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Development of Multi-scale Model in Large-scale HTS Coils with Improved Coupling

Lei Wang, Jinxing Zheng, Quan Li, Yuntao Song, and Yuanxi Wan

***Abstract*—Due to the nonlinear properties of superconductors, modeling of high-temperature superconducting (HTS) devices is difficult, especially for the ones in large scale. In this paper, we develop the multi-scale model from a 2D planar geometry to a 2D axisymmetric geometry to successfully calculate ac loss of large-scale HTS coils. By applying appropriate boundary conditions, the infinite-turn coil is built in the 2D axisymmetric model to produce approximated current density for a HTS coil. Besides, the multi-scale model is improved in terms of coupling, so that sub-models can be connected in one file and the external data processing is not needed. Validation on accuracy and efficiency is demonstrated through a double-pancake coil. Results are compared with a conventional model that considers the superconducting characteristics on each turn of the coil in actual size. Our work shows the development and improvement of multi-scale model in the 2D axisymmetric geometry is feasible. Due to its high accuracy and efficiency, the developed multi-scale model can be a useful approach to analyze electromagnetic characteristics and calculate ac losses in large-scale HTS magnets.**

***Index Terms*—Multi-scale model, Infinite-turn coil, HTS coils, REBCO, AC loss.**

I. Introduction

Due to its high current capacity and good mechanical stability, high-temperature-superconducting (HTS) materials bring new potentials to electromagnetic applications, such as magnets, generators, and motors. In particular, HTS conductors enable compact and strong magnets for nuclear fusion [1]-[4]. The Institute of Plasma Physics, Chinese Academy of Sciences (ASIPP) is currently designing a hybrid superconducting magnet using HTS material as the insert coils for the tokamak [5]. Compared with conventional materials, resistive losses can be avoided with superconductors [3], but ac losses occur through irreversible vortex motion when the HTS coils are ramped up and down [6]. Large ac losses influence the performance and stability of magnets, and more seriously they may cause cooling system overloaded and trigger quench. Hence, accurate evaluation of ac losses in HTS coils is essential in the design and construction of magnets.

However, the loss estimation of large-scale HTS coils is an intrinsically difficult task, due to the huge difference in size between superconductors and practical devices, a large number of turns in a high-field magnet, and the highly nonlinear properties of HTS tapes. Superconducting tapes are within the size of micrometers, while magnets could be up to meters. This huge difference in size causes difficulties in modeling and meshing, and brings enormous degrees of freedoms to be solved. Besides, in order to generate high field, the number of turns in a coil could be up to several thousands, which requires large memory for computation and is tremendously time-consuming. In addition, the HTS materials are usually modeled by *E*-*J* relationship with a power law, which includes the dependence of J_c on the magnetic field. The highly nonlinear property and strong anisotropic field dependence makes it difficult further to implement the related calculations [7].

Several methods to solve electromagnetic quantities and ac losses in HTS conductors have been developed, including the anisotropic homogeneous-medium approximation and the minimum magnetic energy variation (MMEV) method [8-12]. Based on the method of MMEV, a series of systematic studies on pancake coils were conducted by E. Pardo *et al.*, from a few turns to large magnets [13]-[16]. *T*-formulation was developed to investigate current and magnetic field in HTS tapes [17]-[19], which has been frequently applied and validated in quantifying losses [20]-[23]. Besides, the *H*-formulation finite element method (FEM) has also been widely employed [24]-[26], along with which, the anisotropic homogeneous-medium approximation was applied on a stack of REBCO tapes by V. M. Zermeno *et al*. [27] in order to reduce the degrees of freedom and computation time. This method could be extended to a two-dimensional axisymmetric model for REBCO coils with stacks of pancakes and large number of turns [6]. Moreover, L. Queval *et al.* developed a multi-scale model approach, in which

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the background field was obtained first, and then ac loss in each conductor was estimated. This method could be employed to speed up the simulations of superconducting devices with large number of tapes in parallel [28], [29].

In this study, we develop Queval *et al.*'s multi-scale model in a 2D planar model for the HTS coated conductor tapes to a 2D axisymmetric model for HTS coils. The appropriate boundary conditions are developed in the 2D axisymmetric geometry to model the infinite-turn coil to produce the approximated current density for HTS coils. Besides, an improvement in terms of coupling is made between the sub-models of multi-scale model. Through two unidirectional couplings, sub-models can be connected in one computing file, and there is no need to use an external data processing [29]. The more detailed implementations about the development and improvement are presented in Section II. In Section III, we validate the model by comparing the results of a multi-scale model and a conventional model based on a double-pancake coil. Conclusions are given in Section IV.

