makes you have another idea. And you can sometimes pick it up a bit easier like that. But when I first got it I was like, 'Whoa.' (Ethel)

In the above extract Ethel seemed to recognise that, despite the problem being novel, she had the power to explore. At first, she felt like this exploration was 'guessing', perhaps because of the unfamiliarity of the problem or perhaps because of low self-efficacy in this area of mathematics. However, as the exploration continued and fruitful lines of inquiry emerged, Ethel appeared to appreciate that her 'guessing' could equate to new learning and that her struggle might yield success. Ethel's comment, and the many similar comments from other students, could be viewed as an expression of persistence and might also suggest a growth mindset. It is possible that, in this instance, IBI helped to promote persistence and its effect on learning. Ethel expressed these views again when she said:

> It was hard. I think there were a lot of steps to it. But I realised that you kind of try different things out and they might not work, but if you try lots of things out then one of them might work. (Ethel)

Students also expressed the idea that not all problem explorations led to a correct solution. Indeed, it was anticipated that the students would make mistakes while they explored the IBI problems.

> That one [L8], I couldn't quite get. I don't think I got that right, but sometimes it's not about getting it right. It's about doing your best and trying to get better. (Sirena)

In the above except, Sirena expressed the idea that mathematics is not simply about finding correct answers, but rather mathematics includes the exploration of problems that may or may not lead to a correct answer. Kouba and McDonald (1987) suggested that many students feel that mathematics is a 'doing' activity, typically by applying a single procedure to derive a successful outcome. It is possible that the IBI problems helped Sirena challenge this notion and develop her view that success in maths is not about 'doing' a known procedure but can also be about the pursuit of knowledge and deep understanding.

In addition to the idea that mathematics might not solely be about finding solutions, the students also expressed the idea that problems might have many solution pathways.

> There's normally an easier way or a harder way, and if you know more methods and have time to try different ones and figure them out, you can work it out in different ways, so you really know it. (Adele)

Adele appeared to believe that there might be multiple ways to solve IBI problems and being able to look at problems from multiple angles might result in improved understanding.

The previous two quotations suggest that IBI might have helped the students in Ms Silver's class understand that mathematics is not solely about finding one correct solution using one specific procedure. Rather the students seemed to express that problems often have multiple solutions and that exploration of problems is part of the learning process.

The following extract suggests that students might have made another connection during the IBI unit. Namely that solutions to maths problems might not be a simple one-step process.

> Yeah, I would get more used to it and there's more than one step, because when we got the first one, I was sort of expecting just to do one thing and solve it, and then I realised that no, it's more like five or six different things that make you solve the problem. (Ethel)

Through exploring the IBI problems Ethel seemed to become aware of the fact that mathematics problems can sometimes be complex and require multiple steps before reaching a solution.

Another way the students appeared to perceive IBI as a source of empowerment was through a sense of positive self-fulfilment and achievement. This feeling typically came after the students experienced some success while exploring a problem.

> You kind of think that you're proud of yourself because you figured out the question, or at least you got closer or got some bits. And then if you talk it out with Ms you might pick up the bits you didn't get. (Adele)

In the above extract, Adele expressed feelings of pride as a result of working through the IBI problems. It is possible that the IBI created an opportunity for Adele to develop a sense of agency in the lesson and that this led to increased fulfilment. This is consistent with the literature which shows that IBI can lead to a sense of personal empowerment (Hassi & Laursen, 2015). In addition, Adele acknowledged she felt proud even when she did not find a solution but rather got 'some bits'. This suggests that IBI might have helped Adele see value in learning even when that learning was not complete. Furthermore, Adele explained that the follow up instruction helped her to understand the rest of the problem. These views align closely with that of Schwartz and Martin (2004) wherein IBI can help students identify gaps in their knowledge and that this awareness primes the student for more effective follow-up direct instruction.

A related sentiment regarding this increased personal empowerment and sense of agency was expressed by Elva.

> I liked that it [the Ichiro problem] was so clearly laid out to me, and I got it and I felt proud of myself. I liked that one, even though it's in a massive

> paragraph and that was hard to me. I like it because it was clear when I've worked it out. I think that was my favourite. (Elva)

Like Adele, Elva also used the word 'proud' to describe her feelings about the IBI problem, although in this case she used it to define the feeling she got when she found the correct solution. Therefore, it is unclear if the positive feelings are associated with effort and learning or simply with successful outcomes.

As well as the above references to improved agency, some students appeared to understand that the IBI approach may have influenced another metacognitive area, that of self-efficacy.

> I didn't really like it [the IBI unit], because it was a massive change. Because literally every lesson, she was like, 'Okay, you have to do this and that and then that and then this'. Then when we came in, she was like, 'No, just do it yourself'. So we were all like, 'What!' So it was very different. We had to think differently. I think she was at the same time trying to boost our self-confidence. (Elayne)

In the above quote, Elayne described her negative feelings toward the IBI unit. She seemed to view the IBI approach as a big change from how her maths class usually was in which her teacher would tell her exactly what to do ('do this and that'). It is evident from Elayne's quote that this change was initially difficult for her. She described needing to 'think differently'. Elayne doesn't seem to think her teacher has negative reasons for taking such an approach. Instead, she ventured that Ms Silver was using IBI as a means of increasing her self-confidence. This comment is interesting because Ms Silver rarely discussed IBI in the context of self-efficacy with the students, often limiting her introduction to, 'I just want to see how you get on with these'. This effect has been seen within the literature, whereby studies have demonstrated that inquiry based approaches can increase students' self-efficacy (Kogan & Laursen, 2014).

One final way in which students expressed their views of IBI as empowering is best represented in the following extract.

> In the end, all of us did know how to do it. I think it was easier, but it did take me the whole lesson just to do that. … and then I actually did understand it in the end. I think if I did do that topic, I would be able to do it but just a bit quicker this time. (Elayne)

In the above extract, Elayne expressed several views. Firstly, she seemed surprised that a single problem could have taken an entire lesson. It would seem elements of the IBI unit challenged Elayne's view that problems in mathematics should be short. This idea is discussed further in Section 6.4.3.5. Secondly, Elayne said that everyone (herself included) understood the problem in the end, suggesting that the IBI was empowering because of its effectiveness.

Of the 12 students who were interviewed, all 12 expressed the view that IBI was empowering in some way. However, when looking at the intensity with which students expressed these views (see Boyatzis, 1998) there were notable differences between students of different mindset groups. Students who scored as growth mindset on the m-ITIS expressed views of IBI as empowering with greater intensity than those who scored as fixed mindset, as measured by the percentage of transcribed words in a given transcript coded to this theme (please see Section 4.10 for a discussion of intensity scoring and Table 6.3 for the results).

#### 6.4.3.2 IBI as a form of neglect

The previous section explored how students perceived IBI as an empowering method of teaching. Whilst all students expressed this view to some extent there were also times the students interviewed expressed the somewhat contrasting view that IBI was neglectful. This perception was expressed in various ways such as IBI created frustration, made the task more difficult, promoted failure, or resulted in a lack of teacher support. For example:

> C: It was odd because it's the first question and we already haven't been told how to do it. You can't really do them if you've not been shown how.
>
> R: What would your teacher normally do?
>
> C: Ms would normally talk it through on the board.
>
> (Cayleigh and Researcher)

In this excerpt Cayleigh expressed a view that was common amongst those students interviewed, the idea that mathematics instruction should begin with explanation. This is contrary to the principles of IBI which emphasise the importance of exploration before explanation. Cayleigh was able to identify this difference in approach but did not seem to embrace it. Cayleigh expanded her thinking with the following sentiment.

> I think there could have been more explanation right at the start and then throughout. Say if it was like A, B and C. A should be explained well at first because, like, it carries on. The questions would carry on from the first answer and if there was more explaining in A, then B would be easier, C would be easier, D would be easier. Because you kind of get your head around it. (Cayleigh)

Cayleigh explained why she felt that explanation upfront is a superior teaching method. Cayleigh seemed to see mathematics in a linear fashion, with problems clearly connected to each other. When faced with a series of problems 'like A, B and C' Cayleigh suggested that the purpose of mathematics is to progress linearly through these challenges, and this should commence with a thorough explanation which would allow students to move through the remaining problems. In her mind she is doing well in mathematics when she can tackle these problems easily and in order. In many ways, this mirrors some non-IBI approaches in which students are carefully scaffolded through a series of problems, perhaps with increasing complexity.

Even when the IBI approach did result in increased knowledge many students were unable to recognise the change and seemed to feel they had stumbled upon the solution by luck rather than by focused perseverance.

> I don't know how I got there, just lucky I guess. (Karson)

The idea that IBI breaks the students' expectations of how a lesson should flow by neglecting to explain first is also reflected in the following quote.

> You know you've got into the routine of doing the worksheets, and things like this [the IBI unit] are quite different because … sometimes it's not quite straightforward because it's quite wordy. You sort of have to think more about it. But with the worksheet it's normally just doing one-step things, so you kind of – it doesn't involve as much thinking. (Ethel)

In the above quote, Ethel highlighted that complex problems inherent in IBI are different to the normal 'routine' of her mathematics lessons. It is interesting to note that she feels that IBI requires her to think more, which would seem to align with the views of IBI proponents. Despite this, Ethel seemed to have a preference for upfront explanation which 'doesn't involve as much thinking.' Elayne expressed a similar view when suggesting feedback for Ms Silver, explaining that she should explain more and not disrupt the typical mathematics 'experience'.

> R: Would you change anything about the teaching of the [IBI] problem?
>
> E: Probably Ms helping us a bit more or going through it at the beginning of the lesson, instead of just handing us the paper and going, 'There you go, try and work that out.' I think it was a different experience to try and adapt to.
>
> (Researcher and Elayne)

Students expressed a range of reactions to the IBI unit (the words students used to describe maths are presented in Figure 6.6). One of the most common reactions described was of panic, as evidenced by the below extract.

> I was like, 'Oh my God, why aren't we going through it? I don't know why'. (Zelda)

Zelda appeared to be both panicking and also confused. She was unable to understand why a teacher would expect students to do novel exploration without 'going through it' first. From my observations of the class, I was able to see what appeared to be instances of students panicking as a result of the new approach. For example, during L5 the following exchange took place between the teacher and Ekko during a whole class discussion.

E: I had it and then I lost it completely.

T: Keep going then, because I think you've got it. Ekko, don't panic.
(Ekko and Teacher)

In another interview Cayleigh also touched upon a similar but slightly different idea.

> You know when you get a block, it's like there's something there and you can't move. It's hard to understand some things. (Cayleigh)

Listening to the way Cayleigh described her perceptions of IBI one might conclude she was feeling anxiety. Her description of feeling like she 'can't move' is akin to the physiological response demonstrated in those with mathematics anxiety, who have been shown to respond to mathematics much like someone facing a tiger (Carey et al., 2019).

Another, perhaps more extreme, example of this came up in my interview with Eleri.

R: What does that feel like when you're in class and you're stuck on problems like these?

E: It makes you want to cry, and I used to cry a lot … and you try and hide it, but you can't.

R: What happens then? … What do you do?

E: It just embarrasses you a bit, because you can't do it and everyone else can.
(Researcher and Eleri)

During the interview, Eleri described feeling embarrassed and like she wanted to cry whilst working on the IBI problems. Whilst she did not state the specific IBI questions that resulted in this emotional response, she was left with the impression that she was the only person in the classroom who could not solve the problems. But this was never the case. Many students did not solve the problems successfully during the exploration phases of the lessons. It is unclear why Eleri had this impression, but the negative effect it had on her emotions was apparent.

In addition to the above emotional responses to IBI, students often reached a point at which their lack of understanding led to frustration.

It felt frustrating, because I couldn't really understand it, and I didn't know what to do. (Zelda)

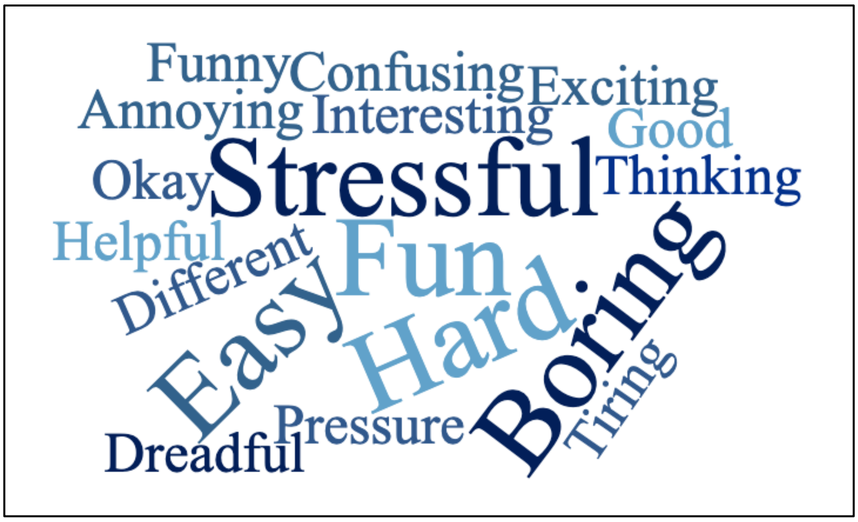


Figure 6.6: 'What three words would you use to describe maths?' at Stratham College[15]

The above discussion, that IBI could lead students to feel a sense of panic or frustration might explain why students also expressed feelings of disengagement.

I didn't know what to do with that one, so I just started doodling. (Adele)

I don't think I've solved that one yet. I just said I don't know, because I just got so distracted … I was just bored, because I couldn't do it. (Elva)

The link between engagement levels and learning has been well covered within the literature (Fung et al., 2018). Therefore, students' descriptions of the IBI unit as disengaging at multiple points is concerning.

A common perception was that IBI simply made the problems harder, as expressed by Karson and Sirena below.

[15] Generated using https://www.wordclouds.com/

> I don't see the point in trying to have a go at something that you haven't learnt about. (Karson)

> I'm a lazy person, so I didn't really think much on it. I just didn't really like the fact that I had to do something that I didn't understand. (Sirena)

In these exchanges both students appeared to express the idea that the purpose of mathematics is to execute what has been taught. IBI challenges this notion. The apparent consequence of this challenge was that Karson felt the activity was pointless and Sirena felt disaffection. These were common views amongst those interviewed and align with the ideas discussed in Section 6.4.2.2.1 that students view mathematics as a 'doing' subject (Kouba & McDonald, 1987). In a similar discussion with Elayne, she said:

> I was just staring at this green sheet and it was not giving me any answers, so I kind of lost patience a bit at the end. I have a very short attention span as well, and I was looking at it and the first five or ten minutes, I was like, 'Okay', and then after that I was like, 'This is too hard!' (Elayne)

Elayne appeared to mirror the idea that mathematics is about applying knowledge that has been directly taught. When that knowledge is not present it leads to disengagement. In addition, she appeared to feel that mathematics problems should be doable within a short amount of time and that when she was unable to answer the question within that short time, she concluded that the problem was beyond her capability. She therefore sought to supplement her knowledge gaps from the expert in the room, the teacher. Elayne did not appear to believe in her ability to derive new knowledge through exploration beyond 'five or ten minutes'.

Elayne's previous comment appeared to indicate a student who sees mathematics as a search for answers rather than a search for understanding. Another, perhaps clearer, example of this came from Elva.

> I thought, 'What's happening here? I don't know the answer. What do I do?' When I ask Courtney, she's trying to work it out as well, and I'm just, I don't

> know, waiting for someone to tell me the answer because I can’t work it out anymore. (Elva)

Elva recounted her feelings during one of the IBI problems. Her comments suggested that Elva was focused solely on obtaining the answer. Without knowing what to do or the procedure to apply, Elva disengaged from the task and possibly missed vital learning opportunities.

In the above two extracts, Elayne expected the answers to emerge from the problem (‘it was not giving me any answers’) and Elva expected to know the answer and what to do before she would engage with the problem. These ideas appeared in other interviews, as students seemed to feel that mathematics was mainly about getting the correct answers. However, this approach may be ill suited for IBI, which requires students to attend more to the *process* of problem solving rather than to the *product*. This conflict may have contributed to Elva and Elayne’s lack of engagement.

> I just want the teacher to show me, because it’s just easier than hints. If you kind of get it, that’s when you could have a hint, but when you don’t get it at all, when you’re completely stuck, that’s when they should help you how to solve it. Like, say there’s multiple questions, they show you how to do the first one, then you can get the rest of them. (Cayleigh)

Cayleigh seemed to feel her role was to ‘get’ the answers. Possibly the more answers Cayleigh gets right the more she feels she is doing well. This score-keeper mentality is in conflict IBI which emphasizes depth over breadth, with students spending longer on fewer problems. Taken to its extreme, this mentality could encourage students to cheat, as in the following example.

> I would think, ‘I don’t know this. Oh look, Courtney has sorted this out. I don’t think she’s got it right, but why not.’ Especially when you don’t have anything else and I’m not that great at equations, so I sort of struggled. Something that I can’t figure out so why don’t I just copy? (Sirena)

Students perceived the neglectful nature of IBI through many lenses. In some cases, they felt failure to provide upfront explanations made the problems unnecessarily difficult and robbed them of the ability to work through problems and get answers. In other cases, IBI failed to adhere to their notion of how a mathematics lesson should flow. The IBI approach led to a range of emotional reactions from students which frequently centred on feelings of frustration and panic. Every student in some way expressed ideas that IBI was neglectful. However, students who scored as fixed mindset on the m-ITIS expressed views of IBI as neglect with greater intensity than those who scored as growth mindset (please see Section 4.10 for a discussion of intensity scoring and Table 6.3 for the results).

#### 6.4.3.3 Importance of the teacher

Students expressed views that related to the role of the teacher within the IBI unit. These perceptions of the teacher included the role of the teacher in making engaging lessons, providing individual and equitable support, and keeping order.

The view that teachers should create interesting and engaging lessons was strongly expressed by all students. Examples of this were numerous but perhaps best summarised by Clay when discussing how Ms Silver taught the IBI lessons:

> She made it fun, and she doesn't tell you off as much if you talk in lessons. She doesn't just give you a sheet and tell you to fill it in. She does it … different and more fun, and you try harder when it's not boring. (Clay)

Clay seemed to express a view that was raised by many students, that by creating lessons which are fun the teacher can generate greater engagement and students will 'try harder'. Clay went on to expand upon this point.

> I liked the stories. They were better than just doing worksheets as you got to read a bit. I like reading. It was hard to find out the bits you needed in the story though. That bit was tricky. (Clay)

Researchers have discussed how teachers can generate higher engagement by creating problems that connect mathematics with real-life situations (Wang et al., 2018). Clay seemed to express this view in the above extract. Several of the IBI problems were set in a story (e.g. L4, L5, L6). Perhaps these IBI stories created greater engagement for some students. It is interesting to note that Clay simultaneously described these problems as 'hard' (because of the difficulty of identifying the important parts of the story), but still seemed to enjoy them. This is in contrast with several of the points discussed in Section 6.4.3.2 (IBI as a form of neglect), wherein students typically felt disengaged when problems were viewed as difficult.

Karson was able to articulate the importance of teachers in creating engaging experiences by contrasting Ms Silver with teachers who failed to do so:

> They just said, 'Copy what was on the board,' and then how are we supposed to learn by copying what he's just writing? He told us to copy it as soon as he'd written it, so we'd be copying and if you were a bit behind, he'd scream at you. And then I just didn't understand how that was helping us learn by us copying what he had written. It just confused me. Ms Silver is much better. She tries to involve you more. (Karson)

Another commonly held view was that the teacher was there to provide support. When asked what Cayleigh thought Ms Silver's role was during the IBI unit, she said:

> I think her role is to help the students that need help when they don't get it. So, if they need explaining again, they should maybe get one-to-one, sort of thing. They can come up to you and help you. (Cayleigh)

Cayleigh's perception of the teacher appears to be one of providing the students with sufficient help. This is perhaps in conflict with the approach of IBI used in this case, where little support was offered until after the exploration phase. Cayleigh's comment extended beyond classroom-wide support and called upon the teacher to provide individualised ('one-to-one') support. This individualised comment was echoed by other students who seemed to perceive that Ms Silver should have dedicated personal time to them whenever they hit an impasse. For example, when explaining effective teaching Zelda said:

> If I don't understand something I'll literally just say, 'Ms, I don't get it,' even if she's addressing the class, and she'll be like, 'Oh, I'll come to you in a

minute.' And she does come to us really quickly. That's another good thing about her, she doesn't make you wait 20 minutes and then forget your question. She just straightaway is like, 'Oh, what's wrong?' And then she does explain it. If you still don't understand it, she's like, 'Try a different method of doing it,' and then she gives a different method as well. So, it's quite helpful. (Zelda)

This view would again conflict with the IBI unit as Ms Silver would not have been able to provide this level of support to students during problem exploration. It is possible that this led to students relying on others for this support or disengaging from the task. It is interesting to consider Zelda's above comment alongside the one she made in Section 6.4.3.4 (Page 184), where she admitted that she relied on her peers to provide support.

In addition to providing individualised support, teachers were also expected to provide this equitably. As can be seen in the below extract, Elva felt that Ms Silver should ensure everyone receives help at some point in the lesson. Again, this would have been hard to achieve within an IBI setting. It is possible a student may feel neglected as a result of the limited instruction implicit in IBI exploration, and this student may misconstrue the teacher's encouragement to one group of students as inequitable support.

But yeah, just help everyone and be equal in the amount of help you give. I know some people are lower than others and can use more help, but I think as long as you're giving help to everyone that needs it, I think I'll just be fine because everyone needs a little bit of help sometimes. (Elva)

Many students discussed the importance of the teacher through a lens of classroom management.

I feel like there is enough people being relocated, though, because they're really annoying and it's not really that helpful when you're trying to do some work or something. Sometimes, in some lessons, they're okay to shout but just most of them, they're not very nice. And they don't usually do any work. I think that they're okay at the moment with what we're doing because it's

> quite enjoyable. They haven't really shouted out for a while or, I don't know, thrown something across the class or something. That's good. (Harper)

Harper's comments seem to align with my own observations of the class, which I discuss in Section 6.4.1. In my view, classroom management was a problem, especially at the start of the IBI unit, in which several students were sent to relocation (meaning dismissed from the class) by the end of each lesson. It is possible Harper saw the role of the teacher to keep the class under control. The IBI unit frequently required students to explore problems in groups or pairs. As such the classroom environment could be noisy and difficult to manage. My observations were that classroom behaviour improved as the unit progressed, which Harper seemed to also recognise. Perhaps the IBI unit created enjoyable experiences which caused classroom behaviour issues to reduce.

When discussing the importance of the teacher within mathematics instruction, Elayne explained her view with the following quote.

> I don't really see her [Ms Silver] as a teacher, more as a supervisor to make sure that we didn't get too rowdy or whatever. I think she was more like a supervisor than a teacher. (Elayne)

It seemed Elayne felt that the teacher's primary role was that of classroom management and keeping order. Perhaps Elayne felt that students should take agency over their learning and that teachers create a suitable classroom environment to facilitate learning.

While the role of the teacher was a prominent theme, there was little evidence to suggest that this frequency was influenced by the students' mindsets as students with both growth and fixed mindsets dedicated similar percentages of their interviews to this theme (please see Table 6.3).

#### 6.4.3.4 Importance of peers

The principal feature of IBI, as opposed to direct instruction, is the exploration of a novel problem before formal instruction. This exploration can take place individually or in groups. Ms Silver used a mixture of whole-class, small-group, and individual work when deploying the IBI unit. During interviews a common theme to emerge revolved around the importance of peers during the exploration phase. The nature of this importance was varied and included the ability to obtain support, share knowledge, be creative, and have fun.

Exploration of novel problems can result in students reaching individual impasses. The awareness of these impasses and subsequent bridging of knowledge to overcome them is put forward as a possible benefit to the IBI approach (Chi et al., 1994; Goldin, 2014; Schwartz & Martin, 2004). The interviews suggested that students were able to use their peers to help them overcome some of these impasses and subsequently advance their understanding of the problems. For example:

> If I've done all I can I ask my neighbour who's sitting next to me and if they don't get it, I will ask the teacher. First of all, I try and work it out myself. (Cayleigh)

Cayleigh seemed to see the role of a peer as someone who could help when she felt she had exhausted her own efforts. In this case, Cayleigh placed the help of her peer before that of the teacher, possibly suggesting that the IBI task promoted peer to peer collaboration and discussion. A similar example was expressed by Elva.

> I found that one [the Fibonacci problem] hard to work out, and Courtney had to sit down with me and explain it to me because she got it. But I was like, 'How did you get that? What's happening?' Eventually it made sense. (Elva)

While students seemed happy to explore a problem with a fellow classmate, it did appear that this classmate needed to be someone that the student trusted (as suggested in the below comment by Sirena). It is possible that when reaching what the student believes is an impasse, they seek someone who will provide them with clarity and new knowledge.

> I don't normally ask someone that I'm not friends with because they could lie to me, and I wouldn't know if they were lying. (Sirena)

In addition to valuing peers as a source of knowledge, some students saw peer interactions as an opportunity to share knowledge.

> I don't think she finds it as easy as I do, so I usually try to help when I've finished doing mine…and then I can help her afterwards. So I get it a little, and then she'll get it as well. (Harper)

In the above quote, Harper described how she helped a fellow student on the IBI problems. Ms Silver encouraged the students to help each other at various times during the IBI unit and this likely created opportunities for Harper to help her classmate. It is also possible that by explaining her thinking to a peer, Harper was able to reinforce her own understanding and more clearly define her own strengths and weaknesses on the topic.

As well as providing a platform for knowledge sharing, the peer networks were also highlighted as a source of support and motivation.

> To be honest, I got some help from my friend Harper [on the Ichiro problem]. I was like, 'Okay, I'll try and do this', and then she explained to me how to look at it and was like, 'You can do it!' I was like, 'Oh okay, I'll just continue it on then.' And then when I got to the answer, 15, I was like, 'Is this the answer? Is it not the answer?' And I asked Harper and she was like, 'Yes, that's the answer.' Like, 'Yes! I did it!' (Zelda)

The above extract highlights how peer interaction supported Zelda. By discussing the problem with Harper, who did not disclose the answer but provided encouragement, Zelda was able to be motivated enough to continue exploring the problem which ultimately led to a successful outcome.

In addition to sharing knowledge, some students also felt that talking with peers was a good way to develop innovative approaches that they might not have considered in isolation.

> I mean, sometimes I sort of asked the people around me what they were thinking to see if they're thinking the same thing as me. I suppose talking to someone about it can kind of help, I guess, because if they've got the same ideas then it's kind of easier to talk to them about it. Or maybe you can talk it over and come up with a new way that you haven't tried. (Ethel)

In the above extract, Ethel suggested that peer discussions were a valuable source of innovation during the IBI unit. By working together, Ethel felt that she was able to develop new approaches to explore the problems that might not have been considered individually. The students within this class had little or no prior exposure to IBI and therefore might be unfamiliar with how to explore a problem in a creative way. The availability of peer networks to supplement this creativity may have been why so many students expressed this theme.

In addition, several students extended the above discussion about group creativity to one of group consensus.

> You might not have thought about it that way. Maybe they have a different way and you can try them both and see which one is the most right, or maybe the best. You can, like, suss it out. (Opal)

Opal appeared to believe that the IBI problems could be solved in multiple ways and that the solutions might be exposed to her though peer cooperation. Through an active dialogue, a consensus might emerge. This is similar to the ideas put forward by D. Johnson and Johnson (2016) who discussed how students coalesce around a single solution during inquiry-based discussions.

Another way in which students discussed the importance of peers was how exploring problems with peers was enjoyable.

