Statistical properties of volume and calendar effects in prediction markets

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*Highlights (for review)

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Highlights

- Calendar effects and stylized facts of volumes are analyzed for prediction markets.
- To conduct the analysis, a dataset of daily prices from 3385 markets is used.
- Volume's statistical properties are different than those observed in financial markets.
- Price does not exhibit any significant calendar effect.
- Volume exhibits some calendar effects that are similar to those of financial markets.

Statistical properties of volume and calendar effects in prediction markets

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Abstract

Prediction markets have proven to be an exceptional tool for harnessing the "wisdom of the crowd", consequently making accurate forecasts about future events. Motivated by the lack of quantitative means of validations for models of prediction markets, in this paper we analyze the statistical properties of volume as well as the seasonal regularities (i.e., calendar effects) shown by volume and price. To accomplish this, we use a set of 3385 prediction market time series provided by PredictIt. We find that volume, with the exception of its seasonal regularities, possesses different properties than what is observed in financial markets. Moreover, price does not seem to exhibit any calendar effect. These findings suggest a significant difference between prediction and financial markets, and offer evidence for the need of studying prediction markets in more detail.

Keywords: Prediction markets; Political markets; Stylized facts; Long memory; Power-law behavior

1. Introduction

Prediction markets are effective tools that harness the *wisdom of the crowd* to make accurate forecasts on a number of events (Berg, Nelson, and Rietz, 2008). Although prediction markets are most famous for allowing anyone to bet on political events, often resulting in better predictions on political election outcomes than polls and experts (Wolfers and Zitzewitz, 2006), they are also used in many other contexts, e.g., to forecast business output by companies such as Google, Intel, and General Electrics, to predict the likelihood of natural disasters, or the future value of macroeconomic parameters (Plott and Chen, 2002; Cowgill, Wolfers, and Zitzewitz, 2009). Moreover, due to features such as possessing a definite end-point, prediction markets represent an ideal test bed

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to study decision making under uncertainty. This allows, opposite to financial markets, to observe the outcome of an event, and all uncertainty is resolved at a fixed point-in-time.

However, historical insufficiency of data has limited the number of empirical studies of prediction markets. Notably, there is no comprehensive work on the empirical regularities observed in prediction markets (or *stylized facts*), whereas in financial markets data-driven analysis has always represented a prominent, valuable field of study (Mantegna and Stanley, 2000; Cont, 2001; Abergel et al., 2016). One of the main consequences is that quantitative models of prediction markets lack an important means of validation.

In this paper, we focus on the analysis of daily volumes (measured as the number of shares traded on a given day), and calendar effects, i.e., regularities that occur during a trading period, such as a week, or a year. We find that volume in political prediction markets shares only few of the characteristics typical of stock market time series. Specifically, we find that some volume properties, including calendar effects, seem to be similar to those observed in the stock market, whereas we find no evidence of any price seasonalities.

This paper provides three main contributions to the literature. First, the analysis of empirical regularities that we present in this paper extends the boundaries of the Econophysics literature beyond financial markets and financial economics, which has historically been the focus of the discipline (Jovanovic and Schinckus, 2017; Richmond et al., 2013; Chakraborti et al., 2011; Chakraborti and Toke, 2011), and show that using the Econophysics methods for new types of markets, such as prediction markets, is as promising, and can help in understanding human behavior and decision making under uncertainty. This is, to the best of our knowledge, the first work, together with Restocchi, McGroarty, and Gerding (2018), that uses Econophyiscs to study prediction markets in a systematic way.

Second, this paper provides a significant advance in the study of prediction markets. Although prediction markets and their mechanisms have been investigated in depth for years (Vaughan Williams, 2011; Chen et al., 2018; O'Leary, 2011; Wolfers and Zitzewitz, 2006; Luckner et al., 2011), a comprehensive analysis of their stylized facts has been done only for price changes (Restocchi, McGroarty, and Gerding, 2018). However, volumes and calendar effects are integral parts of prediction markets, and provide both information upon which build prediction market models and a powerful tool to validate them.

Third, differently from price changes, traded volume and calendar effects are a more direct result of people's behavior, and not just an emerging property of a complex system. For this reason, the regularities we find in these paper can give insights on people's decision making under uncertainty.

The paper is organized as follows. In Section 2 we present the data set and explain how prediction markets work. In Section 3, we perform a statistical analysis of volume, and Section 4 depicts our findings on volume and price calendar effects. Finally, in Section 5 we summarize and discuss our results.