II. Methodology

A. Approximation of infinite-turn coil

Different from the homogenization model simulating a coil as an equivalent homogeneous bulk, the essence of multi-scale model is to obtain the background magnetic field using fast coil model with every individual turns modeled, and then quantify ac loss in each turn with the obtained field. Obviously, the precision of the background field determines its accuracy. However, the current density distribution imposed in the fast coil model is uniform, which is different from that in actual superconductors. Therefore, an approximated current density distribution similar to that in superconductors should be obtained first.

Since a practical HTS coil contains a large number of turns, the distribution of current density would be similar to that in an infinite-turn coil. Based on this approximation, we can build a simplified model with appropriate periodic boundary conditions to simulate the infinite turns. The simplified model only contains the tapes in the axial direction side-by-side and a small air domain. The width of air domain is determined by the gap distance between the adjacent tapes in a pancake. The periodic boundary conditions are imposed on the borders of the air domain in the radial direction, and are realized by Dirichlet boundary conditions with $H_z = 0$. The approximation of infinite-turn coil is presented in Fig. 1.

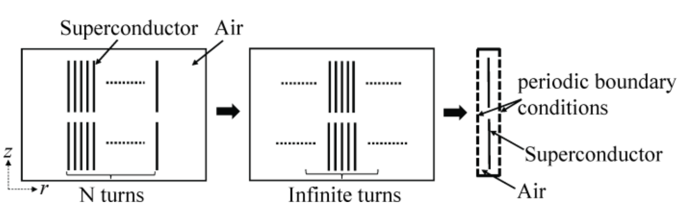

Fig. 1. The approximation of infinite-turn coil. The periodic boundary conditions $H_z = 0$ are imposed on the dashed borders.

After modeling the infinite-turn coil, we can build the computing process of multi-scale model, shown in Fig. 2.

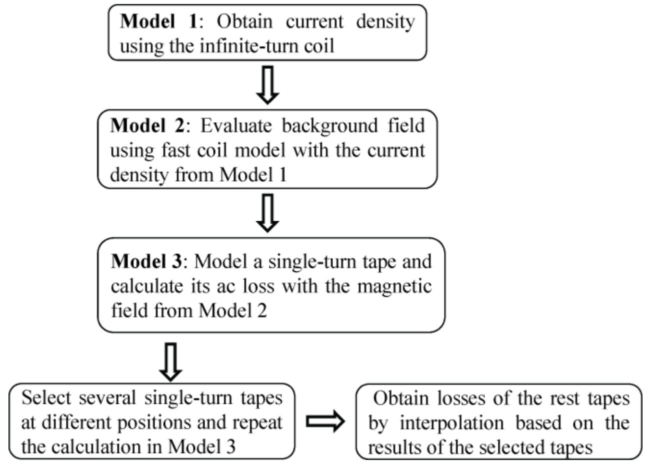

Fig. 2. Flow chart of multi-scale model.

So one can see that the multi-scale model contains three sub-models. Model 1 is that simplified coil model with appropriate periodic boundary conditions, which has been introduced before. The superconducting tapes in it are modeled and calculated by *H*-formulation to get the approximated current density. Model 2 contains the full structure of the coil with all the individual turns built in their actual geometries. Different from the conventional superconducting models, the coil in model 2 is built by an *A*-formulation magnetostatic model [29] without considering the superconducting characteristics on tapes, because the only purpose of model 2 is to obtain the magnetic field of the coil. The imposed current density for coil's magnetic field calculation is from model 1. Model 3 is used to calculate tapes' ac losses with the magnetic field from model 2. Since model 2 contains the whole coil, the magnetic field can be applied for the ac loss calculation of tapes at any positions. In order to improve the calculation efficiency, only one single-turn superconducting tape is modeled each time. Only several tapes need to be selected and calculated by model 3, because the losses of the rest unselected tapes can be obtained by interpolation. The relationship between model 1, 2 and 3 is presented in Fig. 3.

