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The effect of prestressing force on natural frequencies of concrete beams – a numerical validation of existing experiments by modelling shrinkage crack closure

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Abstract

This paper investigates the effect of prestressing force on the natural frequencies of reinforced concrete beams. From a pure theoretical point of view, such effects in different prestressing conditions appear to involve no ambiguity; however, in practice contradictory observations have been reported in existing research publications. Theoretical studies showed that natural frequencies would be decreasing or unchanged in different scenarios. On the other hand, some experiments that were conducted on prestressed concrete beams indicated an increasing trend of the natural frequencies with the prestressing force. This paper is aimed to provide a systematic explanation of the reasons causing the discrepancies and propose a coherent framework for the prediction of the natural frequencies under a prestressed condition. Numerical simulations using finite element model are carried out to simulate the influence of prestressing force on natural frequencies with the existence of the shrinkage cracks. The results demonstrate that such shrinkage-type cracks inside the concrete indeed tend to close when the prestressing force is applied, and this in turn increases the bending stiffness and consequently results in an increase of the natural frequencies of the beams.

Keywords: Natural frequency, Prestressing force, Shrinkage crack

1. Introduction

The natural frequencies of prestressed structural components have been a subject of much research interest, particularly in more recent years as several research studies found that the natural frequencies

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of prestressed reinforced concrete (RC) beams tend to increase with prestressing force, which contradicts the general understanding of classical theories. In light of these observations, a number of experiments have been conducted in attempt to provide insight into the phenomenon (Saiidi et al. [1]; Jang et al. [2]; Wang et al. [3]; Noble et al. [4]).

As a matter of fact, the fundamental mechanisms involved in typical prestressed conditions are well understood and their effect on the natural frequencies of the components can be established without ambiguities. Two bounding conditions can be well explained by the theoretical models proposed by Timoshenko et al. [5] and Hamed & Frostig [6], respectively.

According to Timoshenko's theory [5], under an externally applied axial compression force, the natural frequencies of a beam will decrease with the increase of the compression force. In the context of prestressed beams, this theory would strictly apply only in the prestressing condition where the tendon in anchored at the beam ends, and apart from this, the tendon is not in touch with the main beam. Thus, when the main beam vibrates it is equivalent to a scenario with externally applied compression at the beam ends. For easy reference, we shall call this scenario as Type-A prestressed condition.

Hamed and Frostig [6] considered a prestressed beam with prestressing tendons in curved profiles and concluded that the natural frequencies will not change with the prestressing force in such cases. In fact, the same formulation also applies to cases with a straight tendon if the tendon is perfectly attached to the beam in the transverse direction; a prestressed beam where the tendon is bonded to the main beam clearly belongs to this category, although bonding is not a necessary condition. Besides, a prestressed beam with unbonded tendon which deflects with the main beam in transverse direction belongs to this type even the tendon could slip in the beam. Also, for convenience, we shall call these as the Type-B prestressed condition.

Jaiswal [7] observed that Timoshenko's theory applied well to the prestressed beams with unbonded tendons while Hamed and Frostig's theory applied well to prestressed beams with bonded tendons. Wang et al. [3] made similar observations. However, as mentioned above the fundamental difference between the two ideal conditions is not bonded or unbonded tendons. Timoshenko's theory can only be used to predict the natural frequencies of a beam close to externally applied axial load at two ends which do not change with the deformed shape. When the prestressing force that sticks to the deformed shape, Hamed and Frostig's theory should be used. For example, a prestressed beam with tendons in curved profiles should be fitted into Hamed and Frostig's theory no matter it is bonded or unbonded because the prestressing tendon will deflect with the primary structure even it is unbonded.

Clearly, there are many factors which could influence the prestressing conditions, thus introducing much more complexities than the two idealized situations. For example, the actual installation and arrangement of tendons could result in the real interaction of the prestressing tendon with the primary structure not fitting either of the two conditions. Furthermore, if the material properties are prestress-dependent to a certain degree, for example with an increase in the flexural rigidity (equivalent EI) due to crack closure, this will add further complexity to the final outcome. Therefore, a systematic methodology to deal with the influences of the various factors is needed.

The study will firstly review the representative theoretical models and further clarify their applicability. The study will then look into the various complexities and uncertainties and examine their influences on the natural frequencies in typical prestressing conditions. In particular, the effect of shrinkage cracks in RC beams on natural frequencies is investigated with numerical simulation and in comparison with two sets of laboratory experiments done by Noble et al. [4] and Jang et al. [2]. The results demonstrate that the discrepancy between tests and theories is attributable to the closure of initial cracks in concrete under a post-tensioned prestress condition. A coherent conclusion on the effect of prestressing force on natural frequencies is drawn.

The natural frequencies of beams are basic dynamic parameters in structural design. For standard beams, the exact analytical solution of natural frequencies can be found. For beams with complexities such as beams with varying sections along the span, the analysis of their natural frequencies will generally require numerical approaches using, for example, finite element method. However, when it comes to prestressed beams, it is often not a clear-cut for whether the beam should fit one of the two ideal scenarios (Type-A or Type-B), not to mention additional complexities. Hence, there is ambiguity to calculate the natural frequencies of beams in an actual prestressing condition.

Saiidi et al. [1] conducted dynamic tests on an actual concrete bridge and a laboratory specimen. The bridge was a simply supported post-tensioned concrete box girder structure. A Field test was conducted to the bridge and it was found that the natural frequencies of real bridge decreased with the prestress loss. Since this phenomenon contradicts with the expectation from Timoshenko theory which indicates that the natural frequencies would increase when applied axial force reduces, Saiidi et al. carried out more tests on a simply supported post-tensioned concrete beam in the laboratory. The author mentioned that the specimen developed a small crack at midspan under its own weight during handling. The natural frequencies of dynamic tests showed an increasing trend with the increase of prestressing force. This was consistent with the observations from the actual bridge tests but contradicted with the prediction according to the Timoshenko's theory.

Jang et al. [2] conducted dynamic and static tests on simply supported prestressed concrete beams with the bonded centric tendon. Since the magnitude of the prestressing force cannot be changed for bonded prestressed beams, the authors made six scale-model specimens with different prestressing forces for the tests. It was concluded that the natural frequencies increased with the increase of the prestressing force. Apart from the dynamic tests, the authors also performed static bending tests and found a similar trend that the flexural stiffness increased with the prestressing force.