Table 1: Summary statistics for the distribution of traded volume

N.Observations	Mean	St.Dev.	Minimum	$q_{25\%}$	$q_{50\%}$	$q_{75\%}$	Maximum
112761	3515.68	18950.04	1	43	30[illegible]	1[illegible]61	1388889

2. Data and Methods

Our data set comprises the daily volumes and the OHLC contract prices of 3385 betting markets on political events, provided by PredictIt[1], for a total of 112761 valid observations (i.e., after removing all days in which there was no trading activity). Contracts on the PredictIt exchange market are Arrow-Debreu securities, i.e., contracts which are priced between 0 and 1 dollars, and whose payoff is either 0 or 1 dollars and solely depends on the the outcome of a future event. For instance, one could buy a contract on either *"Trump will lead"* or *"Clinton will lead"* in the market *"Who will lead in Trump vs. Clinton polling on September 14?"* (or sell a contract on *"Clinton will lead"* or *"Trump will lead"*, respectively). Then, one contract *"Clinton will lead"* pays 1 dollar if Clinton will be leading in Trump vs. Clinton polling on September 14, and 0 otherwise. As a consequence, rational traders are willing to buy a contract on a given outcome only if the current price of such a contract is lower than the probability they attach to the respective outcome to occur.

To perform our analysis, we use this data in two ways. To examine the distribution of daily traded shares, we aggregate volumes across all markets, which allows us to have sufficient observations to reconstruct a significant distribution. Conversely, to examine other properties such as calendar effects, we analyze each market separately and then take both the average and the median results among all markets, which allows us to have a more detailed statistical description of these phenomena.

In the next sections, we present our findings and describe in more detail how the results are obtained.

3. Statistical analysis of traded volume

In this section, we analyze the statistical properties of volume, which is measured as the number of daily traded shares, from the PredictIt data set. Specifically, we examine its distribution, its temporal evolution, and its long-term memory.

3.1. Volume distribution

To analyze the distribution of the number of contracts traded each day for each market, we exclude those days in which no contract has been traded, which leaves 3363 markets and a total of 112761 observations (i.e., trading days with positive volume). The summary statistics of the distribution of volumes (shown

[1] www.predictit.org

in Table 1) indicate that most of the markets examined display a small number of daily trades. Specifically, we find that only in half of the days with trading activity the number of transactions is greater than 306, and only during 25% of the active days 1761 or more contracts are purchased. Also, we observe that the mean is one order of magnitude larger than the median, and the kurtosis and skewness values are high. This may indicate that the distribution of volumes is characterized by heavy tails, i.e., most of the trading activity is concentrated in few trading days. Many probability distributions that characterize natural and

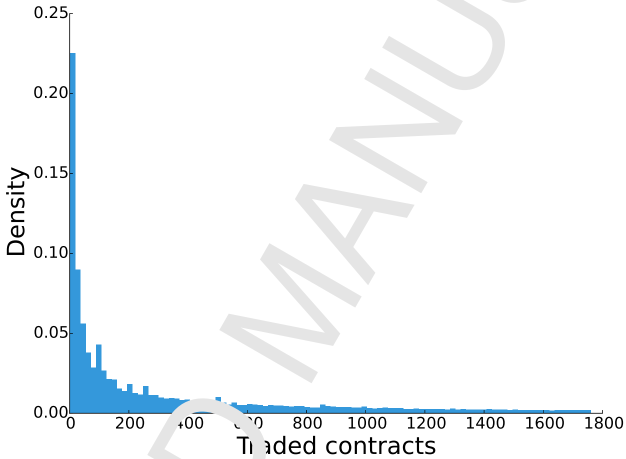


Figure 1: Distribution of the number of daily traded contracts. The distribution is shown only for $v < 1761$, corresponding to the 75% of the observations.

social phenomena display such heavy tails. More specifically, most of these distribution have a power-law like asymptotical behavior Newman (2004); Sornette (2006). In financial markets, the tails of distribution of price changes have been shown to be heavy for most stocks and indexes (Campbell, Lo, and MacKinlay, 1997; Cont, Potters, and Bouchaud, 1997) and, although the exact asymptotic behavior of such tails is still under debate (Schinckus, 2013; Malevergne, Pisarenko, and Sornette, 2005), the power-law decay, given by:

$$p(x) \sim x^{-\alpha} \tag{1}$$

is the most widely used (Gopikrishnan et al., 2000; Plerou et al., 2004) to fit the decay of the tails.

