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### Statistical properties of hybrid estimators proposed for GEDI – NASA's Global Ecosystem Dynamics Investigation

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## Statistical Properties of Hybrid Estimators Proposed for GEDI – NASA’s Global Ecosystem Dynamics Investigation

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## Abstract

NASA's GEDI (Global Ecosystem Dynamics Investigation) mission will collect waveform lidar data at a dense sample of ~25-m footprints along ground tracks paralleling the orbit of the International Space Station (ISS). GEDI's primary science deliverable will be a 1-km grid of estimated mean aboveground biomass density (Mg/ha), covering the latitudes overflown by ISS (51.6°S to 51.6°N). One option for using the sample of waveforms contained within an individual grid cell to produce an estimate for that cell is hybrid inference, which explicitly incorporates both sampling design and model parameter covariance into estimates of variance around the population mean. We explored statistical properties of hybrid estimators applied in the context of GEDI, using simulations calibrated with lidar and field data from six diverse sites across the United States. We found hybrid estimators of mean biomass to be unbiased and the corresponding estimators of variance appeared to be asymptotically unbiased, with underestimation of variance by approximately 20% when data from only two clusters (footprint tracks) were available. In our study areas, sampling error contributed more to overall estimates of variance than variability due to the model, and it was the design-based component of the variance that was the source of the variance estimator bias at small sample sizes. These results highlight the importance of maximizing GEDI's sample size in making precise biomass estimates. Given a set of assumptions discussed here, hybrid inference provides a viable framework for estimating biomass at the scale of a 1-km grid cell while formally accounting for both variability due to the model and sampling error.

## 1. Introduction

The NASA GEDI (Global Ecosystem Dynamics Investigation) mission mounted a full-waveform lidar instrument on the International Space Station (ISS) in late 2018. GEDI is designed to measure forest structure, and one of its primary science deliverables will be a 1-km grid of mean aboveground biomass density (AGBD in Mg/ha) estimates over the forested areas between 51.6° N and S (the range of the ISS). The basic lidar metric supporting these estimates will be waveforms related to the canopy height density profile for 25-m (diameter) footprints, spaced 60-m apart along ground tracks paralleling the ISS orbit. After two years of operation, GEDI will have covered the majority of 1-km cells with two or more ground tracks. Performance of other lidar instruments suggests that canopy height and structure metrics derived from GEDI data will be strongly correlated with AGBD (Zolkos et al. 2013).

This paper concerns the challenge of using spatially discontinuous lidar footprint data acquired along tracks to estimate mean AGBD within each 1-km grid cell. One approach has been to “scale up” from field data to spatially coincident lidar footprints using one level of models, apply those models to all lidar footprints, and then use the lidar-based AGBD predictions to calibrate another level of models that predict AGBD using coarse-resolution optical or radar data (e.g.,(Baccini et al. 2017)). However, while diagnostics such as Root Mean Square Error from the latter model can be used to indicate confidence for predictions at the scale of each coarse-resolution grid cell under such an approach, ignoring residual variance in the field-to-lidar model can hide substantial uncertainty (Saarela et al. 2016), as can discounting the sampling uncertainty involved with associating fine-grain lidar measurements with coarser remote sensing data.

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3 In this paper, we propose the use of hybrid inference ((Fattorini, 2012); (Ståhl et al. 2016)) to  
4 estimate mean AGBD , with associated uncertainty, at the level of the GEDI grid cell while  
5 explicitly accounting for both field-to-lidar model error as well as sampling uncertainty. Hybrid  
6 methods have been used with both spaceborne (Healey et al. 2012; Margolis et al. 2015) and  
7 airborne (Corona et al. 2014; Ståhl et al. 2011) lidar instruments to estimate forest AGBD,  
8 though never for areas corresponding to grid cells. The calibration/application approach we  
9 propose (Figure 1) relies upon lidar datasets contributed from around the world being converted  
10 to GEDI-like waveforms that realistically incorporate several types of uncertainty (Hancock et  
11 al. 2019). Resulting simulated GEDI waveforms are being related to corresponding ground  
12 measurements to create parametric, footprint-level AGBD models (Kellner et al. 2018).

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27 Using hybrid estimation, these models will be applied to all footprints acquired by GEDI, and the  
28 resulting predictions will form the basis of a sample-based estimate of mean AGBD within each  
29 grid cell; no wall-to-wall imagery will be used. Our estimators of mean AGBD and the variance  
30 of that mean are similar to those used by Ståhl et al. (2011), treating GEDI tracks intersecting a  
31 1-km grid cell as a simple random sample of clustered observations. Variance is estimated as a  
32 function of: 1) the number of clustered predicted footprint-level AGBD values and the variability  
33 among them, as well as 2) the uncertainty of the parameter estimates used in the footprint-level  
34 models.

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46 Key assumptions in this approach include the following: (1) the parameter covariance matrix  
47 generated during the footprint modeling process appropriately conveys the uncertainty of  
48 footprint-level AGBD predictions; (2) GEDI waveforms simulated from airborne data adequately  
49 represent signals received from the sensor, and; (3) the expected values of the proposed hybrid  
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estimators are unbiased. This paper evaluates this third assumption.