Wang et al. [3] conducted both dynamic and static tests on simply supported beams with parabolic tendon or eccentric straight tendon. The tendons in their tests were not bonded at first. They measured the natural frequencies of the beam when prestressing force was 1) not applied, 2) applied but tendon was not grouted to concrete and 3) applied and grouted. They concluded from their test results that the natural frequencies of prestressed beams with straight tendons were not affected by prestressing force before grouting the tendon.

Noble et al. [4] tested 9 prestressed simply supported beams. The 15.7 mm diameter post-tensioned unbonded tendons were threaded through 20 mm diameter duct in concrete and cast into the beam. They reported no structural cracks due to the small deflections in the static and dynamic tests. Both dynamic and static tests were conducted on the beams. Neither dynamic tests nor static tests indicated that fundamental natural frequency would decrease as an idealized prediction using Timoshenko model. In the study of Noble et al. [4], no statistically significant relationship between prestressing force and natural frequencies was found because of the large error in the estimation of the natural frequency from the dynamic test data.

As the experiments results listed above contradict to the prediction of classic theories and no satisfactory explanation is proposed to the best of the author’s knowledge, it is important to find out the cause before a coherent conclusion could be given.

2. Basic theoretical models

As far as the prestress loads and interaction with the structure is concerned, it may be stated that all of the prestressed beams should fall between the analytical scenarios of Timoshenko et al. [5] and Hamed & Frostig [6]. The former represents an ideal condition where the prestressing force is equivalent to the external compression force applied at the two ends of the beam and their line of action does not change with the deflected shape. The latter, on the other hand, represents an ideal condition where the prestressing tendon moves with the main structure and therefore does not introduce the so-called “compression (P-Δ) effect”.

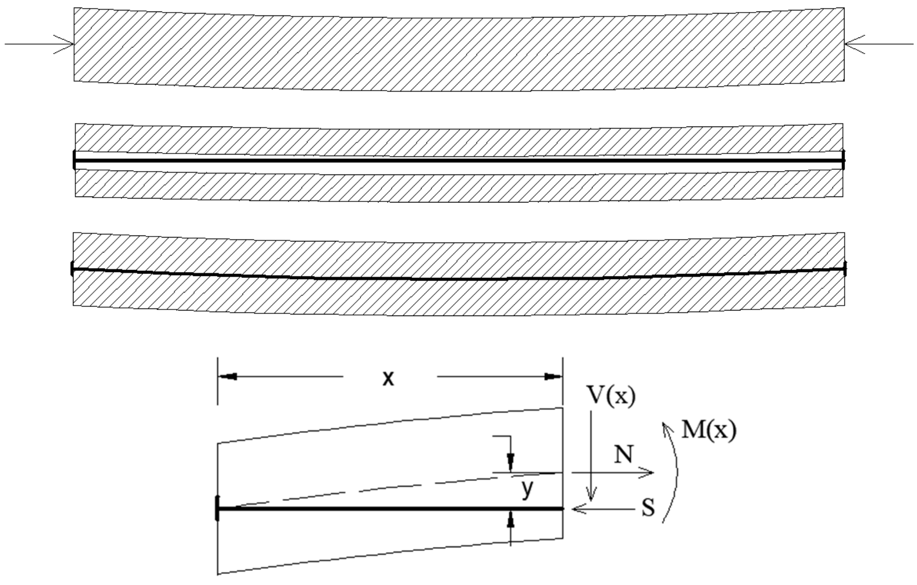
Therefore, the general trend of influence of prestressing force on the natural frequencies should be expected to be a reduction, more or less, from a pure prestressing loading point of view. Anything outside the above expectation will imply involvements of other factors – these could include material consolidation/hardening (for concrete), interruption of boundary or support conditions, tightening of joints, to name a few.

Before interpreting the phenomena in connection with the above complexities, it is useful to reiterate the basic assumptions and the work principles in the two bounding scenarios.

The two contrasting scenarios between Timoshenko's case and Hamed and Frostig's case are illustrated in Fig. 1. In the basic scenario with externally applied compression (Fig. 1a), when the beam deflects, the direction of the axial force does not change. Consequently, an additional moment will be produced causing additional bending deformation and hence the natural frequencies, as will be discussed further later.

A prestressing condition is shown in Fig. 1b, where the prestressing tendon is detached from the beam even under deflection, the effect of the prestressing force is essentially the same as the scenario in Fig. 1a. As illustrated in Fig. 1d, the resultant compression force in main beam *N* does not coincide with prestressing force *S* which induce an additional moment to the beam and enhance the bending deformation. It is noted that the moment discussed here is about the neutral axis of the beam section. Hence the resultant shear force V(x) (in concrete) or $S_y(x)$ (in tendon) do not affect this moment.

When the tendon deflects perfectly with the beam (Fig. 1c), such as in a bonded prestressing condition, the prestressing force will keep alignment with the deflected beam along the beam length. As demonstrated in Fig. 1e, under such a condition, resultant shear force V(x), $S_y(x)$ and axial force in tendon $S_x(x)$ will not produce moment about the neutral axis of the section. No additional moment may be produced by the prestressing force as the beam deflects (vibrates), and consequently, no change of the equivalent flexural stiffness will result.

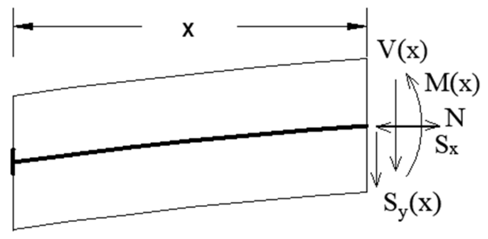


(a)

(b)

(c)

(d)



(e)

Figure 1. Scenarios with different interaction of the prestressing tendon with the primary structure. (a) Beam under externally applied axial compression force - Timoshenko scenario; (b) Post-tensioned beam with tendon detached from beam other than at two ends - Timoshenko scenario; (c) Post-tensioned beam with tendon constrained to adjacent beam nodes in transverse direction (in addition to fixed at two ends) - Hamed and Frostig scenario; (d) Force diagram of prestressed beam - Timoshenko scenario; (e) Force diagram of prestressed beam – Hamed and Frostig scenario.