Both the summary statistics and Fig. 3.1 suggest that this might also be the case of our distribution. We check this by fitting the tail of our distribution by following a procedure which enables us to estimate the power-law exponent for discrete data (Bauke, 2007), and relies on a maximum likelihood estimation. Although there is a variety of methods to fit power law distributions to empirical data (e.g., Clauset, Shalizi, and Newman (2009),Ausloos (2014), this procedure, in contrast to other methods such as graphical methods and linear regression, is found to be more robust and reliable (Bauke, 2007; Deluca and Corral, 2013)).

In more detail, this method, which is essential to fit a PDF to a discrete power-law form (Bauke, 2007; Clauset, Shalizi, and Newman, 2009), consists in finding the value α, such that:

$$p(x) = \frac{x^{-\alpha}}{\Delta\zeta} \tag{2}$$

where x represents the daily volumes, and $\Delta\zeta$ is the difference:

$$\Delta\zeta \equiv \zeta(\alpha, x_{min}) - \zeta(\alpha, x_{max}) \tag{3}$$

where ζ is the Hurwitz zeta function, defined as:

$$\zeta(\alpha, x_{min}) = \sum_{i=0}^{\infty} \frac{1}{(i + x_{min})^{\alpha}} \tag{4}$$

Here, x_{min} is the number of traded shares after which the distribution of volume starts behaving like a power law. The theoretical limit of the distribution, i.e., the largest possible value of x, is denoted by x_{max}. However, for volumes, there is no such a constraint. Indeed, in theory, any number of shares can be exchanged during a single trading day. Therefore, we can assume that $x_{max} = \infty$ and, consequently, $\zeta(\alpha, x_{max}) = 0$.

Given this, it is possible to compute the likelihood function for $p(x)$, which is given by

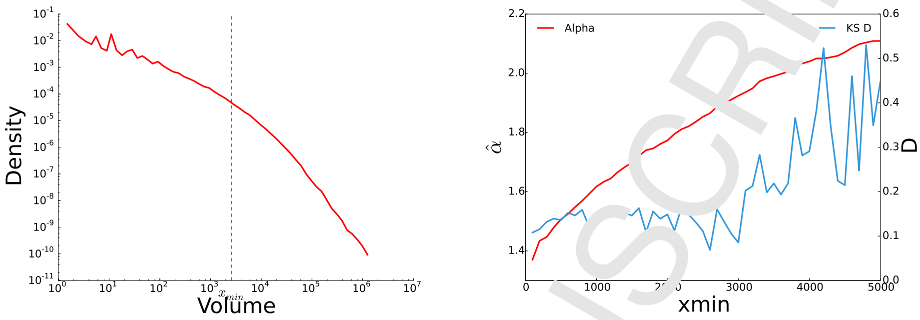
$$L(\alpha) = -\alpha\left(\sum_{i=0}^{N} ln(x_i)\right) - N ln\big(\Delta\zeta\big)\big) \tag{5}$$

Then, the maximum likelihood estimator, $\hat{\alpha}$ is given by:

$$\hat{\alpha} = \underset{\alpha}{\text{argmax}}[L(\alpha)] \tag{6}$$

Since, in this case, there exists no closed-form solution for $\hat{\alpha}$, we find the value that maximizes Eq. (5) numerically.

Finally, the last step required in order to accurately estimate α, is to find the numerical value of x_{min}. To achieve this, we perform a two-sample Kolgomorov-Smirnov test (KS), as suggested by Clauset et al. (Clauset, Shalizi, and Newman, 2009). The procedure they introduce is as follows: first, we fix the value of x_{min}, starting from the smallest possible, and remove from our data all values of x such that $x < x_{min}$, if any. Second, we fit a power-law distribution to these values, and find $\hat{\alpha}$. Third, we perform the KS test between our data and a sample drawn from a power law distribution with exponent $\hat{\alpha}$, hence computing the KS statistic (D). Finally, we increase by the smallest possible increment the value of x_{min}, and we repeat the procedure until all possible values of x_{min} have been considered.



(a) (b)

Figure 2: Figure (a) displays the PDF of volumes in a logarithmic scale. Figure (b) shows the KS statistic and the corresponding values of $\hat{\alpha}$ for different x_{min}

Then, we choose the x_{min} that minimizes the value of D, and take the corresponding $\hat{\alpha}$ as the power-law exponent for our distribution. By following this procedure, we find that the distribution of traded shares follows a power-law with exponent $\hat{\alpha} = 1.865 \pm 0.002$ for values greater than 2600, corresponding to the 20% of the total observations. This value is not distant from the power-law exponent $\gamma_q = 1.53 \pm 0.07$ estimated for financial markets (Gopikrishnan et al., 2000; Gabaix et al., 2007), from which we can conclude that, although in prediction markets volumes are lower than in the stock market, the decay of the number of traded shares is similar.