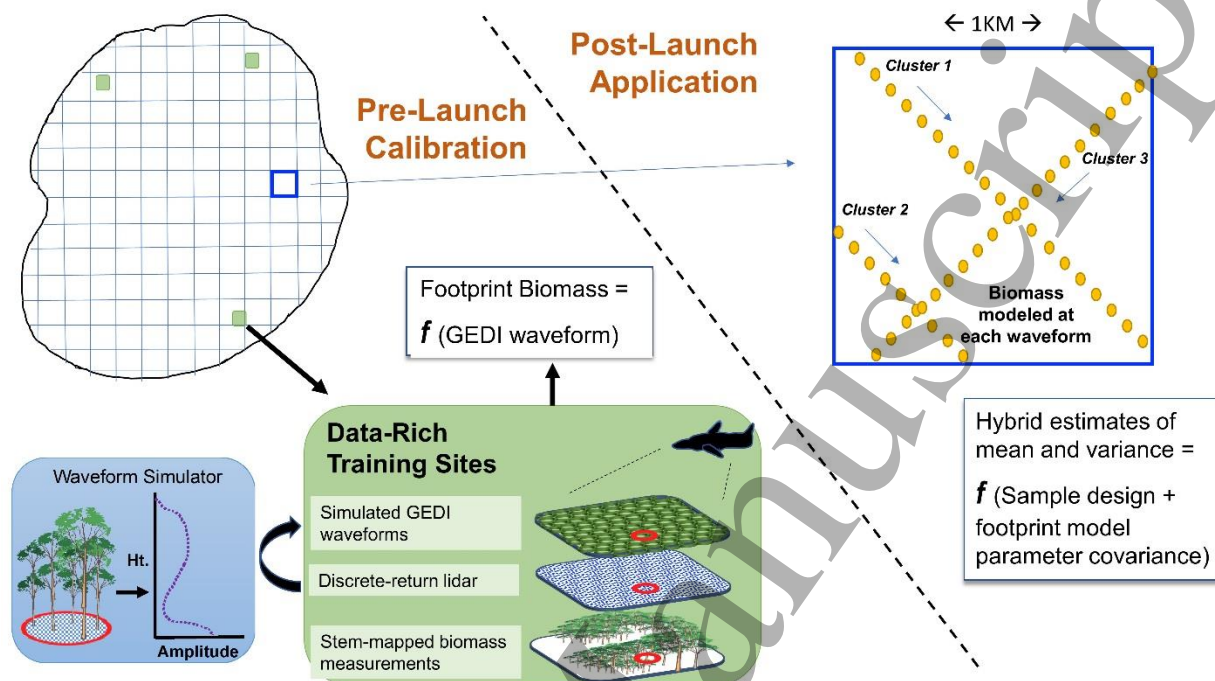


Figure 1. Proposed pre-launch calibration approach, using simulated GEDI waveforms collected over closely geo-registered field plots to develop waveform-to-AGBD relationships that may be applied to all footprints. Hybrid estimators access both properties of how GEDI samples the grid cell as well as the footprint model parameter covariance.

The objectives of this study were to 1) propose a framework through which hybrid inference can be used with GEDI data to estimate mean AGBD and the variance of that mean; and 2) using GEDI waveforms simulated from airborne small-footprint lidar for 60 diverse grid cells, construct a simulation that tests the bias of the proposed estimators. Documenting estimator performance is an important step in evaluating Figure 1's approach to turning GEDI's high-quality, spatially discontinuous observations into directly interpretable gridded estimates of mean AGBD across the globe.

## 2. Methods

This section describes the proposed GEDI sample, estimators and a test of the proposed

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3 estimators. The first subsection describes the proposed sampling framework and our application  
4 of hybrid estimators to estimate both mean AGBD and variance of the mean. An empirical  
5 assessment of the properties of these estimators in the GEDI context is presented in the third  
6 subsection and in Appendix S2, while field and lidar data supporting this assessment are  
7 described in the second subsection and in Appendix S1. Consideration of the role of spatial  
8 autocorrelation, specific to the case of the GEDI 1-km cell is given in Appendix S3, while  
9 footprint-level modeling details are in Appendix S4.  
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## 20 2.1 Estimators

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23 The GEDI instrument will produce 25-m (diameter) footprint observations along tracks  
24 paralleling the ISS orbit. The intersection of a track with a 1-km cell produces a collection of  
25 GEDI footprints, to be denoted here as a cluster. We propose to view each 1-km grid cell as  
26 tessellated into equal-area non-overlapping population elements, each representing a potential  
27 GEDI footprint. GEDI footprints are population elements sampled by the GEDI instrument.  
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Figure 2 illustrates this conception, adapted to the spatial dimensions of lidar and field data available for this study. Footprint (20m) and grid cell dimensions (approximately 800 m) used here (illustrated in Figure 2) were considered sufficiently similar to GEDI to support applied testing of the estimators described below. For simplicity, we consider tracks as having either a 45° or 135° degree inclination for descending or ascending orbits, respectively, although ISS inclination patterns will vary across latitudes. In this idealization, we assert that there are two potential disjoint clusters for the intersection of a track with a grid cell – one starting on the border footprint, and one starting with the subsequent along-track footprint. The sample will be the set of clusters from the tracks that intersect the 1 km grid cell; due to variability of the ISS



orbit this sample can be considered a simple random sample of clusters of GEDI footprints. In this exercise, we do not consider missing data issues that will arise because of clouds.

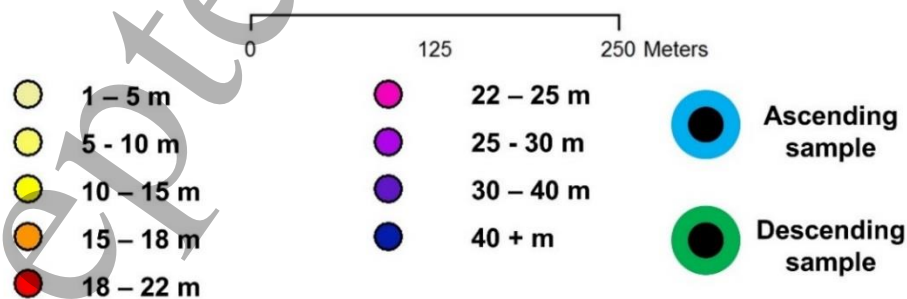
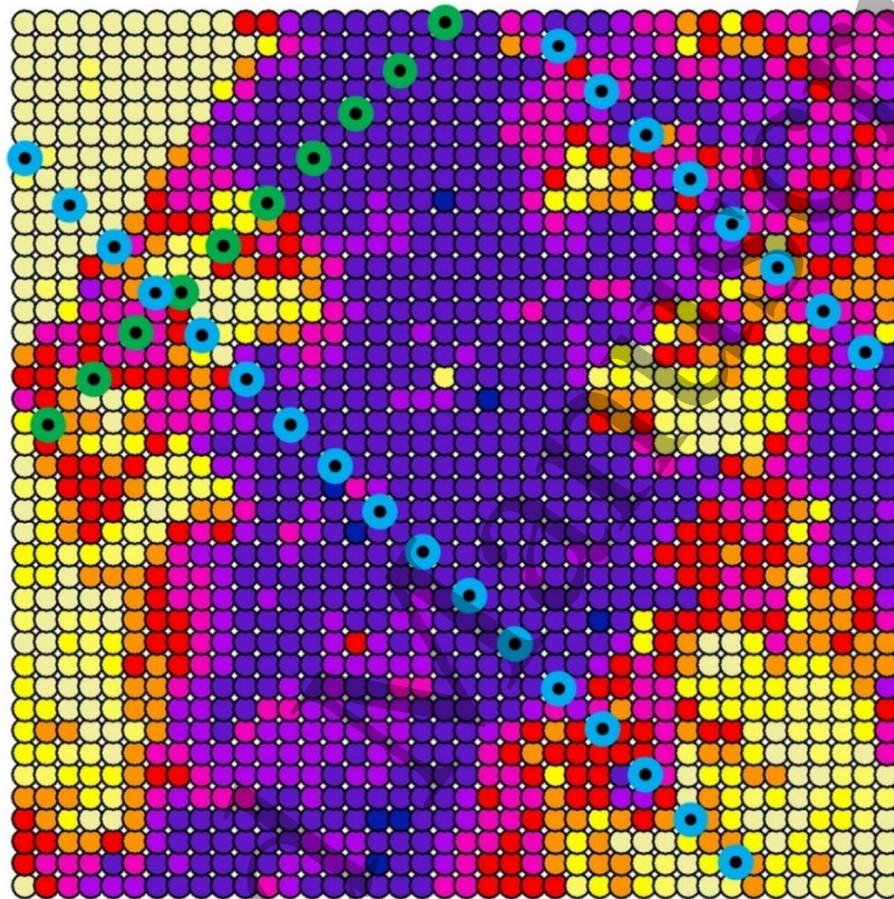