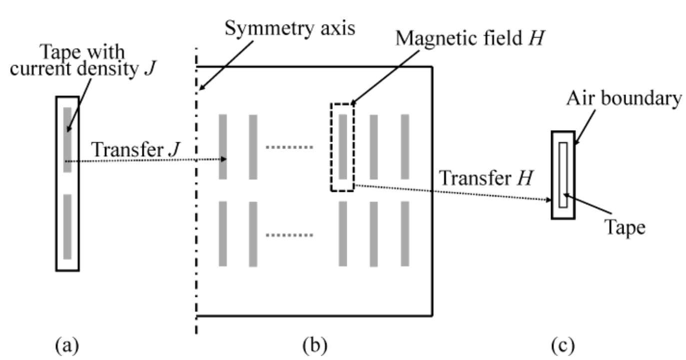

Fig. 3. The relationship between sub-models of the multi-scale model: (a) model 1: infinite-turn coil, (b) model 2: full structure of the coil, and (c) model 3: one single-turn tape.

Data transferring in Fig. 3 is achieved by unidirectional coupling. A time-varying External Current Density on the tapes' cross sections in model 2 and a time-varying Dirichlet

Boundary Condition on the air boundaries in model 3 are employed to receive the current density from model 1 and magnetic field from model 2, respectively.

Through the two couplings, the three sub-models can be connected and united in one COMSOL file. Therefore, the problem that the coupling between the coil sub-model and the tape sub-model could not be performed directly via COMSOL is solved, which means an extra data processing is no longer needed to achieve the coupling between the coil sub-model and the tape sub-model [29]. In our tests, we found that using the extra data processing would result in bad convergence in the calculation of model 3, especially for the complex fit of J_c-B. The improvement we developed has solved the deficiency, and makes multi-scale model more integrated.

The method developed is high in efficiency due to 1) background field of the coils can be obtained simply from a conventional *A*-formulation magnetostatic model without considering the nonlinear superconducting characteristics, and 2) only a single turn with very few degrees of freedom is simulated in model 3 at a time, whilst simulations of multiple turns are independent and can be run in parallel. The accuracy depending on the approximation of infinite-turn coil will be discussed in Section III.

B. *Numerical equations*

In order to model HTS coils, a 2D-axisymmetric model with *H*-formulation is used in the general form PDE module of the finite element software COMSOL Multiphysics. PDE module belongs to Mathematics branch in COMSOL. And PDE stands for Partial Differential Equation, which means that module is efficient at solving partial differential equations. It is widely employed to simulate superconductors by solving the governing equations as follows:

$$\begin{bmatrix} \mu_0 \dfrac{\partial H_r}{\partial t} - \dfrac{1}{r}\dfrac{\partial (rE_\varphi)}{\partial z} \\ \mu_0 \dfrac{\partial H_z}{\partial t} + \dfrac{1}{r}\dfrac{\partial (rE_\varphi)}{\partial r} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = J_\varphi \quad (2)$$

$$E_\varphi = \rho \cdot J_\varphi \quad (3)$$

where H is magnetic field, and r, z and φ represent the radial, axial and angular coordinates of the cylindrical coordinate system, respectively. E_φ and J_φ stand for the electric field and current density. The superconductor resistivity ρ is modeled by:

$$\rho = \left| \frac{J_\varphi}{J_c(B,\theta)} \right|^{n-1} \cdot \frac{E_0}{J_c(B,\theta)} \quad (4)$$

where E_0=10^{-4} V/m is applied as the critical current criterion, n is 25, and $J_c(B,\theta)$ is the critical current density considering J_c-B dependence. B represents the magnitude of the magnetic field and θ represents the angle between the field vector and the c-axis of the superconductor [6].

The critical current $I_c(B,\theta)$ of a given SuperPower-26 tape is taken from D. K. Hilton *et al.* [30], which is a nonlinear least-squares fit for transport critical current versus field magnitude and angle data for REBCO coated conductors at 4.2 K. This fit was widely applied to estimate the hysteresis losses and design magnets of REBCO coated conductors in high fields [6], [31]-[34], which is given by:

$$I_c(B,\theta) = \frac{b_0}{(B+\beta_0)^{\alpha_0}} + \frac{b_1}{(B+\beta_1)^{\alpha_1}} \cdot [\omega_1^2(B)\cos^2(\theta-\varphi_1) + \sin^2(\theta-\varphi_1)]^{-1/2}$$

$$\omega_1(B) = c_1\left[B + \left(\frac{1}{c_1}\right)^{1/\varepsilon_1}\right]^{\varepsilon_1} \quad (5)$$

The parameters are listed as follows: $\alpha_0 = 1.29746$, $\alpha_1 = 0.809120$, $b_0 = 8870.39$, $b_1 = 18456.2$, $\beta_0 = 13.8$ T, $\beta_1 = 13.8$ T, $\varphi_1 = -0.180370$ deg, $c_1 = 2.15$, and $\varepsilon_1 = 0.6$.

