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inlabru: an R package for Bayesian spatial modelling from ecological survey data

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1 Summary

1. Spatial processes are central to many ecological processes, but fitting models that incorporate spatial correlation to data from ecological surveys is computationally challenging. This is particularly true of point pattern data (in which the primary data are the locations at which target species are found), but also true of gridded data, and of georeferenced samples from continuous spatial fields.
2. We describe here the R package `inlabru` that builds on the widely-used R-INLA package to provide easier access to Bayesian inference from spatial point process, spatial count, gridded, and georeferenced data, using integrated nested Laplace approximation (INLA, Rue *et al.*, 2009).
3. The package provides methods for fitting spatial density surfaces and estimating abundance, as well as for plotting and prediction. It accommodates data that are points, counts, georeferenced samples, or distance sampling data.
4. This paper describes the main features of the package, illustrated by fitting models to the gorilla nest data contained in the package `spatstat` (Baddeley & Turner, 2005), a line transect survey data set contained in the package `dsm` (Miller *et al.*, 2018), and to georeferenced sample from a simulated continuous spatial field.

Keywords: Spatial modeling, point process, spatial count, georeferenced data, Bayesian inference

2 Introduction

Many ecological datasets exhibit spatial correlation in observed variables, due to biotic or abiotic processes such as dispersal limitation, social aggregation, and spatial structure in unobserved explanatory variables. Whether the observations are points (e.g. animal locations), counts (e.g. the numbers of animals in spatial samples) or values of some continuous variable (e.g. nutrient levels at sampled points), spatial correlation causes every observation to depend on every other observation within some unknown correlation range. Dealing with this requires models that are mathematically more complex and computationally more demanding than is the case when there is independence among observations.

We account for spatial dependence by incorporating a Gaussian random field (GRF) into models. GRFs are spatially continuous random processes in which random variables at any point in space are normally distributed and are correlated with random variables at other points in space according to a continuous correlation process. GRFs provide a means of modelling the spatial signal in the observations that cannot be accounted for by covariates.

In the case of point data and count data, the GRF is linked to the response variable by a log link function, to give a log Gaussian Cox process (LGCP) model (Møller & Waagepetersen, 2007). (Called “log Gaussian” because the log of the intensity at any point is assumed to be normally distributed, and “Cox process” because this is a Poisson process that has a randomly varying intensity function.) What spatial statisticians call the “intensity” is the density in our context, and we will use the term “density” for this henceforth.

The GRF is approximated by the solution to a stochastic partial differential equation (SPDE; see Lindgren *et al.*, 2011, for details). We do not have space to describe the details of SPDEs, but fortunately the mathematical details need not be understood to use them in `inlabru`. It is sufficient to know that SPDEs provide an efficient way of approximating the GRF in continuous space (Simpson *et al.*, 2016).

Integrated nested Laplace approximation (INLA) Bayesian methods (Rue *et al.*, 2009) are used for inference. INLA is a fast and accurate alternative to Markov chain Monte Carlo (MCMC) for fitting latent Gaussian models, i.e., hierarchical models in which there are unobserved (latent) normally distributed random variables. The models we consider here, in which the GRF is latent, are of this type. We refer the reader to the “Gentle INLA tutorial” at <https://www.precision-analytics.ca/blog-1/inla> for more about INLA, and to the R-INLA project at <http://www.r-inla.org/> for more about the R-INLA package on which the `inlabru` package builds.

The R-INLA package currently requires users to have knowledge of likelihood approximation schemes, and does not allow inference when detection probability is unknown, as is common in many wildlife surveys. The `inlabru` package makes fitting spatial models with INLA more accessible to non-specialist users by employing simpler syntax, and it extends the class of models that can be fitted to include distance sampling.