Figure 2. Near canopy top height (height from below which 96% of lidar energy returns) for GEDI footprints simulated from airborne data in one of six sites in the Pennsylvania/New Jersey study area. A possible sample of three clusters is shown.

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3 The population parameter of interest is the average of the true AGBD over the  $N$  population  
4 elements (potential GEDI footprints), where  $N$  is the total number of population elements in a  
5 grid cell. For a 1-km grid cell,  $N$  is equal to 2,500 20x20-meter population elements, where the  
6 footprints in this realization would have a 20-m diameter. There is a parametric model,  $g(\mathbf{x}, \boldsymbol{\alpha})$ ,  
7 of AGBD in Mg/ha, where  $\mathbf{x}$  are characteristics of the GEDI waveform for the GEDI footprint,  
8 and  $\boldsymbol{\alpha}$  is a vector of parameters. The true AGBD for a footprint is assumed to deviate from the  
9  $g(\mathbf{x}, \boldsymbol{\alpha})$  by a random quantity,  $\varepsilon$ , with an expected sum of zero. Under the assumption that the  
10 model is an unbiased predictor of the true AGBD, and ignoring model misspecification error, the  
11 modeled value  $g(\mathbf{x}, \boldsymbol{\alpha})$  will be used in place of the true AGBD. The mean can be expressed in  
12 terms of GEDI ground clusters. There are  $M$  non-overlapping clusters distributed across all  
13 possible tracks. There are  $T_i$  footprints in the  $i$ th cluster. The reader may have noted that each  
14 population element is in one ascending cluster and one descending cluster. We can express the  
15 population attribute, the average expected predicted AGBD over the  $N$  population elements, in  
16 terms of the clusters; that is:

$$\mu_Y = \frac{\sum_{i=1}^M \sum_{t=1}^{T_i} g(\mathbf{x}_{it}, \boldsymbol{\alpha})}{2N} \quad [1]$$

17 where the modeled AGBD for each population element occurs in the numerator twice.

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47 Equation 1 will be expressed in an equivalent form, for which there is an estimator and a method  
48 of calculating an approximate variance. Let  $G_i = \sum_{t=1}^{T_i} g(\mathbf{x}_{it}, \boldsymbol{\alpha})$ , equal the cluster total of the  
49 predicted AGBD per hectare for the footprints in the  $i$ th cluster. Equation 1 can be expressed as  
50 the ratio of the mean (or sum of) AGBD per cluster and the mean (or sum of) number of  
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elements per cluster:

$$\mu_Y = \frac{\sum_{i=1}^M G_i}{\sum_{i=1}^M T_i} = \frac{M^{-1} \sum_{i=1}^M G_i}{M^{-1} \sum_{i=1}^M T_i} \quad [2]$$

Parameter estimates,  $\hat{\alpha}$ , for the model  $g(\mathbf{x}, \alpha)$  are based on a separate sample,  $S_m$ , that is independent of the 1-km grid cell. In Equation [1] or [2] the parameters,  $\alpha$ , are replaced by the parameter estimates,  $\hat{\alpha}$ , and the population number of clusters,  $M$ , by the sample number of clusters,  $m$ , to get the estimator  $\hat{\mu}_Y$ , i.e.,  $G_i$  with  $\hat{G}_i = \sum_{t=1}^{T_i} g(\mathbf{x}_{it}, \hat{\alpha})$

$$\hat{\mu}_Y = \frac{\sum_{i=1}^m \sum_{t=1}^{T_i} g(\mathbf{x}_{it}, \hat{\alpha})}{\sum_{i=1}^m T_i} = \frac{\sum_{i=1}^m \hat{G}_i}{\sum_{i=1}^m T_i} \quad [3]$$

The second component of the equation expresses the estimator  $\hat{\mu}_Y$ , the sample mean of all predicted AGBD (per sample element) which is an estimator of the population average expected AGBD (per element). The population true average AGBD differs randomly from the expected value  $\mu_Y$  by the population average  $\bar{\epsilon}$  of the  $N$  deviations  $\epsilon_{it}$ . For large  $N$  the contribution of  $\bar{\epsilon}$  to the total random error can be assumed negligible compared to the other two sources of uncertainty (the sampling error and the variability due to the model, see below), even if spatial autocorrelation is present. Hence, it is sufficient to assume the size of the grid cell (and thus  $N$ ) is large enough to imply that the average  $\bar{\epsilon}$  will be close to zero. This assumption, along with the assumptions related to model fit will be covered in the discussion section.