III. VALIDATION

A. *Modeling*

To validate the accuracy of the method, a double-pancake coil with 100 turns in each pancake is modeled with conventional model and multi-scale model, respectively. The conventional model considers superconducting characteristics on each turn of coils in actual size, which is widely recognized as the reference. The reason for choosing a double-pancake coil is that it forms the basis of multi-pancake coil and the implementation of multi-scale model can be demonstrated fully. The sketch and layout parameters of the coil are presented in the Fig. 4 and Table I.

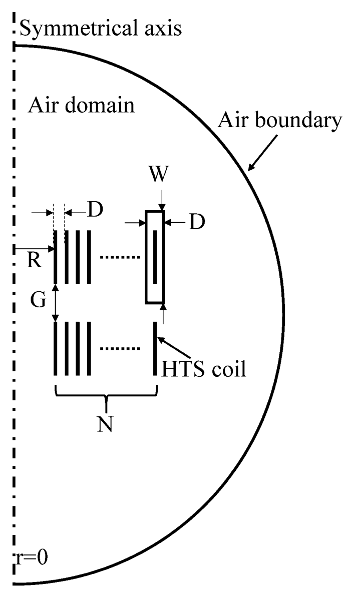

Fig. 4. Sketch of the double-pancake coil model.

TABLE I
LAYOUT PARAMETERS OF THE COIL

Symbol	Description	Value
W_{YBCO}	YBCO width	4 mm
T_{YBCO}	YBCO thickness	1 µm
D	Gap between adjacent turns / Unit cell width	293 µm
W	Unit cell height	4.4 mm
G	Gap between adjacent pancakes	0.4 mm
N	Number of turns	100
R	Inner radius	20 mm

As for a multi-layer YBCO tape, only 1- µm YBCO layer is modeled. The stabilizer, substrate and insulation are regarded as air. As shown in Fig. 4, a small rectangular domain containing a single YBCO layer is modeled and used as a unit cell. This cell can be used to simulate the second coupling on local magnetic field, shown in Fig. 3. In both conventional model and multi-scale model, the same mesh would be used within this unit cell, where the domain of 1- µm YBCO layer is divided into 100 elements symmetrically in arithmetic sequence along the width [29] and 2 elements along the thickness. The detail of mesh is displayed in Fig. 5.

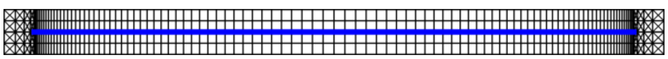

Fig. 5. The mesh in a unit cell. The blue line stands for the superconductor.

The cases of transport currents (with no applied external field) are studied, and ac transport currents at 50 Hz with arbitrary amplitudes of 400 A and 580 A are imposed.

B. *Current density and field distributions*

Because there are large non-critical regions at both current amplitudes in the double-pancake coil, the distributions of current density and field in the case of 580 A are displayed.

Fig. 6 shows the normalized current density distributions at 0.015 second in the conventional model and multi-scale model, respectively. The critical regions, subcritical regions and virgin regions are clearly shown by the conventional model. Whereas in the multi-scale model, the normalized current density distributions of all turns are identical within one pancake, and are simple with just critical and non-critical regions. These are because the current density is obtained from a simplified model with the periodic boundary conditions H_z = 0 (refer to Fig. 1). The distributions from both models will be more similar, if 1-element mesh is used in the thickness of superconductors [29]. However, if we compare the critical regions from the two models, we can observe that the current distributions obtained from the multi-scale model are in good agreement with those from the conventional model in the coil's central parts. And the distributions in the innermost and outermost several turns do not match very well because the multi-scale model is based on the approximation of infinite-turn coil, while real coils have borders.