We illustrate the scope of the package by fitting models to point and count data from a survey of gorilla (*Gorilla gorilla*) nests by Funwi-Gabga (2008), a line transect survey of pantropical spotted dolphins (*Stenella*

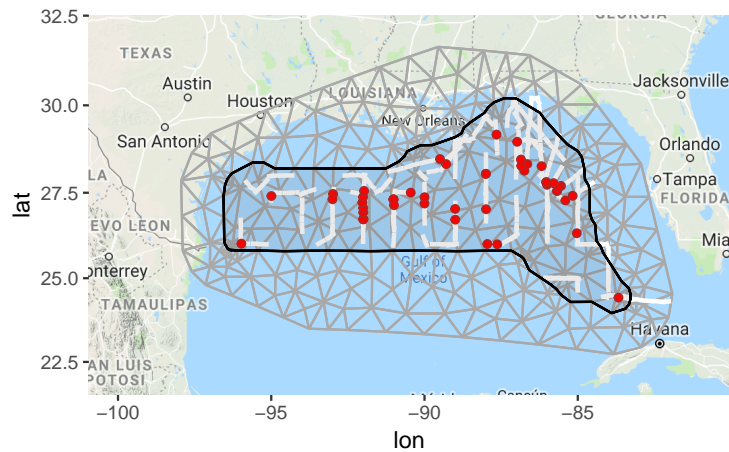


Figure 1: Pantropical dolphin survey data plotted using `ggmap` and the `gm` method. Grey triangles show the `inla.mesh` object. The survey region boundary (black) is held in a `SpatialPolygonsDataFrame`. The line transects (white lines) are held in a `SpatialLinesDataFrame` and the detected dolphins (red points) are held in a `SpatialPointsDataFrame`.

61 *attenuata*)¹, and a simulated survey of a continuous spatial field. Other examples can be found at <http://inlabru.org/tutorials>.

63 3 Data format and visualization

64 The `inlabru` package supports the `sp` package data structures (Pebesma & Bivand, 2005). These are well
 65 documented within `sp`, together with powerful functions for manipulating them. The `SpatialPointsDataFrame`
 66 structure stores spatial points together with spatial covariate data and attributes of points (e.g. size or species).
 67 `SpatialLinesDataFrames` store spatial data for line transect surveys and `SpatialPolygonsDataFrames` are
 68 used to define survey regions and sample plots.

69 Continuous space is approximated in `inlabru` using a “mesh” (a tiling of space with triangular tiles – see
 70 Figure 1 for example). We use the `inla.mesh` class of object from the INLA package for this approximation.

71 Data visualization tools in `inlabru` are built on the `ggplot2` (Wickham, 2009) and `ggmap` (Kahle & Wickham,
 72 2013) packages, with customized `inlabru` functions such as `gg` and `gm` to extend their functionality. Figure 1
 73 shows an example of such a plot generated from a line transect survey of pantropical spotted dolphins in the
 74 Gulf of Mexico.

75 4 Key syntax

76 Models are defined by specifying

- 77 1. a `formula` for the linear or nonlinear predictor that defines the log density function,
- 78 2. the components of this predictor (one of which is typically an SPDE), and
- 79 3. the observed variable distribution.

