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# Field-Based Evaluation of the Effects of Shunt Capacitors on the Operation of Distribution Transformers

L. M. Korunović, *Senior Member, IEEE*, A. S. Jović, and S. Z. Djokic *Senior Member, IEEE*

**Abstract**—This paper analyses the effects of shunt capacitors installed on the low voltage sides of 10/0.4 kV distribution transformers on the operation of these transformers. Using the results of an extensive measurement campaign, the paper compares: real and reactive power losses, secondary-side current and primary-side apparent power before and after the installation of transformer capacitors. The following four cases of reactive power compensation are considered and discussed: adequate, conditionally adequate and total compensation, as well as overcompensation. The presented analysis helps to identify transformer substations where the initial compensation should be changed or adjusted, in order to improve transformers' operational performance. Voltage dependencies of real and reactive power demands of the transformer-supplied loads are taken into account, based on the previously identified load characteristics in the considered networks, as well as voltage dependency of reactive power of shunt capacitors. The result with and without including these voltage dependencies in the analysis are compared and discussed.

**Index Terms**—Capacitor bank, power flow, reactive power compensation, real and reactive power demand, transformer.

## I. INTRODUCTION

REACTIVE power compensation by shunt capacitors is commonly used by medium-size and large customers for reducing their reactive energy consumption, aimed, primarily, at reducing their electricity bills [1]. On the other hand, utilities often install shunt capacitors, as these offer many and varied benefits: reduction of network losses, improvement of network voltage profiles, better voltage regulation, increase of network generation hosting capacity, release of transmission and distribution system capacity, which all typically result in the increased revenue and deferral or full elimination of capital expenditure due to improved network performance.

There are two types of shunt capacitor banks (CBs): fixed and switched [2]–[3]. Injected reactive power of a fixed CB is not controlled to change with load variations, while reactive

power of a switched CB is typically controlled based on the loading conditions. The value of the fixed CBs is most commonly set to maintain a predefined power factor (PF) for specific loading conditions, which is either less than, but close to one ( $PF < 1$  lagging [1]), or almost equal to one ( $PF \approx 1$ ), when approximately zero reactive power demand (“total compensation”) is achieved [4]. The compensation with switched CB is more precise and therefore more efficient, but utilities and customers often chose to install fixed CBs due to their lower costs.

The analysis of operation of distribution networks with CBs was the subject of many books and papers. Most of them deal with financial benefits due to capacitor installations and optimal sizing and location of CBs, such as [3], [5]–[10]. Some references recognize power quality problems related to shunt capacitors, focusing mainly on voltage profile improvement [11], or presenting researches on oscillatory transients due to capacitor switching [12] and resonance phenomena occurring in networks with installed CBs [13]. In [14], it is demonstrated on the basis of real measurements that the transient overvoltages and overcurrents will be limited if the phase-controlled vacuum circuit breakers are implemented for on/off switching of the CBs. It is also shown that a proper phase-selecting control strategy will improve the reliability of power system.

Many papers present simulation results of the effects of shunt capacitors installed in transmission and distribution networks on voltage drops, power factors, power losses, line current and apparent power demand. For example, [15] emphasizes the influence of shunt capacitors installed at high and medium voltage buses on voltage drops and power factors. Operation of shunt CBs and on-load tap-changing transformers for various transmission network conditions aimed at reducing transmission losses and network component loading and for improving voltage control, is presented in [16]. Reference [17] states that CBs are the critical components for increasing the network capacity and improving voltage regulation, while [18] presents the procedure for design calculation of a CB for an industrial plant, in order to improve power factor and consequently to reduce line current and losses.

Although it can be concluded that the reactive power compensation is a mature topic, it should be emphasized that the effects of shunt capacitors on the operation of distribution

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transformers are not considered in any of the previous work. The possible reason is that losses in the transformers are small, but there might be several thousands of MV/LV transformers in a distribution network of a large city, with potentially significant number of dedicated shunt capacitors. In such cases, even very small effects on individual transformers might become significant at aggregate network level.