As noted in the Introduction, the GEDI sample will be treated a simple random sample of clusters of GEDI footprints. The estimator in Equation [3] can be viewed as estimating the mean of cluster totals and mean number of footprints separately, and then combining as a ratio estimator:

$$\hat{\mu}_Y \frac{\sum_{i=1}^m \hat{G}_i}{\sum_{i=1}^m T_i} = \frac{m^{-1} \sum_{i=1}^m \hat{G}_i}{m^{-1} \sum_{i=1}^m T_i} = \frac{\bar{\hat{G}}}{\bar{T}} = \quad [4]$$

This estimation strategy combines a probability sample of auxiliary information (as opposed to wall-to-wall auxiliary information) with a prediction for the population element value, AGBD, instead of directly observing AGBD. Ståhl et al. (2011) proposed an estimator of the form of Equation 4 and derived an estimator of the approximate variance. Due to the small sample size used in that study in relation to the population size, Ståhl et al. (2011) did not include a finite population correction. A finite population correction may need to be taken into account in GEDI's case. Expression 5 below is the variance estimator proposed in Ståhl et al (2011), with the addition of the finite population correction:

$$\begin{aligned} \hat{V}(\hat{\mu}_Y) = & \frac{1}{\bar{T}^2} \left(1 - \frac{m}{M}\right) \frac{\sum_{i=1}^m (\hat{G}_i - \hat{\mu}_Y T_i)^2}{m(m-1)} \\ & + \frac{1}{\bar{T}^2} \sum_{j=1}^p \sum_{k=1}^p \widehat{Cov}_{S_m}(\hat{\alpha}_j, \hat{\alpha}_k) \hat{G}_j' \hat{G}_k' \end{aligned} \quad [5]$$

The first term is due to the sampling error and the second term is due to effects of uncertainty of the  $\hat{\alpha}$  estimates (Ståhl et al. 2011). The  $\widehat{Cov}(\hat{\alpha})$  is the estimated covariance matrix of the  $p$ -parameter estimates, where the estimate is based on the separate set of data,  $S_m$ , that is independent of the 1-km grid cell. For simplicity the first term is referred as the sample component and second term the model component. . If we assume a linear model, that is

$$g(\mathbf{x}_{it}, \hat{\alpha}) = \hat{\alpha}_1 + \sum_{j=2}^p x_{itj} \hat{\alpha}_j, \text{ where } x_{itj} \text{ is the } j\text{th component of } \mathbf{x}_{it}, \text{ then, } \hat{G}_j' =$$

$$\frac{1}{m} \sum_{i=1}^m \sum_{t=1}^{T_i} x_{itj}. \text{ Let } \hat{\mathbf{x}} \text{ be the vector of the means of the cluster totals of the predictor variables,}$$

$$\text{i.e., } \hat{\mathbf{x}}_j = \frac{1}{m} \sum_{i=1}^m \sum_{t=1}^{T_i} x_{itj}, \text{ for } j = 1, \dots, p. \text{ Then the second component on the right side of}$$

Equation 5 can be expressed in matrix notation:

$$\sum_{j=1}^p \sum_{k=1}^p \widehat{Cov}_{S_m}(\hat{\alpha}_j, \hat{\alpha}_k) \hat{G}'_j \hat{G}'_k = \hat{\mathbf{x}}^T \widehat{Cov}(\hat{\boldsymbol{\alpha}}) \hat{\mathbf{x}} \quad [6]$$

Note that  $\hat{\mathbf{x}}_1$  is equal to  $\bar{T}$ . In the construction of Equation [5], a Taylor linearization process is used twice; first with regards to the model parameters of  $g(\mathbf{x}, \hat{\boldsymbol{\alpha}})$  (Ståhl et al. (2011) and second to estimate variance of a ratio estimator (Sändal et al. 1992)

The empirical data described in Section 2.2 were used to calibrate the simulation study described in Section 2.3 to evaluate the statistical properties of these estimators in the GEDI setting.

## 2.2 Supporting datasets

Lidar and field data collected over six diverse areas in the United States supported this study.

These areas were defined by the non-overlapping window (Thiessen Scene Area) of a local

Landsat scene. The six scenes (given by numeric WRS-2 Path/Row) included one each in:

northern Maine (“ME”: 12/28, excluding the Canadian portion); eastern Pennsylvania and central

New Jersey (“PA/NJ”: 14/32); coastal South Carolina (“SC”: 16/37); northern Minnesota

(“MN”: 27/27); northwestern Colorado (“CO”: 35/32); and western Oregon (“OR”: 45/30)

(Figure 3). The selected areas represented a wide range of forest ecosystems and disturbance

processes, as described by Cohen et al. (2017) and Healey et al. (2018). Small-footprint lidar

was collected at each site in the pattern displayed in Figure 3, and a waveform simulator

developed by the GEDI Science Definition Team was used to simulate GEDI waveforms from

the airborne data. Data collection and waveform simulation is described in Appendix S1.