¹see <http://seamap.env.duke.edu/dataset/25>) for details of this survey

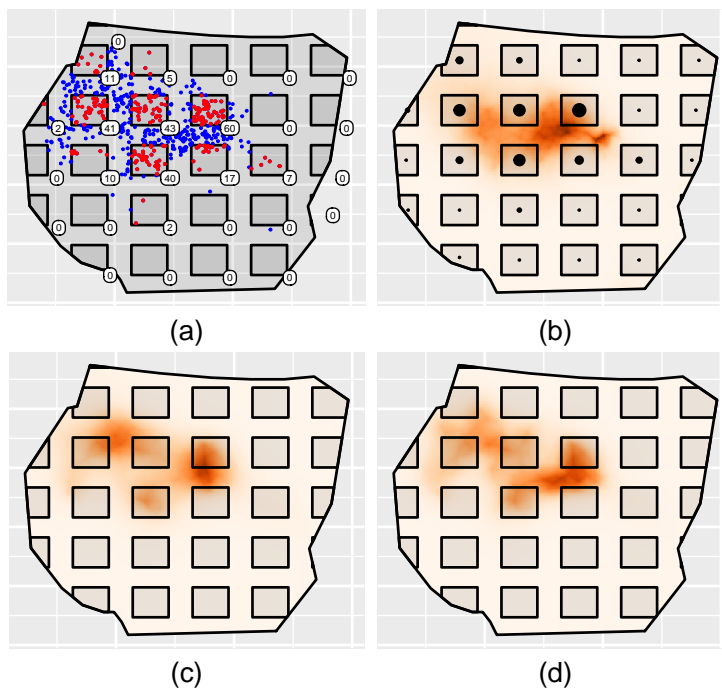


Figure 2: Analysis of gorilla nests as a count and as a point process model. Panel (a) depicts the survey region, search plots, undetected (blue) and detected (red) nests, including the nest counts (white boxes). Panel (b) shows a density fit with `bru` to nest counts, associating counts with the plot centres. Panels (c) and (d) show point process fits obtained with `lgcp` using nests within the plots, and all, respectively.

80 Models are fitted using the function `bru()` or, for LGCP models, `lgcp()`. Examples are given below.

81 5 Spatial count data

82 We begin by using `inlabru` to infer a smooth spatial density surface from plot samples in which the response is
 83 the count of gorilla nests in each plot (see Figure 2 (a)). Although the exact locations of all nests were recorded,
 84 we initially use only nest counts in a sample of plots. The R code showing how to load the package and the
 85 data is provided as Supporting Information S1.

86 The observed response, `count`, is the number of nests in a plot, which we assume to be a Poisson random
 87 variable. We also assume that the log density of the Poisson distribution varies in space and is the sum of an
 88 intercept term (the base log density) and an SPDE (which captures the spatially correlated variation about the
 89 base). We name the SPDE `spat2`. Recall that the SPDE approximates a GRF, and we specify below that the
 90 correlation of this field has a Matérn correlation structure. This correlation model (with unknown parameters)
 91 is specified using the INLA function `inla.spde2.matern`. The SPDE and correlation model are defined on a
 92 `mesh`, which we do not show here because it obscures important elements of the plots (see Figure 1 for an
 93 example of a mesh).

94 The two components of our linear predictor are the intercept and the SPDE. We store these in an object
 95 called `cmp` as follows:

²This name can be chosen by the user.

```
cmp <- count ~ spat(map = coordinates, model = inla.spde2.matern(mesh)) + Intercept
```

96 The syntax for defining SPDEs requires a name for the SPDE (“`spat`” here), followed by specification,
 97 in brackets, of the domain on which it is defined (“`map=coordinates`” here), and its correlation function
 98 (“`model=inla.spde2.matern(mesh)`” here). Note that `coordinates` is a method defined by the package `sp`
 99 to extract locations from `sp` spatial objects. Using it as above specifies that the SPDE applies to spatial
 100 coordinates.

101 We use the `inlabru` function `bru` to fit the model to the gorilla count data `gcounts` (a `SpatialPointsDataFrame`
 102 with a data field `count` containing the nest count data):

```
fit <- bru(components = cmp,
           family = "poisson",
           data = gcounts,
           formula = ~ spat + Intercept,
           options = list(E = gcounts$exposure))
```

103 The `components` parameter specifies the model components. The `family` parameter specifies the probability
 104 density function (PDF) of the response. (All `family` types supported by the `INLA` package are supported by
 105 `inlabru`.) The `formula` specifies how the components are combined to create a linear (in this case) predictor for
 106 density. The parameter `E` in the `options` list sets the “exposure” parameter of the Poisson family, namely the
 107 areas of each searched plot in this example. (The log of the exposure would be an offset in a Poisson generalised
 108 linear model.)

109 We did not need to specify the `formula` above, because `inlabru` assumes that it is the sum of the components
 110 if no `formula` is given. The `formula` is really only required when it is not this sum (see examples in Sections 6.2
 111 and 6.3 below).

112 We can predict any function of any subset of the components of the model specification (`cmp` above) using
 113 `inlabru`’s `predict` function. For example, predictions of the density are obtained as follows:

```
pxl <- pixels(mesh, mask = boundary)
dens <- predict(fit, pxl, formula = ~ exp(spat + Intercept))
```

114 The first line creates a regular grid of locations covering the survey region. The third argument of the
 115 `predict` call specifies what is to be predicted, as a function of the `components`. To predict on the scale of
 116 the linear predictor, for example, we would just replace `exp(spat+Intercept)` with `spat+Intercept`. The
 117 `predict` function estimates the posterior densities of whatever function is specified in its `formula` argument.