3.2. Autocorrelation of volumes

Next, we examine the long-memory properties of volumes. To achieve this, compute the autocorrelation function of the number of traded shares, and fit it to a power-law distribution. To obtain an accurate estimation, we computed the autocorrelation function for lags in the range $1 < \tau < 100$, i.e., we used all markets longer than 100 days, for a total of 236 markets. We find that the volume autocorrelation function can be described as:

$$\langle V(t),\ V(t+\tau)\rangle \sim \tau^{-\lambda} \tag{7}$$

where we estimate the exponent to be $\lambda = 0.094 \pm 0.003$ (see Fig. 3). This result suggests that trading activity behaves in the same way in both prediction and stock markets, in which the power-law exponent is observed to be of the same order of magnitude. More specifically, its value is estimated to be $\lambda = 0.30$ for US stocks (Plerou et al., 2001), and $\lambda = 0.21$ for the Chinese stock market (Qiu et al., 2009), which also suggests that the decay of the volume autocorrelation function is faster the more liquid the market is.

3.3. Temporal evolution of traded volume

An interesting aspect of prediction markets time series (and, more generally, state-contingent claims) is that, in contrast to those of the stock market, they

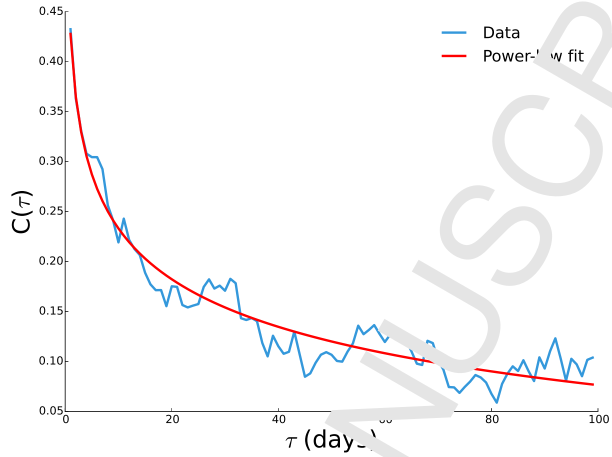


Figure 3: Autocorrelation function of traded volume and the fitted power law with exponent $\lambda = 0.094$.

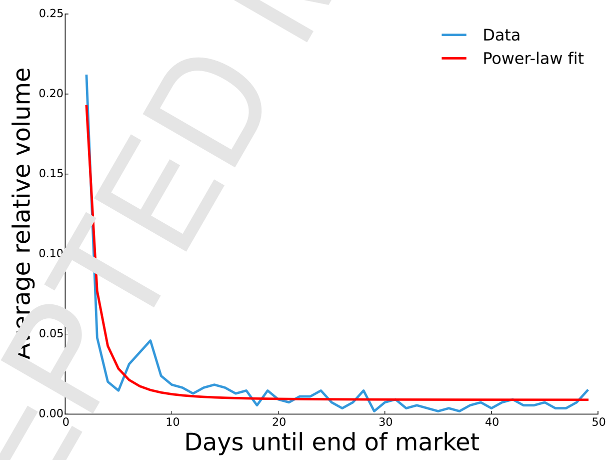


Figure 4: Relative volume depending on the number of days τ until the end of the market.

have a fixed end-point. In this section, we examine this aspect of prediction markets, i.e., the temporal distribution of volume, and find that, towards the end of the market, the average daily volume grows significantly. Specifically, the number of traded shares depends on the number of remaining days τ until the end of the market, and, as shown in Fig. 3.3, this relation follows power-law decay:

$$V(T-\tau) \sim \tau^{-\zeta} \tag{8}$$

where T denotes the final day of the market. We fit this function with a power law, and we estimate the exponent to be $\zeta = 2.44 \pm 0.06$, which suggests that the during the last days of trading, volumes are higher than during all the rest of trading days combined. This result can be explained in several ways. For example, those who invest in prediction markets, may be waiting for a lower uncertainty on the outcome (i.e., waiting for new information to be revealed), or they simply have a higher utility to bet in the days right before the end of the market, hence reducing the time between the investment and the (potential) gain. Either way, we believe this is a crucial result for building realistic models of prediction markets, because this phenomenon may generate non-trivial price dynamics during the last days of trading.