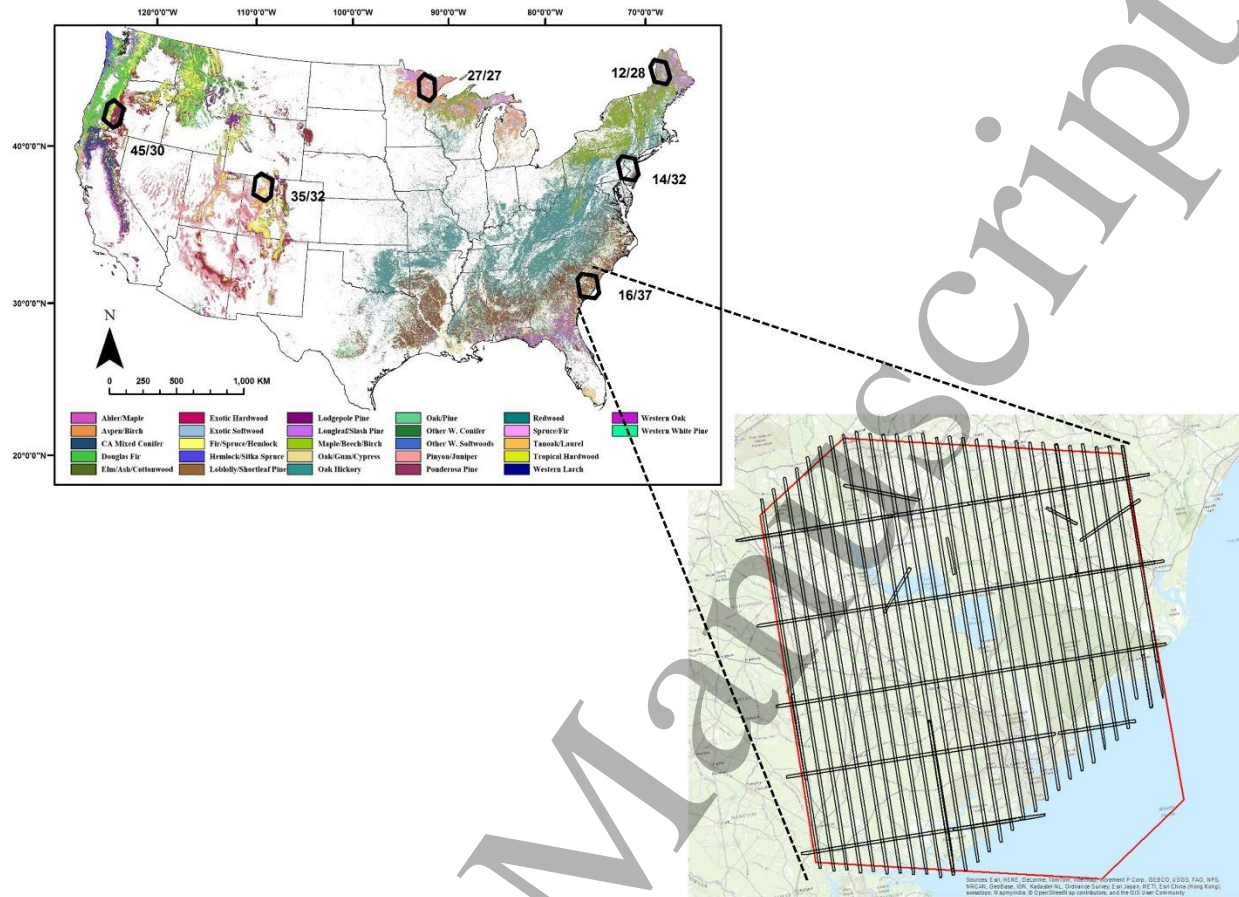


Figure 3. Location of the study's six focal areas, with detail for the SC site (24/34) showing airborne lidar flight lines. The width of each line was variable: ~300-900 m. Adapted from Healey et al., 2018.

### 2.3 Evaluating estimator performance

Simulations were conducted to examine the properties of the above estimators of mean AGBD and variance of the estimated mean. Modeled AGBD values treated as truth for each population element were developed for sixty square grid cells (ten randomly chosen grid cells at each of the six test areas in Figure 3) of dimensions approximately similar to GEDI's product specifications. These populations were created by applying a linear model to the GEDI metrics simulated from actual small-footprint lidar acquisitions (Figure 2). Appendix S2 details the process of both

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3 establishing the true population and simulating different potential cluster patterns and model  
4 parameter combinations to test the proposed estimators under a range of conditions. The  
5 advantage of using real data to develop the “true” population, as opposed to simulating arbitrary  
6 AGBD surfaces, was that it ensured realistic covariance among lidar metrics. To the degree that  
7 lidar metrics are correlated with AGBD (see Appendix S3), this approach also created a realistic  
8 spatial representation of biomass and of residual prediction error when different realizations of  
9 model parameters were applied to the lidar metrics. The swath width of the available lidar data  
10 limited the size of these test cells to between 680x680 m and 800x800 m (instead of 1-km); as  
11 mentioned earlier, this was considered adequate for the tests described below.  
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### 24 **3. Results**

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27 The mean true AGBD of the 60 simulated grid cell-scale populations ranged from 1 to 183  
28 Mg/ha. There was little difference between the true and expected estimated values across  
29 simulated grid cells (Figure 4). Divergence between these quantities centered around zero, and  
30 decreased with more clusters (Figure 5).  
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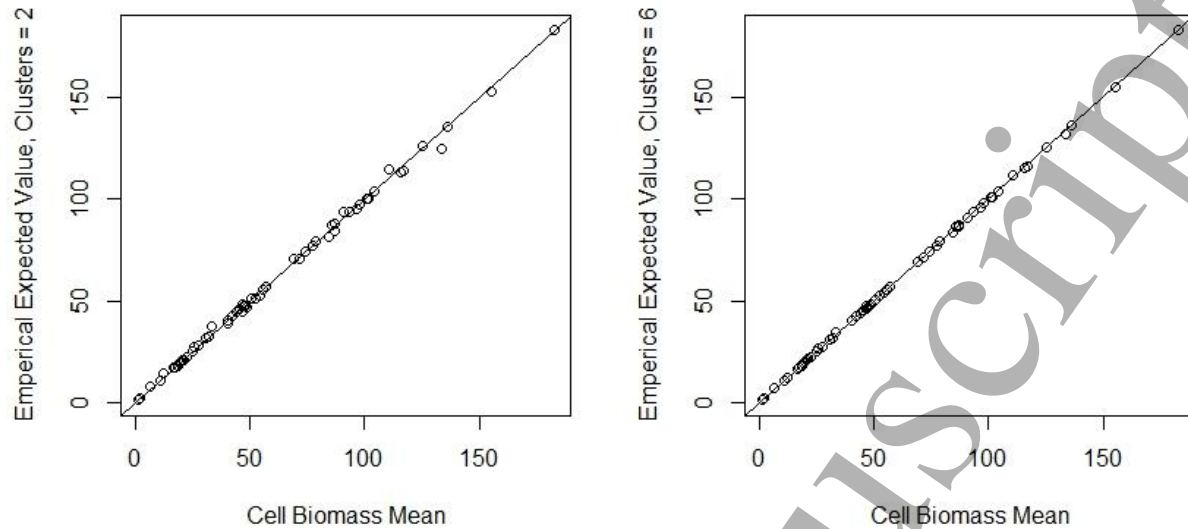


Figure 4: Expected estimate of AGBD (Mg/ha) versus true AGBD (on the X axis) for both two and six tracks. The line represents  $y=x$ .