118 The object obtained from `predict` is a `SpatialPixelsDataFrame`. As with any other spatial object, we can
 119 employ the `gg` function to add it to a blank plot. Hence, calling `ggplot() + gg(dens)` will render the density
 120 shown in Figure 2 (b).

121 6 Fitting point processes

122 We now consider the case in which the data are the *locations* of nests within plots. Some information about
 123 the spatial process governing nest locations is lost when locations are aggregated into counts within plots, and
 124 we would like to use all the information in the data. In this case, the response variables are the coordinates
 125 of the individual nests, and the locations are random variables, whereas with count data the locations of the
 126 plots were fixed and known and the counts were random variables. Spatial point processes models (Møller &
 127 Waagepetersen, 2007; Illian *et al.*, 2008; Baddeley *et al.*, 2015) are used when the points themselves are the
 128 random variables. More specifically, we use an LGCP, in which the log density includes a GRF, to model
 129 overdispersion and clustering that cannot be accounted for by covariates.

130 6.1 Inference for spatial Poisson point processes

131 The work flow of inference in point processes fitting is similar to that described above. We specify the model
 132 by replacing the user-defined response “count” on the left of the component specification, with the key word
 133 “coordinates” to indicate that the responses are spatial coordinates.

```
134 cmp <- coordinates ~ spat(map = coordinates, model = inla.spde2.matern(mesh)) + Intercept
```

134 The R code showing how to load the data is provided in Supporting Information S1. Fitting an LGCP
 135 model is done using `lgcp`:

```
136 fit <- lgcp(components = cmp, data = plotnests, samplers = plots)
```

136 Here `plotnests` is a `SpatialPointsDataFrame` containing the locations of the observed nests. The `samplers`
 137 argument is passed a `SpatialPolygonsDataFrame` called `plots` that specifies the polygons that were searched.
 138 If this argument is left empty, `lgcp` will assume that the whole domain defined by the mesh (contained in the
 139 SPDE specification, `spat`, in `cmp`) was searched, which would result in biased inference if the whole domain was
 140 not searched.

141 Running the code above and then using `predict` and `plot` yields the density plot shown in Figure 2 (c). For
 142 comparison, Figure 2 (d) shows a LGCP fit to the complete gorilla nest data set, which was obtained as above
 143 but with `samplers=boundary` in place of `samplers=plots`, where `boundary` is a `SpatialPolygonsDataFrame`
 144 object defining the survey boundary.

145 6.2 Inference for univariate point processes: distance sampling detection function

146 We illustrate `inlabru`’s ability to model one-dimensional point processes by fitting a detection function to the
 147 perpendicular distances of detected dolphins on the line transect survey shown in Figure 1. The R code showing
 148 how to load and prepare the data is provided as Supporting Information S2.

149 The observed density of distances to detections is the product of the underlying density of distances to dol-
 150 phins ($\lambda(d)$ say, where d is distance) and the probability of detecting a dolphin that is at distance d ($h(d; \log\{\sigma\})$)

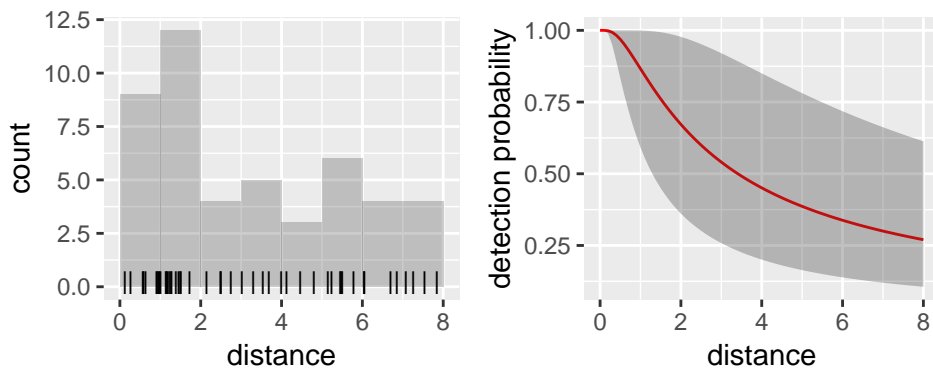


Figure 3: Pantropical dolphin detection distances (left) and fitted hazard rate detection function (right), showing 95% credible region. With adequate fit, the red line is a smooth through the histogram, as is apparent here.