The main contribution of the paper is the development of a calculation procedure for the evaluation of the effects of CBs on distribution MV/LV transformer operation based on the measurements on low voltage sides of these transformers in real networks. Previously published load flow procedures developed for radial distribution networks either use load characteristics on primary-side of MV/LV transformer as input data [19]-[20], or take into account the models of distribution transformers, but do not consider the effects of shunt CBs on transformer operation [21]-[22]. The procedure presented in this paper is the modification of the method from [20], which is applied for transformer operation analysis with and without CB based on transformer secondary measurements. It takes into account both load-voltage characteristics and CB's reactive power-voltage dependence on transformer secondary-side, where the load and CB are connected. The applied load-voltage characteristics of different load classes and for various seasons and day intervals are obtained from unpublished research of the authors, based on the methodology from [23] and field measurements in actual LV networks.

The paper is structured as follows: a short overview of CBs installed on LV side of distribution transformers in considered network is given in Section II; Section III introduces four general compensation cases and illustrates them with examples; Section IV presents procedures for the calculation of transformer characteristics when operating with and without installed CBs for the same loading conditions, as well as with and without taking into account load-voltage and CB's reactive power-voltage characteristics. Comparison of results for considered transformer operating conditions is presented and discussed in Section V, while main conclusions are drawn in Section VI. Some of input data used in the paper are listed in Appendices A and B.

## II. DESCRIPTION OF THE CBs AND CONSIDERED NETWORK

This paper presents the results of analysis of an extensive measurement campaign performed in distribution company ED Leskovac (EDL), where fixed CBs are installed in 177 of 664 secondary (10/0.4 kV) distribution substations, denoted in this paper as "transformer substations" (TSs). Fixed CBs are installed in the period from 2000 to 2006, with main purpose of reducing network losses. The presented measurements cover a period of around two years, from January 2015 to December 2016.

The data used in the presented analysis correspond to the measurements in 10/0.4 kV TSs supplying different load classes over various seasons. The measurements include: real power, reactive power and voltage on secondary (LV-side). The analysis considers the following four general cases of reactive power compensation: adequate, conditionally

adequate and total compensation, as well as overcompensation.

The distribution network area of EDL consists of the networks of town Leskovac and several nearby settlements. The total power of fixed CBs installed on low voltage side of 177 10/0.4 kV transformers in EDL is 6.877 Mvar. Rated powers of TSs with CBs are in the range from 100 kVA to 2x630 kVA. The majority of the installed CBs (71) have rated power 20 kvar, giving total of 1.42 Mvar. However, 3x25 kvar CBs, installed in 38 TSs, inject a higher amount of 2.85 Mvar reactive power into distribution network. The remaining 68 CBs have rated powers in the range from 10 kvar to 4x25 kvar, injecting 2.607 Mvar in total.

For illustration, Fig. 1 presents the parts of EDL MV distribution network, supplied by one TS 35/10 kV (Fig. 1a), and two TSs 110/10 kV transformers (Fig. 1b and Fig. 1c), which supply 10/0.4 kV TSs discussed in Section III. Due to the lack of space, only feeders that supply these TSs, TS-1, TS-4, TS-5 and TS-7, are depicted:  $F_3$  in Fig. 1a,  $F_{13}$  in Fig. 1b and  $F_7$  and  $F_9$  in Fig. 1c, respectively. Additionally, rated powers of TSs and installed CBs are also provided in Fig. 1.