3.4. Volume-volatility correlation

In this section we examine the correlation between volume and price in prediction markets. In the stock market, it has been observed in a number of contexts that volume changes and the volatility of returns are correlated (Chordia, Roll, and Subrahmanyam, 2001; Podobnik et al., 2009). For instance, it is shown that volatility grows disproportionally to the total number of trades in a market (Podobnik et al., 2009). Unfortunately, for the prediction markets, we do not possess order-level data, and hence we show that volume and volatility are correlated on a daily time scale. That is, we compute the correlation coefficient

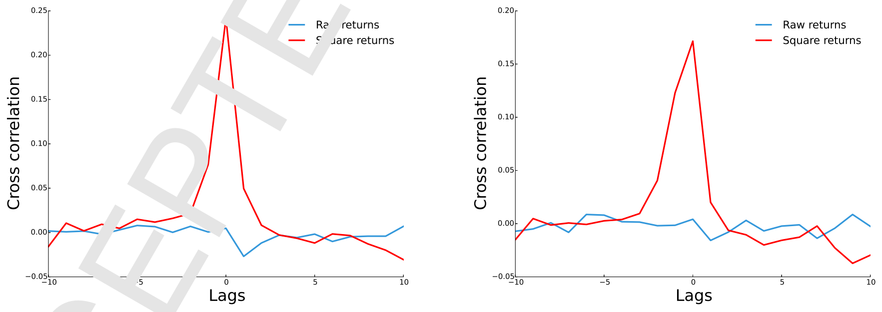


Figure 5: Figure (a) shows the cross-correlation between traded volume and both volatility (square returns) and price changes (raw returns). Figure (b) shows the cross-correlation between changes in traded volume and both volatility (square returns) and price changes (raw returns)

$$C(\tau)^{sq} = \langle r_t^2,\ v(t+\tau)\rangle \tag{9}$$

and find that correlation is significant only for $\tau = 0$. Fig. 3.4 shows the cross correlation function between traded volume and volatility and also between traded volume and raw returns, defined as:

$$C(\tau) = \langle r_t,\ v(t+\tau) \rangle \tag{10}$$

for which the correlation coefficient is insignificant at all lags τ. This implies that volume is only correlated with volatility (at lag 0) but not with price changes, which is a well known fact in financial markets (Podobnik et al., 2009). Interestingly, we find similar results when computing the cross correlation between returns and volume changes (Fig. 3.4). This is in contrast to what is found in the stock market, for which it has been observed that the correlation between volume changes and volatility decays with a power law (Podobnik et al., 2009). Conversely, in our data set we find that volatility is correlated with volume changes only at lag 0.

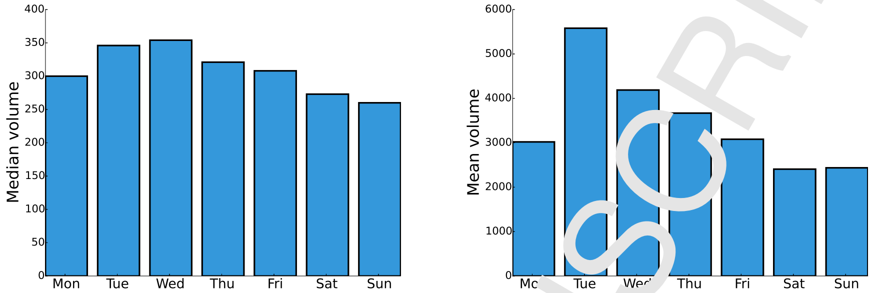
4. Calendar Effects

Calendar effects, or *seasonalities*, are cyclical regularities that occur throughout a trading period, be it a year, a week or a day, and have been observed in both returns and volume by a number of authors who examined international stock markets (Sewell, 2011). In this section we examine some well-known effects that are present in financial markets (Dzhabarov and Ziemba, 2010), and we find that only some of them can be observed in prediction markets. Specifically, we first describe cyclical regularities exhibited by trading activity and then focus on price changes, for which we examine the Weekend and the January effects in detail.