3. Prestressed RC beams: discrepancy of experimental observations and states of concrete

3.1 Effect of cracks on natural frequencies and its closure due to prestressing force

As mentioned in Section 1, the tests on prestressed RC beams conducted by Saiidi et al. [1], Jang et al. [2] and many other researchers showed that the natural frequencies increased with prestressing force, and such results cannot be explained by either Timoshenko's model or Hamed and Frostig's model. According to the theories, the natural frequencies are supposed to reduce by a varying degree with the presence of the prestressing force and should not increase. To the knowledge of the author, there has been no satisfactory theory that can explain an increase in the natural frequencies in a prestressed beam. A logical explanation would have to be in the direction that somehow the "material" has become stiffer with the application of the prestress.

Considering the fact that almost all the tests mentioned above were done on prestressed reinforced concrete beams, the effect of crack should not be overlooked. However, most of the researchers did not mention the existence of cracks and Noble et al. [4] reported that no visible crack was found in their specimens. Nevertheless, cracks could still have existed internally within the concrete of the RC beams, escaping detection from the surface. In fact, the well-known shrinkage cracks tend to occur near the steel bars inside concrete (Chen et al. [8]), as will also be demonstrated in the numerical simulations in the next Section in this paper. These cracks can reduce the natural frequencies of an RC beam in the "original" (un-prestressed) state. Once the prestressing force is applied, the pre-existing shrinkage type of internal cracks in concrete would close, causing the stiffness of the concrete beam to increase and therefore shift the change of the natural frequencies of the beam in a different direction.

In order to verify the above hypothesis, numerical models have been developed to simulate the experiments by Noble et al. [4] and Jang et al. [2]. Since few details about the curing of their concrete beams were mentioned in their papers, it is assumed that their concrete beams were cured with the regular process in the laboratory. It is essential to be able to simulate reasonably the initial state of the test concrete beams, especially the severity and distribution of the shrinkage cracks. To this end, a review of relevant research studies concerning the formation and propagation of the shrinkage cracks is carried out, and the results are applied in the numerical simulation.

3.2 Initial concrete material state and shrinkage cracks

In order to investigate the potential influence of prestressing force on the state of concrete material and the subsequent effect on the natural frequencies, it is essential to be able to describe the initial cracking that could exist in unloaded concrete in a reinforced concrete beam in quantitative terms. In this Section, related research studies are reviewed to estimate the spacing of initial shrinkage cracks. The estimation will be used in the subsequent simulations.

The formation of shrinkage cracks requires energy which can be used to calculate the range of the shrinkage crack spacing (Leonhardt [9]; Chen et al. [8]). In the real specimens, the final spacing of cracks has some uncertainties due to the random defects in concrete materials. For concrete beams with transverse reinforcement, the stirrups induce discontinuity in the concrete and as a result, the shrinkage cracks tend to form from the locations of the stirrups (Rizkalla et al. [10]). After the spacing of shrinkage cracks is determined, the depth of the cracks may be calculated by the Fictitious Crack Model (Hillerborg et al. [11]). The state of shrinkage cracks is described with spacing and depth so calculated.

Leonhardt [9] proposed a method to calculate the minimum spacing of cracks. He pointed out that the concrete starts to crack when the load reaches the tensile strength. The stress in steel will suddenly increase because of the cracking of concrete. Some bond-slip could occur if the increase is large enough. It was assumed that the minimum value for the average spacing of crack was the sum of the length with active bond stress and half of the length with almost no bond stresses. The value of these two lengths could be calculated by the stress in the steel at the crack immediately after cracking, the diameter of longitudinal reinforcement, the percentage of steel reinforcement and a factor depending on the concrete cover and the spacing between longitudinal bars.

Chen et al. [8] proposed a theoretical method to predict the shrinkage crack spacing of concrete pavement. They simulated the concrete as an elastic bar which was restrained by reinforcing bars or subgrade as illustrated in Fig. 2a. The initial strain caused by shrinkage was assumed as constant and the stress in concrete was caused by the reinforcing bars or subgrade which prevented the concrete from free shrinkage. The shear force restraining the concrete equalled to the shrinkage stress and was simulated by a series of springs. The stiffness of these distributed springs was a constant determined by the property of the interface of concrete and reinforcement.

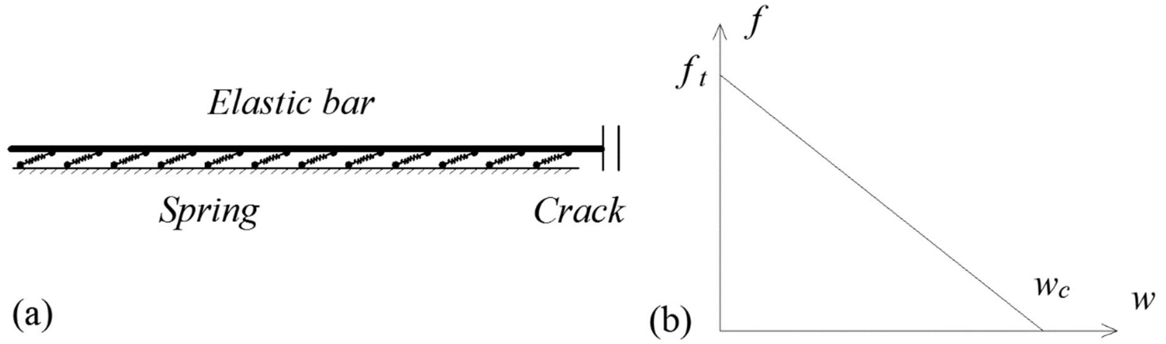


Figure 2. Modelling of crack spacing. (a) Elastic bar simulating concrete; (b) Fictitious Crack Model (Chen et al. [8]).

The calculation procedure is as follows. The displacement, stress and strain of every point of the elastic bar can be found so the strain energy of this elastic bar U_B and the energy stored in the distributed springs U_S can be represented separately. Besides, the energy needed for cracking U_C can be evaluated by Fictitious Crack Model (Fig. 2b). In Fig. 2b, f_t is the tensile strength of concrete and w_c is the width of the fictitious crack when its strength falls to zero at this point. The description in detail can be found in Section 4.2. The displacement of the elastic bar after first cracking can be found considering the unloading near the crack. Then the redistributed stress of the elastic bar after first cracking can be calculated. There could be another crack if this redistributed stress reaches the tensile strength of concrete. In this way, the spacing of cracks can be predicted.