> R: You said it [maths] was fun, exciting and different. When is it most fun?
>
> E: When we're working in groups with your friends, or people you've never worked with before, and you are all getting stuck in. Like, you're talking about work and non-work at the same time, but you are working.
>
> (Researcher and Eleri)

Eleri expressed a preference for group work. She described group work as fun and that it possibly cultivated greater work ethic ('getting stuck in'). Her suggestion that it can be fun to work with new peers ('people you've never worked with before') might suggest that she sees value in obtaining different points of view when exploring novel problems. This idea was expressed again by Eleri later in the interview:

> R: Is there something that could have been done better [during the IBI unit]?
>
> E: If we changed seats more often, because we have other people to work with.
>
> (Researcher and Eleri)

There were occasions in which the importance of peers was discussed in a more negative light. For example:

> Sometimes it's helpful. But to be honest, when you come to do tests it's only you that's doing it. So, from that point of view, it's not that helpful because you haven't got your group to help you on the tests. But sometimes, it's quite fun. (Opal)

There are several comments worth highlighting in the above extract. Firstly, Opal seemed to mirror earlier comments that mathematics is a 'doing' subject, in this case knowing mathematics meant passing tests. Secondly, Opal suggested that understanding was ultimately individual, and the group would not always be around to supplement her understanding. It appeared Opal did not appreciate that exploration with peers might lead to a deeper individual understanding of concepts. Other students also expressed issues with peer exploration of the IBI problems, suggesting that a balance of individual and peer-based work was preferable.

> It depends. Sometimes it's good to work together and try stuff as a group. But sometimes it's distracting, and you want to work alone. (Clay)

When analysing the prevalence of this theme between students of differing mindsets there was no noticeable difference (see Table 6.3).

#### 6.4.3.5 Pace and format

As already noted, the students in Ms Silver's class had little prior experience with IBI in mathematics. Historically, Ms Silver tended to focus on upfront explanations or guided worksheets followed by a series of questions for the students to practice. Therefore, it is not surprising that all students expressed views which suggested they were aware of the change in teaching methodology. These perceptions tended to focus on the structural differences (exploration preceding instruction) and the time allotted to each problem.

Elayne's quote, previously discussed on Page 171, suggests that the students were aware of the difference between their normal mathematics lessons and were tempted to ascribe a motive to the teacher's decision to change (i.e. to 'boost our self-confidence'). Also, Elayne's suggestion that this approach required them to 'think differently' is interesting, as it might be said to align with the views of proponents of IBI who argue that exploring the problem space allows students to develop a deeper understanding of the problem and their knowledge gaps. This idea was also captured in discussion with other students.

> S: It's harder, well maybe not harder but different. Ms was like just keep trying, but we didn't know what to do. That bit was hard for me. Me and Dori worked on it together. We got some bits but then sort of hit a dead end. When we asked Ms for help, she said… 'just keep trying'. We did keep trying a few other ways, but we didn't get it right.
>
> R: Did you understand it in the end?
>
> S: Not really, we didn't get it. Not until Ms explained it at the end when we totally understood it and everything.
>
> R: Do you think it would have been better if Ms Silver had explained it for you at the beginning rather than at the end?
>
> S: I don't know. It's nice to have a go. You can figure out which bits you know well and which bits you don't.
>
> (Sirena and Researcher)

In the above exchange, Sirena seemed to express several views towards IBI. Initially, she seemed to believe that the exploration portion of the lesson was more challenging than her usual maths lessons. Her comment that she was able to try a 'few other ways' suggests she was able to successfully engage with the task but recognised that understanding did not come until 'Ms explained it'. Sirena's final comment, that exploration of a novel problem allows a person to 'figure out which bits you know well and which bits you don't' would suggest that she was able to identify her knowledge gaps and possibly develop a better understanding of the content from the direct instruction that followed. This comment aligns with other researchers' views that awareness of knowledge gaps facilitates learning (Chi et al., 1994; Schwartz & Martin, 2004).

In addition to structure, many students discussed how the pacing of the IBI lessons felt different. There is evidence within the literature that students hold the view that problems should be solved in less than ten minutes (Schoenfeld, 1985). Proponents of IBI often cite the advantages of extended exploration of a novel problem, believing it can lead to greater conceptual understanding and increased engagement. These views would seem to be supported by the following interview extract.

> It was quite good that you … got a lot of time, because usually it's hard to just do it in about ten minutes, because you might not be able to fully do it all and you might still not have finished after 20 minutes or more or the next lesson or something. It's quite hard, and it's good to get that extra time to have a proper go at it before going over it. (Harper)

Initially, it would appear that Harper continued to hold the idea that doing mathematics meant finishing problems. This might be seen to align with the previously discussed point that students see mathematics as a 'doing' subject (Section 6.4.2.2.1). However, later within this extract Harper acknowledged that the goal might actually be to 'have a proper go at it' and that this exploration might not yield the answer. Harper's suggestion that additional time was helpful for the IBI task might indicate she was moving beyond the idea that mathematics has to incorporate short problems that can be solved quickly.

However, not all students agreed with Harper.

> The problems were interesting, but I think after a while you just got sick of looking at the same piece of paper and the same question. At the same time, it did give you enough time to be like, 'Uh, I don't understand this,' but I think

> it was too much time. After a while, I completely zoned out. I don't even know what I'm doing anymore. (Elayne)

Elayne seemed to believe that the class was given too much time to work on the IBI problems, and this broke with her normal expectations. Elayne and her classmates had little previous exposure to IBI and therefore the idea of spending 30 minutes on a single problem was abnormal. Elayne's confession that she zoned out, despite reportedly finding the problems 'interesting', suggests that she might lack the persistence necessary to keep exploring the problem space.

Perceptions of pace and format were expressed by all interviewees to some extent. There was no difference between student mindsets with regard to the amount of time dedicated to this theme (see Table 6.3).

#### 6.4.3.6 Summary of student perceptions

The above sections analyse the results of 12 interviews with students who had taken part in the IBI unit. The interviews suggested that students perceived the unit within five primary themes. Firstly, students felt that the inquiry approach helped to empower them as they expressed the positive effect of exploring challenging problems. Students suggested the IBI increased their self-confidence and agency. Secondly, students seem to suggest that IBI was neglectful, since exploring a problem without prior instruction was a poor way to gain understanding. Thirdly, all students recognised the role that Ms Silver played in providing engaging content. Students also felt that part of the teacher's role in IBI was to provide classroom management. Fourthly, peer networks were an important part of the inquiry approach, with peers providing support, motivation, and innovative ideas. Finally, the inexperience that these students had with IBI possibly led to many students feeling the method disagreed with their understanding of how a mathematics lesson should flow, with instruction typically preceding practice problems. Furthermore, the IBI problems challenged the students' notions that problems should be short.

### 6.4.4 An analysis of student learning outcomes

The overall scores of the students with MD increased from 16.8 to 22.7, their procedural scores increased from 13.1 to 17.0, and their conceptual scores increased from 3.7 to 5.7. Please see Table 6.4.

Table 6.4: Pre- and post-test results for Ms Silver's class

| | Test component | Max | Pre-test | | Post-test | |
|---|---|---|---|---|---|---|
| | | | *M* | *SD* | *M* | *SD* |
| Students with MD (*n* = 18) | Overall | 33 | **16.8** | 3.6 | **22.7** | 3.8 |
| | Procedural | 21 | **13.1** | 2.8 | **17.0** | 1.6 |
| | Conceptual | 12 | **3.7** | 2.2 | **5.7** | 2.7 |
| MD and Fixed (*n* = 5) | Overall | 33 | **15.2** | 3.3 | **22.4** | 3.5 |
| | Procedural | 21 | **12.0** | 3.2 | **17.2** | 2.2 |
| | Conceptual | 12 | **3.2** | 1.1 | **5.2** | 1.6 |
| MD and Growth (*n* = 12) | Overall | 33 | **17.2** | 3.8 | **22.4** | 4.0 |
| | Procedural | 21 | **13.3** | 2.7 | **16.9** | 1.4 |
| | Conceptual | 12 | **3.8** | 2.6 | **5.5** | 2.9 |

Table 6.5: Growth and fixed post-test results for Ms Silver's class

| | Test component | Max | Fixed *n* = 5 | | Growth *n* = 12 | |
|---|---|---|---|---|---|---|
| | | | *M* | *SD* | *M* | *SD* |
| Post-test | Overall | 33 | **22.4** | 3.5 | **22.4** | 4.0 |
| | Procedural | 21 | **17.2** | 2.2 | **16.9** | 1.4 |
| | Conceptual | 12 | **5.2** | 1.6 | **5.5** | 2.9 |

Separating the results of the students with MD by mindset shows that students with both growth and fixed mindsets made improvements over the course of the IBI unit. There was no difference observed on the post-test between those who reported holding a fixed mindset and those who reported holding a growth mindset (see Table 6.5). Furthermore, the post-test scores were not strongly correlated to mindset nor any of the subscales of the ATMI (see Table 6.6).

Table 6.6: Spearman's rank correlation for the post-test, mindset, and ATMI

| | Post-test<br>*n* = 18 |
|---|---|
| | $r_s$ |
| Maths mindset | **-.17** |
| Enjoyment | **-.06** |
| Motivation | **.08** |
| Self-confidence | **.00** |
| Value | **.09** |

Looking at the pre- and post-test by item shows the 18 students with MD improved on seven test items of procedural knowledge and six test items of conceptual knowledge. Please see Table 6.7 for more information.

The procedural test items the students improved most on were test items 1c, 3c, 5a, 9a2, 9b1, 9b2, and 14a (see Table 6.7). Test items 1c, 3c, 9b1, and 9b2 asked students to solve an algebraic equation for a single unknown variable. Each of these equations could be solved in just one step (for example, by multiplying 34 by 13). The students also improved on test items that asked them to evaluate an algebraic expression for a given value (5a and 14a) as well as items that asked them to write an algebraic expression to represent a given quantity (9a2). Improvement on these types of problems might be understandable when considering how much time students spent exploring linear relationships and thinking algebraically.

The conceptual test items the students improved on were test items 6, 7, 8a1, 8b1, 10a, and 15 (see Table 6.7). For test item 6, students were given a diagram of two pouches and told their contents were the same. Each pouch was labelled with a different algebraic expression containing the variable *y*. Students were asked to work out the value of *y*. Essentially this problem required students to solve an algebraic equation in which the unknown quantity appeared on both sides of the equation. By contrast, test item 7 provided students with a system of equations containing two unknown variables, the price of a shirt and the price of a jumper. Students were asked to work out the price of one jumper. Items 8a1, 8b1, and 10a asked students to reason about the relationship between different expressions. Test item 15 asked students to write an algebraic expression to represent the given shaded area. As before, students had many opportunities throughout the IBI unit to think abstractly in a variety of situations, and this may have led to increased conceptual understanding.

Table 6.7: Mean pre-test and post-test scores by test item

| | Test item | Pre-test | | Post-test | |
|---|---|---|---|---|---|
| | | *M* | *SD* | *M* | *SD* |
| Procedural | 1c | **.50** | .51 | **.89** | .32 |
| | 3c | **.78** | .43 | **1** | 0 |
| | 5a | **.78** | .43 | **1** | 0 |
| | 9a2 | **.72** | .46 | **.94** | .24 |
| | 9b1 | **.78** | .43 | **1** | 0 |
| | 9b2 | **.78** | .43 | **1** | 0 |
| | 14a | **.22** | .43 | **.61** | .50 |
| Conceptual | 6 | **.22** | .43 | **.56** | .51 |
| | 7 | **.22** | .43 | **.67** | .49 |
| | 8a1 | **.28** | .46 | **.67** | .49 |
| | 8b1 | **.28** | .46 | **.50** | .51 |
| | 10a | **.56** | .51 | **.83** | .38 |
| | 15 | **.06** | .24 | **.39** | .50 |

This study focuses on the performance of students with MD, whom in this case, happened to also score among the lowest on the pre-test. Therefore, it is possible that the observed changes, described above, are actually a result of the data regressing towards the mean (Marsden & Torgerson, 2012). However, because students in this study were not selected on the basis of their pre-test scores, this effect is thought to be minimal.

## 6.5 Summary of the Case of Ms Silver's Class

The case study of Ms Silver's class was designed to answer two research questions: (RQ1) How do students with mathematics difficulties perceive IBI? and (RQ2) Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties? In this section I summarise how the analysis presented in Section 6.4 addresses these questions.

Both research questions make the assumption that the lessons within the unit were taught to a sufficient quality of inquiry to be called IBI. Therefore, before addressing the research questions, consideration has been given to the quality of inquiry actually delivered. This was assessed using the EQUIP rubric (Section 6.4.1). Taken as a whole, it was determined that the unit met the level of 'proficient inquiry', which meets the threshold laid out in the case study design (Section 4.3). Two small areas within the EQUIP rubric scored below this threshold. The first related to the Teacher's Role, in which Ms Silver maintained the central role and missed opportunities to allow students to drive the flow of the lesson. The second related to communication patterns, in which discussions at a class level were typically didactic between a student and the teacher, providing little opportunity for direct student-to-student interaction. One area that exceeded 'proficient inquiry' related to content depth, in which Ms Silver was able to use her strong background in mathematics to make explicit and clear connections to the big picture.

Over the course of 12 interviews with 12 separate students with MD, combined with my own observations of the lessons, a number of themes emerged from the data regarding students' perceptions of the IBI unit. These themes helped me to answer my first research question, (RQ1) How do students with MD perceive IBI?

The first of these perceptions related to IBI as empowering (Section 6.4.3.1). In this theme students expressed ideas that the IBI unit allowed them to 'engage their brains' and 'get stuck in'. Students also seemed to expand their notion of mathematics beyond the simple application of a procedure. Some students communicated an increased sense of agency and pride.

Secondly, and somewhat contradictory to the previous theme, students perceived IBI as neglectful (Section 6.4.3.2). Students appeared to hold notions that mathematics should be explained first and then practiced. The IBI approach therefore challenged these notions and left some students perceiving the change as an act of neglect. Feelings of panic and distress were expressed as some students struggled to tackle the exploration of problems for which they received minimal prior instruction.

Students who scored as growth mindset expressed perceptions of IBI as empowering with greater intensity than those who scored as fixed mindset on the m-ITIS. The inverse was true for the idea that IBI was neglectful, with fixed mindset students expressing this perception with greater intensity than those holding growth mindsets. Please see Section 4.10 for a discussion of intensity scoring and Table 6.3 for the results.

The third theme that the students' perceptions fell under was the importance of the teacher in creating engaging lessons, providing individual and equitable support, and keeping order (Section 6.4.3.3). Students contrasted the IBI unit with those lessons that followed a more traditional approach. They frequently perceived the IBI tasks to be more 'fun'. In addition, many students appeared to hold the belief that their teacher should provide a high level of individual and explicit instruction. This belief appeared to conflict with the teacher's role during the explore portion of an IBI lesson, in which the teacher refrains from explaining how to solve the problem until after the students have had a chance to explore.

Importance of peers was the fourth theme which students perceived during the IBI unit (Section 6.4.3.4). The nature of this theme was varied. Students reported that they enjoyed collaborating with their peers because it felt good to be able to help, reinforced their own understanding by explaining to someone else, and encouraged them to keep going. Students appeared to perceive that they were able to overcome impasses by sharing knowledge, and that through this sharing they could generate innovative solution pathways to explore.

The final theme to emerge from the case was the students' perceptions of pace and format of the lesson and problems (Section 6.4.3.5). Students perceived the change in lesson format during the IBI unit, recognising that exploration before instruction was 'different'. Some students even felt that this new format was an attempt by the teacher to 'boost their self-confidence'. Students were also aware that the pace of the lesson (as measured by number of problems given), and problem length (as measured by time spent per problem), were much longer under IBI. This contrasted to expressed views that mathematics problems should be short and straightforward.

The second research question, RQ2, seeks to understand whether students' beliefs were associated with the effectiveness of IBI. To answer this, I drew upon the students' pre- and post-test scores (Section 6.4.4) as well as classroom observations (Section 6.4.2). Classroom observations were especially critical in answering this research question since correct answers on the pre- and post-test had the capacity to mask misunderstanding while incorrect answers had the capacity to mask understanding. In this way, the pre- and post-test complemented classroom observations and, crucially, provided triangulation.

An analysis of the pre-test and post-test results shown in Table 6.4 indicates the overall scores for the 18 students with MD improved from 16.8 to 22.7, the procedural scores improved from 13.1 to 17.0, and the conceptual scores improved from 3.7 to 5.7. Looking at these results in consideration of the students' mindsets (Table 6.5) showed that holding a growth or fixed mindset was not associated with pre-test or post-test performance. Furthermore, post-test scores were not strongly correlated with scores of mindset and attitude (see Table 6.6).

The analysis of the lesson observations was done using the McLeod (1992) framework. Firstly, the students appeared to hold various beliefs about the nature of mathematics. These included beliefs such as the idea that mathematics is something one 'does', the belief that mathematics problems should not take long to solve, and the belief that mathematics problems should have a single solution pathway. These beliefs were associated with task disengagement.

Students also appeared to hold beliefs about themselves. These covered the areas of self-efficacy (Bandura, 1997) and student mindset (Dweck, 2017b). Students appeared to have low self-efficacy when they struggled to explore the given IBI task. Holding this belief appeared to hamper the effectiveness of the IBI, with students frequently failing to start the problem at all. The observed effect of mindset on student performance was mixed. Students holding a fixed mindset did tend to struggle on the IBI problems, sometimes blaming their 'brains' or even inherited family traits. However, in some cases holding a fixed mindset suggested a positive impact on performance. Where this occurred, it was associated with students who held the view that they were 'good at maths', and hence, motivated to validate themselves as capable mathematicians.

Several beliefs about the way in which mathematics should be taught seemed to emerge during the observations. As I discussed above, students often felt that mathematics problems should be short (both in length and time), and holding this belief seemed to be negatively associated with IBI's effectiveness during the observations. Some students were able to see the change in teaching approach as a positive, believing that the variety was helpful. However, many students appeared to hold firm beliefs that explicit instruction should precede problem exploration, especially those that performed relatively well under the traditional approach. This belief appeared to hinder the effectiveness of the IBI.

Finally, students held beliefs about the wider social context of mathematics. Where students were able to make connections between the mathematics problems and their real-life social world, the result was increased engagement. However, holding a dim view of the value of mathematics in the real world seemed to lead to task disengagement on the inquiry problems. Such a belief might be obtained from the family social context, e.g. 'My dad always says no one ever uses the maths they learn at school'.

# 7 Discussion

A review of the literature on inquiry-based instruction (IBI), student affect, and mathematics difficulties (MD) is presented in Chapter 2. The literature in these respective areas is well established, and numerous applications to practice have emerged over the last two decades. However, the extent to which these three constructs *interact* has received less attention. Studies that have explored affect in mathematics have usually looked into it as an *outcome* of IBI rather than as a potential moderating factor (Kogan & Laursen, 2014; McGregor, 2014). Furthermore, the literature fails to explore these constructs in the context of students with MD. Therefore, this thesis set out to study two research questions:

RQ1. How do students with mathematics difficulties perceive IBI?

RQ2. Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties?

This chapter combines the analysis of the first case study (Chapter 5) and the second case study (Chapter 6) and offers a discussion of the findings in relation to each research question. I then discuss how these findings make a contribution to the literature. Finally, I conclude the chapter with implications for practice, notable limitations, and directions for future study. Please see Table 7.1 for a summary of the findings from case 1 and 2.

Table 7.1 Summary of findings from case 1 and 2

| RQ | Case 1 findings | Case 2 findings |
|---|---|---|
| 1 | Students seemed to perceive IBI according five themes.<br>• IBI as form of empowerment<br>• IBI as a form of neglect<br>• Importance of the teacher<br>• Importance of peers<br>• Mathematics disaffection | Students seemed to perceive IBI according five themes.<br>• IBI as form of empowerment<br>• IBI as a form of neglect<br>• Importance of the teacher<br>• Importance of peers<br>• Pace and format |

| | | |
|---|---|---|
| 2 | Several student beliefs seemed to be associated with the effectiveness of the IBI unit.<br>• Mathematics is unrelated to reality<br>• Mathematics is something one 'does'<br>• Problems in maths can only be solved in one way<br>• Low self-efficacy<br>• Fixed mindset<br>• A mathematics lesson should include multiple short problems that can be solved in ten minutes or less<br>• The teacher is an important source of knowledge<br>• Bottom set students do not enjoy maths | Several student beliefs seemed to be associated with the effectiveness of the IBI unit.<br>• Mathematics is unrelated to reality<br>• Mathematics is something one 'does'<br>• Problems in maths can only be solved in one way<br>• Low self-efficacy<br>• Fixed mindset<br>• A mathematics lesson should include multiple short problems that can be solved in ten minutes or less<br>• The teacher is an important source of knowledge<br>• Some careers are only accessible to the top set |

**Successful inquiry implementation**

Despite the popularity of IBI, evidence for its effectiveness for students with MD has been mixed, and teachers have demonstrated a reluctance to use these techniques with this group of students (Darragh & Valoyes-Chávez, 2019; Lambert, 2018; Louie, 2017). This reluctance partly comes from the idea that IBI is too discovery-oriented (Carnine, Jones, & Dixon, 1994; Woodward & Baxter, 1997) and that students with MD cannot engage in higher-order thinking (Zohar et al., 2001) or independent exploration (Mazenod et al., 2019). In addition, teachers who wish to implement IBI for the first time face a number of challenges, for instance poor student behaviour (Stylianides & Stylianides, 2014), and they tend to revert to the direct teaching approaches that they used previously (H.-C. Li & Stylianides, 2018).

Despite the teachers of both case studies having little prior experience with IBI, both were able to deliver mathematics instruction to a proficient level of inquiry based on the EQUIP (see Appendix L and M for an analysis of the EQUIP ratings in each case). The data also suggest that, as the study progressed, both teachers improved their proficiency for inquiry instruction, spending more time facilitating discussions and explorations (see Figure 5.3 and Figure 6.3). H.-C. Li and Stylianides (2018) saw a similar effect when studying Taiwanese teachers implementing inquiry-based teaching of mathematics for the first time. In H.-C. Li and Stylianides's study, the teacher initially embraced the facilitator role, then reverted to traditional instruction, before finally embracing the facilitator role again in the final third of the 19-lesson intervention. In my study, both Mr Scott and Ms Silver gained proficiency quickly and sustained this proficiency throughout the IBI unit.

Overall, I draw several conclusions from this. Firstly, teachers new to IBI can implement inquiry to a proficient level with the help of tools such as the EQUIP. Secondly, contrary to some of the literature, both teachers were able to establish proficiency quickly and consistently over the course of the IBI units.

## 7.1 Research questions revisited

In this section I discuss how the analysis across both cases addresses the two research questions.

**RQ1: How do students with mathematics difficulties perceive IBI?**

Across the two cases a total of 22 interviews were conducted (ten in Mr Scott's case and 12 in Ms Silver's case). Numerous themes emerged from these interviews, the analyses of which are presented in Section 5.4 and Section 6.4. By comparing the findings across the cases, I categorise the students' perceptions of IBI into several themes: (1) IBI as a form of empowerment, (2) IBI as a form of neglect, (3) importance of the teacher, and (4) importance of peers.

**IBI as a form of empowerment**

Across both cases, students perceived that IBI was an empowering way of doing mathematics. This perception has been discussed in the literature. For example, Hassi and Laursen (2015) found that mathematics teaching that focused on inquiry and student collaboration resulted in undergraduate students' transformation towards 'personal empowerment' (p. 316). The results of this thesis extend Hassi and Laursen's finding to secondary school students with MD. This finding is also complementary to that of Foster (2013b) who claimed reductionist teaching practices can be 'dangerously disempowering' for students (p. 564).

One of the ways in which students felt IBI was empowering was the way it gave them more confidence ('I think it made me feel more confident in the subject and the topic we're doing', James, Mr Scott's class). This echoes findings of other researchers, such as Kogan and Laursen (2014) who suggested that mathematics courses which focused on IBI increased the confidence of undergraduate students as well as the likelihood they elected to take further maths courses. Kogan and Laursen's work focused on university level students and used students' course selections as a measure of confidence. My finding, that students perceived IBI to improve their confidence, aligns with this literature and extends the finding to include secondary students with MD.

Within the theme of IBI as empowering, students in both cases perceived IBI to give them greater control over their learning and as a result increased their motivation. Increased motivation has been suggested as one of the primary benefits of IBI (Glogger et al., 2013; Hmelo-Silver, 2004). In addition, it has been found that students are more motivated when they believe they have control over their learning (Bandura, 1997). Therefore, the perception of increased motivation across these two cases is consistent with previous research and suggests that students with MD experience this reportedly favourable effect.

Students across both cases felt that IBI promoted a deeper understanding of the mathematical concepts. My observations of the students in their lessons (see Section 5.4.1 and Appendix L.1.5 for Mr Scott's case and Section 6.4.1 and Appendix M.1.5 for Ms Silver's case) as well as my analysis of their pre- and post-tests support this view (see Section 5.4.4 and 6.4.4). Enhanced conceptual understanding is an oft-cited benefit of IBI (Boaler, 1998; Cobb et al., 1991; J. C. Marshall & Horton, 2011). The emergence of this idea within both case studies therefore builds upon this research in the context of secondary students with MD.

Various mechanisms have been put forward to explain why IBI might be effective at promoting a deeper understanding of mathematical concepts. By allowing students to explore the problems it is proposed they become conscious of gaps in their knowledge and that this awareness facilitates the assimilation of the missing pieces (Loibl & Rummel, 2015; Schwartz & Martin, 2004). Studies also highlight that inquiry-based tasks help activate deeper awareness of the learning processes and prepare students for subsequent direct instruction (Kapur, 2010, 2011, 2014; Schwartz et al., 2011; Schwartz & Martin, 2004). I see evidence in both cases which align with the research in this regard. For example, in the following quote, Kevin explains that he finds learning easier if he attempts the problem first with follow-up direct instruction from the teacher.

> Because I like to be able to have a challenge and not just know what it is. Because if he's going to discuss after, like what has happened and all that, then it's a lot easier. And then he can go through it with us after to explain how to do it. And then we could try it again after that to get it right. (Kevin, Mr Scott's case)

**IBI as a form of neglect**

The previous section explored how students perceived IBI as empowering. While all students expressed this view, there were also times the students expressed the somewhat contrasting view that IBI was neglectful. The most common sub-theme to emerge under the theme of neglect was that the IBI approach made learning mathematics more difficult ('Because doing it without knowing what you are doing is hard, and when you know what you are doing it's easier', Lisa, Mr Scott's case). Students seemed to view mathematics as something one 'does' and IBI was difficult partly because the teacher had not told the class what to do. Students did not seem to see the exploration phases of the IBI unit to be part of the mathematical process. Research has shown that many students see mathematics as a subject based purely on rules and procedures that must be memorised (C. A. Brown et al., 1988; Frank, 1988; Kouba & McDonald, 1987). The exploration phase of IBI appeared to conflict with these beliefs as students were expected to explore the problem space without being told what to do (i.e. what rules to follow or calculations to perform). This prevented the students from quickly obtaining the correct answer, which was often viewed by the students as the goal of the lesson. This finding is echoed by previous research (e.g. Schoenfeld, 1985). It seemed this left many of the students in both case studies feeling neglected.

Students across both cases perceived the order of instruction as confusing and unhelpful ('It was confusing, and why would he not explain the topic first, and why would he make us do it first and not explain it to us? It wasn't helpful', Lisa, Mr Scott's case). This type of response might stem from the change in teaching order, from a *tell-and-practice* model of teaching towards an *explore-and-discuss* model of teaching. However, as an alternative explanation, one might argue that the increased confusion stemmed from a fundamental conflict between IBI and Cognitive Load Theory (CLT; Kirschner et al., 2006). CLT states that complex learning situations such as IBI result in excessive extraneous load that make it difficult for students to acquire long term learning. The extraneous load created by the IBI in both cases could account for why some students reported feelings of confusion.

Across both cases, students were asked to describe how they would have taught the IBI lesson or what they would change about the way their teacher taught it. In response, students frequently suggested the neglectful nature of not explaining first. Statements such as Elayne's comment below are illustrative of this perception:

> …going through it at the beginning of the lesson, instead of just handing us the paper and going, 'There you go, try and work that out.' (Elayne, Ms Silver's case)

**Importance of the teacher**

Students across both cases expressed comments which suggested they perceived the teacher to have an important role within IBI. Their perceptions of the teacher's role within IBI was multi-dimensional.