4.1. Trading activity calendar effects

There is evidence that, in financial markets, trading activity significantly varies depending on the time of the day and the day of the week. The first comprehensive study of volume calendar effects (Jain and Joh, 1988) examines several years of NYSE-listed stock data and find that liquidity is lowest on Monday, peaks on Wednesday, and drops until Friday. A similar, more recent study (Chordia, Roll, and Subrahmanyam, 2001), which analyzes U.S. stocks between 1988 and 1998, find that the volume peak has shifted to Tuesdays, whereas Fridays have become the days with the lowest liquidity. In this section we analyze trading activity in our data, and find that it significantly varies across days of the week and across months of the year. Although this behavior is similar to that of the U.S. stock market, this is a non-trivial result, since prediction markets possess two main differences compared with stock markets. Specifically, in prediction markets, it is possible to trade during weekends. Also, since liquidity in prediction markets is much lower than in financial markets, we find that the average number of traded shares is significantly affected by those markets in which volumes are largest. Specifically, to overcome this issue, we present our results using both the average and the median volumes.

Despite these differences, we find that most of our results are comparable with those of U.S. stocks. In fact, we conclude that, in our data, trading activity



(a) (b)

Figure 6: Figure (a) and Figure (b) display the median and the mean, respectively, number of contracts traded by day of the week.

Table 2: This table displays summary statistics of the trading activity (expressed as the number of contracts traded) across the days of the week. The t statistic is used to either accept or reject the null hypothesis that the mean volume value of a given day of the week is the same as the mean value for the other days.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Mean	3019.45	5580.16	[illegible]	3667.96	3079.47	2406.12	2436.10
St. Dev.	10550.43	36248.83	[illegible]	19204.85	11540.58	10730.70	8266.70
Median	300	346	354	321	308	273	260
t-stat.	-5.50*	8.55*	5.09*	1.11**	-4.62*	-11.85*	-13.34*

* corresponds to a significance level of 0.01%.
** indicates that the result is not significant.

is lowest during weekends, but otherwise shows a trend similar to that found in the U.S. stock market (see Fig. 6). Table 2 shows that the average volume is low on Mondays, peaks on Tuesdays, and then decreases gradually for the rest of the week, and it reaches its lowest value during weekends, which agrees with the analysis by Chordia, Roll, and Subrahmanyam (2001). The analysis of the median number of traded shares (Fig. 6, and Table 2) shows a similar pattern, although the volume differences across the days of the week become less pronounced compared to the average value, and the number of traded contracts has a high on Wednesdays instead. We repeat the analysis for the months of the year, and we find that, although the differences between mean and median are more pronounced than in the weekly analysis, both measures show similar trends (see Fig. 7). First, January and December are the months with the least trades in both cases. Second, both the mean and the median volumes increase from January to Spring (April and March for the median and the mean value, respectively), then have a local low in August, and then a new high in Autumn (October for the median volume, November for the mean volume). These findings suggest that, despite the structural differences, volume temporal regularities in prediction markets are similar to those found in stock markets.

However, there is an important difference in the implications that volume

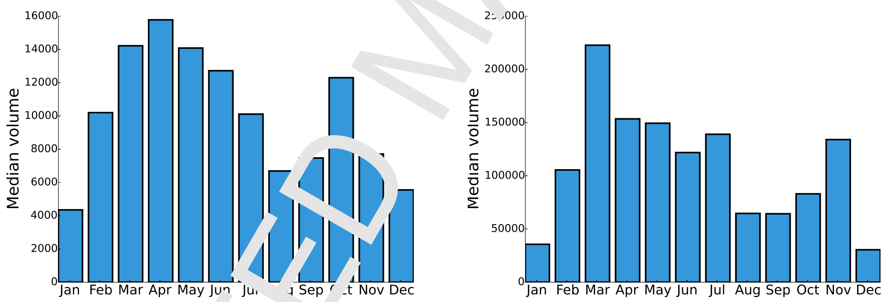
Table 3: This table displays summary statistics of the trading activity (expressed as the number of contracts traded) across the months of the year. The t statistic is used to either accept or reject the null hypothesis that the mean volume value of a given day of the week is the same as the mean value for the other days.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	35621.59	105487.06	222737.46	153514.76	149391.7	121854.35	139041.75	64646.92	[illegible]4314.74	83033.[illegible]	134040.53	30493.27
St. Dev.	18903.99	66802.39	156852.56	140245.76	183499.35	112634.55	118178.35	42586.1[illegible]	45412.31	[illegible]532.94	151787.61	13093.59
Median	4350.0	10200.0	14220.0	15780.0	14085.0	12720.0	10110.0	6690.0	7470.0	[illegible]300.0	7710.0	5550.0
t-stat.	-35*	0.02**	12.39*	5.83*	4.46*	2.79*	5.65*	-15.35*	-15.37*	[illegible].63*	3.2*	-40.36

* corresponds to a significance level of 0.01

** indicates that the result is not significant

seasonalities have on these two types of markets, which arises from the fact that prediction markets have a significantly lower liquidity compared to financial markets. In fact, although low liquidity does not necessarily imply lower market efficiency, it leaves price open to possible manipulations by malicious parties, which are not necessarily pecuniary, but may be simply introduced to bias public opinion about the realization of a particular event (Goodell, McGroarty, and Urquhart, 2015).