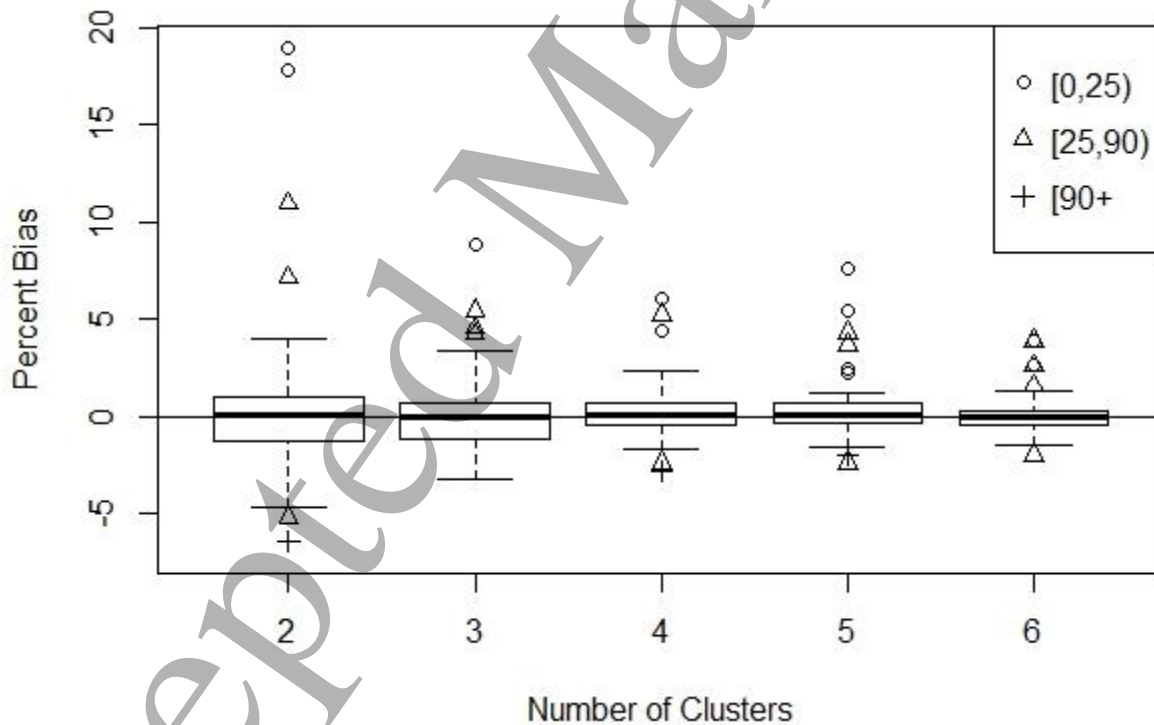


Figure 5: Boxplot of percent bias for ABGD estimates (Equation 4) at the grid cell level. Outliers are divided into categories defined by the mean AGBD for the simulated cell (0-25, 25-90, and 90+ Mg/ha).

The empirical percent bias of the variance estimator (Equation 5) as a function of number of



clusters is shown in Figure 6. The estimator appears to be only asymptotically unbiased, with mean underestimation of variance at approximately 21% with two clusters and 2% with six clusters. Outliers in Figures 5 and 6, which measure bias as a percent of the mean AGBD, tended to be low-biomass locations.

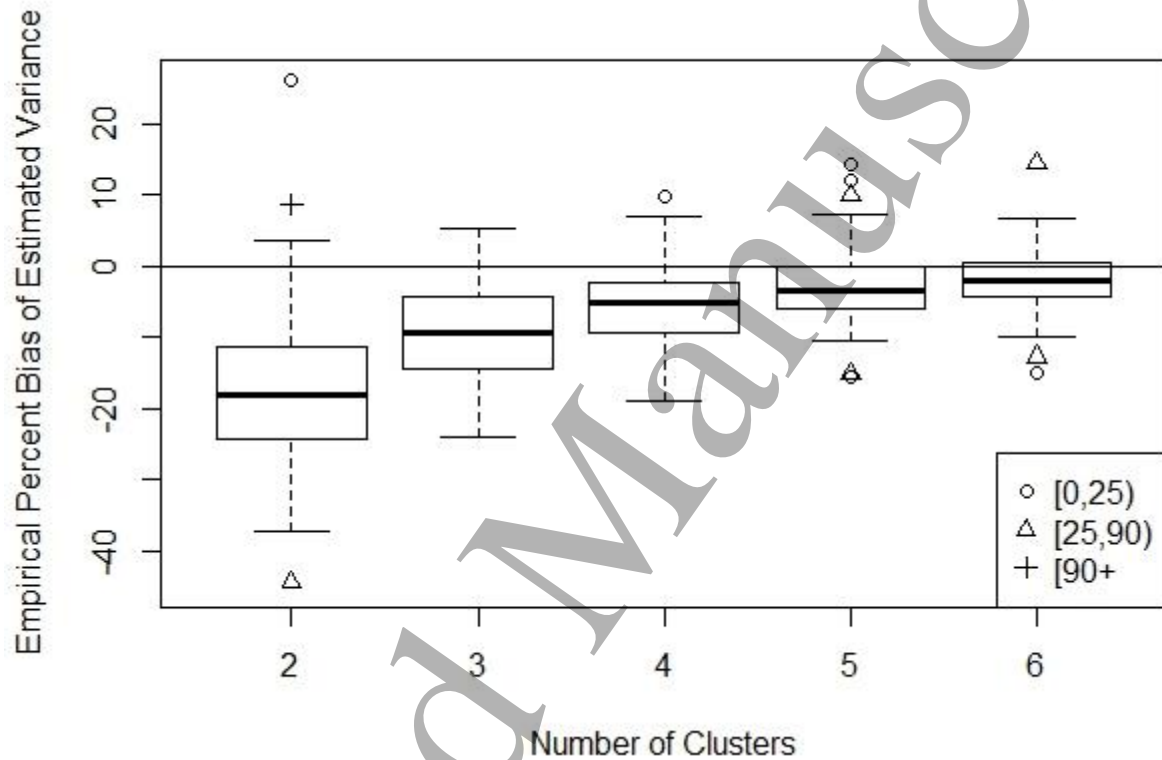


Figure 6: Boxplot of the empirical percent bias in the variance estimator for each grid cell (Equation 5). Outliers are divided into categories defined by the mean AGBD for the simulated cell (0-25, 25-90, and 90+ Mg/ha).

The variance under-prediction at low numbers of clusters may be traced to the sample component of Equation 5, not the model component (Figure 7).