With this theoretical model, the stress in concrete can be calculated by the distance of the point to the adjacent crack. When it reaches the tensile strength of concrete, the crack tends to form at this point and this distance is the minimum spacing of shrinkage cracks.

Rizkalla et al. [10] first reviewed many papers studying the spacing of cracks and compared their results with experiments. They concluded that predictions using the method by Leonhardt [9] tend to match well test results when no transverse reinforcements existed in the specimens. They then conducted their own experiments with nine specimens with transverse reinforcement, i.e. stirrups. It was found that in these tests the cracks always propagated from the location of the stirrups. In the specimens where the spacing of stirrups was much larger than the crack spacing predicted using Leonhardt's method, additional cracks would form between stirrups.

The above tests indicated that while Leonhardt's method is generally applicable, the discontinuity induced by transverse reinforcement can disrupt and control the spacing of cracks. It should be noted that the tests by Rizkalla et al. [10] were conducted on reinforced concrete slabs in tension; however, the influence of stirrups on the spacing of cracks is similar to the situation with shrinkage cracks.

On the basis of the above review, in the numerical simulations which will be described in the next Section, the spacing of shrinkage cracks will be predicted first using both the models by Leonhardt [9] and Chen et al. [8]. The final spacing will be determined considering the predictions and the spacing of stirrups. When the predictions are closed to the spacing of stirrups or multiple of the spacing of stirrups, the cracks would form from the location of these stirrups.

4. Numerical simulation on the effect of shrinkage cracks in prestressed RC beams

4.1 General model considerations

This Section presents the numerical simulation of prestressed RC beams to investigate the natural frequencies due to the effect of 1) shrinkage cracks and 2) the closure of cracks due to prestressing.

The finite element model used in the simulation is developed with ABAQUS. The simulation process is carried out with the following steps. Firstly, the shrinkage cracks in the reinforced concrete beams before prestressing are simulated. Then, the prestressing tendon is tensioned. The shrinkage cracks will close to different degrees due to different levels of the prestressing force. The natural frequencies of the beams before and after the application of the prestressing force are compared, and the results are discussed in conjunction with the experimental observations.

4.2 Modelling cracks of concrete in FE model

Many crack theories have been studied and the Fictitious Crack Model proposed by Hillerborg et al. [11], Modéer [12], Petersson [13] is widely used (Dong et al. [14]). According to linear elastic theory, there is infinite stress at the tip of a crack, but this is not the case for concrete because the material in the crack tip zone will be partially damaged before a high level of stress could develop. The Fictitious Crack Model, therefore, defines a fracture zone at the crack tip. Numerous micro-cracks exist in this fracture zone and this zone actually shows a plastic behaviour. In the analysis, this fracture zone is replaced by a slit that is able to transfer stress (Fig. 3) and the stress transferring capability depends on the width of the slit in the Fictitious Crack Model as illustrated in Fig. 2b. The width of the slit is varying along the fracture zone and consequently, the stress transferring capability is changing at a different point in the slit as shown in Fig. 3.

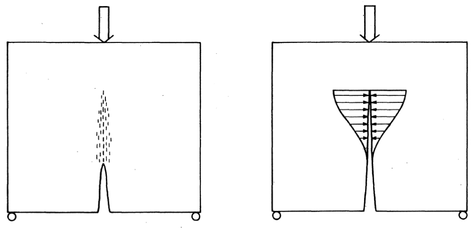


Figure 3. Fracture zone and Fictitious Crack Model (Petersson [13])

The crack will propagate when the stress transferring capability at the tip shown in Fig. 3 falls to zero. The displacement at the tip now is noted as w_c. This parameter w_c can be calculated using fracture energy G_F by Eq. (1) (Petersson [13]). Both w_c and G_F are material properties of concrete.

$$w_c = \frac{2G_F}{f_t} \tag{1}$$

In the FEM, the shrinkage of concrete is simulated indirectly by inducing a temperature drop. The temperature field is only applied to the concrete while reinforcement bars are not affected. Hence the bars will prevent the concrete from shrinkage. With the restraint from reinforcement bars, tensile stress will occur in concrete, similar to the model used by Chen et al. [8]. When the tensile stress is large enough, the shrinkage cracks will form and propagate.

The crack propagation is controlled by the displacement near the tip of the crack, such that when the tip displacement reaches w_c the crack will propagate. The fracture energy G_F for normal concrete is in the range of 70 - 140 N m^{-1} (Petersson [13]), so the critical value w_c can be calculated by Eq. (1) as 4.67×10^{-5} - 9.33×10^{-5} m. In the numerical simulation of this paper, w_c is set as 5×10^{-5} m. With these settings, an initial crack would appear at those locations where tensile stress reaches tensile strength.

In tests, the shrinkage strain of a concrete beam largely depends on the exposed drying surface area-to-volume ratio and it varies from 100×10^{-6} (all sealed) to 500×10^{-6} (not sealed) (Zhou et al. [15]). It is not mentioned by most researchers doing prestressed beam tests because the shrinkage factor has not been taken into account. The shrinkage strain in the present numerical simulation is set as 200 $\times 10^{-6}$ for all models assuming 3 sides are sealed during casting and curing of the concrete. Some concrete beams were simulated considering the influence of stirrups on cracks which showed that this level of shrinkage strain was large enough for the formation and propagation of cracks in concrete. By setting an expansion coefficient and a temperature drop in concrete, this shrinkage process can be simulated precisely.

4.3 Modelling of shrinkage cracks and model validation

In the present numerical simulation, the extended finite element method (XFEM) is employed to simulate the formation and propagation of “shrinkage cracks” in RC beams.

XFEM uses enriched shape functions with special characteristics to bring the discontinuous information to the computational field and it is an efficient numerical method to handle problems with discontinuities (Xu et al. [16]). It allows cracks to form and propagate within an element.