(a) (b)

Figure 7: Figure (a) and Figure (b) display the median and the mean number of contracts traded by month of the year, respectively.

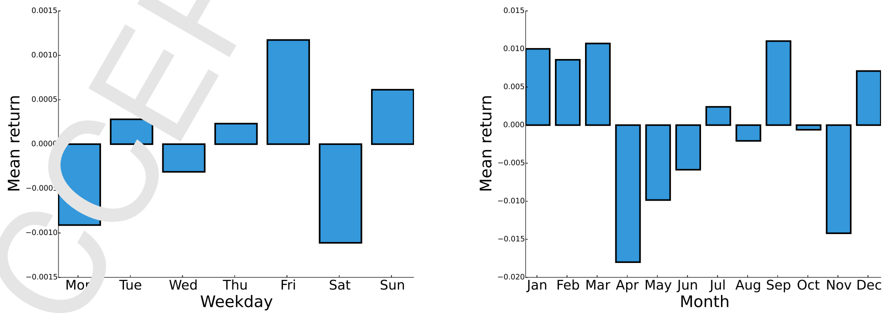
4.2. Price calendar effects

In this section, we examine price changes across days of the week and months of the year. We first introduce these regularities, also presenting the results found in financial markets, and then show that these two patterns are not exhibited by our data. Indeed, we find that, opposite to volume, price in prediction markets does not follow the same behavior as in the stock market and, more generally, does not seem to exhibit any regularity. Conversely, in numerous stock markets, it has been observed that prices display more calendar regularities than volume, and the study of this topic has generated a large body of literature (Thaler, 1992; Constantinides, Harris, and Stulz, 2003). After their discovery, many of these anomalies have reduced or even disappeared (Mclean and Pontiff, 2016), but some of the most important calendar effects, among which the *January effect* and the *Weekend effect* are the most documented (Sewell, 2011), are still present in many stock markets (Dzhabarov and Ziemba, 2010).

4.3. The Weekend and the January effects

The weekend effect (sometimes referred to as *Monday effect*) is an empirical regularity by which average returns on Mondays are significantly lower than those of the rest of the week, and is often regarded as the strongest of calendar effects (Rubinstein, 2001). This anomaly was firstly observed in the 1930s (Fields, 1931), but the first comprehensive discussion was provided by Kenneth French (French, 1980), who analyzed more than twenty years of stock returns in the U.S. market to test two hypotheses. The first, called calendar time hypothesis, states that the expected returns on Mondays should be three times those for the other days of the week, since the risk accumulated during weekends should be reflected in Monday's returns. The second, named trading time hypothesis, states that, if only trading time matters to generate returns, there should be no distinction between Mondays and other days. However, French found that neither of these hypotheses were true. In fact, he found that, on average, Mondays display lower returns than all of other days of the week and, more specifically, Monday is the only day of the week during which average returns are negative.

Lakonishok and Maberly (Lakonishok and Maberly, 1990) provide an explanation of the weekend effect based on the analysis of trading patterns of individual and institutional investors. First, they find that, on Mondays individual investors tend to trade more compared with the rest of the week, and also that the number of sell transactions relative to buy transactions increase significantly. Second, they observe that, in their data, the traded volume by institutional investors was the lowest on Mondays. They claim that these two regularities combined provide a partial explanation for the weekend effect.



(a) (b)

Figure 8: Figure (a) and Figure (b) display the mean return across days of the week and months of the year, respectively.

Table 4: This table displays summary statistics of the returns for each day of the week. The t statistic is used to either accept or reject the null hypothesis that the mean return of a given day of the week is the same as the mean return for the other days.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Mean	-0.0009	0.0003	-0.0003	0.0002	0.0012	-0.0011	0.0006
St. Dev.	0.11	0.11	0.11	0.11	0.14	0.08	0.07
Median	0	0	0	0	0	0	0
t-stat.	-1.11*	0.34*	-0.41*	0.3*	1.21*	-1.75*	1.1*

* indicates that the result is not significant.