say, where $\log\{\sigma\}$ is an unknown parameter). Under the usual line transect assumption that animals are uniformly distributed with respect to distance from the line, $\lambda(d) = \lambda$ so that the density of the observed distance process is $h(d; \log\{\sigma\})\lambda$. Hence the log density can be written as $\log[h(d; \log\{\sigma\})] + \beta_0$, where $\lambda = e^{\beta_0}$.

We specify the (nonlinear) predictor for this model, and its components, as follows:

```
fml <- distance ~ log(h(distance, lsig)) + Intercept
cmp <- distance ~ lsig + Intercept
```

where $h(\text{distance}, \text{lsig})$ is $h(d, \log\{\sigma\})$ and `Intercept` is $\beta_0 = \log(\lambda)$. To complete the specification we need to define the function $h(\text{distance}, \text{lsig})$. We define it to be the hazard-rate detection function of Hayes & Buckland (1983), with shape parameter 1, as follows:

```
h <- function(distance, lsig){ 1-exp(-(distance/(exp(lsig)))^-1)}.
```

Because one of the components (the parameter `lsig`) enters the linear predictor for log density via a nonlinear function, $\log[h(d; \log\{\sigma\})]$, we need to specify the `formula` explicitly, rather than have `inlabru` construct it by default as the sum of the components. This model is fitted using `lgcp` as follows:

```
fit <- lgcp(cmp, mexdolphins$points, formula = fml).
```

where `mexdolphins$points` is a `SpatialPointsDataFrame` with a variable `distance` for every point.

After fitting the model, predicting the detection function for distances 0 to 8 (the maximum distance considered) is straightforward using

```
pts <- data.frame(distance = seq(0,8, by = 0.1)),
dfun <- predict(fit, pts, formula = ~ h(distance, lsig))
```

while `plot(dfun)` plots it with 95% credible interval (as shown in Figure 3).

We note in passing that `inlabru` can be used to estimate *any* PDF using commands similar to those above, if we consider the intensity of a Poisson process to be an unnormalized PDF.

167 6.3 Inference for thinned Poisson processes: distance sampling

168 We now use `inlabru` to estimate the density and distribution of dolphin groups with the conventional distance
 169 sampling assumption of uniform group distribution within searched strips. This assumption is tenable because
 170 the searched strips have negligible width compared to the size of the survey region (see Figure 1) and were
 171 laid down with random start location. We implement the assumption by simultaneously modelling the spatial
 172 distribution of detected points (as in Section 6.1) and the PDF of distances of detections from the lines, assuming
 173 uniform distribution of these distances (as in Section 6.2). The R code for this is provided as Supporting
 174 Information S4.

175 An analysis of these data (also assuming uniform group distribution within searched strips) using the R pack-
 176 age `dsm` is available at <http://distancesampling.org/R/vignettes/mexico-analysis.html>. The methods
 177 implemented in `inlabru` and `dsm` differ in a number of ways, including that `inlabru` implements a fully-Bayesian
 178 approach, so one can specify priors on parameters (not illustrated here), and `inlabru` estimates detection prob-
 179 ability and the density surface simultaneously, while `dsm` estimates detection probability in one step and the
 180 density surface conditional on this estimate, in another.