As pointed out by Rizkalla et al. [10], the presence of stirrups is important on crack initiation. For simplicity, in the present simulation of the shrinkage-induced cracks, the stirrups are not explicitly modelled but they are represented by introducing some defects at the corresponding locations. The defects replacing stirrups are simulated by an array of shell element embedded in the model. They act as slits which cannot transfer tensile stress and can transfer compressive stress when setting them as “crack” in interaction module in ABAQUS. The depth of them is set to be identical to the corresponding stirrups they are representing. Hence the effect of stirrups on concrete continuity is simulated by these slits. When the temperature of concrete drops and shear (bond) stress transferred from reinforcement bars prevents the shrinkage of concrete, cracks will propagate from these slits.

As the above mentioned “defects” are used to represent stirrups, their spacing should be equal to the spacing of stirrups or the multiple of the spacing of stirrups unless the predicted crack spacing is much smaller than the spacing of stirrups.

To verify the modelling of the cracks, a simply supported concrete beam tested by Shardakov et al. [17] is modelled. The dimension, loading and boundary condition of the beam are illustrated in Fig. 4. The material is set to be concrete whose Young’s Modulus is 35 GPa, density is 2400 kg m^{-3} and tensile strength is 3 MPa.

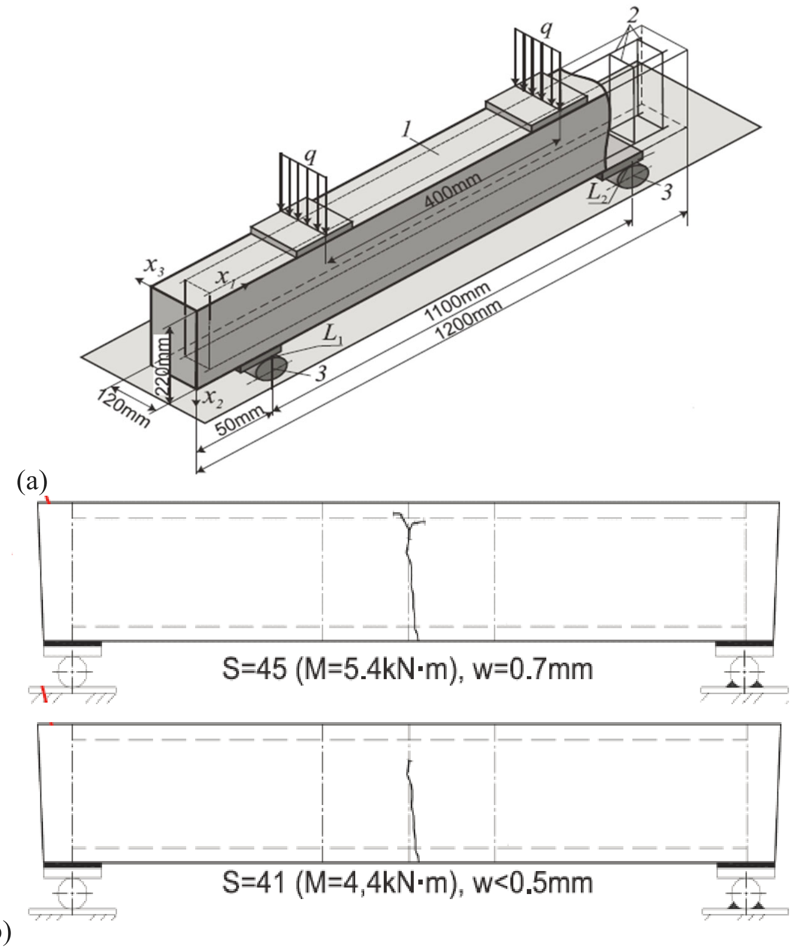


Figure 4. Test of Shardakov et al. [17]. (a) Dimension of specimen; (b) Pattern of crack.

Shardakov et al. [17] conducted 4-point bending on the beam illustrated in Fig. 4a. A crack occurred and propagated with the increase of load as shown in Fig. 4b. The fundamental natural frequency of the beam without crack was 2069 Hz and it decreased to 1921Hz (a reduction of 148Hz) when crack propagated under a moment of 5 kN m. They also developed a numerical model and the decrease of the fundamental natural frequency with crack is 109 Hz in their model.

In this Section, the crack modelling technique introduced in Section 4.2 is used for a validation purpose. One 1 mm depth defect is embedded in the mid-span at the bottom.

The natural frequencies (fundamental frequency in particular) are determined in the FE analysis using a free vibration procedure in a way analogous to what is commonly used in physical experiment. In this process, a concentrated load is applied at the midspan of the beam first, and this load is then released, causing free vibration of the beam. The vibration response of the beam is calculated in a general step-by-step manner. The deflection time history at the midspan is recorded, and it is subsequently analysed using FFT to determine the resonance (natural) frequencies. In this way, any nonlinearity or other complexities (e.g. crack closure) is automatically represented in the vibration response and hence in the extracted natural frequencies.

According to the FE model in this paper, the fundamental natural frequency of the beam before cracking is 2041 Hz and it changed to 1860 Hz after cracking (a reduction of 181Hz). The pattern of crack simulated in the model is illustrated in Fig. 5.