Firstly, students felt that the teacher's role was to create engaging lessons. Typically, this meant working on real-world problems. This seemed to translate into students feeling the problems were more fun and resulted in increased persistence and effort. This finding, while not previously explored for students with MD in the literature, is perhaps unsurprising given the literature linking engagement with the use of real-world mathematics problems (Boaler, 1998; Wang et al., 2018).

Another commonly held perception was that the teacher should provide individual support and help when students reach an impasse ('I think her role is to help the students that need help when they don't get it', Cayleigh, Ms Silver's case). This perception has the potential to conflict with the exploration phase of IBI, in which help from the teacher is meant to be delayed. This finding could be argued to overlap with the previously discussed perception of IBI as neglectful, suggesting that these themes may not be entirely mutually exclusive. Furthermore, this finding might also align with what others have found when exploring students' views about mathematics teaching, for example, the view that the teacher's role is to help the students learn mathematics (Kloosterman, Raymond, and Emenaker, 1996), or the view that the teacher's role is to transmit knowledge whilst that of the student is to receive knowledge (Frank, 1988).

In addition to providing individualised support, teachers were also expected to provide this equitably. This perception was commonly expressed negatively, meaning students felt that the teacher was inequitable in how they assisted students ('Just help everyone and be equal in the amount of help you give', Elva, Ms Silver's case). It is possible a student may misconstrue the teacher's encouragement to one group of students as inequitable support. Overall, this finding is interesting, as the literature has tended to suggest that IBI leads to increased equity (Boaler, 2002). Laursen and Rasmussen (2019) argue that one of the four pillars of effective IBI is that instructors foster equity in their design and facilitation choices. The finding that students might view IBI as inequitable is somewhat at odds with this literature and a potential area for further study.

In both cases, students perceived the role of the teacher was to provide classroom management and maintain order. The emergence of this perception is interesting as previous literature has highlighted that teachers can struggle to maintain good classroom management when implementing IBI, and this leads to many teachers avoiding this pedagogical practice (Stylianides & Stylianides, 2014). Students' perceptions of this issue may demonstrate their awareness of classroom management concerns and see this as the teacher's responsibility. The EQUIP describes the role of the proficient inquiry teacher as one that is primarily facilitative. It therefore does not explicitly address the teacher's role in classroom management, and this presents one area in which the EQUIP might be developed.

**Importance of peers**

Cross-case analysis of the data suggests a common set of perceptions that students held about their peers. These seemed to fall into subthemes of (1) peers as a source of knowledge; (2) peers as a source of motivation; and (3) peers as a source of innovation.

Some researchers have suggested that students can hold the belief that mathematics learning is done individually (Kloosterman et al., 1996). However, across both of the case studies students tended to perceive their peers as important sources of knowledge ('If I've done all I can I ask my neighbour who's sitting next to me', Cayleigh, Ms Silver's case). Researchers have suggested that students' early epistemological beliefs about mathematics are that knowledge is transmitted from an authority figure (Perry, 1970; Schommer, 1990). Therefore, the common perception of peers as a source of knowledge is somewhat surprising. Moreover, students seemed to view this knowledge sharing as bi-directional. Students suggested they could share knowledge with peers as well as receive. D. Johnson and Johnson (2016) might have considered this bi-directional sharing of knowledge as an example of positive social interdependence, with groups believing they can reach their goals if others within their group also meet their goals.

Across both cases, peers were frequently cited as a source of enjoyment and motivation when exploring IBI problems. This is especially positive given increased motivation has been associated with greater learning outcomes (Belenky & Nokes-Malach, 2012).

Finally, many students felt that peer discussions were a valuable source of innovation during the IBI unit. By working together, students felt they were able to develop new approaches to explore the problems. This finding connects with the work of D. Johnson and Johnson (2016) who assert that outputs from group work are greater than what can be achieved individually. Even though my study did not include a control group to contrast students that worked together versus students that worked individually, the students' perceptions of greater innovation as a result of group work could be an indication of the kind of enhanced outputs described by D. Johnson and Johnson (2016). The students within these cases had little to no prior exposure to IBI and therefore might be unfamiliar with how to explore a problem in a creative way. The availability of peer networks to cultivate this creativity (D. Johnson & Johnson, 2016) may explain why so many students expressed this theme.

**Other perceptions**

Two further themes emerged when analysing the students' perceptions of IBI across the two case studies, although each theme emerged within only one of the cases. The first is mathematics disaffection, which was a theme from Mr Scott's case, and the second is pace and format, which was a theme from Ms Silver's case. Therefore, a cross-case comparison suggests a lack of triangulation for these findings. However, it is worth touching briefly on these and considering why they may not have been expressed as strongly across both cases.

Perceptions of mathematics disaffection emerged strongly within Mr Scott's case, although it was not clear whether this was a result of the IBI or prior experiences with mathematics.

Most comments referenced a general sense of disaffection with the way mathematics is taught, such as mathematics is boring and all about completing worksheets and seatwork. This finding is not surprising given others have argued students view mathematics as something that is undertaken individually (Kloosterman et al., 1996) and involves mainly seatwork (Stodolsky, 1985). It is unclear why this theme was so strong within Mr Scott's case but less so in Ms Silver's. My observations of the lessons do not lead me to conclude that either teacher was more or less engaging, and both teachers reached a proficient inquiry level on the EQUIP rubric. I note that, within Mr Scott's class, many of the comments regarding mathematics disaffection seemed to draw from prior experiences that extended beyond their time with Mr Scott. This might suggest the historic classroom experiences of the students in this case led them to express views of disaffection more than students in Ms Silver's case. The approaches of different teachers is known to play a meaningful role in students' overall attitudes towards mathematics (Boaler, 2002; Boaler & Greeno, 2000; De Corte et al., 2010).

With regards to the theme of pace and format, students felt that IBI lessons took longer than they expected based on past experiences. This is consistent with the literature that students believe mathematics problems should be solvable in less than ten minutes (Schoenfeld, 1985). In addition, students in Ms Silver's class perceived the change in the order of instruction within the IBI lessons. Students were aware that the exploration seemed to precede direct instruction. It is possible that students were more aware of the change in format given Ms Silver's more demonstrative teaching style prior to enacting the IBI unit (discussed in Section 6.1.1). This contrasts with Mr Scott's alleged preference for experiential learning prior to beginning the IBI unit (discussed in Section 5.1.1).

**The association of mindset and perceptions of IBI**

The analysis of student interviews was essential in evaluating the students' perceptions of IBI. The students selected to partake in these interviews were broadly balanced across the mindset categories (see Table 5.3 and Table 6.3). Perceptions of the importance of the teacher and the importance of peers were expressed to similar extents by students with growth, mixed and fixed mindsets. However, this did not appear to be the case for the themes of IBI as empowerment and IBI as neglect. Students with growth mindsets expressed the theme of IBI as empowerment with greater intensity (Boyatzis, 1998) than those with fixed mindsets in both cases. Conversely, students who held fixed mindsets expressed the theme of IBI as neglect with greater intensity than those with growth mindsets in both cases.

This finding is potentially important as the ideas contained within the theme of IBI as empowerment include self-efficacy, motivation, agency, and engagement, all of which are associated with improved mathematical performance in the literature (Bandura, 1997; Boaler, 1998; Dweck, 2017b; Glogger et al., 2013; Hmelo-Silver et al., 2007; Kogan & Laursen, 2014). Conversely, the ideas within the theme of IBI as neglect have been negatively associated with performance (Kouba & McDonald, 1987; McLeod, 1992).

The number of students interviewed with mixed mindsets was small (only two from Mr Scott's case and none from Ms Silver's). These two students expressed IBI as empowering with greater intensity than those with fixed mindsets but to a similar intensity as those with growth mindsets. The same two students tended to express views of IBI as neglectful with less intensity than those with fixed mindsets but to a similar intensity as those with growth mindsets. It appears students with mixed mindsets perceived IBI in a similar way to their peers with growth mindsets. Future research should seek to further understand the views of students with mixed mindsets.

Previous research has not thoroughly explored how students with MD perceive IBI, and the finding that students holding different mindsets can develop quite different perceptions is one of the key contributions to the literature this thesis makes. In Section 7.3, I discuss the implications of this finding to practice.

In addition, while the intensity of IBI as empowerment and IBI as neglect varied by mindset it is important to note that all students, regardless of mindset, expressed both perceptions at some point during the interviews. This might be an example of the state-type versus trait-type dispositions discussed in Section 2.8. Under Hannula's (2011) framework, trait-type affect constructs describe stable dispositions whilst state-type constructs describe more dynamic and rapidly changing dispositions. My assessment of mindset, which largely relied upon the use of questionnaires, assumes mindset as mainly a trait-type construct (Di Martino, 2019). However, the observation that all students expressed these perceptions at some point during their interviews could be indicative of mindset as a changing, state-type construct. This can present issues with the classification of students as fixed, mixed, or growth. I acknowledge these limitations in Section 7.4.

**Summary of findings for RQ1**

The exploration of how students with MD perceive IBI has not been reported based on a systematic search of the literature. The findings of this multiple case study suggest that students perceive IBI through several common themes. These are: (1) IBI as a form of empowerment, (2) IBI as a form of neglect, (3) Importance of the teacher, and (4) Importance of peers.

Students brought several existing beliefs with them when they entered the IBI unit. One of these was their belief about intelligence. I found that when students held growth mindsets, they expressed the perception of IBI as empowerment with greater intensity (Boyatzis, 1998) than those with fixed mindsets. Conversely, when students held fixed mindsets, they expressed the perception of IBI as neglect with greater intensity than those with growth mindsets. Please see Section 4.9 for a discussion of intensity scoring.

**RQ2: Are students' beliefs (e.g. mindset) associated with the effectiveness of IBI for students with mathematics difficulties?**

Questionnaires designed to determine students' mindsets and attitudes towards mathematics were used alongside pre- and post-tests to address this research question. These tests were used to measure student learning. In addition, classroom observations and interviews allowed for a qualitative view of the effectiveness of IBI in each case study. Altogether, these data sources enabled triangulation.

Much literature puts forward the view that direct instruction is the most effective technique for students with mathematics difficulties. Kroesbergen and van Luit (2003) reviewed 58 intervention studies in which researchers studied the efficacy of various teaching methods on the learning of students with special needs, including those with mathematics difficulties, and concluded that direct, explicit instruction was the most effective approach. Dennis et al. (2016) echoed this finding in a meta-analysis of 25 experimental and quasi-experimental studies of mathematics learning difficulties.

My study was not designed to evaluate the effectiveness of inquiry instruction versus direct instruction for students with MD, and therefore it is not possible from my data to suggest either approach is superior; to do so would have required a different methodology (possibly one that incorporated a control group). However, it is worth highlighting that the findings of the case studies presented in Chapter 5 and Chapter 6 provide some evidence that IBI was effective at increasing students' understanding in mathematics (as measured by my observations and the post-tests). In the case of Mr Scott, test scores for students with MD improved from 5.8 to 6.3 out of a maximum score of 12. In the case of Ms Silver, the test scores for students with MD increased from 16.78 to 22.67 out of a maximum score of 33.

The absence of a material increase in the case of Mr Scott (compared to Ms Silver) may be due to several factors. The first is that the IBI intervention simply did not result in much learning. However, given my observations of students' enhanced understanding (see Section 5.4.1 and Appendix L.1.5), there may be other factors to consider which mask the magnitude of the increase. Several factors that might be worth considering include the small class size, the length of instruction, and the length or nature of the assessment given. Even so, the absence of a larger increase in the case of Mr Scott's class calls into question the effectiveness of the IBI approach, and therefore further research is required.

When evaluating the effectiveness of IBI, the literature suggests its most significant benefit is developing students' conceptual understanding. Cobb et al. (1991) demonstrated this effect in the field of mathematics through a year-long controlled study of ten second-grade classes. Cobb et al. (1991) and similar studies reported in the literature have tended to focus on general education students. The results of my thesis are mixed for students with MD. In the case of Ms Silver's class, the scores on conceptual understanding increased from 3.7 to 5.7 (Table 6.4) out of a maximum score of 12. However, in the case of Mr Scott's class, test scores only increased from 3.5 to 3.8 (Table 5.4) out of a maximum score of 6. A similar situation is seen with regards to procedural knowledge, in which the students in Ms Silver's case increased their scores from 14.4 to 16.5 (Table 6.4) out of a maximum score of 21, and the students in Mr Scott's case increased their scores from 2.3 to 2.5 (Table 5.4) out of a maximum score of 6. Again, the results from the case of Mr Scott show only a small increase. Therefore, as discussed above, more research is needed to determine the effectiveness of IBI for enhancing conceptual and procedural knowledge.

Turning to the effect of mindset, many studies put forward the idea that holding a growth mindset results in greater academic performance than holding a fixed mindset (Aditomo, 2015; Aronson et al., 2002; Blackwell et al., 2007; Claro et al., 2016; Rattan et al., 2012; Sahlberg, 2011). So numerous are the studies to this effect that the view has almost acquired consensus. However, to my knowledge, no study has tested the effect of mindset for students with MD in an inquiry-based environment. Therefore, this thesis makes a contribution to the literature. The results of my multiple case study were unable to replicate the finding that students with growth mindsets outperform those with fixed mindsets. There was only a small difference on the post-test between students holding a fixed mindset and students holding a growth mindset in the first case (Table 5.5) and no difference in the second case (Table 6.5). Admittedly, the size and scope of this case study limits the generalisability of this finding, and concerns of whether the mindset instrument actually reported 'false growth mindsets' (Gross-Loh, 2016, para. 2) as opposed to true growth mindsets cannot be rejected. Nonetheless, I am unable to conclude that holding a fixed mindset is negatively associated with learning during IBI for students with MD, or that holding a growth mindset is favourably associated with learning based on the results of the pre- and post-tests. It is surprising that this effect failed to take place in an inquiry-based learning environment which, based on mindset theory, should have exaggerated the negative effects of holding a fixed mindset (see Section 4.11 for the ethical considerations taken into account for this study). Accepting the literature as true, my findings bring up an interesting question. Could IBI *temper* the effects of holding a growth or fixed mindset for students with MD? This presents an area for further research.

Differentiating the above analysis to inspect conceptual and procedural knowledge separately provides a similar result. Students with growth mindsets performed similarly to students with fixed mindsets on these measures (Table 5.5 and Table 6.5). Students of all mindsets improved their scores on both the procedural and conceptual items (Table 5.4 and Table 6.4), but there was no evidence from either case study that students with certain mindsets made greater improvements. Again, these results are interesting, as the literature would have predicted that students with growth mindsets should outperform those with fixed mindsets (Dweck, 2013). One possible explanation of this may lay in the fact that, despite their low attainment, students in both cases presented with a mixture of mindsets. It is therefore possible that existing compensatory factors helped to buffer against potential positive or negative effects of mindset.

Nevertheless, the evidence suggests holding a growth versus fixed belief of intelligence was not associated with the effectiveness of IBI for students with MD, at least not from the perspective of the overall test results nor from the results of the procedural and conceptual items.

One of the greatest benefits to employing a multiple case study approach is that it allows the researcher to look beyond purely quantitative features. This discussion now turns to the qualitative data collected through detailed observations of the students during the IBI lessons. These lesson observations were analysed using McLeod's framework of affect (1992). This framework considers beliefs as the most stable and most cognitive of the three domains of affect (the other two being attitudes and emotions). Under the framework, beliefs in mathematics are broken into four categories: beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the social context.

Looking first at the *beliefs students held about mathematics*, a cross-case comparison of Mr Scott's and Ms Silver's cases suggest a common view among the students that mathematics is something one 'does'. Comparing two examples:

L: What does area mean?
E: Pretty sure that's when you add up all the sides.
(Linda and Erica, L1, Mr Scott's case)

R: How are we getting on here?
C: We know we have to do something with this number and this one [pointing to circled numbers on the worksheet], and it has something to do with brackets, but we can't remember how to solve these.
(Researcher and Cayleigh, L5, Ms Silver's case)

Some students in both cases appeared to believe mathematics was nothing more than applying procedures with given numbers. The literature on this view goes back some time, with research suggesting students believe all word problems can be solved in just three-steps (Frank, 1988). First, the student selects an operation (e.g. division) using keywords within the text, then they use this operation on numbers within the text, and then finally they report an answer (Greer, 1997; Sowder, 1988). The findings of this multiple case study suggest this belief has a role in how students with MD tackle IBI problems. In both cases, the students were unfamiliar with IBI in mathematics, so perhaps exposure to years of problems in which students are taught to follow stepwise solutions has led to this non-availing belief (Muis, 2004). Non-availing beliefs are discussed in Section 2.7.

In many cases, holding the above view led students to fail to engage in an exploration of the IBI problem, frequently suggesting that they would await direct instruction from the teacher ('let Ms explain it', Karson, L7, Ms Silver's case). Dweck (1975) describes this issue under the term 'learned helplessness', in which children's experiences in school have trained them to expect teacher assistance when encountering difficulties. This effect has been frequently observed in mathematics (Yates, 2009). As such, students struggle to engage with inquiry-based environments that expect them to explore challenging problems unaided. My observations from across the two cases seem to support the literature in this regard.

One of the advantages of inquiry-based instruction is that the problems create opportunities for students to connect mathematics with their real-world, out-of-school lives. In their review into the literature of inquiry-based pedagogy in mathematics, Artigue and Blomhøj (2013) highlight that teaching should focus on helping the student 'construct the meaning of the abstract concepts and methods gradually through mathematization of meaningful, real-life situations' (p. 804). A cross-case analysis of the two case studies (Section 5.4.2.2.1 and Section 6.4.2.2.1) suggests a consistent belief that the mathematics and selected problems were unrelated to reality ('This is pretty useless. When are we ever going to need this?', Irene, L1, Mr Scott's case). The observations that evidenced this belief coincided with reduced performance on the part of the student (e.g. talking with a peer about an unrelated topic), suggesting the belief that mathematics is unrelated to reality was negatively associated with the effectiveness of IBI.

However, this belief may not have been stable, and lessons which more clearly made connections with the real-world coincided with observed increases in task engagement. An example of this was the Basketball Court lessons (L2 and L3) for Mr Scott's case (see Section 5.3 for lesson overview). Hannula (2011) discusses the idea that beliefs can be 'rapidly changing affective states vs. relatively stable affective traits' (p. 43). Observation-based data can tend to provide the researcher with insights of state-type beliefs rather than the more stable trait-type beliefs (Di Martino, 2019). This might provide an explanation for the observed belief changes that occurred in both cases. The introduction of real-world problems possibly led to a state-type shift in beliefs about the abstractness of mathematics problems for students with MD. When students in both case studies were able to connect problems to their real-world lives, they typically engaged better and attempted a greater repertoire of solution pathways.

One final belief about mathematics emerged from across the two cases. Frequently students demonstrated a reluctance to try alternative solutions, especially when they felt they had already answered the question or were on the right path. When encouraged to search for alternative or confirmatory solution pathways students seemed either confused or fixated on making their chosen pathway work. For example, during the interaction with Edith and her group in Section 6.4.2.2.1, the students focused on validating their answer by repeatedly asking the teacher if it was correct as opposed to self-validating via another solution pathway. Students who held this view missed opportunities to engage further with the problems. A common observation I made across both case studies was that a student would try a single pathway and, if unsuccessful, they would give up. C. A. Brown et al. (1988) reported that the majority of secondary school students believe that mathematics is primarily based on rules that have to be memorised. The observations made in both case studies, that students were reluctant to attempt multiple solution pathways, might stem from this belief. Other research has argued that inquiry teaching methods that encourage students to explore multiple solution pathways allow students to be more innovative and to think about a wider range of alternatives (Makar, 2012). Overall, my observations within these two cases suggest that holding a belief that mathematics problems can only be solved in one way seemed to be negatively associated with how well a student undertook the exploration portion of IBI, at least within the context of this study.

Turning to *beliefs about self,* data from both case studies presents evidence of the students' mindsets. As previously discussed, students with a growth mindset did not outperform those with a fixed mindset on the pre- and post-test, despite previous literature suggesting they would. However, a qualitative comparison across the two cases suggests that mindset did impact upon the way students engaged with the IBI problems. I discuss this comparison below.

Students typically indicated their mindset by making reference to their brains ('I don't know … my brain's not very good at this', Jackie, L4, Mr Scott's case; 'I'm rubbish at this stuff. Harper has got the brains', Zelda, L2, Ms Silver's case). These views align with what Dweck

(2017b) describes as a fixed mindset. Observed student engagement and effort on the IBI problems suggested that holding a fixed mindset normally resulted in worse academic performance (see Sections 5.4.2.2.2.2 and 6.4.2.2.2.2 for these analyses). However, there were times when holding a fixed mindset appeared to actually help the students perform better. Students who held a fixed mindset *and* believed they were innately good at maths showed perseverance in the face of challenges. In her summative book, Dweck (2017b) explains that fixed mindset, where the individual holds the view that they are innately good at something, can be a positive motivator. Holding a view that one is good at mathematics can help motivate one to tackle challenging maths problems. However, this fixed view of intelligence apparently fails to motivate students at more advanced levels, when it could 'reveal the limits of the student's ability' (Dweck, 2013, p. 12). Evidence of this did appear within the case studies ('I've always been quite good at maths', Clay, L4, Ms Silver's case). In this example, Clay had successfully tackled the inquiry problem. It is therefore too blunt to say that fixed mindset beliefs always result in poor performance as the actual situation is more nuanced.

Another aspect of *beliefs about self*, which is related to mindset, is self-efficacy. Schunk (2012, p. 146) defines self-efficacy as the 'personal beliefs about one's capabilities to learn or perform actions at designated levels'. Expanding the definition to mathematics, Ashcraft and Rudig (2012) describe self-efficacy as 'an individual's confidence in his or her ability to perform mathematics' (p. 249). Cross-case analysis suggests that the students with MD in both case studies often held a negative view of their self-efficacy. Comments such as 'I don't get any of it' (Sirena, L1, Mr Scott's case) and 'I'm not good at these' (Kevin, L4, Ms Silver's case) are representative of comments from both cases. Such non-availing beliefs were negatively associated with the effectiveness of the IBI.

Bandura (1997) suggests that there are four sources of self-efficacy. The first of these is mastery experiences, meaning first-hand experiences of success or failure. Often the students within the case studies exhibited high self-efficacy in 'doing' mathematics as it pertains to procedural knowledge but low self-efficacy in understanding mathematics on a deeper conceptual level. Looking at this observation through the lens of Bandura's first source of self-efficacy, it is possible that students' prior direct instruction experiences in mathematics provided them with mastery experiences of the procedural aspects of problem solving but with little mastery experiences of inquiry-based exploration.

The overall observations of self-efficacy are consistent with the literature, which has demonstrated the correlation between beliefs about self-efficacy and student performance (Bandura, 1997; Gao, 2019). The same patterns were observed in both cases of students with MD in an IBI context.

Next, I address *beliefs about mathematics teaching.* McLeod argued that students hold many beliefs about how mathematics should be taught, which they develop through exposure to mathematics teaching in school (1992). The research in this area is perhaps not as developed for some of the other beliefs. One such non-availing belief is that the role of the teacher is to transmit knowledge and that of the student is to receive knowledge (Frank, 1988). The cross-case analysis suggests that this belief was present within students with MD during IBI ('I'll get it better when he [the teacher] explains it', Lisa, L1 Mr Scott's case; 'If he's not gonna bother explaining it then why should we bother trying to work it out?', Timothy, L5, Mr Scott's case). These observations support the literature in this regard, although extending it into the context of students with MD undertaking IBI.

Perhaps the belief that the role of the teacher is to explain and the role of the student is to memorise led students to conclude that, when learning via IBI, they were not *really* learning (at least not until the explanation phase). Others have noted this behaviour, whereby students require new classroom norms (e.g. risk-taking) in order to make the most of inquiry-based lessons (Makar & Fielding-Wells, 2018). Perhaps, with further exposure to IBI, these students would shed such non-availing beliefs and begin to see their role as broader than a recipient of knowledge administered by the teacher.

Makar (2012, p. 371) argues that the problems presented to students in school mathematics lead them to believe that mathematics teaching is based upon isomorphic problems focused on canonical solutions, and mathematics problems are 'clearly stated, take only a few minutes to answer, include little or no use of context and have a single correct answer'. Therefore, students come to expect teaching that follows this model. Schommer and Walker (1995) refer to such beliefs as 'naïve' and generally unhelpful for the learning of mathematics. IBI problems are more open-ended and invite multiple solution pathways. Cross-case analysis suggests the above beliefs about teaching were present within both cases, and students were aware of the differences to their normal mathematics lessons. There were positive examples of this belief ('It's good they [the problems] are always different', Edith, L4, Ms Silver's case; 'It's better than bookwork', Maddie, L1, Mr Scott's case). Frequently, however, these beliefs interacted in a non-availing way with the exploration aspects of IBI ('I think it's a waste of time to keep trying stuff and then you never get it right', Opal, L7, Ms Silver's case).

The analysis of the two case studies suggests that students held a view that mathematics problems should be short and not take much time ('Do these always take so long? We've only done one problem', Kevin, L3, Mr Scott's case; 'But these ones take ages', Dax, L7, Ms Silver's case). Comments like these are indicative of this belief and appeared to be associated with poor engagement and perseverance when solving extended inquiry problems. The literature is rich with research suggesting students often hold the non-availing belief that problems in mathematics should be short, and that this belief emerges as a result of the types of problems presented in school (Makar, 2012; McLeod, 1992; Schoenfeld, 1985). Schoenfeld (1985) suggested that students believe that all problems in mathematics can be 'solved in less than ten minutes, if they are solved at all' (p. 43) and, moreover, those students that held this view struggled to persevere when solving inquiry problems. This thesis extends this observation to students with MD learning in an inquiry-based environment. This finding suggests the belief that maths problems should be short may be typical of students with MD as well.

The next belief within the analysis framework relates to *beliefs about the social context.* McLeod suggested that the beliefs that students hold about the social context of mathematics impacts upon the way students tackle problems. Op't Eynde, De Corte, and Verschaffel (2002) argue that the social context includes beliefs that students hold about handling disagreements and making decisions with their peers. A cross-case analysis of this study suggests that students held various beliefs about the role of peers in making decisions. Some believed it was important to build consensus in their groups while others believed it was best to take a democratic approach.

In addition, students with MD in both cases believed that their peers could provide insight into mathematics. I also saw this idea emerge within the student interviews ('If I've done all I can I ask my neighbour who's sitting next to me', Cayleigh, Ms Silver's case). Kloosterman et al. (1996) conducted a 3-year study into students' mathematics beliefs and suggested that students' beliefs about the role of peers varied depending upon classroom experiences. They found that many students felt mathematics was individual work. By contrast, I saw little evidence of this belief across the analysis of both cases.

The students also seemed to hold beliefs about the social context of their schools. In both cases the classes were 'lower set' and students seemed aware of their social positioning within the pool of mathematics students at the school. Cross-case analysis suggested this awareness led to certain beliefs about the social context, specifically that they were not expected to perform as academically as the 'top set'. For example, students felt that enjoyment for mathematics was reserved for 'top set' students who were 'swotty' (Maddie and Irene, L7, Mr Scott's case). The deleterious effect of attainment grouping on students' self-esteem, engagement and perceived 'ability' has been well documented (Braddock & Slavin, 1992; Francis, Archer, et al., 2017; Marks, 2013; Mazenod et al., 2019). The cross-case analysis suggests that the students held non-availing beliefs about themselves within the academic hierarchy of their schools. It is possible these beliefs negatively impacted their exploration of the IBI problems.

Students held beliefs about the broader social context beyond the school. These varied from the belief that mathematics has little value ('This is pretty useless. When are we ever going to need this?', Irene, L1, Mr Scott's case) to the belief that mathematics was not an important part of the student's home life ('These types of problems just really confuse me. My mum said she was the same way when she was in school', Eleri, L7, Ms Silver's case). These beliefs interacted with how students viewed the IBI problems. In the case of Irene, this was associated with poor engagement on the task, and in the case of Eleri, she appeared reluctant to try out the problem and seemed to almost give up as soon as she began. No doubt numerous other factors could have led to these students' poor engagement with the IBI problems, but the expression of non-availing beliefs regarding the social context is consistent with previous research (Schoenfeld, 1983). Schoenfeld argued that the beliefs of problem solvers '(not necessarily consciously held) about what is useful in mathematics may determine the set of "cognitive resources" at their disposal as they do mathematics' (p. 329).