181 The key to simultaneous estimation of detection probability and the density surface is the fact that if
 182 the locations of points arise from a Poisson process, then the locations of the *detected* points arise from a
 183 *thinned* Poisson process. “Thinning” involves detecting points with some probability (h , say) that is less than
 184 1. The density (intensity) of a thinned Poisson process is the unthinned density D , multiplied by the thinning
 185 probability h . For example, if $h = 0.5$ so that half the points are detected on average, then the density of
 186 detected points is half that of the all points: $Dh = D/2$. On a line transect survey, the probability of missing a
 187 point depends on its distance d from the line, so that h is a function of distance ($h(d)$) and the density of the
 188 thinned Poisson process at the point’s location is $Dh(d)$, where D is the underlying density at this location.
 189 Writing D as $D = \exp(\text{Intercept})$ and noting that $Dh(d) = \exp(\text{Intercept} + \log(h(d)))$, we see that the log density
 190 of the thinned Poisson process is equal to the log density of the underlying process plus the log of the detection
 191 probability. This is convenient, because it means that we can do inference for thinned LGCPs by simply adding
 192 a term for the thinning probability to the log density.

193 With this in mind, and noting that the thinning probability has an unknown parameter that we call `lsig`,
 194 we specify our model by combining the `components` specification and `formula` specifications from Sections 6.1
 195 and 6.2.

```
cmp <- ~ spat(map = coordinates, model = inla.spde2.matern(mesh)) + lsig + Intercept
fml <- coordinates + distance ~ spat + log(h(distance, lsig)) + log(1/8) + Intercept
```

196 The left hand side of the formula (`coordinates + distance`) tells `inlabru` that we are modelling both the
 197 spatial point process governing dolphin group locations, and the detection distances. The right hand side says
 198 that the log density of this process is the sum of the log detectability and the spatial process composed of the
 199 spatial SPDE, and the `Intercept`. The offset term `log(1/8)` specifies that the density of distances is assumed

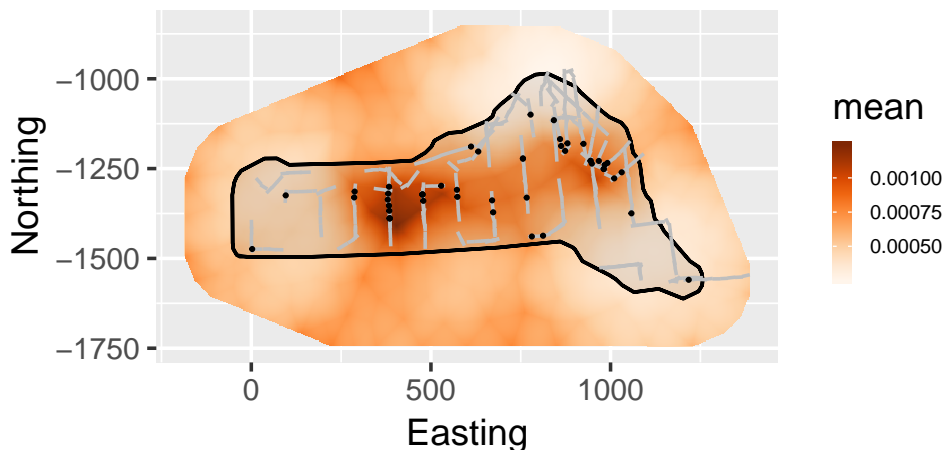


Figure 4: Predicted density surface (in counts per square km) of a point process model for dolphin groups fitted jointly with a hazard rate detection function (not shown).

200 to be constant on the distance interval $(0, 8)$ – as the transect half-width is 8 km.

201 With the above definitions, fitting the model is straightforward using the same syntax as shown in Section 6.1,
 202 where now the `samplers` argument is a `SpatialLinesDataFrame` storing the survey’s ship transects. The
 203 prediction code introduced in Section 5 is then used to estimate the spatial density surface shown in Figure 4(a).

204 We can add further processes, such as a group size probability model. This allows us to make detection
 205 probability depend on group size and to model a spatially varying group size distribution. We do not illustrate
 206 this here for lack of space.

207 7 Georeferenced data from a continuous spatial field

208 We illustrate spatial modelling from a continuous spatial field by sampling the simulated field (which might cor-
 209 respond to a soil nutrient level, for example) shown in Figure 5(a), at the locations of the crosses in that figure.
 210 Having specified a Matérn correlation function using `inla.spde2.matern` in a similar way to that shown pre-
 211 viously, and given that the sampled observations are in the `observed` data field of a `SpatialPointsDataFrame`
 212 named `geosamp`, the model is fitted as follows, assuming a Gaussian error model:

```
cmp <- observed ~ field(map = coordinates, model = inla.spde2.pcmatern(mesh)) + Intercept
fit <- bru(components = cmp, data = geosamp, family = "gaussian")
```

213 (Here we have named the SPDE “`field`” rather than “`spat`”.)