The January effect is another important calendar regularity, whereby returns on January are significantly higher than in other months. It has been first observed in the U.S. and Australia stock markets (Wachtel, 1942; Praetz, 1972; Officer, 1975; Rozeff and Kinney, 1976), and in several international stock markets afterwards (Gultekin and Gultekin, 1983; Agrawal and Tandon, 1994). Similarly to the weekend effect, the January effect has proven to be a regularity whose causes are puzzling (Haugen and Lakonishok, 1988). There are many competing explanation attempts, but most of these theories revolve around small firms. Indeed, there is evidence that this phenomenon is related with the capitalization of firms, and then that it is likely to be a consequence of a small-firm effect (Reinganum, 1983), low share prices (Bhardwaj and Brooks, 1992), or tax-motivated trading (Sias and Starks, 1997; Poterba and Weisbenner, 2001).

4.4. Analysis of returns

In this section we examine the seasonality of returns, to find whether the Weekend and the January effects exist in prediction markets. To achieve this, we follow the same procedure employed to analyze calendar effects on volume, and take into account both the mean and the median return. However, in contrast to traded volume, returns do not seem to possess any significant differences across days of the week (see Fig. 8). Mean daily returns, as it is shown in Table 4, lie between -0.001 and 0.001 for all days of the week, i.e., they are one order of magnitude smaller than the minimum possible raw return $|r_t| = 0.01$, and these small differences disappear completely when considering the median returns. Accordingly, we find that all the p-values from the t-test are greater than 0.7, and hence the null hypothesis that average returns are the same across the days of the week cannot be rejected. Similarly, we find that monthly returns do not display any significant difference (see Table 5). These findings are consistent with the hypothesis that the January effect is due to smaller-capitalization stocks and tax loss selling (Roll, 1983). Indeed, in prediction markets, there is no equivalent of capitalization since contract prices purely reflect the likelihood of a given event to occur as perceived by market participants. Also, losses from these markets do not impact on fiscal contribution, since prediction markets fall under the gambling legislation in most countries and, importantly, volumes are too low to affect fiscal contribution whatsoever.

Table 5: This table displays summary statistics of the returns for each month of the year. The t statistic is used to either accept or reject the null hypothesis that the mean return of a given month of the year is the same as the mean return for the other months.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	0.01	0.0086	0.0107	-0.018	-0.0098	-0.0059	0.0024	-0.0021	0.011	-0.0006	-0.0142	0.0071
St. Dev.	0.498	0.579	0.699	0.545	0.563	0.579	0.593	0.614	0.558	0.582	0.655	0.549
Median	0	0	0	0	0	0	0	0	0	0	0	0
t-stat.	0.34*	0.25*	0.25*	-0.55*	-0.32*	-0.2*	0.08*	-0.07*	0.4*	-0.03*	-0.37*	0.21*

* indicates that the result is not significant

5. Conclusions

We analyzed calendar effects and several statistical properties of volumes in prediction markets, by using a data set comprising 3385 time series of security prices and trading volumes on political events. First, we find that volume seasonalities are similar to those found in financial markets. Given the fact that prediction markets possess a structure which is significantly different from that of financial markets, and far lower liquidity, these results suggest that some market properties, such as volume calendar effects, could be exogenous to the markets themselves, and are not an emerging property of a complex system (in which traders are interacting). Rather, they seem to be regularities that belong to the sphere of investors' decision making under uncertainty, regardless how much money they are trading, or what the investment time horizons are. Second, our results show that price seasonalities, as well as volume regularities, are different from those observed in financial markets. Although the different mechanisms of prediction markets, and in particular their limited time horizons, make the few differences we observed in the properties of traded volume somewhat expected, the absence of price seasonalities, compared with those of financial markets (and volume seasonalities in both financial and prediction markets) suggest that price calendar effect may be an emerging phenomenon caused by the interaction of traders, rather than an effect produced by exogenous causes such as volume seasonalities. This difference has two interesting implications: First, it suggests that the two processes are different in nature, and are worth of more investigation to better understand the decision making reasoning behind them. Second, it implies that volume calendar effects could be used directly as a feature to model prediction markets, rather than to validate them.

Overall, our results suggest that studying prediction markets could provide additional insights on people's individual and collective behavior when trading under uncertainty, and we advocate the use of our results to build and validate new models of prediction markets.

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