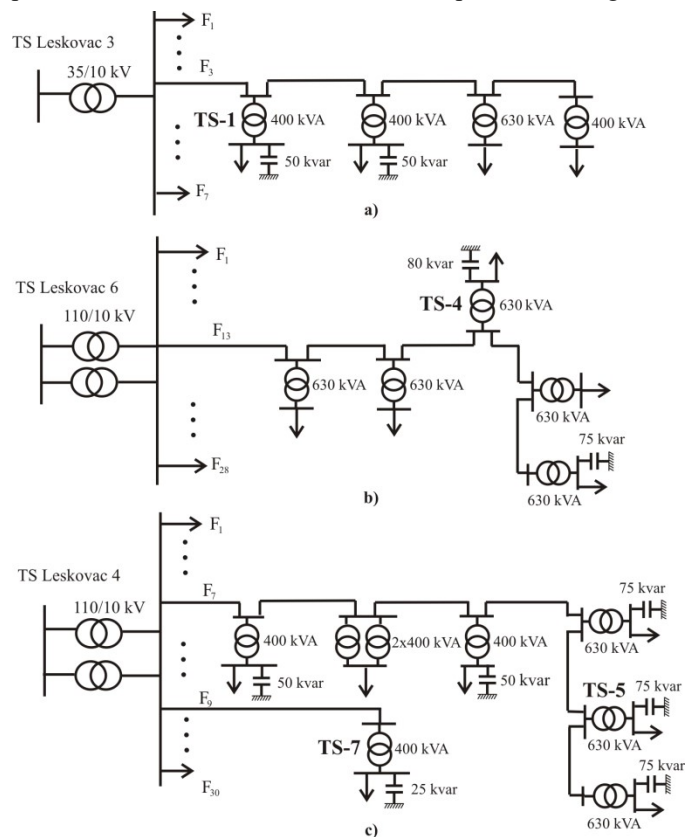


Fig. 1. The parts of the considered MV network where TSs discussed in the paper are located: a) TS-1, b) TS-4, and c) TS-5 and TS-7.

The TSs with installed CBs supply different load classes: detached privately built houses without central heating, flats in tenement buildings with and without central heating, administrative buildings, commercial buildings and rural load class, as well as a mixed load. After checking the operational status of CBs, it was found that none of the CBs was intentionally switched off and that about 91 % of them are working ten and more years after their installation. A three-

phase power analyser was used for field measurements and data acquisition on transformer secondary, as depicted in Fig. 2. The current inputs (three channels) of the recording equipment were connected to X/5 A current transformer secondaries, while the voltage inputs (four channels) were connected directly to the low voltage busbars and data were recorded every 5s.

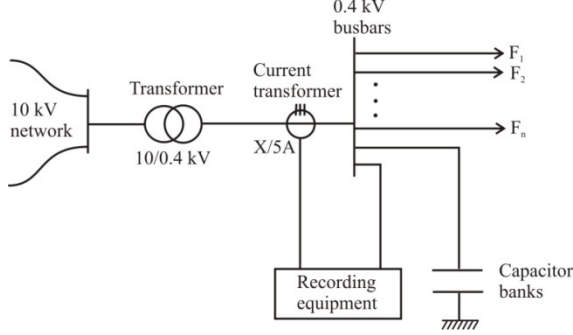


Fig. 2. Field measurement diagram.

### III. FOUR GENERAL COMPENSATION CASES

Previously published literature and papers dealing with reactive power compensation consider mainly excessive reactive energy consumption and suggest the use of CBs to reduce this consumption, i.e. to maintain PF at the predefined values [1], [2]. In considered EDL network, the fixed CBs are installed for the reduction of reactive load and therefore network loss reduction. Accordingly, values of both PF and reactive power demand are analysed in this paper as the “measures of compensation quality”, where four general compensation cases identified in measurements are described in Subsections III.A - III.D using representative examples from the measurements. A more detailed analysis of the measurements is performed for 21 of 177 TSs with installed fixed CBs, revealing that 2, 4, 5 and 9 TSs were with adequate compensation, overcompensation, total compensation and conditionally adequate compensation, respectively, while in one TS with two transformers, one transformer was adequately compensated and another was overcompensated. Under the assumption that all other TSs with CBs will have the same distributions of compensation cases as the considered sample of TSs, it can be estimated that around 12 %, 21 %, 24 % and 43 % of all TSs with fixed CBs in EDL network are with adequate compensation, overcompensation, total compensation and conditionally adequate compensation, respectively. The rest of the paper focuses on transformer operating conditions including loss reduction for different compensation cases.