**Summary of findings for RQ2**

The effect of student beliefs (such as mindset) on performance in mathematics has been discussed in the literature. However, little research has focused specifically on students with

MD in an inquiry-based environment. My analysis across these two case studies serves to address this need, especially in the area of mindset, which has received much attention recently in practice as well as research. Specifically, in relation to RQ2, I found the following:

- Students with MD were able to improve their procedural and conceptual knowledge over the course of the IBI unit
- There was no difference in how well students with MD and growth mindsets performed as compared to students with MD and fixed mindsets on the pre- and post-test
- Students with MD held beliefs that were associated with poor performance and diminished engagement with the IBI unit
- These included the belief that mathematics is something one 'does', mathematics problems should be short, there is only one correct way to solve a given problem, the teacher is an important source of knowledge, and mathematics is unrelated to reality. In addition, students held potentially negative self-beliefs including low self-efficacy, fixed mindset, and that students in lower sets are not meant to excel in mathematics.

## 7.2 Key contributions to literature

In Section 7.1 I discuss the key findings from two case studies, as well as how these findings interact with the literature. In this section, I summarise the principal contributions that this thesis makes to the existing literature. My research questions touch upon several areas. These are:

1) Inquiry-based instruction
2) The affective domain, specifically the construct of beliefs
3) Mathematical difficulties

The largest contribution this paper makes to the literature is to explore the intersection of these three stands of research into a single study. Figure 7.1 shows this visually, with the nexus of the three fields representing a gap in the literature. It is here that the findings of this research have made the greatest contribution.

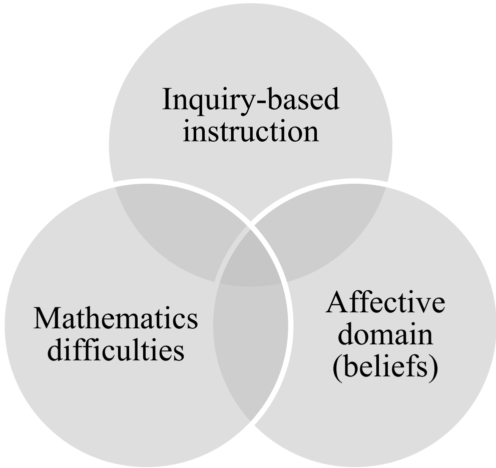


Figure 7.1: Three areas of research this thesis makes a contribution to

The first contribution to the area of inquiry-based instruction is that IBI can be effectively delivered to students with MD. Numerous researchers have highlighted the challenges that teachers face when implementing IBI, such as classroom management challenges and poor engagement from students (Stylianides & Stylianides, 2014). My findings add to the literature and demonstrate that teachers who have little history of IBI can effectively administer this type of instruction to students with MD with the help of EQUIP and a skilled observer.

The second contribution to the area of inquiry-based instruction is that students with MD can improve their procedural and conceptual understanding through IBI. Aspects of the literature put forward the view that direct instruction is the most effective technique for students with mathematics difficulties (Dennis et al., 2016; Kroesbergen & van Luit, 2003). Whilst this study does not include a comparison group, my findings add weight to the literature that says students with MD can benefit from an inquiry-based approach to learning mathematics. In addition, it is interesting to note the observed increase in knowledge was greater for one of the cases (Ms Silver's) compared to the other (Mr Scott's). Future research should seek to further explore the contextual factors that contribute to the effectiveness of inquiry instruction for students with MD. Moreover, the size and scale of this study limit the generalisability of this findings and further research is recommended (see Section 7.5).

In the area of affect, the primary contribution of this thesis is an exploration of the beliefs that students with MD can hold and whether these beliefs were associated with the effectiveness of the IBI. Previous research has suggested that students hold various beliefs towards mathematics and themselves and that holding inappropriate beliefs negatively impacts academic performance (Di Martino, 2019; Dweck, 2017b; McLeod, 1992; Schoenfeld, 1987; Schommer, 1990). However, prior research has neglected to explore this construct for students with MD, and therefore, the findings of this study make a contribution.

The results of this multiple case study show that students believed mathematics is something one 'does' (Frank, 1988; McLeod, 1992), mathematics problems should be short (Schoenfeld, 1985), there is only one correct way to solve a given problem (McLeod, 1992), the teacher is an important source of knowledge (Frank, 1988; Kloosterman et al., 1996), and mathematics is unrelated to reality. Students also showed low self-efficacy (Bandura, 1997; Schoenfeld, 1987), fixed mindset (Dweck, 2013, 2017b), and the expectation that students in lower sets are not meant to excel in mathematics. The apparent result of holding these views was reduced observed engagement (e.g. putting head on the desk, off-task discussions with peers). Taken together these findings paint a picture of the beliefs students with MD may hold and the extent to which these are associated with performance during IBI. Implications for practice are discussed in Section 7.3. I acknowledge that the size and scope of this study limit the generalisability, and therefore, future research is needed to support these findings.

A further contribution to the area of affect relates to the role of mindset. Mindset theory (Dweck, 2013) suggests that students with growth mindsets typically outperform those with fixed mindsets, especially when experiencing IBI which relies on exploration. The findings of this study are consistent with this literature as it relates to the students' *observed* engagement and persistence on the IBI problems. However, my finding that students with growth mindsets did not outperform students with fixed mindsets as measured by the post-test is surprising. This might suggest that the theory of mindset does not generalise to the context of students with MD. Further research is recommended.

Across a series of interviews, the students expressed several ways in which they perceived IBI. These broadly fell into four themes: (1) IBI as a form of empowerment, (2) IBI as a form of neglect, (3) Importance of the teacher, and (4) Importance of peers.

Several aspects of these perceptions have been explored within the literature. For example, within IBI as empowerment, Hassi and Laursen (2015) argued university students perceive IBI as personally empowering. In addition, Kogan and Laursen (2014) reported IBI led to increased student self-efficacy and Boaler (1998) argued that IBI creates opportunities for deeper engagement with mathematics. Within the theme IBI as neglect, others have shown that students see mathematics as a ‘doing’ subject based on rules and memorisation (C. A. Brown et al., 1988; Kouba & McDonald, 1987; McLeod, 1992). The perception that the teacher’s role is to teach and the student’s role is to learn has also been explored (Kloosterman, Raymond, and Emenaker, 1996; Frank, 1988). However, no prior research (to my knowledge) has explored how students with MD perceive IBI. Therefore, this thesis provides valuable insight into this gap.

Finally, across both case studies students with growth mindsets expressed the theme of IBI as empowerment with greater intensity (Boyatzis, 1998) than those with fixed mindsets. Conversely, students with fixed mindsets expressed the theme of IBI as neglect with greater intensity than those with growth mindsets. This finding presents a central contribution to the literature. Research suggests that many teachers shy away from using IBI for students with MD, favouring direct, explicit instruction (Kroesbergen & van Luit, 2003; Maccini & Gagnon, 2002; Zohar et al., 2001). This finding may help to shed some light on why some students seem to engage more with IBI than others. I believe educators can incorporate these findings into their instruction when teaching inquiry. Implications for practice are discussed in Section 7.3.

## 7.3 Implications for practice

The main implication for practice that I would like to address is the use of different teaching practices for different groups of students (or rather, groups of students that are *perceived* to be different).

The Department for Education (DfE) in the U.K. released a report titled, ‘Teacher’s Standards’ (2011) and it gives the following guidance to school leaders, staff, and governing bodies:

> Set *high expectations* which inspire, motivate and challenge pupils. …Set goals that stretch and challenge *pupils of all* backgrounds, *abilities*, and dispositions. (p. 10; emphasis added)

The report goes on further to recommend:

> Adapt teaching to respond to the strengths and needs of all pupils. … Have a clear understanding of the needs of all pupils, including those with special educational needs; *those of high ability*; those with English as an additional language; *those with disabilities*; and be able to use and evaluate *distinctive teaching approaches* to engage and support them. (pp. 11–12; emphasis added)

The above recommendation by the DfE is somewhat dissonant. It begins with the goal of setting high expectations for *all pupils,* but it then somewhat contradicts this recommendation by suggesting those with ‘high ability’ should receive ‘distinctive teaching practices’ from those of presumably ‘low ability.’ In this way, the document seems to support the practice of teaching students in lower sets in a different way to those in top sets. The DfE likely intends this as a call for skilled differentiation, but such guidance could actually be one explanation for why research has shown lower sets receive distinctively shallow teaching compared to top sets (Mazenod et al., 2019). As this study has shown, students with mathematics difficulties (MD) *can* develop understanding from inquiry-based practices if they are given the opportunity.

This study brings to light several implications for practice.

- Teacher professional development in inquiry-based instruction should specifically address students with MD and work to dispel myths that these students cannot succeed in inquiry-based mathematics lessons
- Teachers of students with MD should incorporate problems into their lessons that have multiple solution pathways or multiple answers as a means to expand their students’ conception of mathematics
- Teachers should decentralise their role in the classroom by incorporating meaningful opportunities for students to share their knowledge with classmates, for example through cooperative learning or whole-class dialogic teaching
- It has been shown that students in lower set classes can receive fixed mindset messages as a result of disparate teaching practices. Given the observed association between holding a fixed mindset and performance on IBI problems, teachers should work to remove fixed mindset messages from their instruction.

- Teachers should be explicit about the choices they make in an inquiry classroom and communicate the intended benefits to their students to help dispel any perceptions that inquiry teaching is neglectful.
- Teachers should engage in ongoing critical reflection of their implementation of IBI to identify areas for increased proficiency, possibly through the use of an observation protocol such as EQUIP.

## 7.4 Limitations

The purpose of this section is to identify the primary limitations of this research. These limitations constrain the interpretation and generalisability of the findings. Future research may seek to address some of these limitations.

**Small sample size**

The nature of case study methodology implies a deep focus on a smaller sample size (Yin, 2017). The most common criticism of this approach is the lack of generalisability of the findings (Merriam, 2016). However, in response to this limitation, I note that it is not the purpose of a case study to build a generalisable model but rather to deeply explore a phenomenon in context (Yin, 2017). Learning is idiosyncratic and therefore the extent to which my findings are generalisable to other contexts is limited and applications should be undertaken cautiously.

Furthermore, this thesis makes use of a mixed methods approach by supplementing qualitative data with quantitative methods. This presents another limitation. Quantitative data such as test scores and mindset questionnaires are helpful in understanding these cases, but it is difficult to draw definitive conclusions from such small samples. A study with an increased sample size would help to add weight to the above findings.

**Challenge in measuring beliefs**

A substantial part of this research relies upon the identification of student beliefs. Historically questionnaires have been used to measure students' beliefs, but more recently, observations and interviews have been used to supplement these. This poses numerous limitations. Firstly, it assumes that internal student beliefs can somehow be measured through external behaviours and that students express their beliefs in observable ways. Secondly, it assumes that I, as the researcher, am able to assume an unbiased interpretation of these beliefs as a result of the observations. Both of these issues are implied within most studies that approach beliefs from a qualitative standpoint.

In addition, beliefs, like other affect constructs, are not necessarily stable (Hannula, 2011). A distinction is made between trait-type affect constructs describing stable dispositions and state-type constructs describing more dynamic and rapidly changing dispositions. Different research methodologies suit the measurement of different temporal aspects. Quantitative methods can suggest stable trait-type constructs, whereas qualitative observational methods may allow the researcher to study more volatile state-type constructs (Di Martino, 2019). My analysis does not attend to this distinction and the extent to which the observed beliefs are state-type or trait-type is a potential area for future study.

**Short duration**

This study explored students' perceptions, beliefs, and performance throughout an inquiry-based teaching unit of a relatively short duration. The was done in an effort to minimise any potential harm (see Section 4.11). Given it appears no harm was caused by this short study, future research should seek to understand students' experiences with inquiry-based instruction over a longer period of time so that longitudinal analyses can be made possible.

**Exploration of large qualitative data sets**

This study consisted of 15 inquiry lesson observations and 22 student interviews. This level of data presents the researcher with a challenge. Effectively balancing the need to synthesise, whilst also ensuring the authentic voice of each case is maintained, is challenging. There is an obligation upon researchers to attend to all the data collected. However, given its volume I am not able to present the true depth of the data within this thesis. In this way, I have attempted to balance my report between 'the rocks of oversimplification and the rapids of the too-entangled-to-be-useful' (Davis, 2019, p. 152). In doing so, I made choices about what to report and what to exclude. Therefore, this thesis is limited by my own participation in it. Adopting a reflexive view, I acknowledge my effect on the process and outcomes of this thesis, including but not limited to my choice of research focus, observation notes, and framing of key contributions to literature.

### 7.5 Directions for future research

Future research should seek to further integrate views from generalist mathematics education (the body of research which speaks to the teaching and learning of mathematics for students in *general*) and specialist mathematics education (the body of research which speaks to the teaching and learning of mathematics for students with MD). At the moment, these two fields have different recommendations for best practice. Scholars of generalist mathematics education recommend students undertake challenging problems rooted in inquiry-based pedagogies. Scholars of specialist mathematics education, on the other hand, often recommend students receive direct, explicit instruction. These contrasting points of view can be problematic when terms such as mathematics difficulties, mathematics learning disability, dyscalculia, and low attainment are ill defined. Future research should seek to clarify under what conditions it is appropriate to use inquiry and non-inquiry methods in a way that moves beyond oversimplified notions of student 'ability'.

This study suggests that students with MD who hold fixed mindsets underperform during IBI compared to their peers with growth mindsets, based on in-class observations. However, this did not translate to the students' performance on the pre-test and post-test. Further research should explore this finding across several paths. Firstly, additional observations of students with MD undertaking IBI may help extend my research into different contexts. Secondly, large sample studies which follow a longitudinal design could explore further whether holding a fixed mindset results in lower academic performance in inquiry-based environments.

This research demonstrates the importance of beliefs for students with mathematics difficulties when taught using IBI. My study did not seek to modify these beliefs, but rather to observe them under inquiry conditions and identify any associations with performance. Given the observed deleterious effects of holding some of these beliefs, future research might seek to develop belief interventions targeted at changing these views in students with MD. Furthermore, my observation that students of differing mindsets perceive IBI as empowering or neglectful is novel within the literature and may support further research which aims to improve teacher implementation of IBI.

Finally, I reiterate that my research focused on students with MD taught using IBI. Future research should explore my findings in more contrasting studies and contexts. For example, a broader multiple case study could include different classes experiencing different teaching approaches, varying from direct instruction to IBI. Research of this kind may allow for a deeper understanding of the association between the beliefs of students with MD and the effectiveness of different teaching approaches.

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# 9 Appendices

## Appendix A: Electronic Quality of Inquiry Protocol (EQUIP)

| ***IV. Instructional Factors*** | | | | | |
|---|---|---|---|---|---|
| *Construct Measured* | | *Pre-Inquiry (Level 1)* | *Developing Inquiry (2)* | *Proficient Inquiry (3)* | *Exemplary Inquiry (4)* |
| I1. | Instructional Strategies | Teacher predominantly lectured to cover content. | Teacher **frequently lectured** and/or used demonstrations to explain content. Activities were **verification only**. | Teacher **occasionally lectured**, but **students were engaged in activities** that helped develop conceptual understanding. | Teacher occasionally lectured, but students were engaged in investigations that **promoted strong conceptual understanding**. |
| I2. | Order of Instruction | Teacher explained concepts. Students either did not explore concepts or did so only after explanation. | Teacher asked **students to explore concept before receiving explanation**. Teacher explained. | Teacher asked students to **explore before explanation**. **Teacher and students explained**. | Teacher asked students to explore concept before explanation occurred. Though perhaps prompted by the teacher, **students provided the explanation**. |
| I3. | Teacher Role | Teacher was center of lesson; rarely acted as facilitator. | **Teacher was center** of lesson; **occasionally** acted as **facilitator**. | Teacher **frequently** acted as facilitator. | Teacher **consistently and effectively** acted as a facilitator. |
| I4. | Student Role | Students were consistently passive as learners (taking notes, practicing on their own). | **Students were active to a small extent** as learners (highly engaged for very brief moments or to a small extent throughout lesson). | **Students were active as learners** (involved in discussions, investigations, or activities, but not consistently and clearly focused). | Students were **consistently and effectively active as learners** (highly engaged at multiple points during lesson and clearly focused on the task). |
| I5. | Knowledge Acquisition | Student learning focused solely on mastery of facts, information, and/or rote processes. | Student learning focused on **mastery of facts** and process skills without much focus on understanding of content. | Student learning required **application of concepts** and process skills in new situations. | Student learning required **depth of understanding** to be demonstrated relating to content and process skills. |

| **V. *Discourse Factors*** | | | | | |
|---|---|---|---|---|---|
| *Construct Measured* | | *Pre-Inquiry (Level 1)* | *Developing Inquiry (2)* | *Proficient Inquiry (3)* | *Exemplary Inquiry (4)* |
| D1. | Questioning Level | Questioning rarely challenged students above the remembering level. | Questioning rarely challenged students above the **understanding level**. | Questioning challenged students **up to application or analysis levels**. | Questioning challenged students at various levels, **including at the analysis level or higher; level was varied to scaffold learning.** |
| D2. | Complexity of Questions | Questions focused on one correct answer; typically short answer responses. | Questions focused **mostly on one correct answer;** some open response opportunities. | Questions **challenged students to explain, reason, and/or justify**. | Questions required students to explain, reason, and/or justify. **Students were expected to critique others' responses**. |
| D3. | Questioning Ecology | Teacher lectured or engaged students in oral questioning that did not lead to discussion. | Teacher occasionally **attempted to engage students in discussions** or investigations but was not successful. | Teacher **successfully engaged students** in open-ended questions, discussions, and/or investigations. | Teacher **consistently and effectively engaged students** in open-ended questions, discussions, investigations, and/or reflections. |
| D4. | Communication Pattern | Communication was controlled and directed by teacher and followed a didactic pattern. | Communication was **typically controlled and directed by teacher** with occasional input from other students; mostly didactic pattern. | **Communication was often conversational** with some student questions guiding the discussion. | Communication was **consistently conversational with student questions often guiding the discussion**. |
| D5. | Classroom Interactions | Teacher accepted answers, correcting when necessary, but rarely followed-up with further probing. | Teacher or another student **occasionally followed-up student response** with further low-level probe. | Teacher or another student **often followed-up response** with engaging probe that required student to **justify reasoning or evidence**. | Teacher **consistently and effectively facilitated rich classroom dialogue** where evidence, assumptions, and reasoning were challenged by teacher or other students. |

| **VI. *Assessment Factors*** | | | | | |
|---|---|---|---|---|---|
| *Construct Measured* | | *Pre-Inquiry (Level 1)* | *Developing Inquiry (2)* | *Proficient Inquiry (3)* | *Exemplary Inquiry (4)* |
| A1. | Prior Knowledge | Teacher did not assess student prior knowledge. | Teacher **assessed student prior knowledge but did not modify instruction** based on this knowledge. | Teacher assessed student prior knowledge and then **partially modified instruction** based on this knowledge. | Teacher assessed student prior knowledge and then **modified instruction** based on this knowledge. |
| A2. | Conceptual Development | Teacher encouraged learning by memorization and repetition. | Teacher encouraged **product- or answer-focused** learning activities that **lacked critical thinking**. | Teacher encouraged **process-focused** learning activities that **required critical thinking**. | Teacher encouraged process-focused learning activities that involved **critical thinking that connected learning with other concepts**. |
| A3. | Student Reflection | Teacher did not explicitly encourage students to reflect on their own learning. | Teacher explicitly encouraged students to **reflect** on their learning but only at a minimal **knowledge level**. | Teacher explicitly encouraged students to **reflect** on their learning at an **understanding level**. | Teacher consistently encouraged students to reflect on their learning at multiple times throughout the lesson; **encouraged students to think at higher levels**. |
| A4. | Assessment Type | Formal and informal assessments measured only factual, discrete knowledge. | Formal and informal assessments **measured mostly factual, discrete knowledge**. | Formal and informal assessments **used both factual, discrete knowledge and authentic measures**. | Formal and informal assessment methods **consistently and effectively used authentic measures**. |
| A5. | Role of Assessing | Teacher solicited predetermined answers from students requiring little explanation or justification. | Teacher **solicited information from students to assess understanding**. | Teacher solicited explanations from students to **assess understanding and then adjusted instruction accordingly**. | Teacher frequently and effectively assessed student understanding and adjusted instruction accordingly; **challenged evidence and claims made; encouraged curiosity and openness**. |

| ***VII. Curriculum Factors*** | | | | | |
|---|---|---|---|---|---|
| *Construct Measured* | | *Pre-Inquiry (Level 1)* | *Developing Inquiry (2)* | *Proficient Inquiry (3)* | *Exemplary Inquiry (4)* |
| C1. | Content Depth | Lesson provided only superficial coverage of content. | Lesson provided **some depth** of content but with **no connections made to the big picture.** | Lesson provided depth of content with **some significant connection** to the big picture. | Lesson provided depth of content with **significant, clear, and explicit connections** made to the big picture. |
| C2. | Learner Centrality | Lesson did not engage learner in activities or investigations. | Lesson provided **prescribed activities** with anticipated results. | Lesson allowed for **some flexibility during investigation** for student-designed exploration. | Lesson provided **flexibility for students to design and carry out** their own investigations. |
| C3. | Integration of Content and Investigation | Lesson either content-focused or activity-focused but not both. | Lesson provided poor integration of content with activity or investigation. | Lesson **incorporated student investigation that linked well with content**. | Lesson **seamlessly integrated the content and the student investigation**. |
| C4. | Organizing & Recording Information | Students organized and recorded information in prescriptive ways. | **Students had only minor input as to how to organize and record information**. | Students regularly organized and recorded information in **non-prescriptive ways**. | Students organized and recorded information in **non-prescriptive** ways that allowed them to **effectively communicate their learning.** |

Appendix B: Implicit Theories of Intelligence Scale (ITIS)

Name________________________________________________________

### SECTION 1

**<u>Directions</u>: Please rate how strongly you agree or disagree with each of the following statements by circling the appropriate number. There are no right or wrong answers.**

**1. You have a certain amount of intelligence, and you really can't do much to change it.**

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Strongly Agree | Agree | Mostly Agree | Mostly Disagree | Disagree | Strongly Disagree |

**2. Your intelligence is something about you that you can't change very much.**

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Strongly Agree | Agree | Mostly Agree | Mostly Disagree | Disagree | Strongly Disagree |

**3. You can learn new things, but you can't really change your basic intelligence.**

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Strongly Agree | Agree | Mostly Agree | Mostly Disagree | Disagree | Strongly Disagree |

**4. You have a certain amount of MATHS intelligence, and you really can't do much to change it.**

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Strongly Agree | Agree | Mostly Agree | Mostly Disagree | Disagree | Strongly Disagree |

**5. Your MATHS intelligence is something about you that you can't change very much.**

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Strongly Agree | Agree | Mostly Agree | Mostly Disagree | Disagree | Strongly Disagree |

**6. You can learn new things, but you can't really change your basic MATHS intelligence.**

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Strongly Agree | Agree | Mostly Agree | Mostly Disagree | Disagree | Strongly Disagree |

→ ***Continue***

Appendix C: Attitudes Towards Mathematics Inventory (ATMI)

**ATTITUDES TOWARD MATHEMATICS INVENTORY**

*Name* ____________________________ *School* ____________________________

*Teacher* ____________________________

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Enter the letter that most closely corresponds to how each statement best describes your feelings. Please answer every question.

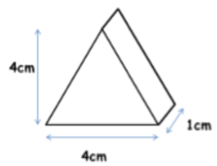
PLEASE USE THESE RESPONSE CODES:

A – Strongly Disagree
B – Disagree
C – Neutral
D – Agree
E – Strongly Agree

| | | |
|---|---|---|
| 1. | Mathematics is a very worthwhile and necessary subject. | |
| 2. | I want to develop my mathematical skills. | |
| 3. | I get a great deal of satisfaction out of solving a mathematics problem. | |
| 4. | Mathematics helps develop the mind and teaches a person to think. | |
| 5. | Mathematics is important in everyday life. | |
| 6. | Mathematics is one of the most important subjects for people to study. | |
| 7. | High school math courses would be very helpful no matter what I decide to study. | |
| 8. | I can think of many ways that I use math outside of school. | |
| 9. | Mathematics is one of my most dreaded subjects. | |
| 10. | My mind goes blank and I am unable to think clearly when working with mathematics. | |
| 11. | Studying mathematics makes me feel nervous. | |
| 12. | Mathematics makes me feel uncomfortable. | |
| 13. | I am always under a terrible strain in a math class. | |
| 14. | When I hear the word mathematics, I have a feeling of dislike. | |
| 15. | It makes me nervous to even think about having to do a mathematics problem. | |
| 16. | Mathematics does not scare me at all. | |
| 17. | I have a lot of self-confidence when it comes to mathematics. | |
| 18. | I am able to solve mathematics problems without too much difficulty. | |
| 19. | I expect to do fairly well in any math class I take. | |
| 20. | I am always confused in my mathematics class. | |
| 21. | I feel a sense of insecurity when attempting mathematics. | |
| 22. | I learn mathematics easily. | |
| 23. | I am confident that I could learn advanced mathematics. | |
| 24. | I have usually enjoyed studying mathematics in school. | |
| 25. | Mathematics is dull and boring. | |
| 26. | I like to solve new problems in mathematics. | |
| 27. | I would prefer to do an assignment in math than to write an essay. | |
| 28. | I would like to avoid using mathematics in college. | |
| 29. | I really like mathematics. | |
| 30. | I am happier in a math class than in any other class. | |
| 31. | Mathematics is a very interesting subject. | |
| 32. | I am willing to take more than the required amount of mathematics. | |
| 33. | I plan to take as much mathematics as I can during my education. | |
| 34. | The challenge of math appeals to me. | |
| 35. | I think studying advanced mathematics is useful. | |
| 36. | I believe studying math helps me with problem solving in other areas. | |
| 37. | I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math. | |
| 38. | I am comfortable answering questions in math class. | |
| 39. | A strong math background could help me in my professional life. | |
| 40. | I believe I am good at solving math problems. | |

Appendix D: Sample observation notes from Harrison School

**Location:** Harrison School
**Date:** 1st December (L6)
**Length of Observation:** 60 minutes

| **Descriptive Notes** | **Reflective Notes** |
|---|---|
| Teacher (T) welcomes students and starts to explain the task. T tells a story about a Toblerone chocolate bar and then passes out the worksheet.<br>Students listen and laugh to T's story. | The students are especially attentive today (it's a Friday afternoon). Students enjoy T's stories and find him funny. |
| T says, 'I want you to read the directions on the sheet and try to answer the question. I can give you guys some clues, but I want you to try and figure it out on your own. Remember area is a 2D measurement.'<br>Students begin working with their table-group and read the problem aloud to each other. | T sets expectations by telling students to read the directions and try the problem.<br>The students are responsive and on task. During past lessons students have been slow to start, but today they are quick.<br>K is more focused than I've seen him before. |
| The problem describes a single segment of a Toblerone bar as having a base of 4 centimetres, a height of 4 centimetres, and a width of 1 centimetre. The Toblerone bar has a total of 12 segments. The question is, 'How many square centimetres of cardboard are necessary to wrap the Toblerone bar?' | It's not possible for an equilateral triangle to have both a side length of 4 and a height of 4. T is okay with this since non-integer sides lengths would make the problem 'too difficult' for the students.<br> |
| One group discuss how a Toblerone bar has gaps between the segments and wonder whether or not this affects the answer. Another group discuss the difference between the base, height, and width and how they should label their sketch of the Toblerone. | Students are paying attention to details and asking each other good questions. They aren't simply guessing at what to 'do' with the numbers. They are thinking carefully about the shape of a Toblerone bar and how this might need to be taken into account. |

Appendix E: Consent documents

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**Mathematics Research Study**
**Information Sheet for Students**

**What is the purpose of the study?**
The purpose of this study is for the researcher to gain a better understanding of how you and the other students in your class experience instruction in mathematics.