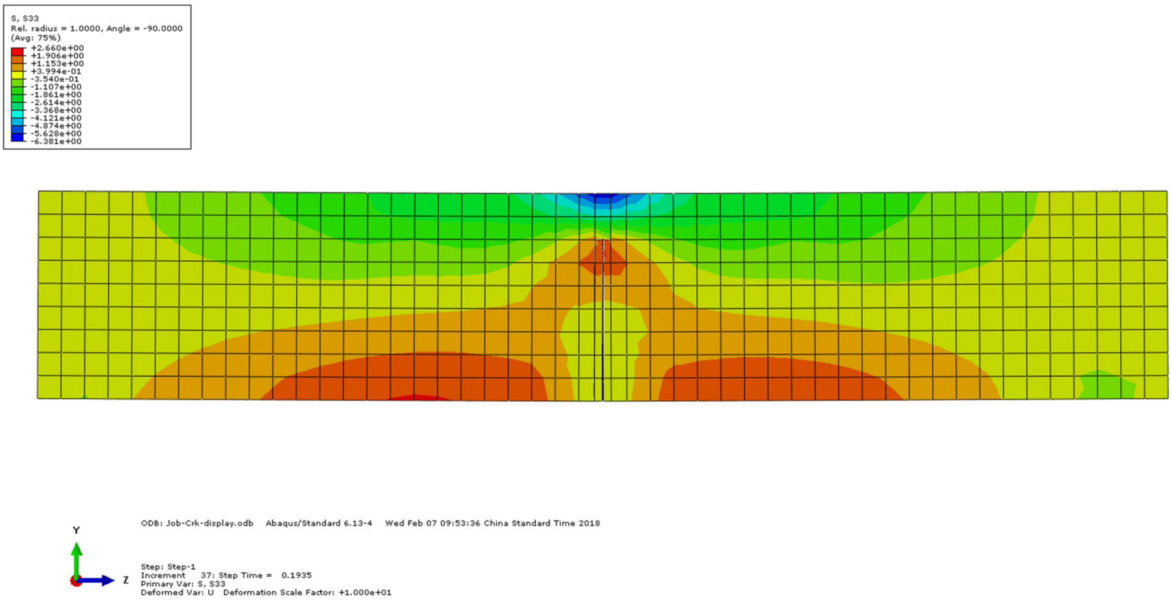


Figure 5. Pattern of crack in numerical simulation

The change of fundamental natural frequency due to crack in the model (181Hz) is compared with tests (148Hz) and they agree well, hence it is reasonable to believe that the crack modelling technique used in this paper is reliable to evaluate the effect of cracks on natural frequencies.

4.4 Modelling of prestress effect and model validation

This Section describes the modelling process for the installation of prestressing tendon. Models without shrinkage cracks are employed for the illustration. The same procedure is applied in later analysis simulating experimental tests with the incorporation of shrinkage cracks.

Two FE models of simply supported prestressed beams are developed. The length, height and width of both beams are 4 m, 100 mm and 75 mm respectively. The material of the main body of beams is concrete whose Young's Modulus is 26.9 GPa and density is 2300 kg m^{-3}. A straight steel prestressing tendon with 8 mm diameter is arranged at the centre of each beam and is simulated by the beam element. The Young's Modulus of the steel is 210 GPa and the density is 7850 kg m^{-3}. The tendon in the first model is only tied to the beam at both ends while it is detached from beam along its length to simulate Type-A prestressing scenario. The tendon in the second model is tied to the beam at every node to simulate Type-B prestressing scenario. Lateral deflection of the beams is restricted to prevent the vibration in the horizontal direction.

The prestressing force of 0 kN, 45 kN (43.4% of Euler's critical load) and 90 kN (86.9% of Euler's critical load) are applied also through a temperature scheme. A temperature descent of 100 □ in the tendon is applied while the expansion coefficient of tendon material is set to be 4.7×10^{-5} and 9.4×10^{-5} for the target prestressing force of 45 kN and 90 kN, respectively. The deformation of concrete causing prestress loss is considered when calculating the expansion coefficient. The compressive stress in concrete is 6 MPa and 12 MPa for the above two levels of prestressing force, respectively.

Three groups of models are developed to verify the mesh convergence. The average mesh sizes are 16.7 mm, 12.5 mm and 6.25 mm for the three models, respectively. The percentage of changes in frequencies due to mesh refinement is illustrated in Table 1. It can be seen that the results indicate that the mesh resolution (16.7 mm) is good enough and its results are shown in Table 2.

Table 1. Error induced by mesh of numerical simulation (compared to mesh size 16.7 mm)

Magnitude of prestressing force (kN)	Timoshenko scenario		Hamed and Frostig scenario	
	Mesh size 12.5 mm	Mesh size 6.25 mm	Mesh size 12.5 mm	Mesh size 6.25 mm
0	0.33%	0.66%	0.44%	0.87%
45	0.53%	1.06%	0.44%	0.87%
90	0.74%	1.48%	0.44%	0.87%

Table 2. Comparison of numerical simulation (mesh size 16.7 mm) and theoretical solution without considering the effect of shrinkage cracks

Magnitude of prestressing force (kN)	Timoshenko scenario		Hamed and Frostig scenario	
	Theoretical solution (Hz)	Numerical simulation (Hz)	Theoretical solution (Hz)	Numerical simulation (Hz)
0	9.688	9.719	9.688	9.740
45	7.287	7.353	9.688	9.740
90	3.513	3.659	9.688	9.740

As it can be seen, the results of the numerical model and theoretical solution have a good agreement which indicates that the FE model can be used to calculate the natural frequencies of prestressed beams with acceptable precision.

5. Simulation of two sets of previous experiments and comparisons

The tests done by Noble et al. [4] and Jang et al. [2] are simulated in this Section. The prestressing tendon is detached from the beam in tests of Noble et al. which makes it similar to Type-A. Meanwhile, the prestressing tendon is fully bonded to the beam in tests of Jang et al. hence its results will be compared with Type-B. Both of their test results contradict the corresponding theories and they will be simulated considering the effect of shrinkage cracks to explain this discrepancy.

5.1 Noble et al.'s experiment

Noble et al. [4] tested 9 concrete beams using both static method and dynamic method. Since their static tests were done to measure flexural stiffness and then calculated the natural frequency with it, their dynamic tests results were more straightforward because the natural frequency was directly measured in the dynamic tests. A finite element model is developed with the dimension of tests of Noble et al. [4]. All the beams are simply supported and have prestressing tendons with different eccentricity. The specimen simulated in this paper is the one with the centred prestressing tendon. The length of the beam is 2 m and the height and width of the beam section are 200 mm and 150 mm respectively. The concrete beam is reinforced by two D8 bars at the top and two D12 bars at the bottom. There are 11 H8 stirrups in the beam having an interval of 200 mm. A 15.7 mm diameter prestressing tendon is placed in a 20 mm diameter tube in the centre of concrete beam and the prestressing force increase gradually from 0 to 200 kN in 20 kN increments.

As mentioned in Section 4.3, slits are embedded at stirrup positions for the initiation of "shrinkage" cracks in the beams. After the initiation of cracks, the propagation of cracks is calculated by the software automatically following the theory of Fictitious Crack Model as described in Section 4.2.