#### A. Adequate Compensation

In some TSs with dedicated CBs, measured reactive power was always small and positive, i.e. average PF was  $0.95 \leq \text{PF} < 1$  lagging. This is denoted as “adequate compensation” (AC), as the reactive energy consumption is not excessive, although the installed CBs are not capable of adjusting a full and continuous reactive power compensation of the connected load.

The TS-1 is used as an example of AC. It supplies mixed

load class, consisting of: 45 % commercial load (administrative buildings), 35 % residential load (households in buildings without central heating) and 20 % other commercial load, i.e. commercial buildings. The TS-1 has a rated power of 630 kVA and a 50 kvar CB installed. Fig. 2 illustrates recorded three-phase real and reactive power demands on the LV-side of TS-1 over the course of four days in winter season.

Lower curve in Fig. 3 depicts reactive power demand,  $Q$ , which starts to increase from a relatively small value of  $\sim 10$  kvar around 7 a.m. and again reduces to that value after 8 p.m. These reactive power variations are characteristic for commercial load class, which represent the majority of TS-1 load. In the period from 8 p.m. to 7 a.m., the existing CB installation almost fully compensates reactive power demand of supplied load. However, during the working hours, a large number of electrical devices that consume reactive power is connected (e.g. magnetic-ballast fluorescent lighting, computers, monitors, printers, etc.), resulting in the maximum and recorded reactive power value of 41.7 kvar, while the average value is 22.9 kvar. As the average real power ( $P$ ) demand is 134.2 kW, the commonly used condition for reactive power compensation,  $\text{PF} \geq 0.95$ , is fulfilled, since the average  $Q/P$  ratio during the measurements was  $\leq 33$  %, i.e. it was actually around 17 %. Furthermore, 99.9 % of time PF was greater than 0.95 lagging.

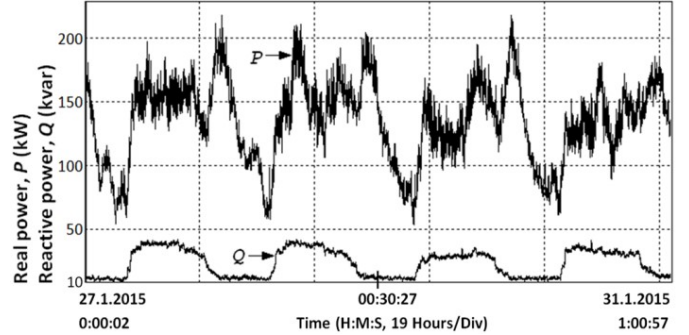


Fig. 3. Real and reactive power demands in TS-1: AC case.

#### B. Overcompensation

The next considered compensation case is denoted as “overcompensation” (OC). It is related to situations when a CB is not only supplying all reactive energy demanded by the connected load, but it also provides a surplus reactive energy, which is exported back towards the distribution transformer.

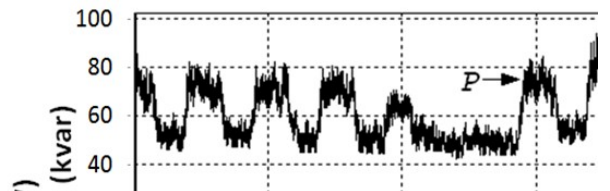


Fig. 4. Real and reactive power demands in TS-4: OC case.

Fig. 4 illustrates an OC example, related to TS-4, which supplies commercial load class, i.e. administrative buildings. Rated power of TS-4 is 630 kVA and transformer has a CB of 80 kvar installed on the LV-side. The real power (upper curve) and reactive power (lower curve) are recorded over a period of 13 days in autumn. Reactive power is negative during the whole period: minimum, average and maximum reactive power values are -65.5 kvar, -47.1 kvar and -14.3 kvar, respectively. Thus, PF was leading during the whole measurement period and varied within a relatively wide range,  $0.55 \text{ leading} \leq \text{PF} \leq 0.99 \text{ leading}$ , while only 2 % of time it was greater than 0.95 leading. Furthermore, Fig. 4 shows that reactive power variations are closely related to the changes of real power, indicating that TS-4 load consists of a significant number of electrical devices consuming both real and reactive powers: air-conditioners, computers, monitors, printers, magnetic-ballast fluorescent lamps, etc.