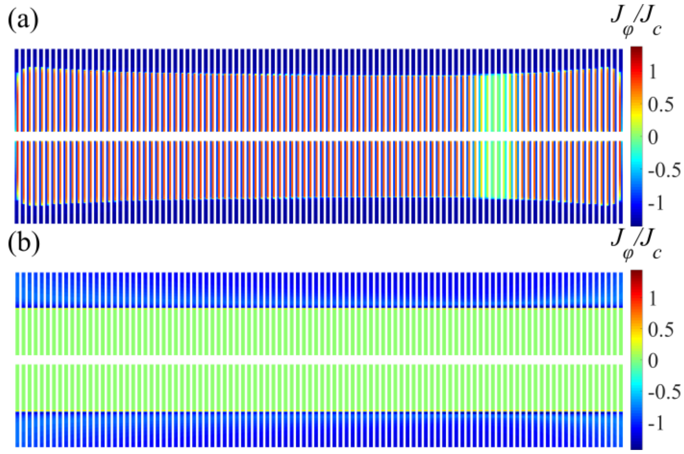

Fig. 6. The normalized current density distributions in (a) conventional model and (b) multi-scale model (580 A, t = 0.015 s).

Fig. 7 shows the distributions of magnetic flux density corresponding to Fig. 6. We can see that the multi-scale model produces very similar results as the conventional model, while slightly different distributions can be observed at the non-critical regions (refer to Fig. 6). The streamlines are parallel to the surfaces of superconducting coils in the conventional model, but deviations occur with the flux density decreasing in the multi-scale model. However, considering that there is little penetration in the regions with deviations, the total ac losses should be hardly affected.

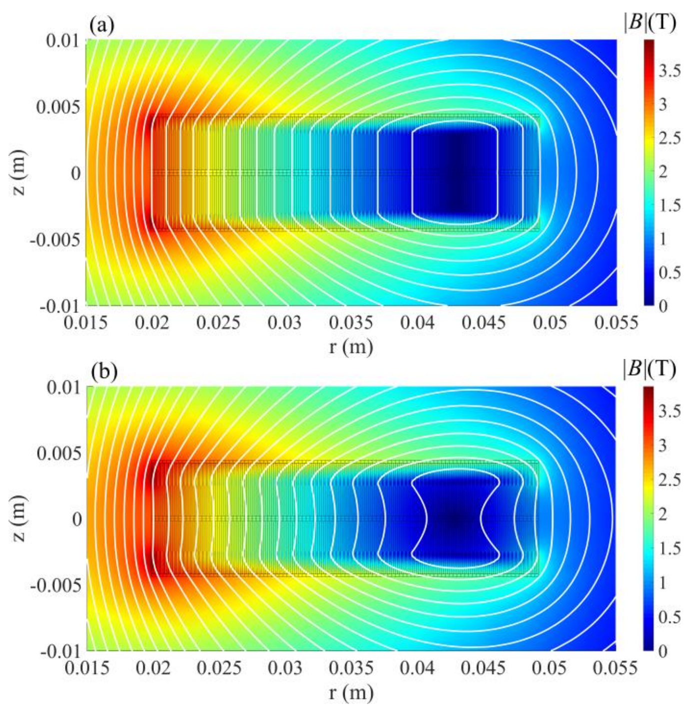

Fig. 7. The magnetic flux density distributions in (a) conventional model and (b) multi-scale model (580 A, t = 0.015 s).

C. *AC loss*

Out of the symmetry of the double-pancake coil, we choose only certain turns of coils in the upper pancake when applying the multi-scale model to calculate ac losses. The turns in the upper pancake are numbered from 1 to 100, counted from the most inside one. 21 turns are selected, one in every five turns from the 1st to the 96th and the 100th turn. In [29], more tapes are chosen in the regions where the field spatial variation is high. But in coils, ac losses are closely related to the radius of each turn. In this case, loss is more sensitive to the turns selected, which will be discussed later. We calculated the ac loss of each turn in the upper pancake from the conventional model, so that loss in each turn can be compared. In this way, the accuracy of multi-scale model can be examined. The results are compared and presented in Fig. 8.