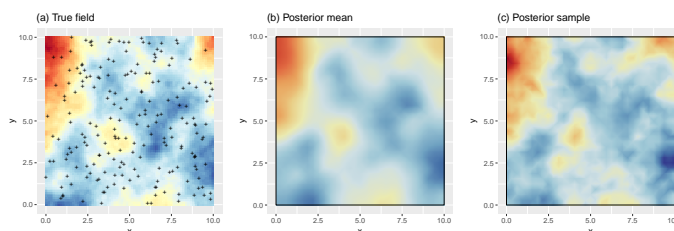


Figure 5: (a) A simulated continuous spatial field, showing sample locations, (b) the posterior mean of the model fitted to the sample data, and (c) a sample from the posterior distribution of the field.

214 The mean of the fitted model is shown in Figure 5(b), while a sample from the posterior distribution of the
215 field is shown in Figure 5(c). Note that the mean surface is necessarily smoother than the true field (which is
216 conceptually a draw from a random field with the given mean), while the posterior sample better reflects the
217 fine-scale structure of the true field.

218 8 Discussion

219 The `inlabru` package makes Bayesian spatial modelling with INLA, including point process modelling, more
220 accessible to ecologists. It allows one to model species distribution and estimate density and abundance with
221 data that are (a) complete spatial maps of the locations of individuals or groups, (b) counts in plots, (c) points,
222 and (d) distance sampling data.

223 It is distinguished from methods and software that fit density surfaces to count data in that it can deal with
224 points as responses in continuous space and does not require that space be discretised (although `inlabru` can
225 deal with such data, as illustrated in Section 5 above). Nor does it require a neighbourhood structure to be
226 defined, as is required for conditional autoregressive models or simultaneous autoregressive models, for example.

227 It also provides a means of doing Bayesian spatial modelling with distance sampling data. Its distance
228 sampling capabilities are not as well developed as those of the frequentist package `dsm` (Miller *et al.*, 2018),
229 and unlike `dsm`, it estimates the detection probability and density surface simultaneously. It shares this feature
230 with the frequentist package `unmarked` (Fiske & Chandler, 2011), although `unmarked` has no spatial modelling
231 capabilities. Simultaneous estimation of detection probability and the density surface is conceptually satisfying,
232 but the jury is out on whether this, or estimation of the two in separate steps, is preferable in practice.

233 Features of `inlabru` that we do not have space to describe include its ability to do temporal and spatio-
234 temporal modelling and its ability to simultaneously estimate the density of a point process and the spatially-
235 varying density of what spatial statisticians call “marks” on points (dolphin group size, being an example) as
236 well as its impact on the shape of the detection function.

237 Features under development include point transect data, modelling multi-species density when there is
238 spatial interaction or common explanatory environmental data for the distribution of different species sharing
239 a habitat, and modelling of habitat preference based on telemetry data. There are some technical obstacles to
240 implementing spatial capture-recapture methods (Efford, 2004; Borchers & Efford, 2008; Royle & Young, 2008)
241 in `inlabru`, but work in this area is ongoing.

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244 Supplementary Material

245 RMarkdown scripts:

- 246 • Supporting Information S1: `1_spatial_gorilla_models.Rmd`. Code for spatial Poisson count and LGCP
247 inference.
- 248 • Supporting Information S2: `2_dfun_univariate.Rmd`. Code for detection function inference.
- 249 • Supporting Information S3: `3_distsamp.Rmd`. Code for line transect models.
- 250 • Supporting Information S4: `4_georefsim.Rmd` Code for models for georeferenced data.

251 Author contributions statement

252 FB, DB, JI and FL conceived the ideas and designed methodology; FB and DB analysed the data; FB and
253 FL wrote the code, with a minor contribution from DB; FB led the writing of the manuscript, with major
254 contributions from all authors.

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