The spacing of stirrup is 200 mm which is closed to the predicting spacing of cracks (210 mm and 184 mm in models of Chen et al. [8] and Leonhardt [9] respectively), it is reasonable that cracks initiate from every stirrup location. Hence the spacing of embedded defects introduced in Section 4.3 is 200 mm. Applying 200×10^{-6} (3 sides sealed) shrinkage strain by defining the temperature drop and the expansion coefficient of concrete, 11 cracks are found to propagate from the stirrups and the crack length is up to 21 mm as Fig. 6.

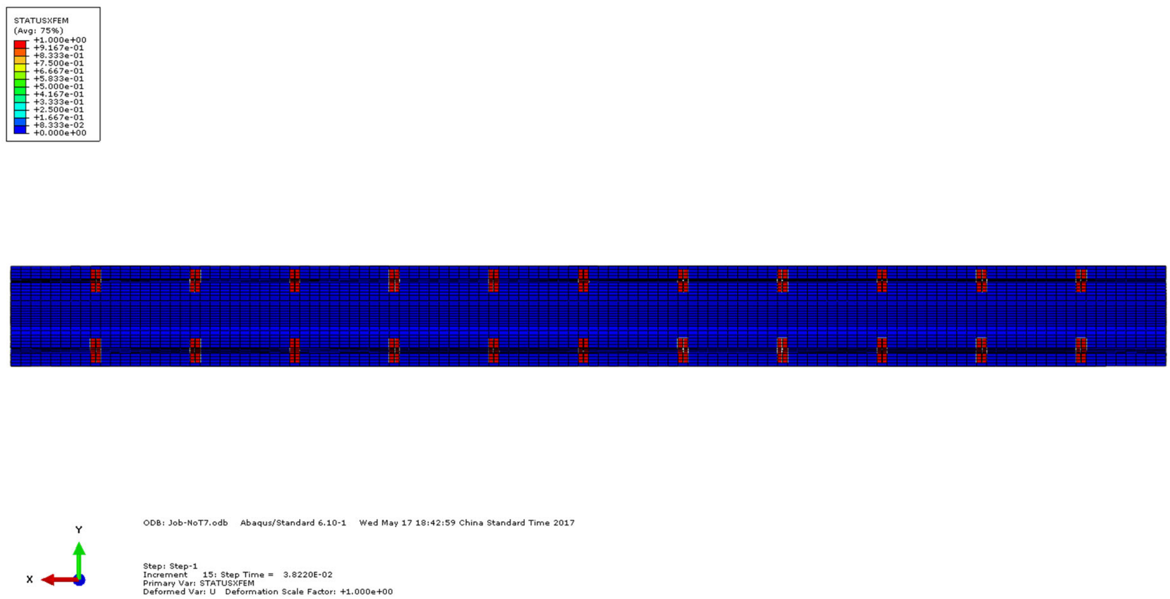


Figure 6. Cracks due to shrinkage in Noble’s specimen

Two scenarios are simulated when the beam is under 200 kN prestressing force and no prestressing force. The fundamental natural frequency of the beam without prestressing force is 68.5 Hz while it is 69.4 Hz when prestressing force increases to 200 kN. The change rate is 1.39%.

In the paper of Noble et al., 30 signals for each post-tensioning load level were presented. Mean value was calculated and regression analysis was conducted as the experimental uncertainty was large. From the linear regression line proposed in their paper, the fundamental natural frequency increased 0.96% when prestressing force increased to 200 kN. The results are listed in Table. 3:

Table 3. Comparison of Noble’s test and simulation

	Noble's test result		Simulation results	
Prestressing force (kN)	Magnitude (Hz)	Change rate (%)	Magnitude (Hz)	Change rate (%)
0	68.66	0	68.49	0
200	69.32	0.96	69.44	1.39

The simulation showed a reasonable agreement with tests results. Noble et al. [4] concluded that no statistically significant relationship between prestressing force and natural frequencies could be established and this contradicted to the expectation from Timoshenko’s theory under an ideal condition. The agreement between numerical model and experiment indicated that the above discrepancy could be caused by the effect of the closure of shrinkage cracks because this influence was not considered in the theoretical solution based on Timoshenko's theory.

One more validation numerical model is made to validate the previous results. In this model, the shrinkage cracks are not considered, and the prestressing force is zero. The other parameters are unchanged. The fundamental natural frequency is 69.93 Hz. It is higher than the model with cracks and zero prestressing force for both test results (68.66 Hz) and simulation results (68.49 Hz) which indicates that the cracks indeed deteriorate the flexural stiffness of the beam.

5.2 Jang et al.'s experiment

Jang et al. [2] conducted experiments and the increment of natural frequencies in their results is larger compared to tests of Noble et al. [4]. They tested six bonded pre-tensioned concrete beams with different magnitude of prestressing force and its fundamental natural frequency showed an increment of 15%. It is an 8 m simply supported beam with beam section of 300 mm × 300 mm. The beam is reinforced by four D16 bars at four corners. D10 stirrups are arranged with 100 mm interval at two ends and 150 mm interval in mid-span. The tendon consists of three strands with a diameter of 15.2 mm and is put in the centre of the beam section. The sketch of their test is illustrated in Fig. 7.

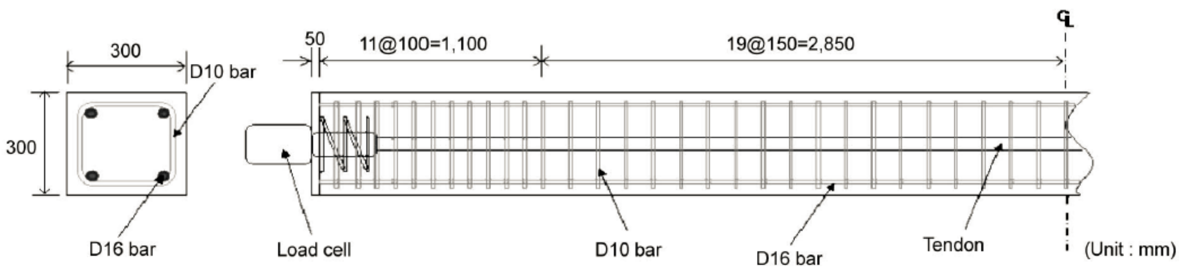


Figure 7. Sketch of test of Jang et al. [2]

They are pre-tensioned first and then the grouting material is injected to constrain the tendons and concrete beams. The prestressing forces are 0 kN, 146 kN, 264 kN, 356 kN, 465 kN and 523 kN in six beams and the corresponding natural frequencies are 7.567 Hz, 8.190 Hz, 8.498 Hz, 8.672 Hz, 8.690 Hz and 8.757 Hz respectively.