**What are the possible benefits of taking part?**
You will have an opportunity to share your opinions about mathematics and classroom teaching. Your contributions may influence teaching practices in the UK and abroad. Also, you may understand your own relationship with mathematics better.

**Why have I been chosen?**
You have been chosen to participate in this study because you are a member of [TEACHER'S NAME] class.

**Do I have to take part?**
Taking part is entirely voluntary, and you may refuse or withdraw at any time.

**What will happen to me if I take part?**
I will observe you and your classmates during your regular mathematics lesson with [TEACHER'S NAME]. There will be five to six observations and these may be video recorded. You may be invited to an interview after the observation to share your opinions, lasting about 30 minutes. The interview may be audio recorded. You will also take part in a brief survey and two assessments.

**What do I have to do?**
You will participate in a series of mathematics lessons that will be observed. Then you may be asked to sit for an interview with the researcher lasting about 30 minutes. You will also complete a survey and two assessments. Your participation will not affect your academic standing at [SCHOOL NAME] in any way.

**What are the possible risks of taking part?**
If you choose to participate in the interview it will require a small time commitment (30 minutes).

**Will my taking part in this project be kept confidential?**
All information will be kept confidential and video/audio files will be identified only by a code, with personal details kept in a secure computer with access only by the researcher.

**What will happen to the results of the pilot study?**
Results may be presented at conferences and published in academic journals and theses. If any individual data are presented, the data will be anonymous, without any means of identifying the individuals involved. To obtain a copy of any publications please email jr701@cam.ac.uk.

**Who is organizing and funding the research?**
My name is Jennifer Rice (jr701@cam.ac.uk) and I am a PhD student at the University of Cambridge. My supervisors are Dr Keith Taber (kst24@cam.ac.uk) and Dr Julie Alderton (jha32@cam.ac.uk). My research is funded by the Cambridge Trust.

**Contact for further information**
If you have any concerns about the way the research is being conducted please contact [TEACHER NAME, EMAIL]. If you have any further questions please contact me at jr701@cam.ac.uk.

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**Thank you for taking the time to read this form.**

## Student Record of Consent Form

**Research project title:** Inquiry based instruction in mathematics classrooms

**Researcher:** Jennifer Rice, Faculty of Education, Cambridge University, jr701@cam.ac.uk

I, the undersigned, confirm that (please tick box as appropriate):

| | | |
|---|---|---|
| 1. | I have read and understood the information sheet for the above study and have had the opportunity to ask questions. | ☐ |
| 2. | I agree to take part in the study. | ☐ |
| 3. | I understand that my participation is voluntary and that I am free to withdraw at any time, without giving reason. | ☐ |
| 4. | I agree that my data gathered in this study may be stored (after it has been made anonymous) and used in publications. | ☐ |

I, the undersigned, confirm that (please tick box as appropriate):

| | | |
|---|---|---|
| 5. | I agree to participate in an interview. | ☐ |
| 6. | I agree to the interview being audio recorded. | ☐ |

| | | |
|---|---|---|
| ____________________________ | ____________________ | ____________________________ |
| **Name of student** | **Date** | **Signature** |
| ____________________________ | ____________________ | ____________________________ |
| **Name of parent/guardian** | **Date** | **Signature** |

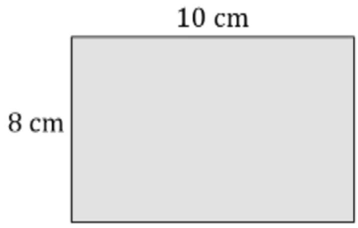
Appendix F: Pre-test and post-test used with Mr Scott's class

Name________________________________________ Date________________

**Measurement Pretest**

**<u>Directions</u>**: Please read through and answer each of the following questions. If you are unsure how to do a problem, write down your best guess. You may use a calculator.

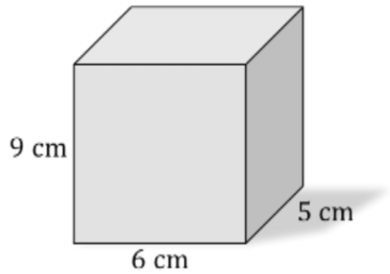
1. The **rectangle** below has a length of **10 cm** and a width of **8 cm**.




What is the perimeter? __________ cm

What is the area? __________ $cm^2$

2. The **rectangular prism** below has a length of **6 cm**, a width of **5 cm**, and a height of **9 cm**.

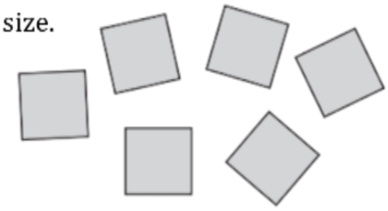



What is the volume? __________ $cm^3$

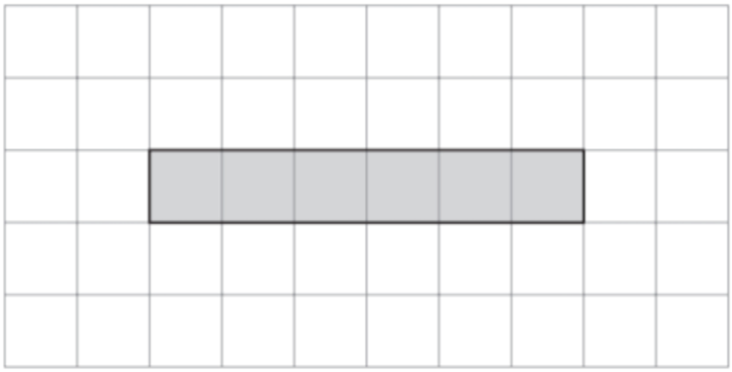
What is the surface area? __________ $cm^2$

3.

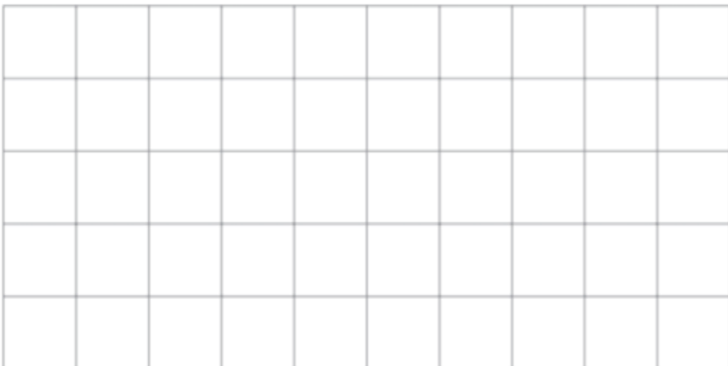
a. Stacey has **6** square tiles that are all the same size.



She joins the 6 tiles to make this rectangle.



On the grid below **draw a different** rectangle she could make using the 6 tiles.



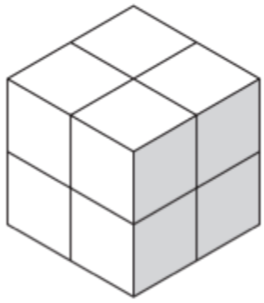
b. Stacey wants to make a **square** using **more than her 6 tiles**.

How many **more** tiles does she need?

_______________ more

4.

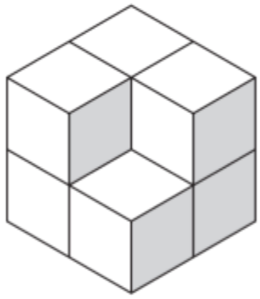
a. Eight small cubes of side length 1 cm are used to make a larger cube.



Complete the table to show the information for the larger cube.

| Larger cube | |
|---|---|
| Volume | ____________ |
| Surface area | ____________ |

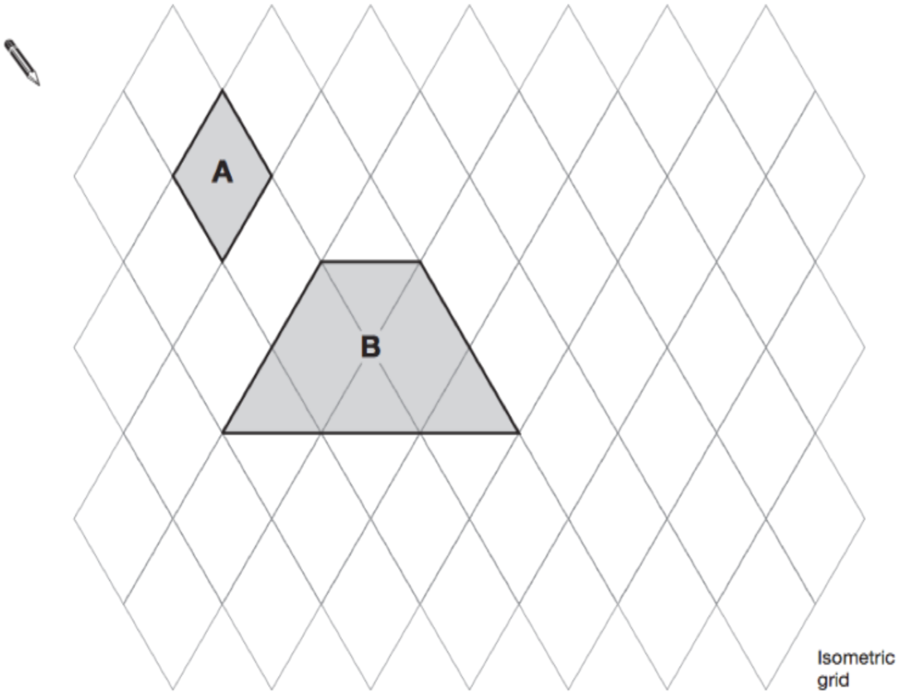
b. One of the small cubes is removed to make this new shape.



Tick (✓) the correct box in each row below.

| | Has increased | Has stayed the same | Has decreased |
|---|---|---|---|
| Volume | | | |
| Surface area | | | |

5. Look at the shaded shapes.




a. The area of shape **A** is **3 cm²**

What is the area of shape **B**?

 ____________ cm²

b. On the grid, draw a **triangle** that has an area of **6 cm²**.

Appendix G: Pre-test and post-test used with Ms Silver's class

Name________________________________________ Date________________

**Algebra Pretest**

**Directions**: Please read through and answer each of the following questions. If you are unsure how to do a problem, write down your best guess. You may use a calculator.

1. Find the values of $w, x, and\ y$

   (a) $2(w+3)=10$ $w=$ ________

   (b) $694+396+x=1742$ $x=$ ________

   (c) $y \div 13=34$ $y=$ ________

2. (a) I think of a number.
   I **double** my number and the answer is **178**
   What is my number? ________

   (b) I think of a different number.
   I **double** my number, then I **double again**.
   The answer is **312**
   What is my number? ________

3. Write the missing numbers.

   $962+$ ________ $=1898$

   ________ $-403=982$

   $51\times$ ________ $=2397$

   ________ $\div 23=828$

4. (a) When $\boldsymbol{y}=\mathbf{1}$, which expression below has the **largest value**? Put a ring round it.

   $3+y$ $10-y$ $3y$ $\frac{y}{2}$

   (b) When $\boldsymbol{y}=\mathbf{4}$, which expression below has the **largest value**? Put a ring round it.

   $3+y$ $10-y$ $3y$ $\frac{y}{2}$

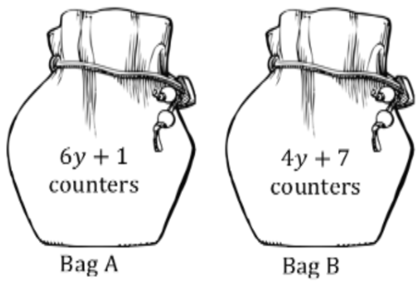
**5.** A boat can be hired for children's parties.
The formula below shows the cost.

$$Cost = £13.50 \times the\ number\ of\ children + £23$$

(a) What is the cost of a party for **8 children**? £ __________

(b) A different children's party cost **£225.50**
How many children were at the party? __________ ***children***

**6.** Bags A and B contain some counters.




The number of counters in each bag **is the same**.
Work out the value of $y$ $y =$ __________

**7.** A shop sells school uniform.

| |
|---|
| Two shirts and one jumper cost **£29** |
| One shirt and one jumper cost **£21** |

How much does **one jumper** cost? £ __________

**8.**

(a) Look at this equation:
$c + 3 = d - 4$

Which of $c$ and $d$ is greater, and by how much? __________, by __________

(b) Look at this equation:
$3 - e = 4 - f$

Which of $e$ and $f$ is greater, and by how much? __________, by __________

**9.** A teacher said:

| |
|---|
| Choose values for $a$ and $b$<br>Use the letters to make expressions for the numbers 1 to 8 |

(a) One group of pupils chose $a = 2$ and $b = 3$
Complete their table.

| $\boldsymbol{a = 2 \quad b = 3}$ |
|---|
| $b - a = 1$ |
| $a = 2$ |
| $b = 3$ |
| $2 \times a = 4$ |
| $= 5$ |
| $a \times b = 6$ |
| $2 \times a + b = 7$ |
| $= 8$ |

(b) Here is part of the table from a **different** group of pupils.

| |
|---|
| $2 \times a = 6$ |
| $a + b = 7$ |

What values did they choose?

$a =$ ___________

$b =$ ___________

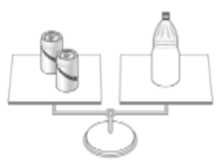
**10.** Look at the equation.

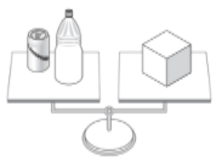
| |
|---|
| $14n = 98$ |

(a) Work out the value of $\boldsymbol{140n}$ ___________

(b) Work out the value of $\boldsymbol{14(n + 1)}$ ___________

**11.** 2 tins balance 1 bottle.
1 tin and 1 bottle balance 1 box.





(a) How many **bottles** do **6 tins** balance? ___________

(b) How many **boxes** do **6 tins** balance? ___________

**12.** I **add** the expressions $\boldsymbol{n}$ and $\boldsymbol{n+2}$. Put a ring round the expression that shows the result.

$2n$ $\quad$ $4n$ $\quad$ $2n+2$ $\quad$ $n(n+2)$

**13.**

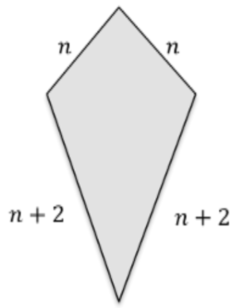
| |
|---|
| $(y+3)$ is always **5 more** than $(y-2)$<br>so $(y+3)-(y-2)=5$ |

**Complete the following**.

$(y+4)-(y-3)=$ ___________

$(y-2)-(y-3)=$ ___________

**14.** The diagram shows a kite.
The side lengths are in centimetres.



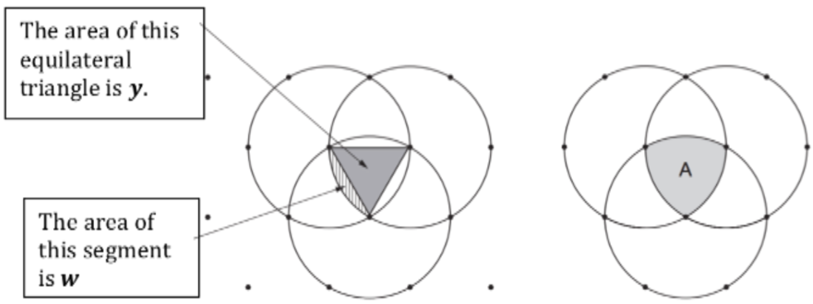

(a) When $\boldsymbol{n=9}$, what is the perimeter of the kite?

___________

(b) When the perimeter of the kite is **100**cm, what is the value of $n$?

___________

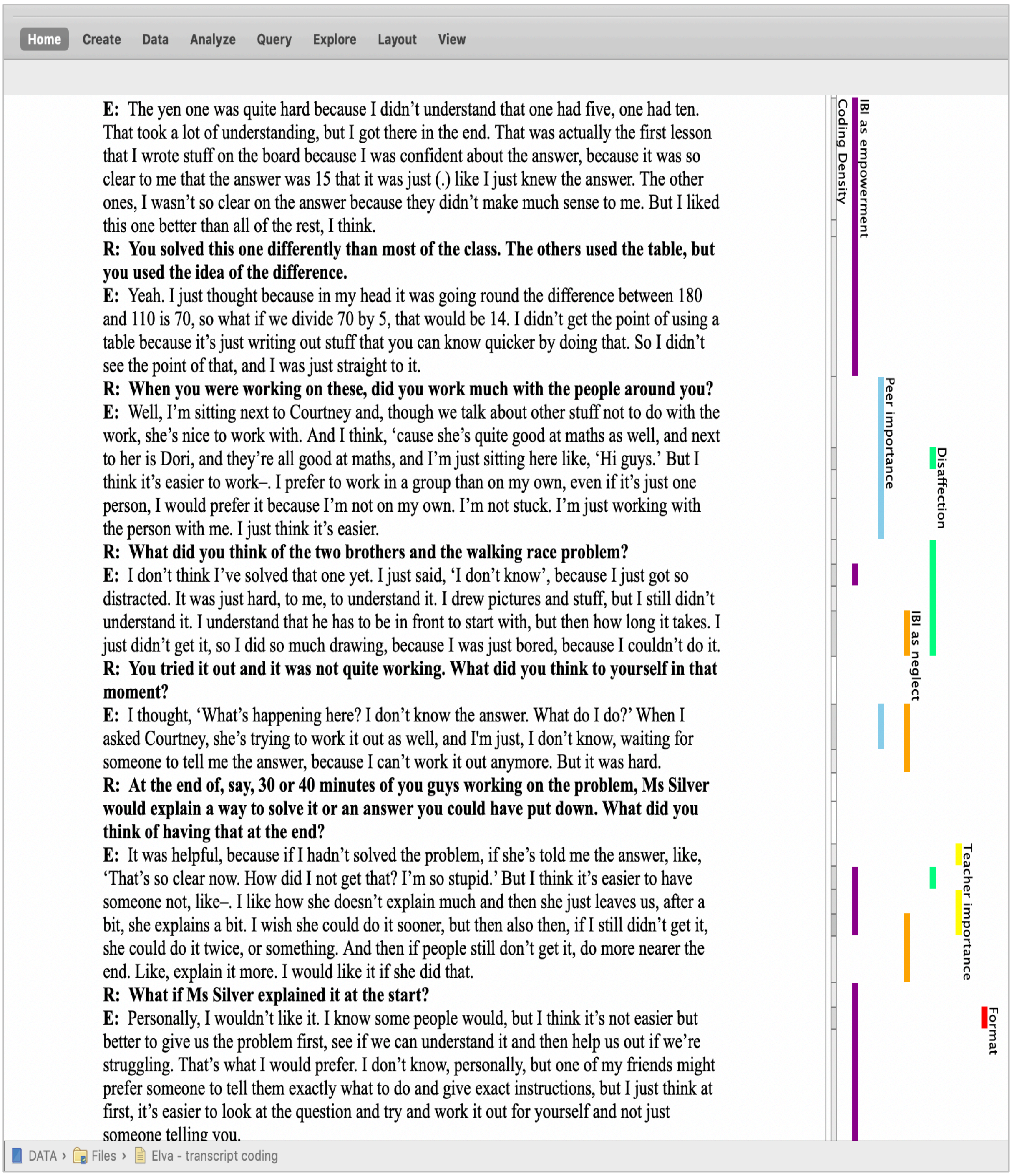
**15.** The diagram shows three congruent circles drawn on isometric grid.




Write an expression, using $\boldsymbol{y}$ and $\boldsymbol{w}$, for area A.

Area A = ______________________

Appendix H: Example of interview transcript coded using Nvivo



**E:** The yen one was quite hard because I didn't understand that one had five, one had ten. That took a lot of understanding, but I got there in the end. That was actually the first lesson that I wrote stuff on the board because I was confident about the answer, because it was so clear to me that the answer was 15 that it was just (.) like I just knew the answer. The other ones, I wasn't so clear on the answer because they didn't make much sense to me. But I liked this one better than all of the rest, I think.

**R: You solved this one differently than most of the class. The others used the table, but you used the idea of the difference.**

**E:** Yeah. I just thought because in my head it was going round the difference between 180 and 110 is 70, so what if we divide 70 by 5, that would be 14. I didn't get the point of using a table because it's just writing out stuff that you can know quicker by doing that. So I didn't see the point of that, and I was just straight to it.

**R: When you were working on these, did you work much with the people around you?**

**E:** Well, I'm sitting next to Courtney and, though we talk about other stuff not to do with the work, she's nice to work with. And I think, 'cause she's quite good at maths as well, and next to her is Dori, and they're all good at maths, and I'm just sitting here like, 'Hi guys.' But I think it's easier to work–. I prefer to work in a group than on my own, even if it's just one person, I would prefer it because I'm not on my own. I'm not stuck. I'm just working with the person with me. I just think it's easier.

**R: What did you think of the two brothers and the walking race problem?**

**E:** I don't think I've solved that one yet. I just said, 'I don't know', because I just got so distracted. It was just hard, to me, to understand it. I drew pictures and stuff, but I still didn't understand it. I understand that he has to be in front to start with, but then how long it takes. I just didn't get it, so I did so much drawing, because I was just bored, because I couldn't do it.

**R: You tried it out and it was not quite working. What did you think to yourself in that moment?**

**E:** I thought, 'What's happening here? I don't know the answer. What do I do?' When I asked Courtney, she's trying to work it out as well, and I'm just, I don't know, waiting for someone to tell me the answer, because I can't work it out anymore. But it was hard.

**R: At the end of, say, 30 or 40 minutes of you guys working on the problem, Ms Silver would explain a way to solve it or an answer you could have put down. What did you think of having that at the end?**

**E:** It was helpful, because if I hadn't solved the problem, if she's told me the answer, like, 'That's so clear now. How did I not get that? I'm so stupid.' But I think it's easier to have someone not, like–. I like how she doesn't explain much and then she just leaves us, after a bit, she explains a bit. I wish she could do it sooner, but then also then, if I still didn't get it, she could do it twice, or something. And then if people still don't get it, do more nearer the end. Like, explain it more. I would like it if she did that.

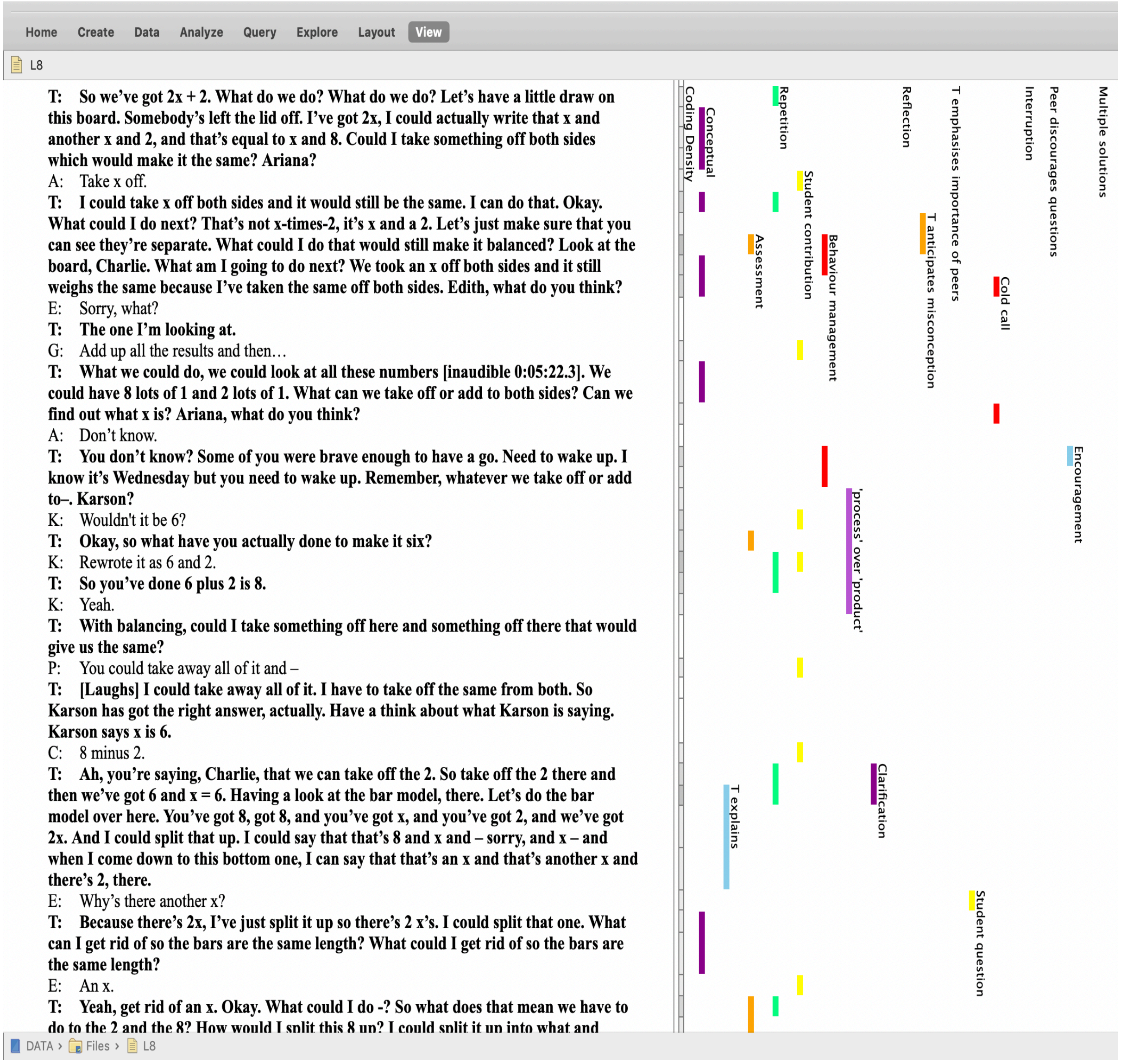
**R: What if Ms Silver explained it at the start?**

**E:** Personally, I wouldn't like it. I know some people would, but I think it's not easier but better to give us the problem first, see if we can understand it and then help us out if we're struggling. That's what I would prefer. I don't know, personally, but one of my friends might prefer someone to tell them exactly what to do and give exact instructions, but I just think at first, it's easier to look at the question and try and work it out for yourself and not just someone telling you.