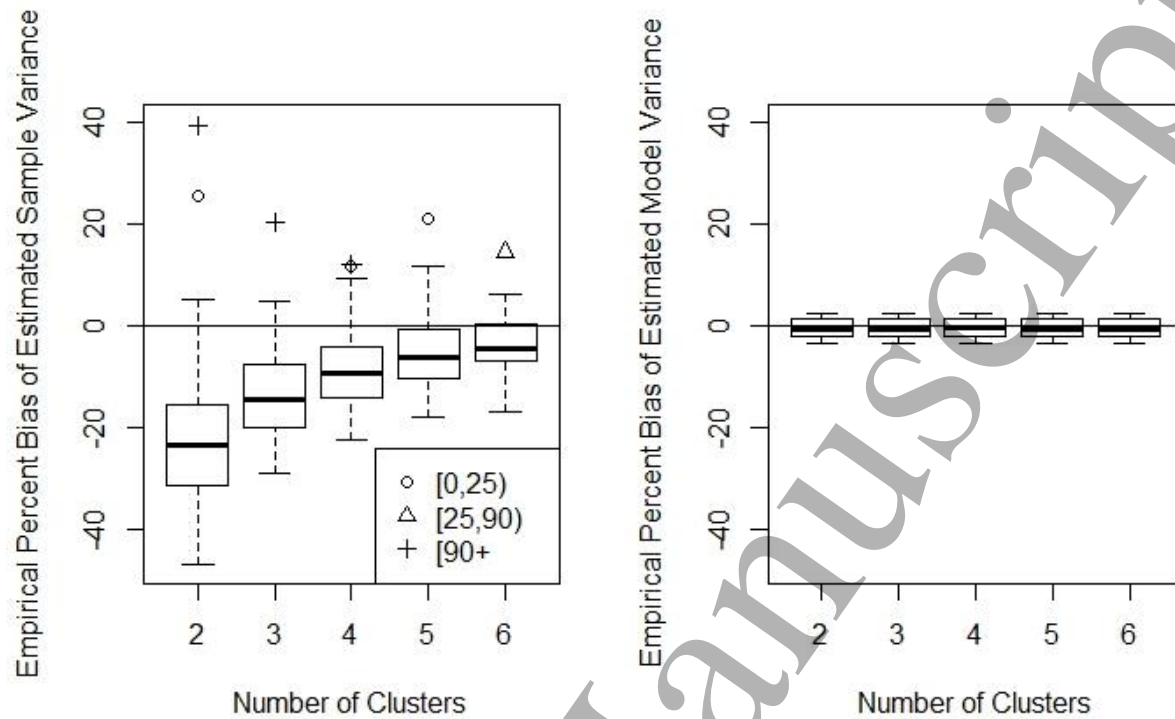


Figure 7. Decomposition of bias in overall grid cell-level variance estimates (Figure 6) into sample and model contributions.

In most cases, the sampling component of the variance was larger than the model component (Figure 8). With more tracks, overall variance of the estimate went down, and model variance accounted for a larger percentage of that variance.

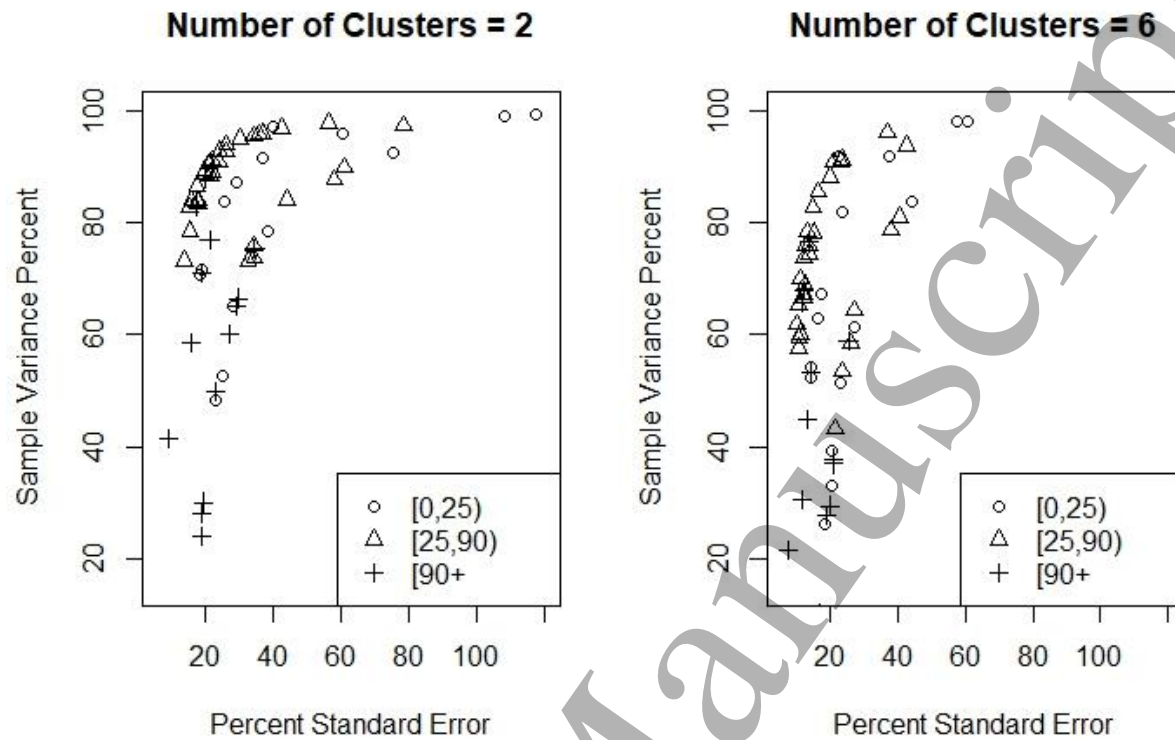


Figure 8: Sample variance as a percentage of the total variance versus the percent standard error (square root of the variance) for 2 clusters and 6 clusters. More clusters reduced both overall variance and the importance of the sample component of the variance.

## 4. Discussion

### 4.1 Advantages and assumptions

Earlier work with NASA's GLAS (Geoscience Laser Altimeter System) instrument pioneered the use of spaceborne lidar to make forest AGBD maps, often employing models that related field biomass to lidar and then, in a separate stage, related lidar-modeled biomass to synoptic data from sources like the MODIS satellites. While some work has attempted to impose a statistical framework on the use of such maps (e.g., (Nelson et al. 2017)), most have employed ad hoc approaches to uncertainty. Mitchard et al. (2014) pointed out two such ad hoc error budgeting approaches that yielded estimates across the Amazon that did not overlap with each