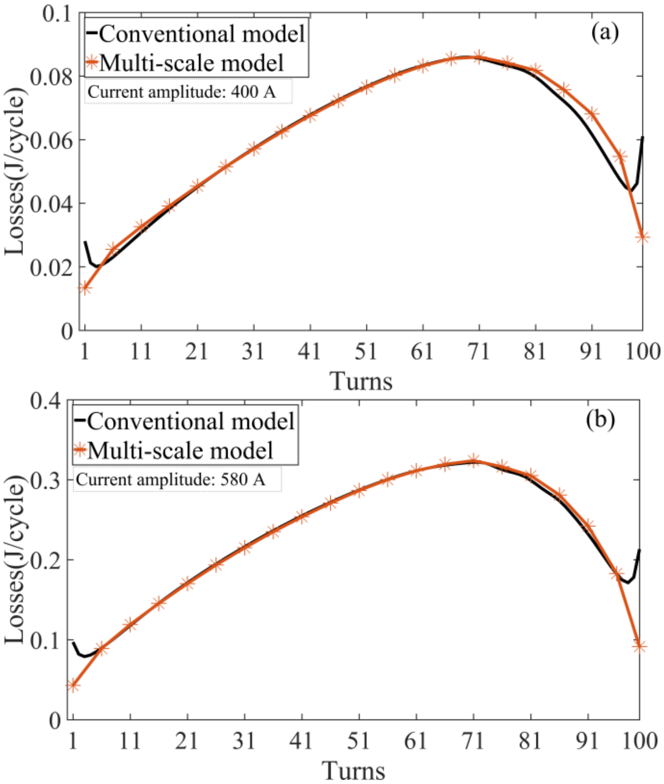

Fig. 8. AC loss of each turn in the upper pancake of the coil in the cases of (a) 400 A and (b) 580 A. The markers represent the selected turns.

From Fig. 8, we can see that the ac loss in each turn from multi-scale model matches with that from the conventional model perfectly, except the innermost and outermost turns because of the approximation of infinite-turn coil and the borders of real coils. This result is consistent with the current density and magnetic field shown in Fig. 6 and Fig. 7. In addition, we found that peak values of losses in each turn appear in the outer parts of coils due to field penetration and turn radius within a cycle (0.02 second).

But one can also find that, the goodness of matching curve in the case of 580 A is better than that in the case of 400 A. This is because with the current increasing, the tapes will be penetrated with deeper critical regions. And this will happen both in the conventional model and multi-scale model. Therefore, the magnetic field caused by penetrated current will be more similar.

To present the explanation quantitatively, we extract the perpendicular magnetic field (B_r) at z = 0.0044 m, which is one side of the unit cells. As for YBCO tapes, ac loss is mainly affected by the perpendicular field. Therefore, it can be used to reflect the goodness of matching curves in Fig. 8. Considering that the differences of losses at the innermost and outermost several turns are caused by the intrinsic deficiency of the infinite-turn approximation, the B_r on the positions corresponding to the turns from the 6th turn to 96th turn are extracted and compared to present the difference between the two current amplitudes, shown in Fig. 9.

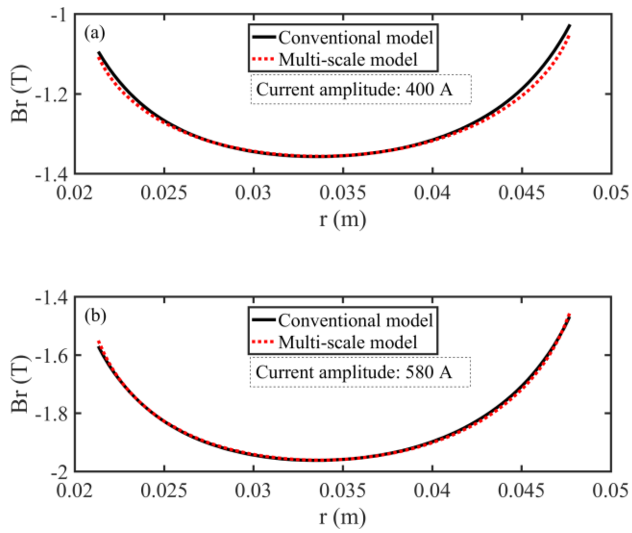

Fig. 9. The perpendicular field on one side of the unit cells at z = 0.0044m from conventional models and multi-scale models at (a) 400 A and (b) 580 A (t = 0.015 s).

Obviously, the matching of B_r from multi-scale model and conventional model in the case of 580 A is better than that in the case of 400 A, which gives the reason for the matching differences caused by different currents shown in Fig. 8.

In addition to the comparison of the total losses in each turn, the instantaneous losses of the 1st, 36th, 71st, and 100th turns at 580 A are calculated and plotted, shown in Fig. 10. These results explain how losses vary along time in detail. We can see that in the coil's central parts (the 36th and 71st turns), the infinite-turn coil is definitely a good approximation for large-scale HTS coils, but errors occur near the borders (the 1st and 100th turns) of real coils.