It is obvious that the increment of natural frequency is much smaller after the prestressing force reaches 356 kN. Hence 356 kN as a special checkpoint along with 0 kN and 523 kN and this situation is simulated in this paper.

Again, the position of stirrups should be considered according to the research of Rizkalla et al. [10]. Since the spacing of stirrup is 150 mm at both ends of beam and 300 mm in midspan while the spacing of cracks predicted by models of Chen et al. [8] and Leonhardt [9] is 300 mm and 319 mm, respectively, it is reasonable that cracks propagate at every two stirrups at ends of beam and they propagate at every stirrup in midspan.

Similarly, 200×10^{-6} (3 sides sealed) shrinkage strain is applied to the beam with temperature field. The fundamental natural frequency of the beam is 7.81 Hz, 8.93 Hz and 8.93 Hz when the prestressing force is 0, 356 kN and 523 kN respectively. The change rate is 14.3% when the prestressing force increases from 0 to 356 kN and it remains stable after that while it is about 15% in tests of Jang et al. [2]. The results are listed in Table. 4 and Fig. 8:

Table 4. Comparison of Jang's test and simulation

	Jang's test result		Simulation results	
Prestressing force (kN)	Magnitude (Hz)	Change rate (%)	Magnitude (Hz)	Change rate (%)
0	7.567	0	7.81	0
146	8.190	8.1	8.10	3.7
264	8.498	12.3	8.54	9.3
356	8.672	14.6	8.93	14.3
465	8.690	14.8	8.93	14.3
523	8.757	15.7	8.93	14.3

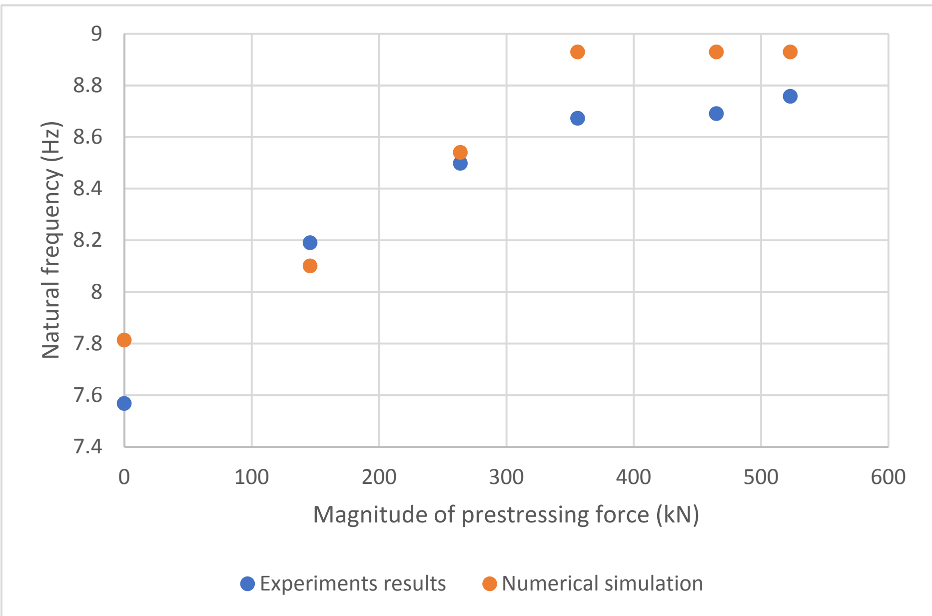


Figure 8. Comparison of Jang's tests and simulations

When the prestressing force increases from 0 to 356 kN which is 68% of the maximum prestressing force (523 kN), the natural frequency increases 1.105 Hz which is 92.9% of the total natural frequency change. When the prestressing force keeps climbing, the natural frequency remains stable. Both tests results and numerical simulation showed the same trend. This phenomenon could be explained that almost all the cracks are closed when the prestressing force reaches 356 kN. Hence, keep increasing the prestressing force will not promote the natural frequency infinitely. Again, the results of experiments and simulation fit well.

One more validation model is made for tests of Jang et al. [2] too. No crack is considered in this model, and the prestressing force is zero. The other parameters are identical to the previous model. The fundamental natural frequency is 8.93 Hz which equals to the cracked beam with a high level of prestressing force. Hence it is reasonable to believe that the increasing trend of natural frequencies unexpected by Hamed and Frostig's theory is caused by the closure of shrinkage cracks inside the concrete.

6. Conclusions

A systematic investigation on the effect of prestressing force on natural frequencies of reinforced concrete beams is conducted using theoretical analysis and numerical simulation with the comparison to existing experimental studies in the literature. It is found that

1) The different applicability of the classical theories for the two bounding cases, one with the prestressing tendon detached from the body of the beam (analogous to the Timoshenko beam with external compression), and another with the tendon fully attached to the beam, is highlighted. Theoretically, provided that there is no change in the mechanical properties of the beam, the former case will lead to a decrease of the natural frequencies with the increase of the prestressing force, whereas no change of the natural frequencies will result in the latter scenario regardless of the level of the prestressing force.

2) Discrepancies observed in the previous experimental studies on prestressed RC beams, i.e. the natural frequencies of the beams tended to increase with the prestressing force, can be qualitatively attributed to the closure of initial cracks such as shrinkage cracks in the beams when prestressing force is applied.

3) Quantitative analysis of the effect of shrinkage cracks is carried out using numerical simulation. The shrinkage cracks are simulated using the Fictitious Crack Model in ABAQUS and through a

temperature change scheme. The simulation model is validated by comparing with an experiment concerning both simulation of the cracks and their effect on natural frequencies.

4) Using the validated numerical model, selected experiments from two reference publications (Noble et al. [4], Jang et al. [2]) are simulated. The results further confirm the effect of shrinkage cracks and that the closure of these cracks due to prestressing force can cause varying degrees of increase of the natural frequencies with the increase of the prestressing force.

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