*Names have been changed in the transcript to protect the students' identities

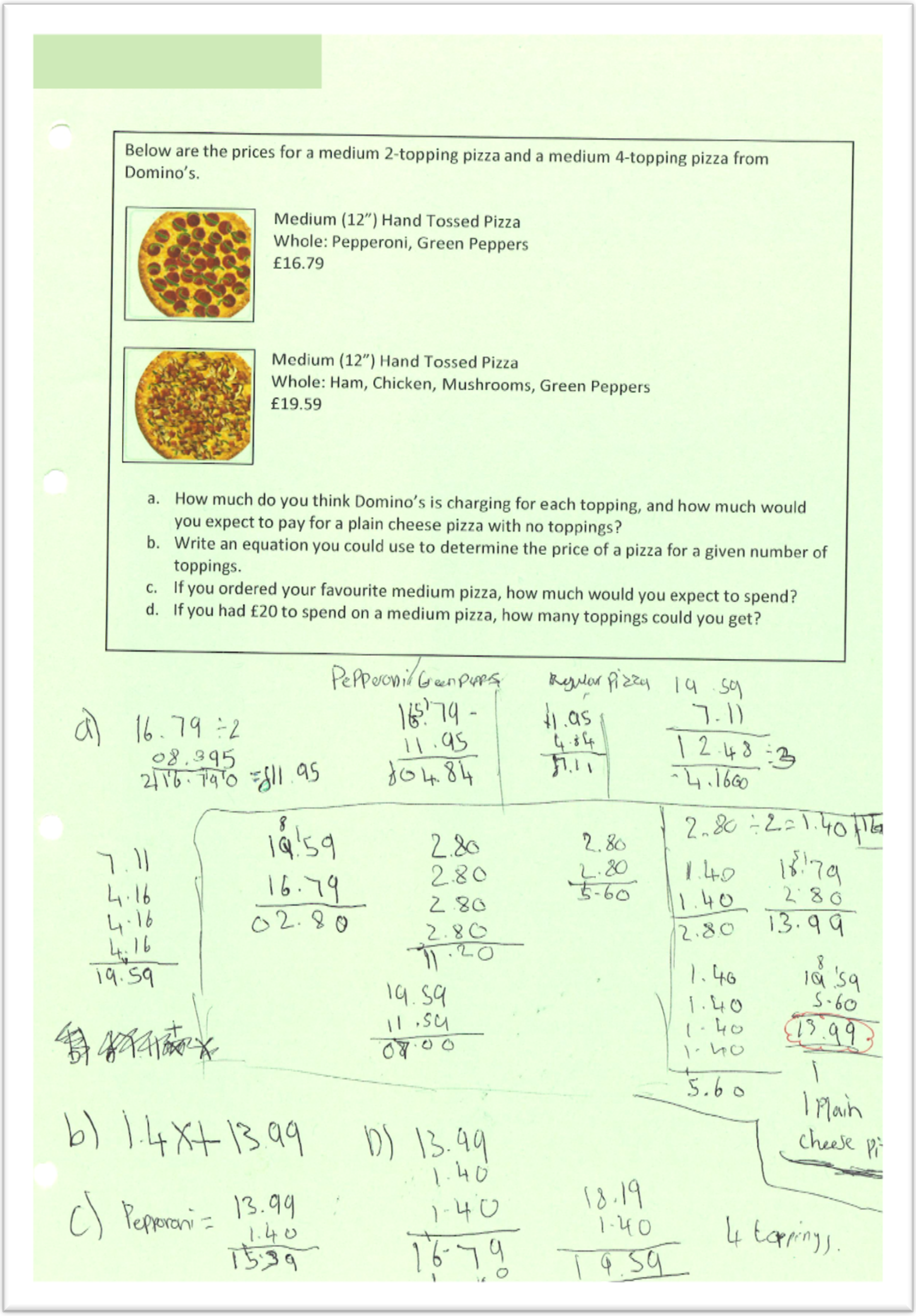
# Appendix I: Example of lesson transcript coded using Nvivo




**T: So we've got 2x + 2. What do we do? What do we do? Let's have a little draw on this board. Somebody's left the lid off. I've got 2x, I could actually write that x and another x and 2, and that's equal to x and 8. Could I take something off both sides which would make it the same? Ariana?**
A: Take x off.
**T: I could take x off both sides and it would still be the same. I can do that. Okay. What could I do next? That's not x-times-2, it's x and a 2. Let's just make sure that you can see they're separate. What could I do that would still make it balanced? Look at the board, Charlie. What am I going to do next? We took an x off both sides and it still weighs the same because I've taken the same off both sides. Edith, what do you think?**
E: Sorry, what?
**T: The one I'm looking at.**
G: Add up all the results and then…
**T: What we could do, we could look at all these numbers [inaudible 0:05:22.3]. We could have 8 lots of 1 and 2 lots of 1. What can we take off or add to both sides? Can we find out what x is? Ariana, what do you think?**
A: Don't know.
**T: You don't know? Some of you were brave enough to have a go. Need to wake up. I know it's Wednesday but you need to wake up. Remember, whatever we take off or add to–. Karson?**
K: Wouldn't it be 6?
**T: Okay, so what have you actually done to make it six?**
K: Rewrote it as 6 and 2.
**T: So you've done 6 plus 2 is 8.**
K: Yeah.
**T: With balancing, could I take something off here and something off there that would give us the same?**
P: You could take away all of it and –
**T: [Laughs] I could take away all of it. I have to take off the same from both. So Karson has got the right answer, actually. Have a think about what Karson is saying. Karson says x is 6.**
C: 8 minus 2.
**T: Ah, you're saying, Charlie, that we can take off the 2. So take off the 2 there and then we've got 6 and x = 6. Having a look at the bar model, there. Let's do the bar model over here. You've got 8, got 8, and you've got x, and you've got 2, and we've got 2x. And I could split that up. I could say that that's 8 and x and – sorry, and x – and when I come down to this bottom one, I can say that that's an x and that's another x and there's 2, there.**
E: Why's there another x?
**T: Because there's 2x, I've just split it up so there's 2 x's. I could split that one. What can I get rid of so the bars are the same length? What could I get rid of so the bars are the same length?**
E: An x.
**T: Yeah, get rid of an x. Okay. What could I do -? So what does that mean we have to do to the 2 and the 8? How would I split this 8 up? I could split it up into what and**

*Names have been changed in the transcript to protect the students' identities

Appendix J: Clay and Simon's worksheets for the Pizza Problem of L4



Below are the prices for a medium 2-topping pizza and a medium 4-topping pizza from Domino's.

Medium (12") Hand Tossed Pizza
Whole: Pepperoni, Green Peppers
£16.79

Medium (12") Hand Tossed Pizza
Whole: Ham, Chicken, Mushrooms, Green Peppers
£19.59

a. How much do you think Domino's is charging for each topping, and how much would you expect to pay for a plain cheese pizza with no toppings?
b. Write an equation you could use to determine the price of a pizza for a given number of toppings.
c. If you ordered your favourite medium pizza, how much would you expect to spend?
d. If you had £20 to spend on a medium pizza, how many toppings could you get?

Below are the prices for a medium 2-topping pizza and a medium 4-topping pizza from Domino's.

Medium (12") Hand Tossed Pizza
Whole: Pepperoni, Green Peppers
£16.79

Medium (12") Hand Tossed Pizza
Whole: Ham, Chicken, Mushrooms, Green Peppers
£19.59

a. How much do you think Domino's is charging for each topping, and how much would you expect to pay for a plain cheese pizza with no toppings?
b. Write an equation you could use to determine the price of a pizza for a given number of toppings.
c. If you ordered your favourite medium pizza, how much would you expect to spend? - 13.99
d. If you had £20 to spend on a medium pizza, how many toppings could you get?

– 4 toppings

pepperoni, peppers

16.79 ÷ 2 = 8

19.59 - 16.79 = 2.80 ÷ 2 = 1.40

16.79

02.80

14 + 1.40 =

15.40 + 1.40 =

16.80 + 1.40 =

18.2

+ 1.40 =

19.60 Dominos charging about 1.40 for each topping

16.79
− 1.40
15.39

x no of toppings

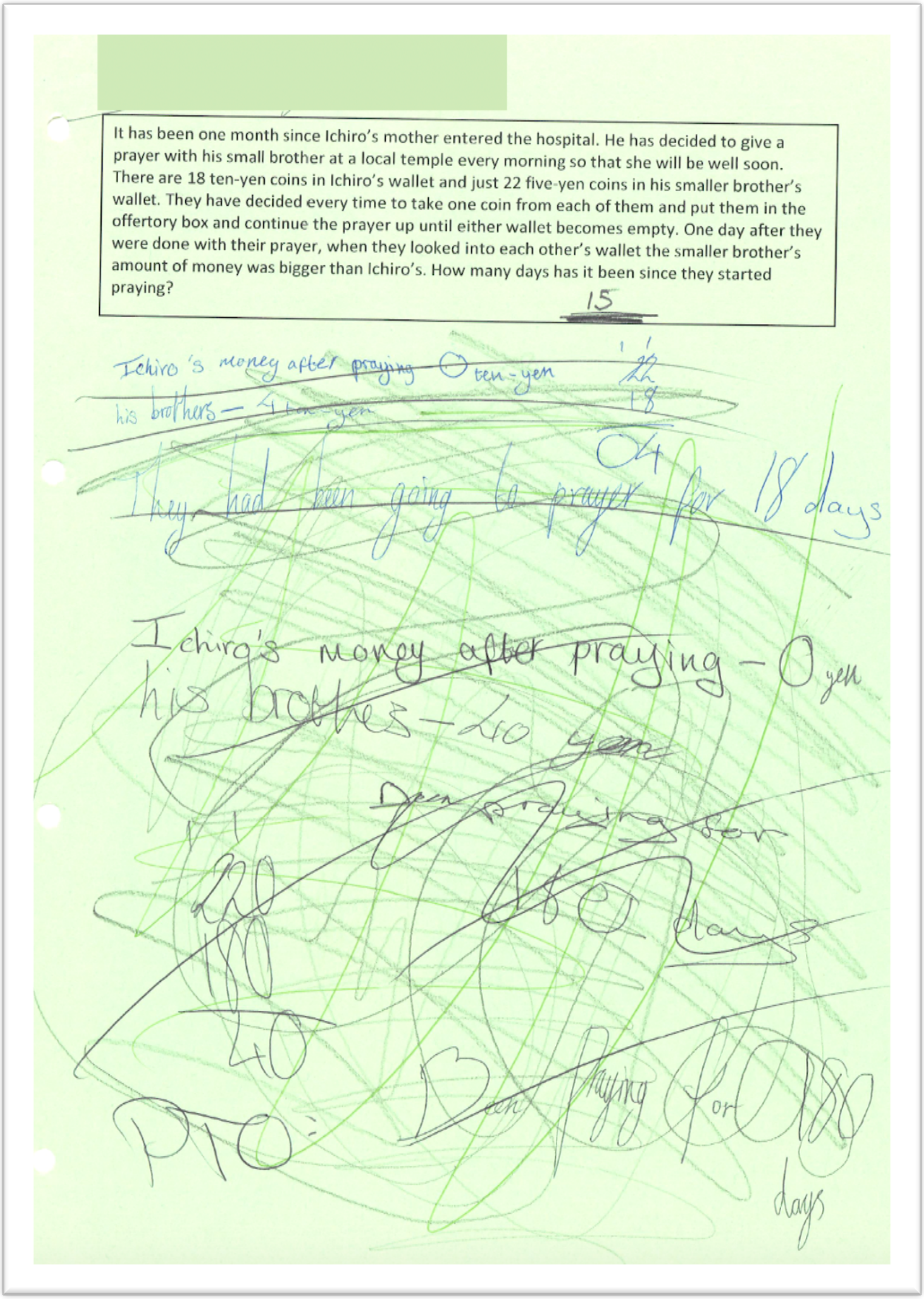
H, C, M, P, P

x + £13.99 – b/c £14

16.79
− 2.80
13.99

1.40x + 13.99

Appendix K: Sample worksheet showing signs of student frustration


It has been one month since Ichiro's mother entered the hospital. He has decided to give a prayer with his small brother at a local temple every morning so that she will be well soon. There are 18 ten-yen coins in Ichiro's wallet and just 22 five-yen coins in his smaller brother's wallet. They have decided every time to take one coin from each of them and put them in the offertory box and continue the prayer up until either wallet becomes empty. One day after they were done with their prayer, when they looked into each other's wallet the smaller brother's amount of money was bigger than Ichiro's. How many days has it been since they started praying?

15

Appendix L: An analysis of the quality of inquiry instruction for Mr Scott's case

With a view towards understanding students' perceptions of inquiry instruction in mathematics, it is important to first establish whether the observed teaching unit could be characterised as inquiry. In the following section, I assess the quality of the overall IBI unit, which consisted of seven lessons. I do so according to the four factors of EQUIP: Instructional factors, Discourse factors, Assessment factors, and Curriculum factors. As previously discussed in Section 4.3, the EQUIP is an instrument that has been validated for use in mathematics classrooms. For a copy of the EQUIP, please see Appendix A.

It is important to state that the lesson plans used during the IBI unit were planned in collaboration with Mr Scott and with reference to the EQUIP. Therefore, this analysis is focused on the *implementation* of the IBI lessons.

### L.1 Instructional factors

This section discusses the Instructional factors of the EQUIP rubric. The Instructional factors are broken into five constructs which look at how the teacher designed and implemented lessons to develop students' procedural and conceptual understanding. The constructs measured are Instructional strategies, Order of instruction, Teacher role, Student role, and Knowledge acquisition. Also included in this section is the extent to which students' procedural and conceptual understanding of the topic of measurement changed throughout the IBI unit.

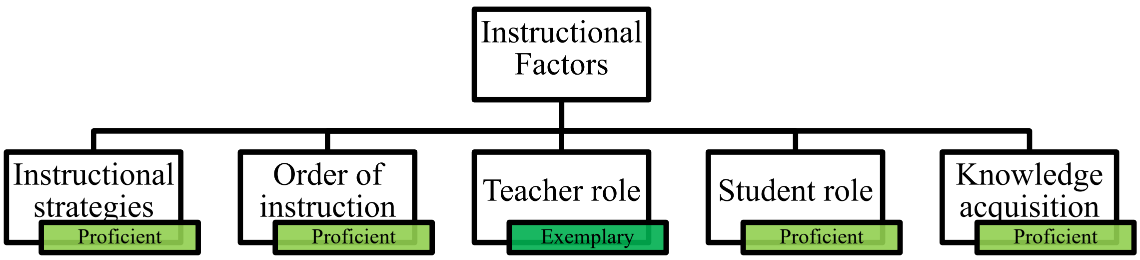



Figure 9.1: EQUIP ratings for five Instructional Factors at Harrison School

#### L.1.1 Instructional strategies

In assessing proficient inquiry, the EQUIP describes high quality inquiry lessons as those that help students 'develop conceptual understanding' (see Appendix A). This was observed in several ways. Mr Scott selected a variety of nonroutine problems that required students to apply their knowledge in new ways. For example, in L2 the students were asked to develop a method to measure the perimeter of their school basketball court, and in L7 students were asked to estimate the number of 500ml water bottles it would take to fill their classroom, both of these problems were nonroutine for this class. These nonroutine problems provided the students with opportunities to develop greater conceptual understanding. Please see 'Knowledge acquisition' below for a description of how student's conceptual understanding may have changed over the course of the unit.

EQUIP states that during proficient inquiry teachers 'only occasionally lecture' (see Appendix A). During the seven lessons observed, Mr Scott allowed students to explore the problems (either individually or with a group) for a substantial part of the lesson time, with the exception of L4 since this lesson was intended to serve as the 'explanation phase' of the two previous lessons. In some lessons he lectured a lot, such as L4, and in others the class spent nearly the entire time exploring such as in L2.

Figure 5.3 shows how the lesson time throughout the IBI was divided among administration, explanation, and exploration. Administration was considered to be tasks that the teacher and students completed in order to prepare to begin or progress the lesson, for example, taking the register or passing out papers. Explanation was considered any time a person (the teacher or a student) spoke to the entire class. This included explanations of the task or solution methods. Exploration was considered any time the students freely explored the IBI task.

To obtain a rubric score of proficient inquiry required students to be 'engaged in activities' (see Appendix A). During the early lessons, students were observed to be cooperative, but often lacked engagement. It was common to see students spend almost the entire lesson in off-task activities with little evidence of learning occurring. For example, a group of five pupils (Daniel, David, Kevin, Timothy, and Tyler) were off-task for nearly all of L1. However, as the unit progressed the engagement levels seemed to increase, and whilst students would still demonstrate off-task behaviours, the frequency observed was lower. This was evidenced by Mr Scott not having to redirect the students' attention as much, coupled with an increase in the number of 'house points' awarded per lesson (none in the first two lessons compared to four in the final two lessons). The observed improved behaviour might be a result of students gaining familiarity with the new ways of working.

In consideration of the above discussion as well as my observations, the instructional strategies were rated as proficient inquiry (level 3): 'Teacher occasionally lectured, but students were engaged in activities that helped develop conceptual understanding.'

### L.1.2 Order of instruction

Not all of the classroom time during the IBI unit was used toward student exploration. Rather, student explorations were typically followed by explanations given by either the teacher or the students. What's important to emphasise is that these explanation phases of instruction were always preceded by substantial student exploration, either on the same day or on a previous lesson (see Figure 5.3). For example, in L3 Mr Scott and several students explained their approach to the Basketball problem done in L2.

> We went around the basketball court and like counted how many times David could lie end to end all around it. (Daniel, L3)

> We had Erica walk end to end with her feet like this. [Jackie demonstrates walking heel to toe along a straight line]. We counted 350 steps all the way around. (Jackie, L3)

In L4, Mr Scott led the students through a PowerPoint presentation he had prepared to review the key definitions and formulas for perimeter and area only after several lessons of exploration had preceded it. It is thought these explanations phases allow the students to formalise newly learned concepts (see Section 2.4). Mr Scott consistently followed this pattern throughout the IBI unit by allowing students time to explore new ideas prior to explanation.

I rated the order of instruction as proficient inquiry (level 3): 'Teacher asked students to explore before explanation. Teacher and students explained.'

### L.1.3 Teacher role

Mr Scott's role throughout the IBI unit was usually that of a facilitator. Beyond creating the investigations and introducing the task, Mr Scott primarily acted as a facilitator by visiting the students as they worked individually or in groups. He spent most of the classroom time probing the students' thinking in order to check their understanding and to also keep the students engaged and on-task. Students' perceptions of the teacher's role are discussed in Section 5.4.3.3.

Given the above, this component of the IBI unit was rated as exemplary inquiry (level 4): 'Teacher consistently and effectively acted as a facilitator.'

### L.1.4 Student role

The students' role throughout the unit was that of active learners. Even during whole-class discussions, which were largely controlled by the questioning of Mr Scott, students were highly engaged and participative. At times, students would get off task (especially at the beginning of the unit or when posed with a challenging question), but Mr Scott demonstrated skill in finding ways to rephrase or adjust his questioning in order to maintain their attention. The delimitation between the teacher role and the student role throughout the IBI unit was in part maintained by Mr Scott's frequent descriptions of how both he and the students should be acting at any given time. For example, in L1 Mr Scott gave the following directions:

> Okay, while you work, I'm going to be around if you have any questions about what to do. But again, I want to see what you can do, so I'm not going to spoil it and tell you exactly how to solve the problems. Once you have gotten around to each of the problems, we will go over the answers and maybe even have some of you present your solutions at the end. (Teacher, L1)

These directions helped to set clear expectations for how the teacher's and the students' roles would differ. However, because the students were not consistently and clearly focused throughout each lesson this component could not receive a rating of 4, exemplary inquiry.

I therefore rated the IBI unit as proficient inquiry (level 3): 'Students were active as learners (involved in discussions, investigations, or activities, but not consistently and clearly focused).'

### L.1.5 Knowledge acquisition

The final construct measured under the Instructional factors section of the rubric is Knowledge acquisition. This construct looks at the depth of understanding students were required to demonstrate throughout the IBI unit. The IBI unit was designed to meet the learning objectives set out by the school which were aligned to the National Curriculum for Key Stage 3 (see Section 5.2). However, since this particular group still struggled with some of the foundational ideas which were first introduced in Key Stage 2, Mr Scott decided to set flexible learning objectives for this unit that could be easily adapted for more advanced students. The learning objectives for the unit were: (1) recognise perimeter as a 'distance' (in cm for instance), (2) recognise area as a 'space' (in cm2 for instance), (3) calculate perimeters and areas of rectangles, and (4) calculate volumes and surface areas of cuboids. In reaching these objectives, the students were consistently required to apply their knowledge in new situations in order to demonstrate high levels understanding.

Some of the greatest improvements in understanding that took place over the course of the IBI unit included the concepts of perimeter and area. In the beginning of the unit, the students attempted to define perimeter and area only in terms of mathematical operations. To them, the concepts of perimeter and area were something you *do* by either adding, subtracting, multiplying, or dividing. For instance, in L1 Linda asked her group members what the word 'area' meant, and Erica answered by saying she was 'pretty sure that's when you add up all the sides.' Later, during the same lesson, Mr. Scott asked the class what area meant, and a few students responded at the same time that it means 'to multiply'. In both instances, the students viewed the concepts only in terms of mathematical operations. However, as the unit progressed the students' understanding of perimeter and area became more sophisticated. In L4 Jenna described her method for finding the perimeter as 'I added it up like I was walking around it [the rectangle]'. While Jenna still partly focused on the operation that was required, her understanding of perimeter was now coupled with the conceptual idea of walking around the shape. Likewise, in the same lesson, when James was asked about area he responded, 'The area is 12 because there's 12 boxes covering the shape'.

In addition to classroom observations, the students took a pre- and post-test to measure their learning across the IBI unit. The results of these are discussed in Section 5.4.4. The IBI unit reached the level of proficient inquiry (level 3): 'Student learning required application of concepts and process skills in new situations'.

**L.2 Discourse factors**

In this section I discuss the Discourse factors of the EQUIP rubric. These are aimed at looking at the ways in which the teacher used questioning and discussion to promote student understanding. The constructs measured are Questioning level, Complexity of questions, Questioning ecology, Communication pattern, and Classroom interactions.

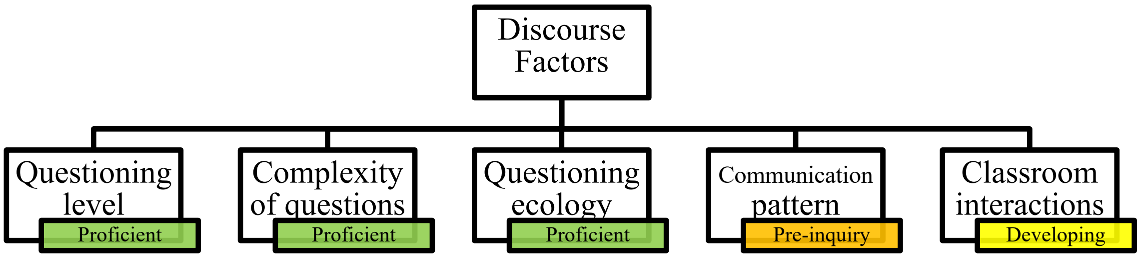



Figure 9.2: EQUIP ratings for five Discourse Factors at Harrison School

**L.2.1 Questioning level**

Mr Scott's questioning level throughout the IBI unit was variable. Using the revised Bloom's Taxonomy (Krathwohl, 2002), I coded the questions Mr Scott asked the students (see Table 9.1). To do so I used the lesson transcripts, which primarily included only those questions Mr Scott posed to the entire class. Because of the noise level in the room it was not always realistic to capture the questions the teacher asked to small groups or individual students. Therefore, it is important to note that the distribution of the questioning levels presented might provide an incomplete picture.

Mr Scott's questioning typically met the 'apply' level or lower. However, several lines of questioning did reach the level of 'evaluate' and 'create' which is in line with exemplary inquiry (level 4) defined as, 'Questioning challenged students at various levels, including at the analysis level or higher; level was varied to scaffold learning' (see Appendix A). Higher level questions were typically used to help frame the entire IBI lesson, whereas lower level questions were used in order to scaffold student learning as each lesson progressed.

I concluded that the questioning level achieved an EQUIP rubric rating of proficient inquiry (level 3): 'Questioning challenged students up to application or analysis levels.'

Table 9.1: Mr Scott's questioning level according to the revised Bloom's Taxonomy

| Questioning level | n | % | Example |
|---|---|---|---|
| Create | 2 | 4% | 'Is it possible to construct a shape in which the numerical values of its area and perimeter are the same?' (L5) |
| Evaluate | 4 | 8% | 'Jackie's group counted 350 steps, but Lisa's group counted 398. That's a difference of almost 50 steps. Why do you think they got such different answers?' (L3) |
| Analyse | 8 | 16% | 'Imagine with me for a moment, how many of these water bottles do you think it would take to fill up our classroom?' (L7) |
| Apply | 10 | 20% | 'How many square centimetres of cardboard are necessary to wrap the Toblerone bar?' (L6) |
| Understand | 13 | 26% | 'Why does it make sense to multiply the two sides to find the area of a rectangle?' (L1) |
| Remember | 13 | 26% | 'Who can tell me what we worked on when I saw you Wednesday?' (L2) |

**L.2.2 Complexity of questions**

Mr Scott's questions typically had one correct answer, however he often made use of additional questioning that would challenge students to explain and justify their work. This was especially evident during whole class discussions in which students were asked to present their work to the class and in turn evaluate their peers' reasoning. For example, in L6 Mr Scott invited David to draw his net for the triangular prism on the white board. Once David had completed his drawing, Mr Scott asked the class what they thought of David's sketch. Henry raised his hand and said, 'You're missing something.' Mr Scott then looked over at David and waited for his response. David took a moment to look over his drawing, and then begun to talk out loud. 'Well, this rectangle bit is the bottom. And I drew these two to be the long sides. …Oh, hold on. And I have this triangle for the front of the chocolate but I'm missing the other side.' David then added the missing triangle face to his net. Exchanges such as these were common during student presentations.

Though many of the questions only had one correct answer, all had multiple solution pathways. As students worked on problems during the lesson, Mr Scott would walk around the room and make note of unique solutions to share and discuss during the whole class discussion. For example, in L4 Mr Scott had students share how they found the perimeter of a square. Most students added up each side while some in the class multiplied the side length by four. During the ensuing discussion, students were asked to reflect on the different solution methods their peers presented and evaluate which one they thought was the 'best' (e.g. most efficient, most memorable, most beautiful).

By routinely asking the class if anyone had solved the problem in a different way, Mr Scott communicated the value of different solutions.

I rated the complexity of the questioning as proficient inquiry (level 3): 'Questions challenged students to explain, reason, and/or justify.'

### L.2.3 Questioning ecology

Another way to describe 'questioning ecology' is 'questioning climate' (Smart & Marshall, 2013). It refers to the complexity and variety of student responses that are elicited by the teacher's questioning (e.g. discussion, investigation, reflection).

At the beginning of the IBI unit students demonstrated discomfort with class discussions. For example, during L1 only one student (Lisa) volunteered to present her work, and none voluntarily offered feedback. By the end of the unit, however, students were observed to be more active in volunteering, for instance by sharing their methods and offering feedback to each other. Open discussion and reflection were a bigger part of the class routine than it had been previously, and students seemed to adjust well to this change (though this adjustment took place gradually). This was best evidenced by an increase in the number of students volunteering to answer questions and share their ideas.

Mr Scott in part achieved this by teaching the students how to give each other feedback. For example, in L3 when students were sharing how their group measured the perimeter of the basketball court, Mr Scott called on students to offer feedback. He had written a list of sentence frames on the board, such as 'I like how your group did ______ because _______' and 'I think it would have been better if your group did _______ because _______.' This strategy helped the students offer each other feedback more effectively. For example, in L3 a group of boys shared with the class their plan to measure the area of the court using David's body. When Mr Scott asked the rest of the class what they thought, Jackie said 'That's creative, but how would you make sure to cover the whole thing and not have any gaps?' This led to a discussion about the importance of no gaps when measuring area, an essential feature of the canonical idea of tiling (also known as tessellation).

Additionally, Mr Scott's use of cold calling (when a teacher calls on a student whose hand is not raised) helped to communicate to the students that everyone's input is expected and valued.

For the above reasons I felt that the questioning ecology was proficient inquiry (level 3): 'Teacher successfully engaged students in open-ended questions, discussions, and/or investigations.'

### L.2.4 Communication pattern

The communication pattern was a weakness of the overall IBI unit, with discussion being controlled and directed by Mr Scott in a mostly didactic pattern. Occasionally students spoke directly to one another in whole class discussions, but only after the teacher spoke and then called on a student. For example, in L5 Jackie suggested a square with a side length of zero as a solution.

> T: What's the problem with choosing zero? I like that you thought to try something different, but what's the issue with zero?
>
> J: Well if you do a number times zero the answer is always zero.
>
> H: [speaking directly to Jackie] There's no area.
>
> (Teacher, Jackie, Henry, L5)

In this instance, Henry spoke directly to Jackie rather than to the teacher, but this was out of the norm. Typically, the group spoke to each other using a hand raising pattern which was always mediated by Mr Scott. Changing classroom discourse patterns in this way can be challenging since it is one of the most well-rehearsed school norms students experience (Drageset, 2015).

Overall, I felt that the communication pattern was best rated as pre-inquiry (level 1): ‘Communication was controlled and directed by teacher and followed a didactic pattern.