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3 other's relatively narrow confidence intervals, suggesting unacknowledged uncertainty in one or  
4 both estimates.  
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8 Saarela et al. (2016) concluded that ignoring error occurring in the field-to-lidar step, as is often  
9 done, can lead to underestimation of variance by a factor of three. The spatial mismatch inherent  
10 in modeling AGBD in large pixels from constituent smaller-footprint measurements can also  
11 introduce uncertainties not clearly addressed in variance estimates (Réjou-Méchain et al. 2014).  
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13 The proposed hybrid approach does explicitly account for field-to-lidar model error, and it also  
14 accounts for sample error as footprints are combined to infer mean 1-km AGBD.  
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23 However, the hybrid approach summarized in Figure 1 is subject to at least three important  
24 assumptions. The first is that the GEDI waveform simulator (Hancock et al., 2019; described in  
25 Appendix S2) correctly translates pulse counts to waveform data while representing the  
26 important sources of error. The benefit of using simulated GEDI data at precisely known  
27 locations to build footprint-level models is that it obviates potential error related to the positional  
28 uncertainty of GEDI footprints (currently estimated to be on the order of 10 m, post-processed).  
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30 The risk is that any systematic simulator error may be propagated through footprint-level models  
31 to bias the 1-km mean AGBD estimates. However, the simulator is based on well-validated work  
32 by Hofton and Blair (1999) and it is unlikely violations of this assumption are substantial.  
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44 A second assumption is that one square kilometer is a large enough domain, given spatial  
45 autocorrelation of the population, that residual model error may be considered negligible. Mean  
46 residual error in the footprint AGBD model is presumed to tend toward zero over a large number  
47 of predictions. In an area that is small, particularly if spatial autocorrelation of residual error is  
48 high, residual errors may not sum to zero, introducing additional error. While GEDI's 1-km  
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3 spatial domain is smaller than other applications of hybrid inference (e.g. Ståhl et al., 2011),  
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5 initial exploration using the data from this study suggests that spatial autocorrelation at GEDI's  
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7 25-m grain size is relatively low, as is the effect of omitting residual error (also called individual  
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9 error in Appendix 3) from the proposed hybrid variance estimator (Appendix 3). In some cases,  
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11 however, the impact of residual error may reach 15% or more of the variance, so further work is  
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13 needed to develop ways to include it in the hybrid approach.  
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17 Finally, it is assumed that the footprint-level AGBD model is correctly specified and “applies” to  
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19 the population in the cell. This notion implies that the number and order of terms are correctly  
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21 specified, and that the parameter covariance matrix used in the variance calculation (Equations 5  
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23 and 6) adequately reflects uncertainty in the relationship between lidar and AGBD within the  
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25 cell. Since even small model difference can generate strongly divergent population estimates  
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27 when multiplied over large areas, this is a critical assumption. Although the GEDI team is  
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29 collating a comprehensive global database of training data, the reality is that ground data are  
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31 sparse in some areas. In such areas, model misspecification is an important risk. In this respect,  
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33 hybrid inference is no different from any other remote sensing approach that relies upon field  
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35 data to calibrate remotely sensed observations.  
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#### 41 4.2 Performance of hybrid estimators in the context of GEDI's sample

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44 Properties of the hybrid estimators proposed for the GEDI mission were evaluated here using  
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46 simulations in which thousands of potential GEDI cluster patterns were tested in the context of  
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48 model covariance across forests in 60 diverse grid cells. Bearing in mind the above assumptions  
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50 associated with the approach outlined in Figure 1, Figures 4 and 5 suggest that the proposed  
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52 estimator for mean AGBD exhibits negligible bias across cells. Our tests and the results in  
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3 Figures 4 and 5 presumed proper model specification, an assumption of model-based inference,  
4 although residual plots (Figure S4-1) suggested that trends in model residual error may exist in at  
5 least one study region. In such cases, corrective measures such as variable transformations may  
6 be required.  
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14 The variance estimator, on the other hand, appeared to be only asymptotically unbiased (Figure  
15 6); that is, bias approached zero as larger numbers of cluster samples were acquired. The Taylor  
16 linearization used to compute the sample component of the variance (as opposed to the model  
17 component) likely contributed to this underestimation. While Taylor linearization simplifies  
18 calculation of the variance, it is known to lead to variance underestimation for small sample sizes  
19 ((Särndal et al. 1992), p. 176). One area of future work may center around methods proposed to  
20 adjust for under-estimation ((Cochran 1977), p 156). Linearization was also used in deriving the  
21 model component of the variance estimator (Equation 5), but the linear model used in this study  
22 for footprint-level prediction likely eliminated the impact of linearization in the model  
23 component.  
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39 Negative bias in the variance estimates at low numbers of clusters should be considered when  
40 hybrid methods are used with GEDI data, and information about the number of clusters will  
41 accompany GEDI hybrid estimates for each cell. Low sample numbers will occur both at the  
42 beginning of the mission and in equatorial regions where the ISS orbit provides fewer “looks”  
43 and where persistent cloud cover may frequently obscure the surface. An advantage of hybrid  
44 estimation is that the same methodology applied at the 1-km scale may also be applied to much  
45 larger, irregularly shaped areas such as entire countries; instead of having to combine grid cell  
46 estimates, all clusters intersecting a given country may be used in the context of a single hybrid  
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3 estimate. GEDI country-scale sample size will be very large, leading to greatly diminished  
4 design-based contribution to variance estimates. The apparent dominance of the sample  
5 component in the variance estimator (Figure 8) may be relevant to GEDI operational planning.  
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7 There are signal-to-noise parameters that may allow GEDI to consider a larger sample of noisier  
8 footprints (and models) or a smaller sample of footprints collected under ideal atmospheric  
9 conditions. It should be recognized that the relative contribution of the sampling and model  
10 components of the variance estimator may differ in different settings as properties of the model  
11 change due, for example, to the size of  $S_m$  or complexity of the underlying lidar/forest structure  
12 relationship. Across the diverse ecosystems we studied, though, results suggest that including a  
13 larger sample will have a bigger impact on the magnitude and unbiasedness of estimated  
14 variance than a marginal improvement in model fit.  
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## 30 **5. Conclusions**

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33 The GEDI mission was conceived to support estimation of forest biomass at greater accuracy and  
34 resolution than has previously been possible by greatly increasing the number of available  
35 observations of forest structure. The sampling pattern employed by GEDI is largely constrained  
36 by its technology: its deployment platform on the ISS, the number and strength of its lasers, and  
37 its mission duration. Central to the GEDI approach has been the creation of a framework within  
38 these observational constraints for properly estimating both mean AGBD and the variance  
39 around that estimated mean.  
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50 Our research suggests that a hybrid approach that accounts for uncertainty due both to the model  
51 and the sampling design is appropriate and effective. Analysis here focused on 1-km cells, but  
52 hybrid inference using GEDI data could likewise be applied over any political or ecological units  
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3 of interest, subject to minimum size and model applicability assumptions cited above. Our results  
4 confirm the importance of having abundant field plot estimates of biomass and associated  
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6 airborne lidar, from which representative models that relate lidar metrics to AGBD may be  
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8 created. The creation of a global database of such field and lidar data has been a priority of the  
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10 GEDI mission before launch, and its continued expansion should both reduce the model  
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12 uncertainty carried into 1-km variance estimates and support the local applicability assumptions  
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14 that underlie model-based inference.  
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20 Lastly, our research also suggests that maximizing the number of clusters within a cell is key  
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22 towards providing approximately unbiased variance estimates. While observational strategies to  
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24 increase cluster density are limited, options such as purposefully targeting important areas,  
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26 lengthening mission duration, or increasing the size of grid cells to encompass more clusters are  
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28 worth exploring.  
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