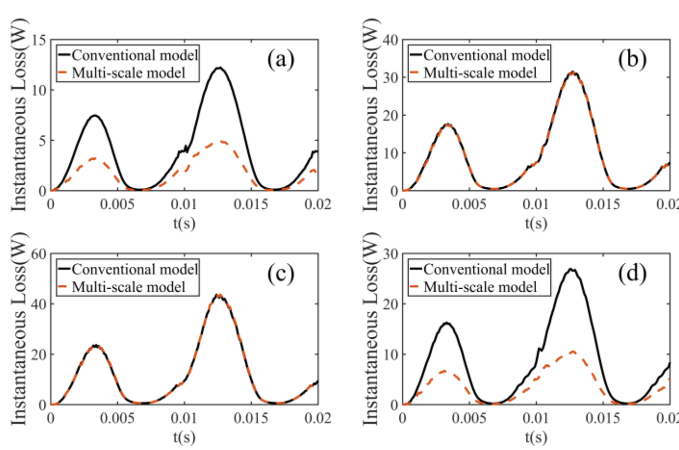

Fig. 10. Instantaneous losses in four specific turns: (a) the 1st turn, (b) the 36th turn, (c) the 71st turn, and (d) the 100th turn (580 A).

Besides the turns calculated, losses in other turns are obtained by interpolation, and the total ac losses are listed in Table II. Computing times and speedup factors are counted and shown in Table III.

TABLE II
AC LOSSES DURING ONE CYCLE

	Average Losses (J/cycle)	
	Conventional	Multi-scale
400 A current	12.35	12.46 (100.9%)
580 A current	46.50	46.12 (99.2%)

TABLE III
COMPUTING TIMES AND SPEEDUP FACTORS

	Computing times (h)*			Speedup factor
	Conventional	Multi-scale		
400 A	48.66	Series	1.56	31.19
		Parallel with 3 cores	0.58	83.90
		Parallel with 7 cores	0.30	162.20
		Full parallel	0.17	286.23
580 A	65.87	Series	1.87	35.22
		Parallel with 3 cores	0.69	95.46
		Parallel with 7 cores	0.36	182.97
		Full parallel	0.19	346.68

*Calculated on a desktop with Intel i5-2500 CPU @ 3.30 GHz, and RAM 8 G. The times in parallel are estimated.

Table II shows that results from conventional models and multi-scale models are very close with a difference around 1%, which means the multi-scale model has high accuracy in ac loss calculation. However, as shown in Table III, the computing times of 21 turns in series are much less than those taken by conventional models. Furthermore, the speedup increases with the number of CPU cores used in parallel computation, and full parallel of the approach allows faster simulations of the models by 2 orders of magnitudes.

It is proved that the multi-scale model is both accurate and efficient to simulate HTS coils, in spite of the less accurate results for the innermost and outermost turns due to the approximation of infinite-turn coil. Since only one turn of coil is simulated each time, the advantages of few degrees of freedom and parallelization of the method are outstanding. Moreover, by applying multi-scale model we can study specific positions in a coil without calculating the entire coil.

IV. CONCLUSIONS

It is always a difficult task to simulate superconductors accurately and efficiently, especially for large-scale HTS devices. Although the conventional model for superconductors has a good reliability, too many degrees of freedom need to be solved make it extremely time-consuming and difficult to be applied.

In this paper, we develop Queval *et al.*'s multi-scale model from the 2D planar geometry to the 2D axisymmetric geometry, to solve electromagnetic quantities of HTS coils. Besides, we improve the computing process of multi-scale model with two couplings connecting sub-models and transferring data. Due to the improvement, multi-scale model can be more easily implemented in one COMSOL file, and the external data processing is no more needed. Our developed model is more practicable in calculation with high convenience and convergence.

In the study, detailed implementations of calculating ac loss with multi-scale model are presented, including those of modeling the infinite-turn coil, three sub-models and two unidirectional couplings. The validation is demonstrated by a double-pancake coil. Through the results from the multi-scale models and conventional models, the high accuracy and efficiency of multi-scale model in simulating HTS coils are validated. Based on the parallel computation, multi-scale model is particularly useful for extremely large HTS magnets, in which a whole system simulation is practically impossible.

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