**L.2.5 Classroom interactions**

The classroom interactions throughout the IBI unit varied. Often, Mr Scott accepted correct answers without further questioning. These may represent missed opportunities for students to engage in deeper discussion about why an answer was correct, and possible alternative approaches to the same answer. The fact Mr Scott missed these opportunities is perhaps due to his lack of experience teaching using an IBI approach or perhaps due to time pressures.

This pattern was not persistent however, as at other points Mr Scott would follow up student responses with further probing questions. He either asked the student who provided the answer to give an explanation or he would ask another student in the class to explain. For example, in L4 when the students were finding the perimeter of a rectangle, Steven provided a correct answer of 26 centimetres. Instead of accepting this answer and moving on, Mr Scott asked, ‘How do you think Steven got 26 centimetres? Jenna?’ This follow up question pushed students to evaluate their classmate’s thinking and achieve a higher level of classroom interaction. This also helped the teacher check for understanding.

Most lessons Mr Scott designed incorporated features of cooperative learning. Students often worked in groups of two to four towards developing a solution to a novel problem. In L1 students moved around the classroom in groups of around four to solve problems posted at each of four stations. Each of the four stations contained a problem unfamiliar to the students. Mr Scott frequently encouraged groupwork when giving instructions. When introducing the task in L1, Mr Scott stated, ‘What you are going to do is try them [the problems] out, ask each other for help, and see how you do.’ Later on, when noticing a group of students were not working cohesively, Mr Scott said to one of the idle group members, ‘It's important that you understand how to do it too. Make sure you are talking and checking with each other please.’ As a result, Mr Scott emphasised both individual and group accountability (D. Johnson & Johnson, 2016).

Therefore, in the area of classroom interactions I rated the IBI unit as developing inquiry (level 2): ‘Teacher or another student occasionally followed-up student response with further low-level probe.’

### L.3 Assessment factors

This section discusses the Assessment factors of the EQUIP. These look at the ways in which the teacher evaluated the students' knowledge acquisition throughout the unit. The constructs measured are Prior knowledge, Conceptual development, Student reflection, Assessment type, and Role of assessing.

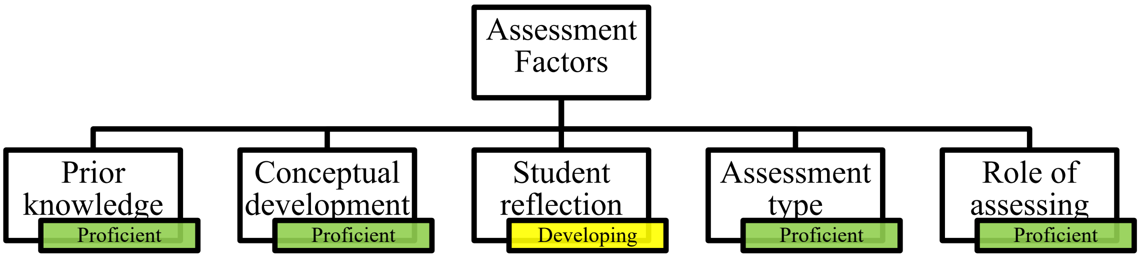



Figure 9.3: EQUIP ratings for five Assessment Factors at Harrison School

### L.3.1 Prior knowledge

Mr Scott assessed students' prior knowledge in every lesson. For instance, the majority of L1 was designed to assess prior knowledge of perimeter, area, and volume. In response, Mr Scott chose to deemphasise circumference and area of circles in the lessons that followed, since students demonstrated significant difficulty with perimeter and area of more simple shapes, such as squares. Another example was observed in L6, when Mr Scott assessed students' prior knowledge of wrapping a present. He asked the class how many of them had wrapped a present before. Since only a few students raised their hands, Mr Scott chose Timothy to share his experience wrapping a present. Mr Scott followed this by describing a time he saw volunteers wrapping Christmas presents at a local shop. These lucid descriptions may have better prepared students for the surface area question that followed. Assessment of prior knowledge was integrated into every lesson. Mr Scott adjusted his questioning in light of student performance regularly.

Therefore, I rated this component as proficient inquiry (level 3): 'Teacher assessed student prior knowledge and then partially modified instruction based on this knowledge.'

### L.3.2 Conceptual development

The lessons of the IBI unit were designed to encourage a focus on 'process' rather than 'product', meaning students were encouraged to attend more to the way (the 'process') they solved the problem than to the answer (the ‘product’) itself. This was frequently evidenced when Mr Scott would ask, ‘Has anyone solve this problem in another way?’ or ‘How do these two solution methods differ?’

However, despite Mr Scott’s encouragement, students frequently focused their energies on producing a correct answer rather than sound reasoning. This was often demonstrated when students would point to their work and ask Mr Scott or myself, ‘Is this right?’ It is possible that class or school norms were driving this behaviour, however it is indicative of a focus on outcomes over understanding. At times, students were happy to let other students do the work for the group and just copy the answer. This is discussed in more detail in Section 5.4.2 in which I describe the impact of the students’ beliefs on their performance during the IBI unit.

Occasionally though, students did demonstrate genuine interest in the process without prompting from the teacher. For example, in L5, after Heather shared a solution, David asked, ‘Is it because it’s a square?’ Despite having an answer, David appeared curious to know more. Mr Scott and the rest of the class subsequently carried this curiosity forward via a further investigation into squares and other regular polygons (e.g. an equilateral triangle).

In light of the above discussion as well as my observations, I rated this component as proficient inquiry (level 3): ‘Teacher encouraged process focused learning activities that required critical thinking.’

### L.3.3 Student reflection

Mr Scott occasionally asked students to reflect on their learning. For example, at the end of L5, he asked students to write (1) What they did well, (2) What they did not do well, and (3) What they could do to improve. To help the students in this process Mr Scott had students show their reflections to him when they were finished. Another way Mr Scott encouraged student reflection was by having the students show their current level of understanding with their thumbs: thumbs up for ‘good’, thumbs to the side for ‘okay’, and thumbs down for ‘bad’. Students explicitly reflected on their progress only a few times throughout the IBI unit. One such lesson was in L1 when students were asked to respond to how well they did on the circle problem. Nearly all students gave a thumbs down.

Therefore, since student reflection was not implemented frequently enough, I rated this component as developing inquiry (level 2): 'Teacher explicitly encouraged students to reflect on their learning but only at a minimal knowledge level.'

**L.3.4 Assessment type**

Students were frequently assessed throughout the IBI unit. For instance, in L5 the students searched for examples of a rectangle in which the numerical value of its perimeter and area were the same. Instead of giving the students a group of rectangles to find the perimeter and area of, this problem gave students an authentic reason to generate rectangles of their own making and then find the perimeter and area in order to test their developing ideas. In this lesson Mr Scott assessed student understanding by reading their written work, asking follow-up questions, and listening to students' conversations with each other. Nonroutine problems with context were used to assess students as well, for instance in L6 when students were asked to determine how much cardboard would be needed to wrap a chocolate bar shaped like a triangular prism. Students were also assessed through the use of the pre- and post-test discussed in Section 5.2.1.

Mr Scott made use of frequent informal assessment by asking questions and observing work. By walking around the classroom whilst the students worked Mr Scott was able to quickly identify when a student or group of students were struggling. This was done by listening to the level of discussion, observing the work being written down, and responding to student prompts. This informal assessment provided real time feedback to Mr Scott, who adapted the lesson in response.

Assessment type was therefore rated as proficient inquiry (level 3): 'Formal and informal assessments used both factual, discrete knowledge and authentic measures.'

**L.3.5 Role of assessing**

Mr Scott routinely assessed the students' understanding and regularly adjusted the pace, and sometimes the direction, of the lesson in response to the students' understanding. This was perhaps best observed in L3 when Mr Scott discussed the area of the school basketball court.

> What I want you to do now is talk with your table about how you might go about finding the area of the basketball court. We know the perimeter of the

> basketball court is about 88 meters. The length is about 28 meters. And the width is about 15 meters. What about the area? Come up with a few different ideas of how you might do that. I'll give you about 15 minutes or so to think it through, and then you can share out some of your ideas. (Teacher, L3)

However, as the students discussed there was a lot of off-task behaviour and conversations did not seem to be about the area problem. Of the on-task conversations I heard, the students seemed to be guessing at what operations to do with the numbers (e.g. 88 divided by 15) rather than thinking conceptually about the problem. As Mr Scott made his way to each group, he adjusted his instruction and instead encouraged the students to ignore the quantities 88, 28, and 15 he had emphasized earlier. Instead, he asked them to think more creatively, like they had when they were thinking up ways to measure the perimeter of the basketball court without measuring tools. This represented a turning point in the students' group talk. Mr Scott's decision to change his question made the problem more accessible to the students and resulted in greater engagement.

In light of the above, this component was rated as proficient inquiry (level 3): 'Teacher solicited explanations from students to assess understanding and then adjusted instruction accordingly.'

### L.4 Curriculum factors

This section discusses the Curriculum factors of the EQUIP. These look at the ways in which the chosen curriculum flexibly supports student exploration and understanding of the 'big picture'. The constructs measured are Content depth, Learner centrality, Integration of content and investigation, and Organising and recording information.

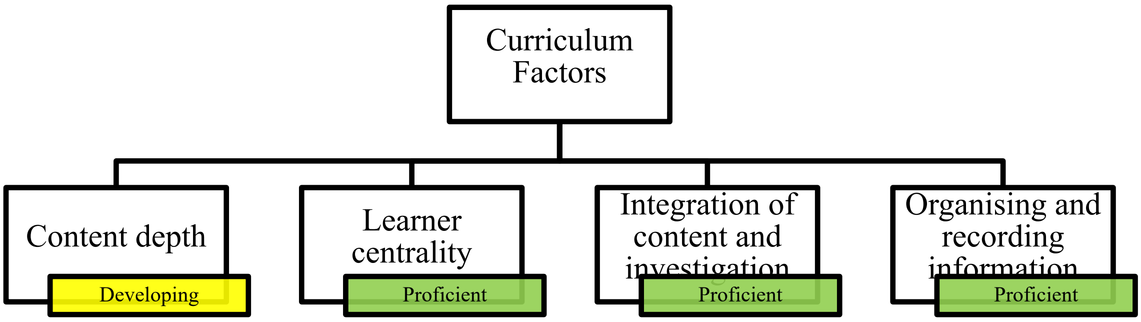



Figure 9.4: EQUIP ratings for five Curriculum Factors at Harrison School

**L.4.1 Content depth**

Mr Scott designed and implemented the IBI lessons in a way that provided depth of content, but few connections were made to the big picture. Opportunities to connect to big ideas were missed, such as in L7 when estimating the volume of the classroom. Not enough explicit connections were made between estimating the area using water bottles and the canonical solution of multiplying the length, by the width, by the height. In the end, it seemed as though students had just been given the formula without there being much questioning as to why that is the formula.

Even though Mr Scott did make some connections to the big picture at points (for example, during L4 when discussing why the area formula for rectangles makes sense), these connections were not frequent enough to achieve a rating of proficient.

Content depth was therefore rated as developing inquiry (level 2): 'Lesson provided some depth of content but with no connections made to the big picture.'

**L.4.2 Learner centrality**

Each of the lessons were designed to be flexible and responsive to the students' needs. Many of the lessons allowed the students to explore the problem in the way that felt best to them, with some resources and guidance provided by the teacher. For instance, in L2 Mr Scott took the students to an outside basketball court to measure its area and perimeter. The students were not given traditional measuring instruments and had to be creative by using the materials they had available to them (e.g. the long edge of an A4 paper or the length of their shoes by walking heel-to-toe). One group decided to have one of their peers lay down and use his height to measure the basketball court heel-to-head.

Mr Scott took student curiosities seriously throughout each IBI lesson. For example, in L3 two groups had measured the basketball court's perimeter using their feet but had obtained different results. Upon noticing this, the groups showed interest in understanding why. Mr Scott then facilitated a discussion about why the two groups could have differed so much, resulting in the realisation that the lower step-count must have been the result of their classmate's larger shoe size. The amount of time Mr Scott dedicated to this exploration demonstrated his commitment to learner centrality.

Learner centrality was also displayed in L7 when Mr Scott asked students to measure the volume of the classroom. This time students had 30-centimetre rulers available but, having previously experienced the basketball court problem, decided to use more creative solutions (e.g. a peer's height). By contrast, the teacher could have limited the students' participation and asked for only three student volunteers to measure the classroom while everyone else watched and recorded. This would have likely been quieter and less chaotic. But the decision to give students the permission to explore as they wished during investigations arguably allowed for greater engagement than would have been achieved otherwise.

Learner centrality was rated as proficient inquiry (level 3): 'Lesson allowed for some flexibility during investigation for student designed exploration.'

### L.4.3 Integration of content and investigation

The lessons of the IBI unit incorporated student investigation that linked well with content, though on occasion these links could have been made more explicit. For instance, in L2 the class investigated how to measure the perimeter and area of their school's basketball court and made sketches. In L4 students were shown a line drawing of a rectangle. Mr Scott led a discussion with the class about how the line drawing of the rectangle is similar to the sketches they made during the basketball court investigation. This discussion helped the students to interpret the diagram and successfully linked the investigation of L2 with the more abstract content of L4. Such examples were numerous.

I rated the integration of content and investigation as proficient inquiry (level 3): 'Lesson incorporated student investigation that linked well with content.'

### L.4.4 Organising and recording information

Throughout the IBI unit students typically organised and recorded information however they saw fit, such as in L7 when students measured the dimensions of the classroom. Only during more direct teaching elements of the unit were students provided with specific diagrams and solution methods to copy, such as L4 when students were asked to copy problems from the SmartBoard into their notebooks. Typically, students were simply provided with paper, pens, and measuring instruments (among other class materials) and ultimately free to record information however they liked.

Given the above, this component was rated as proficient inquiry (level 3): 'Students regularly organised and recorded information in non-prescriptive ways.'

## L.5 Summary of quality of the IBI unit

The most common score assigned to the different components of the EQUIP was that of proficient inquiry, therefore the unit as a whole could be best described as meeting the requirements of proficient inquiry. Mr Scott was successful in leading instruction, discourse, assessment, and curriculum that met many of the goals of inquiry. Students were routinely asked to explore problems before instruction and discuss their ideas with peers. The classroom was highly student-centred, and Mr Scott assessed students' understanding frequently. To have achieved a higher level of inquiry, Mr Scott could have provided more opportunities for students to interact directly with one another during whole-class discussions (without these interactions needing to be mediated by the teacher). In addition, students could have had more opportunities to reflect on their learning.

Appendix M: An analysis of the quality of inquiry instruction for Ms Silver's case

In the following section I assess the quality of the overall IBI unit. I do so according to the four factors of EQUIP: Instructional factors, Discourse factors, Assessment factors, and Curriculum factors. As previously discussed in Section 4.3, the EQUIP is an instrument that has been validated for use in mathematics classrooms. For a copy of the EQUIP please see Appendix A.

It is important to state that the lesson plans used during the IBI unit were planned in collaboration with Ms Silver and with reference to the EQUIP. Therefore, this analysis is focused on the *implementation* of the IBI lessons. Please see Appendix I for an example of the lesson analysis.

**M.1 Instructional factors**

This section discusses the EQUIP Instructional factors of the IBI unit. The Instructional factors are broken into five constructs which look at how the teacher designed (in collaboration with me) and implemented lessons to develop students' procedural and conceptual understanding. The constructs measured are Instructional strategies, Order of instruction, Teacher role, Student role, and Knowledge acquisition. Also included in this section is the extent to which students' procedural and conceptual understanding of the topic linear relationships changed throughout the IBI unit.

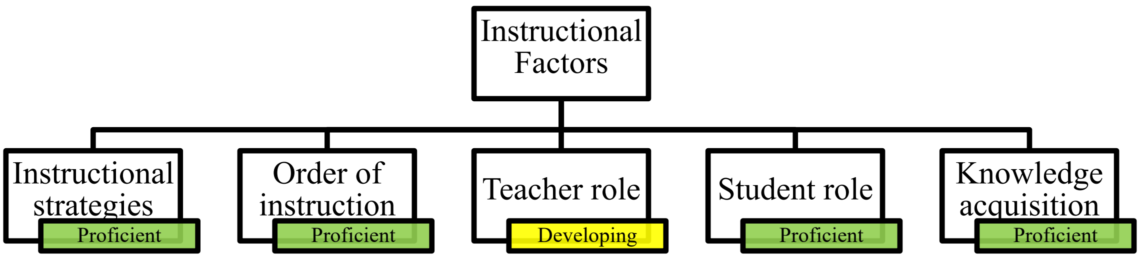



Figure 9.5: EQUIP ratings for five Instructional Factors at Stratham College

**M.1.1 Instructional strategies**

EQUIP describes instructional strategies as principally the amount of time the teacher spends lecturing versus facilitating student exploration.

At the beginning of the IBI unit Ms Silver frequently used a lecturing approach. In L1 and L2 for instance, the students spent large parts of the lesson listening to Ms Silver's explanations or copying notes from the board. Opportunities to invite student explanations were mostly overlooked. For example, during L2 only once was a student asked to explain his thinking aloud to the rest of the class. However, as the IBI unit progressed Ms Silver adapted well to the inquiry style and allowed the students more time for exploration and also invited more students to provide explanations.

Figure 6.3 illustrates how the lesson time throughout the IBI unit was divided between administration, explanation, and exploration. Administration was considered to be tasks that the teacher and students completed in order to prepare to begin a lesson, for example, taking the register or passing out papers. Explanation was considered any time a person (the teacher or a student) spoke to the entire class. This included explanations of the task or solution methods. Exploration was considered any time the students freely explored the IBI task. Distinctions between explanation and exploration in a classroom context are not clear cut. It is possible that elements of exploration took place during the phases I coded as 'explain' and elements of explanation took place during the phases I coded as 'explore'.

As mentioned above, it is clear to see that the time allotted for student exploration increased over the course of the IBI unit. At the same time, administration seemingly disappeared. Rather than taking the register at the start of the lesson Ms Silver took the register whilst the students were working on the IBI task. This helped to free up more time for exploration and subsequent explanation.

In assessing proficient inquiry, EQUIP also states that high quality inquiry lessons help students 'develop conceptual understanding' (see Appendix A). Ms Silver helped students develop conceptual understanding in several ways throughout the IBI unit. Firstly, Ms Silver taught and emphasised the importance of being flexible and applying different solution methods. For instance, in L2, Ms Silver taught the students a bar model, balance method, and abstract method for determining unknown quantities. These three different methods were frequently referred to throughout the IBI unit (see Figure 9.6 for an example).

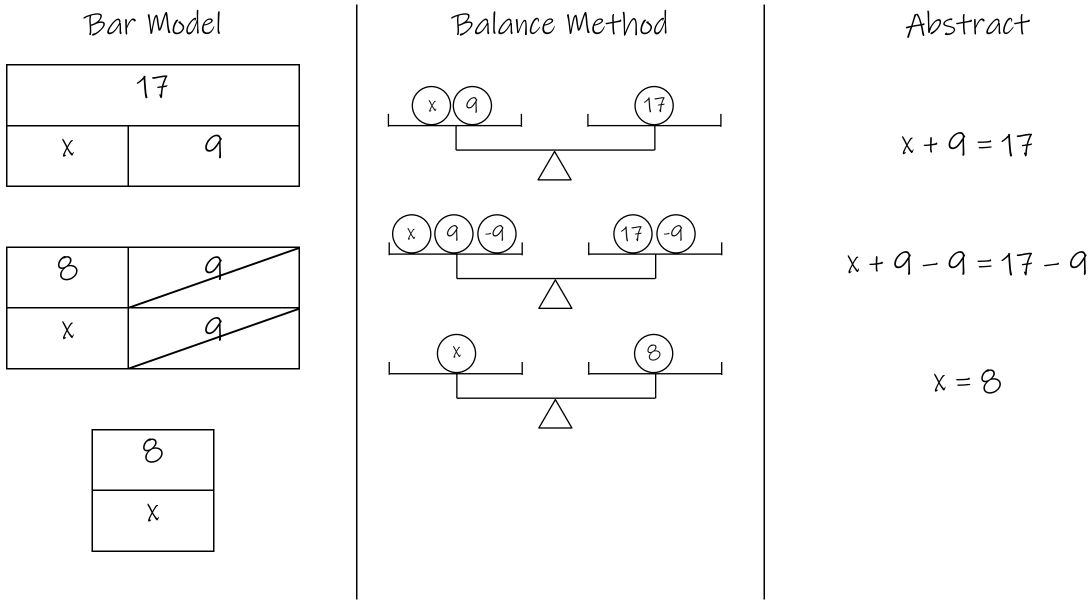



Figure 9.6: Ms Silver's bar, balance, and abstract method for solving equations

Ms Silver also helped students develop conceptual understanding by selecting nonroutine problems for the students to work on. These problems were complex because each was unfamiliar to the students. The problems either had unfamiliar contexts (e.g. the Denarii problem [L3]) or required unfamiliar solution methods (e.g. the Expressions problem [L2]). The selected problems had a number of possible solution pathways, ranging from simple to advanced. For instance, the Ichiro problem used in L7 could have been solved using at least five different methods: a picture, a table, a number sentence, an equation, or an inequality. Likewise, the Expressions problem [L2] asked students to write an algebraic expression given two nearly identical verbal descriptions. Students had to compare and contrast the two verbal descriptions to explore how the language in each connoted a different algebraic expression. Nonroutine problems such as these provided the students with opportunities to develop greater conceptual understanding of the topic of linear relationships.

To obtain a rubric score of proficient inquiry required students to be 'engaged in activities.' For the most part, the students' levels of engagement were good throughout the duration of the IBI unit. However, at the beginning of the IBI unit poor behaviour was occasionally a challenge. For example, Ms Silver sent four students out of the room at the end of the third lesson (L3) for being disruptive. She speculated that the students were acting out because of the change in routine. However, over time, the students appeared to become more comfortable with the new way of working. No students were sent out of the room for the last five lessons (when previously at least one student was sent out for bad behaviour during each of the first three lessons). This was noted during one of the student interviews.

> I think it [the IBI unit] was better as you got used to it. I kind of forgot that there was any change, so I got used to it really quickly. But I think everyone got used to it, and I think everyone found it easier as time went on and we were used to not relying on Miss. (Elayne)

In consideration of the above discussion as well as further observations, the instructional strategies for the overall IBI unit were rated as proficient inquiry (level 3): 'Teacher occasionally lectured, but students were engaged in activities that helped develop conceptual understanding'.

### M.1.2 Order of instruction

Ms Silver consistently allowed her students time to explore new ideas prior to discussion and explanation. Often the discussion of the inquiry problems did not come until the next lesson, as the teacher felt this better suited the students (such as L4). Although this was not the original plan, it had the added benefit of extending the explore portion of each of the IBI tasks beyond the boundaries of a single lesson. On one occasion, a keen student arrived at the next maths lesson with a new solution to the problem he had come up with at home.

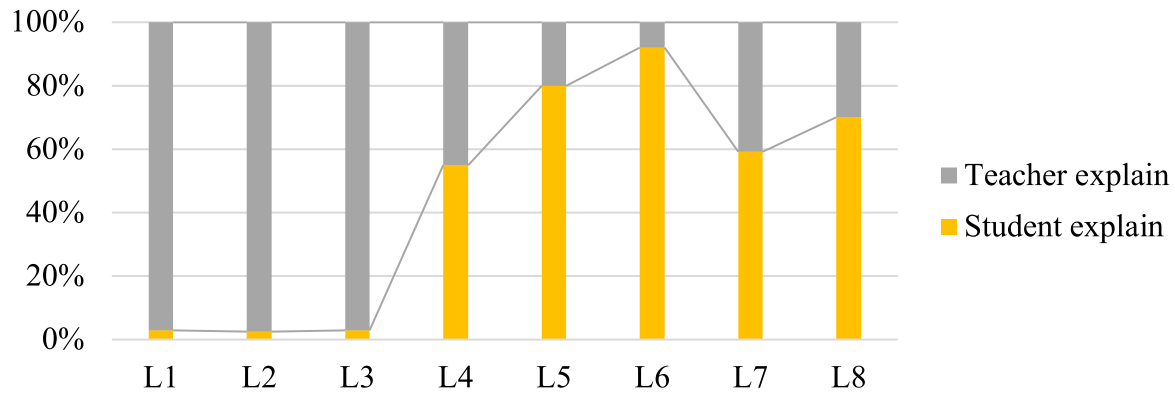



Figure 9.7: Percentage of explanation time by the teacher versus a student

Explanations of the solutions to the IBI tasks, as well as new concepts that emerged from the task, were always provided *after* the students had adequate time to explore the problem. These explanation phases were initially teacher directed with Ms Silver providing the majority of the explanations. However, over time, more explanations came from the students (Figure 9.7).

For example, in L4 Ms Silver began the lesson by leading a discussion about the previous problem (the Fibonacci problem). Rather than explaining the solution herself, Ms Silver invited Charlie to write her solution on the whiteboard and then explain it to the class. Charlie organised her work into a table and backtracked from the end of the story to the beginning (See Figure 9.8).

| | Money | Workings |
|---|---|---|
| Pisa | 0 | 0 + 12 = 12 |
| | 12 | 12 ÷ 2 = 6 |
| Florence | 6 | 6 + 12 = 18 |
| | 18 | 18 ÷ 2 = 9 |
| Lucca | 9 | 9 + 12 = 21 |
| | 21 | 21 ÷ 2 = 10.50 |

Figure 9.8: Charlie's solution to the Fibonacci problem

> I looked at the question, and I saw that he doubled his money. So, use the opposite of double, which is to halve. And then he spent 12 something, so add 12 because that's the opposite of minussing 12, which is spending something. (Charlie, L4)

After Charlie had explained her work, Ms Silver invited the class to ask her questions. This prompted Charlie to show how she checked her work by starting at the beginning of the story when the man had 10.50 denari and following him through Lucca, Florence, and Pisa until he had nothing left.

I rated the order of instruction as proficient inquiry (level 3): 'Teacher asked students to explore before explanation. Teacher and students explained'.

### M.1.3 Teacher role

Ms Silver's role developed over the course of the IBI unit. At the start of the IBI unit Ms

Silver provided most of the explanations. However, as is shown in Figure 6.3 and Figure 9.7, the teacher gave more control to the students over time. This can be seen by the increase in class time devoted to student exploration as well as the increase in student provided explanations. Even though more time was devoted to student exploration as the unit progressed, the teacher still remained central to each lesson in the way that she controlled and directed the pace and student participation. Students' perceptions of the teacher's role are discussed in Section 6.4.3.3.

Given the teacher's role throughout the IBI unit was highly central to each lesson, I rated this component as developing inquiry (level 2): 'Teacher was centre of lesson; occasionally acted as facilitator'.

**M.1.4 Student role**

The students' roles throughout the unit were variable. At the beginning of the IBI unit the students were mostly passive. Students listened to the teacher and copied notes from the board. In L2, for example, the students were shown problems on the white board and then after a brief pause were shown the solution and asked to copy it. Over time, however, the students' roles in the classroom changed and they were increasingly being asked to explore inquiry problems for extended periods of time. As shown in Figure 6.3 the amount of time dedicated to student exploration generally increased over the course of the unit, and as shown in Figure 9.7 the number of explanations provided by students also increased.

Ms Silver did well to set expectations regarding how the students were to behave during the explore portions of the lessons. An example of this can be seen in the following extract.

> I've got another problem for you. I'd like you to think about it, do your best with it, don't mess about, have a read. Don't say, 'I can't do it,' before you've had a go, because you can. (Teacher, L4)

I therefore rated the IBI unit as proficient inquiry (level 3): 'Students were active as learners (involved in discussions, investigations, or activities, but not consistently and clearly focused)'.

### M.1.5 Knowledge acquisition

The final construct measured under the Instructional factors section of the rubric is Knowledge acquisition. This construct looks at the depth of understanding students were required to demonstrate throughout the IBI unit. The IBI unit was designed to meet the learning objectives set out by the teacher which were aligned to the National Curriculum for Key Stage 3 (see Figure 6.2). The learning objectives that Ms Silver chose are described in Section 6.2. In reaching these objectives, the students were consistently required to apply their knowledge in new situations in order to demonstrate high levels of understanding.

One of the greatest improvements observed was the students' knowledge of writing and solving linear equations. When the unit first began students were reluctant to seek out solution methods using algebra. Most of the time students chose to use less abstract and more concrete solutions, such as the table Charlie used to solve the Fibonacci problem (Figure 9.8). However, Ms Silver encouraged students to think about the problems algebraically in order to meet the learning objective for each lesson. For example, in the Fibonacci problem Ms Silver asked the students what equation could be written to help solve the problem algebraically. After some discussion it was determined the expression would be $2[2(2x - 12) - 12] - 12 = 0$. Then Ms Silver walked the students through the steps to solve the equation for $x$, demonstrating its similarities and differences to Charlie's table method. Ms Silver then had the students discuss whether they preferred Charlie's table or the algebraic equation. Most students at that time said they preferred Charlie's method. However, by the end of the unit students were increasingly using algebra to solve problems by writing linear equations, though not always successfully.

In addition to classroom observations, the students took a pre- and post-test to measure their learning across the IBI unit. The results of these are discussed in Section 6.4.4.

The IBI unit reached the level of proficient inquiry (level 3): 'Student learning required application of concepts and process skills in new situations'.

### M.2 Discourse factors

In this section I discuss the Discourse factors of the EQUIP. These are aimed at looking at the ways in which the teacher used questioning and discussion to promote student understanding.

The constructs measured are Questioning level, Complexity of questions, Questioning ecology, Communication pattern, and Classroom interactions.

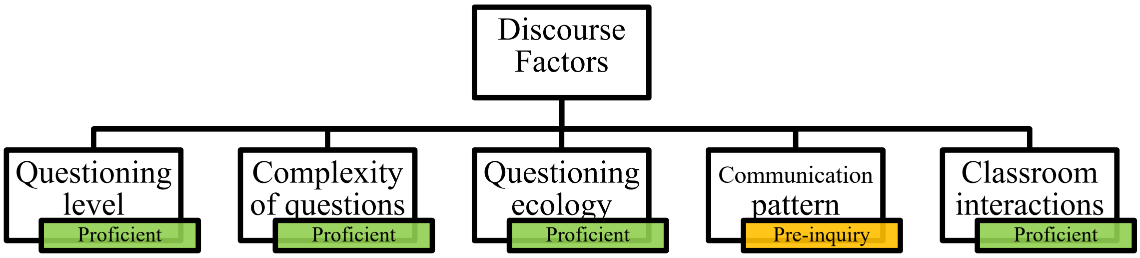



Figure 9.9: EQUIP ratings for five Discourse Factors at Stratham College

**M.2.1 Questioning level**

The questioning levels referred to in EQUIP come from the revised Bloom's Taxonomy (Krathwohl, 2002). The purpose of this taxonomy is to classify questions into different levels of difficulty. I therefore placed the questions Ms Silver asked her students into the taxonomy's six categories: remember (the lowest level), understand, apply, analyse, evaluate, and create (the highest level). Please see Table 9.2 for examples of the questions of her questioning at each level.

This categorisation was undertaken with reference to the lesson videos and subsequent transcripts. Since the classroom was too noisy during the explore phases to realistically capture the questions Ms Silver asked to small groups and to individual students, only questions Ms Silver posed to the entire class were considered. Since it is possible the types of questions Ms Silver asked to small groups and to individual students differed from those she posed to the entire class, the distribution shown in Table 9.1 should be considered incomplete.

Although only about 20 percent of Ms Silver's questions were at the 'apply' level or higher, these more complex questions were given the majority of the lesson time for students to explore and respond to. Therefore, the IBI unit's questioning level could best be described as proficient inquiry (level 3): 'Questioning challenged students up to application or analysis levels'.

Table 9.2: Ms Silver's questioning level according to the revised Bloom's Taxonomy

| Questioning level | n | % | Example |
|---|---|---|---|
| Create | 2 | 1.6% | Can you create an equation that has an integer solution? a decimal solution? no solution? (L8) |
| Evaluate | 4 | 3.2% | Somebody other than those people who wrote on the board, have a look at what's written and tell me what you think is going on and how it's going to work out. (L6) |
| Analyse | 9 | 7.3% | What's the difference between Ariana's idea and Donald's idea? (L6) |
| Apply | 12 | 9.7% | If I had a two-digit number–I didn't know what it was but I knew that one digit was 'a' and one digit was 'b'–what could I say about it? (L7) |
| Understand | 39 | 31.5% | How many metres do you think he would have walked in four seconds? He walks 2.5 metres every one second. How many metres would he walk in four seconds? (L5) |
| Remember | 58 | 46.8% | What is an expression? (L1) |

**M.2.2 Complexity of questions**

EQUIP distinguishes 'Complexity of questions' from 'Questioning level' since 'Questioning level' pertains more to the teacher's *questions* whereas 'Complexity of questions' pertains more to the students' *responses* that are elicited from the questions (ranging from one correct answer to an extended response).

Opportunities for students to explain and justify their work appeared to increase as the unit progressed (see Figure 6.3). Ms Silver invited students to share their solution at the front of the room by first writing out their solution and then explaining it to the rest of the class. For example, in L6 after the students had explored the Henri and Emile problem, Donald wrote his solution on the white board (Figure 9.10) and then explained to his peers what he had done.

> I knew Emile took 1 second to walk 2.5 metres, so I timesed that by 5 and went up until I met where Henri was going, because I knew Henri was to have

a 45-metre head start and could walk 5 metres in 5 seconds. So I saw that they'd meet after 75 metres. So the race should be 74 metres. (Donald, L6)

| Emile | Henri | |
|---|---|---|
| 0 | 45 | head start |
| 12.5 | 50 | |
| 25 | 55 | |
| 37.5 | 60 | |
| 50 | 65 | |
| 62.5 | 70 | |
| 75 | 75 | |

Figure 9.10: Donald's solution to the Henri and Emile problem

After Donald explained his work Ms Silver invited students to ask him questions and offer feedback. One student reminded the group that Emile had said, 'I would like him [Henri] to win, but *just by a little*'. What students considered to be winning 'just by a little' was then debated. For this reason, this question and others like it throughout the IBI unit served as a basis for student explanation and justification.

In addition, Ms Silver walked around the room while students worked on the IBI tasks in order to review their work and to invite them to share their solutions on the board. As a result, multiple solutions to the same problem were presented side by side. Later on, during the explanation portion of the lesson, Ms Silver would ask students to compare and contrast the solutions on the board. If no student had come up with an idea using algebra Ms Silver would demonstrate how it could have been used. As a result, students had ample opportunities throughout the IBI unit to share their reasoning in a variety of ways, whether at their desk with a partner or at the front of the classroom with the whole class.

I rated the complexity of the questioning as proficient inquiry (level 3): 'Questions challenged students to explain, reason, and/or justify'.

### M.2.3 Questioning ecology

Questioning ecology refers to the diversity of questioning used within each lesson of the IBI unit in order to stimulate student discussion, investigation, and reflection.

Ms Silver used a variety of questioning techniques over the course of the IBI unit. As shown in Table 9.2, her questions ranged from Bloom's lowest level of 'remember' up to the highest level of 'create'. Ms Silver's use of challenging open-ended questions and appropriate time for exploration led to opportunities for class discussion. Usually, though, only a few students would raise their hands during these discussions. In order to get additional perspectives, Ms Silver cold called, that is, she called on students who were not raising their hands. For example, after Ekko shared her reasoning with the class, Ms Silver cold called Kent and asked, 'Do you agree?' This increased the number of opportunities students had to share their thinking.

For the above reasons I felt that the questioning ecology was proficient inquiry (level 3): 'Teacher successfully engaged students in open-ended questions, discussions, and/or investigations'.

### M.2.4 Communication pattern

The communication pattern throughout the IBI unit was controlled and directed by Ms Silver in a didactic pattern. Even when students were presenting their work Ms Silver acted as a mediator between them and the rest of the class. For instance, in L4 when Charlie was explaining her solution to the Fibonacci problem the following exchange took place.

> C: Yeah. And then halved, so times by two.
> K: It says divided by 2.
> C: No, because the opposite of halving is timesing.
> K: What?
> T: Okay, keep going.
> (Charlie, Karson, Teacher, L4)

The above exchange is the only time recorded in the IBI unit that students spoke directly to one another during a whole class explanation. But as the above excerpt shows, Ms Silver asked Charlie to 'keep going' despite her peer not understanding her. This was possibly a missed opportunity for Ms Silver to increase the inquiry level of the class communication pattern.

In all, the communication pattern left room for improvement. Therefore, I rated this aspect of discourse as pre-inquiry (level 1): ‘Communication was controlled and directed by teacher and followed a didactic pattern’.

**M.2.5 Classroom interactions**

Classroom interactions refers to the extent the teacher or another student challenged students to provide a follow-up explanation or justification for their answers. Ms Silver often asked the students to justify their work. The students also had opportunities throughout the unit to seek justification from each other.

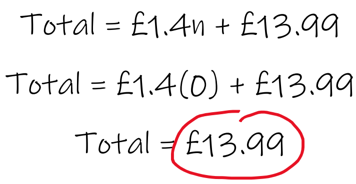



Figure 9.11: Harper's solution to part c of the Pizza problem

For example, in L4 Harper wrote her solution to part c of the Pizza problem on the whiteboard at the front of the room. Part c asked, ‘If you ordered your favourite medium pizza, how much would you expect to spend?’ Since Harper prefers no toppings on her pizza, she solved the equation she wrote by replacing the variable for number of toppings with 0 (see Figure 9.11 ).

Instead of having Harper explain her work, Ms Silver opted to have other students in the class explain what Harper had done. This helped Ms Silver to assess understanding. Assessment factors are further discussed in Section M.3.

> In Harper’s equation what does the ‘n’ represent? What does the question say? Somebody read it. [Harper raises her hand]. I know you know, Harper, because you’ve thought of it. I’m just going to see if I can tease it out of everybody else. (Teacher, L4)

This led on to a class discussion of how to label Harper's equation. They discussed how 'n' must represent the number of toppings on the pizza, the 1.4 must be the price per topping (£1.40), and the £13.99 must be the price for the base. In addition to allowing Ms Silver to gauge the students' levels of understanding, this approach allowed for greater student interaction. Ms Silver emphasized the importance of such interactions, such as the below example.

> Listen very careful when people are contributing because you might learn something. (Teacher, L8)

As can be seen from the classroom layout in Figure 6.1, Ms Silver chose to arrange her classroom in rows. Consequently, this arrangement somewhat hindered cooperation among the students as it was only realistic for students to work with the one or two people who sat next to them. Despite this desk configuration, students often worked together and there were several points when Ms Silver explicitly supported this.

> Put your hand up if you're a person that likes to work on your own. [Several students raise their hands]. And put your hand up if you'd quite like to work with the person next to you and get ideas. [Most of the students raise their hands]. So, we'll do exactly that. You have a go now. (Teacher, L7)

Ms Silver often asked for follow up explanation and discussion from the students throughout the IBI unit. Therefore, in the area of classroom interactions I rated the IBI unit as proficient inquiry (level 3): 'Teacher or another student often followed-up response with engaging probe that required student to justify reasoning or evidence'.

### M.3 Assessment factors

This section discusses the Assessment factors of the EQUIP. These look at the ways in which the teacher evaluated the students' knowledge acquisition throughout the unit. The constructs measured are Prior knowledge, Conceptual development, Student reflection, Assessment type, and Role of assessing.

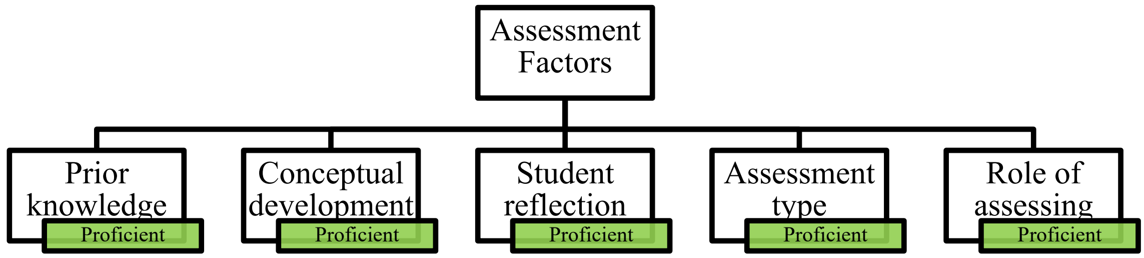



Figure 9.12: EQUIP ratings for five Assessment Factors at Stratham College

**M.3.1 Prior knowledge**

Ms Silver frequently assessed the students' prior knowledge through verbal questioning. For example, at the start of the unit (L1) Ms Silver asked the students for the definition of an expression. Upon realising many of the students struggled to distinguish an expression from an equation, Ms Silver decided to do additional practice problems in which students matched expressions and equations to their appropriate category. This was not the original plan, but Ms Silver felt it necessary since the IBI task which followed required students to know the definition of an expression.

Ms Silver also assessed prior knowledge during several IBI problems in which the context may have been unfamiliar to the students. For example, in L7 with the Ichiro problem Ms Silver asked students if they thought there was a difference between ten-yen and five-yen coins versus ten-pence and five-pence coins. It became evident that most of the students in the class had not heard of yen before and found it useful to think of the problem in terms of pence (their native currency). Ms Silver also assessed prior knowledge during the Henri and Emile problem when she asked the students if they were familiar with the concept of a 'head start'. The concept of a head start is key to understanding the context of the Henri and Emile problem, and it was useful to talk through what the term meant.

Given Ms Silver's regular assessment of the students' prior knowledge as well as her willingness to somewhat modify instruction based on this assessment, I rated the Prior knowledge component as proficient inquiry (level 3): 'Teacher assessed student prior knowledge and then partially modified instruction based on this knowledge'.

**M.3.2 Conceptual development**

Conceptual development was valued over procedural development over the course of the IBI unit. This is perhaps best evidenced by Ms Silver's careful attention to multiple solution methods. Rather than showing the students just one way to solve an equation, Ms Silver regularly emphasized the usefulness of other methods including the 'bar model' and the 'balance method.' By comparing and contrasting these three methods side by side Ms Silver was able to illustrate the importance of being flexible in approach when solving problems. Please see Figure 9.6 for an example of Ms Silver's bar, balance, and abstract method.

The importance of multiple methods was further emphasised throughout the unit when Ms Silver invited students to share different solution methods. Students compared and contrasted their classmates' approaches. Such discussions called on students' critical thinking skills.

Despite Ms Silver's efforts to show the value of process over product, many students still seemed preoccupied with just getting the answer. For example, in L7 there were a number of students who used a table method to determine when Ichiro would have less money in his wallet than his brother, and they did so successfully. Since there was additional time both Ms Silver and I encouraged these students to see if they could arrive at the same solution using a different method. However, in nearly every case the students instead engaged in off-task behaviour.

Occasionally though, students did show curiosity in the process of problem solving even though they knew they already had a correct answer. For example, in L4 when writing the equation to solve the Fibonacci problem a student asked the teacher, 'Why do you have brackets inside brackets?' This showed the student was curious in understanding algebraic notation. Another example of this appeared in L8 when reviewing how a bar model could be used to solve linear equations. A student asked, 'Why is the 3 at the bottom [of the bar model]?' Even though the class had already reviewed the answer this student showed interest in understanding how the bar model could be applied.

In light of the above discussion as well as my observations, I rated this component as proficient inquiry (level 3): 'Teacher encouraged process focused learning activities that required critical thinking'.

**M.3.3 Student reflection**

Ms Silver encouraged students to reflect on their own learning a number of times throughout the IBI unit. The principal method by which Ms Silver achieved this was through use of several coloured sheets in the students' planners. For example, in L2 when Ms Silver first introduced the bar model, balance method, and abstract method, she asked students to display the colour in their planners that best represented their present level of understanding.

> Now I would like you to … get your planners out. If you understand everything we're doing, you put it on green. If you think 'I'm getting there', it's on orange. If you think 'Ms is talking complete twaddle', it's on red. (Teacher, L2)

All the students in the classroom then opened their planners to the colour that best represented how well they understood the material. Figure 9.13 provides a visual of the classroom after Ms Silver had asked the students to display their colours. In response to the number of oranges and reds displayed by the students, Ms Silver chose to present several more examples.

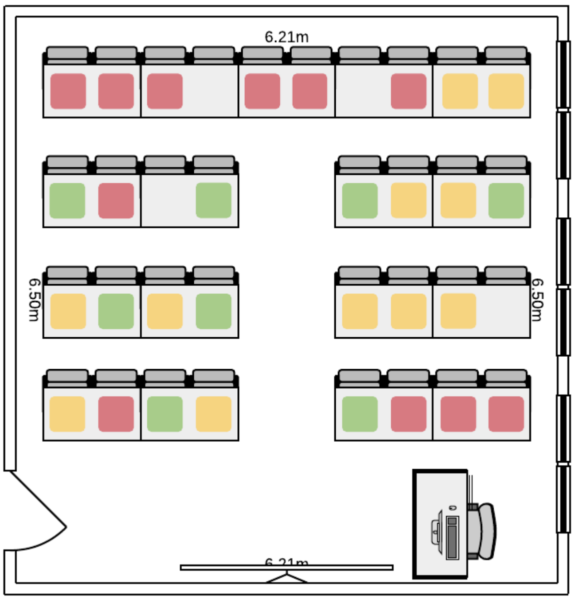



Figure 9.13: Ms Silver's student response system using coloured papers

In addition to displaying the coloured sheets, Ms Silver also encouraged student reflection throughout the IBI lessons by asking students to give a thumbs up if they agreed to another student's answer or if they were ready to move on.

Ms Silver encouraged student reflection throughout the problem-solving process. For example, in L7 Ms Silver asked, 'Out of you people who decided 16 first, who changed their minds? Why did you change your mind?' These quick assessments were useful in helping students frequently monitor their understanding.

I rated this component as proficient inquiry (level 3): 'Teacher explicitly encouraged students to reflect on their learning at an understanding level'.

### M.3.4 Assessment type

Ms Silver assessed the students in multiple ways throughout the IBI unit. For example, as previously discussed in Appendix M.2.1, Ms Silver employed a range of questions at different levels of complexity in order to assess the students' understanding of factual as well as abstract knowledge. She also spoke with the students individually and in small groups during the explore portions of the IBI lessons as a means of gauging progress. After each IBI lesson Ms Silver also collected their worksheets and reviewed their written work. These measures were mostly informal, however formal tests were also employed (e.g. the pre-test and post-test).

Assessment type was therefore rated as proficient inquiry (level 3): 'Formal and informal assessments used both factual, discrete knowledge and authentic measures'.

### M.3.5 Role of assessing

Assessments were used to inform both the pace and (to a lesser extent) the content of each lesson. For example, in L6 Ms Silver noticed, through speaking with students, that many were still struggling to visualise what the race would look like in the starting position. In response Ms Silver chose to play the accompanying video for the problem again even though students had already watched it during the previous lesson. She also asked for a volunteer to sketch what the race would look like on the board. In this way, Ms Silver adjusted her lesson in response to student assessment.

A further example of the role of assessing took place in L2. Upon reviewing the worksheets of the students from L1 it became clear to Ms Silver that the students did not yet know the difference between an expression and an equation. The task asked students for an expression, but many had written an equation (see Appendix M.3.1). Therefore, Ms Silver chose to begin L2 with a discussion about what makes an expression different from an equation. In this way student assessment helped to determine the content of the lesson.

In light of the above, this component was rated as proficient inquiry (level 3): 'Teacher solicited explanations from students to assess understanding and then adjusted instruction accordingly'.

### M.4 Curriculum factors

This section discusses the Curriculum factors of the EQUIP. These look at the ways in which the chosen curriculum flexibly supported student exploration and understanding of the 'big picture'. The constructs measured are Content depth, Learner centrality, Integration of content and investigation, and Organising and recording information.

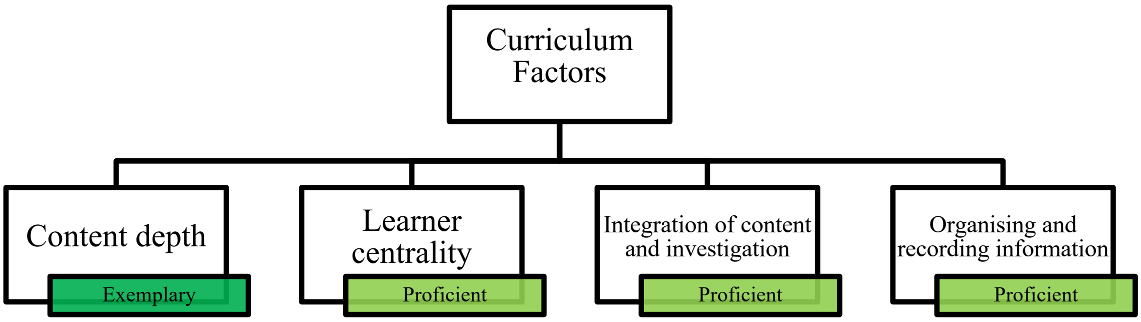



Figure 9.14: EQUIP ratings for four Curriculum Factors at Stratham College

#### M.4.1 Content depth

Content depth was an overall strength of the IBI unit. This is perhaps due to Ms Silver's strong background in mathematics (Section 6.1.1). Ms Silver demonstrated excellence in content knowledge and was skilled at connecting content to the bigger picture.

For example, much of the topic of linear relationships and of the skill of solving equations relies on an understanding of equality. Ms Silver made frequent, explicit connections to the idea of equality and what the equal sign represents. She stressed to the students that an equal sign does not merely denote 'the answer' but instead indicates that the quantity on the left is the same as the quantity on the right. This was further stressed through the use of the bar model (same lengths) and the balance method (same weights). For example, when discussing how to solve an equation Ms Silver made frequent references to the idea of an equation as a set of balancing scales.

> Remember, if we're balancing scales, what we take off one side we've got to take off the other. What we add to one side, we've got to add to the other. Before we took an $x$ off both sides and it still weighed the same, because I've taken the same off both sides. (Teacher, L8)

Furthermore, when talking about equations Ms Silver repeatedly made sense of the meaning of the terms, for instance by describing $2x$ as '$x$ and $x$' or $8x$ as '8 lots of $x$'. Or when the problem had context, such as in the Pizza problem, $1.4x$ as '£1.40 per topping' where $x$ is the number of toppings.

Another insightful teaching moment occurred in L6 when reviewing the students' solutions to the Henri and Emile problem. Each of the students had come up with an answer (e.g. the race should be 74 meters). But Ms Silver recognised that the problem could actually have a range of reasonable answers which could be best represented with an inequality. Thanks to this, the students had an opportunity to stretch their thinking into the realm of inequalities thus deepening the content achieved during the lesson. Instead of saying the race should be 74 meters the students said the race should be less than 75 metres ($x < 75$ where $x$ represents the length of the race in metres).

Given some extra time in one lesson, Ms Silver introduced a problem to the students in which they decomposed two-digit numbers into tens and ones. Ms Silver then connected this idea to writing algebraic expressions. She asked the students, 'What is an expression I could write to represent any two-digit number? Let's say one digit is $a$ and the other is $b$.' This problem was excellent in both reviewing basic number composition as well as writing expressions.

Examples such as these were numerous. Content depth was therefore rated as exemplary inquiry (level 4): 'Lesson provided depth of content with significant, clear, and explicit connections made to the big picture'.

**M.4.2 Learner centrality**

Learner centrality grew over the course of the IBI unit, as evidenced by the increase in time for student exploration as well as the increase in student generated explanations (please see Figure 6.3 and Figure 9.7). Apart from being given the IBI task, students were free to explore the problem in any way they saw fit. However, given the size of the classroom, and the school rules, students were not allowed to freely move about the classroom or the school, so student exploration had its boundaries.

Ms Silver supported learner centrality in her classroom by allowing students to write on the whiteboard whenever they felt like they had an idea they wanted to share. This technique facilitated student autonomy.

> I've had an idea. I'm going to put the problem on the board. Now, if you think of a really good idea, you could come up and write that idea on the board. If you think, 'You know what, I think I know how to solve that bit, I'm going to write down my idea on the board.' Now, not everybody will be solving it the same way as you, so it's not for you to look and think, 'Oh, I'll do it that way.' It's just if you think of a good idea. … So you could look up and think, 'What did that person do? What could I get from that?' (Teacher, L4)

The construct of Learner centrality was therefore rated as proficient inquiry (level 3): 'Lesson allowed for some flexibility during investigation for student designed exploration'.

**M.4.3 Integration of content and investigation**

Each investigation was well integrated with the content. Some of the IBI tasks made explicit use of algebraic concepts, such as in L2 when students were asked to write an expression. In other lessons though, the connection to the topic of linear relationships was more subtle. For example, in L3 with the Fibonacci problem and in L5 with the Henri and Emile problem not all students attempted a solution pathway that used an equation even though this was the primary learning objective for these lessons. To help focus the lesson back on the learning objective, Ms Silver would walk around the room and look for students who had solved the problem using algebra, such as in L7 when Ms Silver asked Elva to share her solution (Figure 9.15). Elva was the only student to use algebra successfully. Most of the students had attempted using algebra, but upon failing to do so successfully, used a table to arrive at an answer. It was therefore valuable to have Elva present her solution.

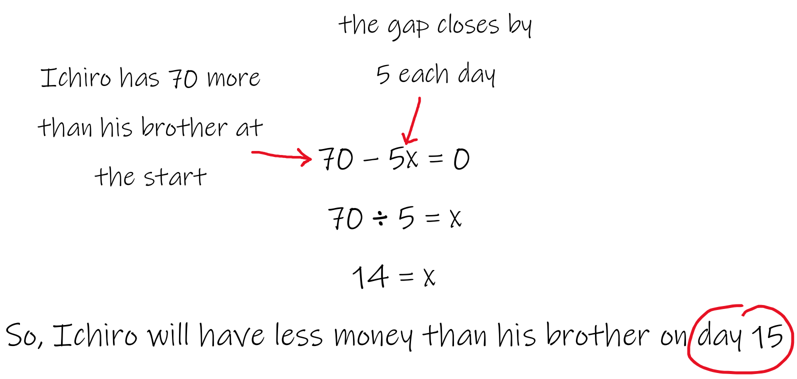



Figure 9.15: Elva's solution to the Ichiro problem

To have achieved the level of exemplary inquiry, Ms Silver could have presented an additional algebraic approach alongside it. She could have shown the students how the equation $180 - 10x = 110 - 5x$ could have also been used to determine the day in which the two brothers would have had the same amount of money in their wallets. This would have provided students with an opportunity to solve an equation in which the unknown appears on both sides (one of the stated learning objectives for the unit).

I rated the integration of content and investigation as proficient inquiry (level 3): 'Lesson incorporated student investigation that linked well with content'.

### M.4.4 Organising and recording information

Throughout the IBI unit students were free to organise and record information as they saw fit. For each task the problem was briefly stated at the top of the sheet, but the rest of the paper was left blank. For example, when solving problems some students chose to draw a graphic while others preferred to organize the information into a table. The openness of the blank paper allowed for this level of flexibility.

Given the above, this component was rated as proficient inquiry (level 3): 'Students regularly organised and recorded information in non-prescriptive ways'.

### M.5 Summary of the quality of the IBI unit

The most common score assigned to the different components of the EQUIP was that of proficient inquiry, therefore the unit as a whole could be best described as meeting the requirements of proficient inquiry.

Ms Silver was successful in leading instruction, discourse, assessment, and curriculum that met many of the goals of inquiry. Students were always given time to explore the IBI problems before receiving instruction. In addition, students were given many opportunities to explain and justify their ideas both with a partner and with the class. Ms Silver assessed students' understanding frequently, and she was especially skilled at providing depth of content.

To have achieved a higher level of inquiry, Ms Silver could have developed more in the area of communication pattern. Most notably, the teacher could have encouraged more student-to-student talk during whole class discussions by asking students to address each other directly.