### C. Total Compensation

The “total compensation” (TC) case is assumed when CB-compensated reactive power demand varies around 0 kvar and its average value, as selected in this paper, is within  $\pm 5$  kvar limits. The main reason for selecting that range of average reactive power variation is because the switched CBs commonly applied in industry typically use a regulation step of 5 kvar. All TSs considered in this paper are with fixed CBs and the range of  $-5 \text{ kvar} < Q < +5 \text{ kvar}$  is not achieved at all times due to variations of reactive load. In some examined TSs, however, it was found that the average  $Q$  is within this range. Accordingly, it is assumed that TC is achieved if average  $Q$  of TSs is in the range  $-5 \text{ kvar} < Q < +5 \text{ kvar}$ .

One of the TSs with TC is TS-5, with rated powers of transformer and fixed CB of 630 kVA and 75 kvar ( $3 \times 25 \text{ kvar}$ ), respectively. This TS supplies individually built residential load (households without central heating). Real and reactive power demands recorded over 14 days in December in TS-5 are illustrated in Fig. 5, showing large variations of real power (from minimum 156 kW to maximum 403 kW), which are expected, as electrical resistive load are used for heating. Reactive power changed from minimum -21.3 kvar to maximum 23.9 kvar, while its average value was around only -2.1 kvar. Thus, PF belongs to the narrow range, from 0.99 leading to 0.99 lagging, during the measurements.

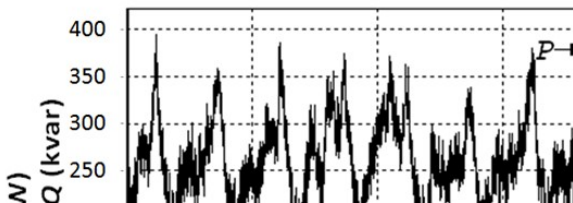


Fig. 5. Real and reactive power demands in TS-5: TC case.

### D. Conditionally Adequate Compensation

The compensation cases for which reactive power demand

was both positive and negative, but the average  $Q$  variations were outside  $\pm 5$  kvar range (limit for TC case), are denoted as “conditionally adequate compensation” (CAC). In these cases, reactive power also changes direction.

The TS-7, consisting of a 400 kVA transformer (supplying predominantly commercial load) with installed CB of 25 kvar is used as a CAC example. Fig. 6 illustrates real and reactive power demands recorded during 15 days of December. Changes of real and reactive powers generally follow characteristics of commercial load class. The significant impact of resistive load on real power profile is evident, indicating that this load is used for heating during working hours in considered winter period. On the other hand, the participation of loads that consume both real and reactive powers (mentioned in Sections III.A and III.B as typical for commercial load) has strong influence on reactive power profile. Reactive power increases notably during working hours and decreases in other time periods, reaching even negative values during the night. The minimum and maximum recorded  $Q$  values are -19.6 kvar and 65.4 kvar, respectively, while average value is relatively large (20.5 kvar), which is the characteristic of CAC. During the measurements, PF changed from 0.93 leading to 0.88 lagging, while 81% of time PF was between 1 and 0.95 lagging.

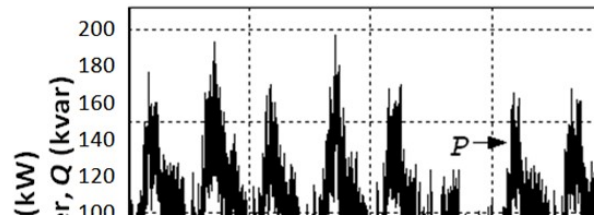


Fig. 6. Real and reactive power demands in TS-7: CAC case.

## IV. ANALYSIS OF TRANSFORMER OPERATING CONDITIONS

When connected, shunt capacitors will influence not only reactive power flows in transformers, but also other parameters and quantities important for transformer operation. This section presents power flow equations for a transformer with a CB installed on LV-side and compares them, for the same loading conditions, with the results without CB. The analysis includes results for voltage-dependent load characteristics and voltage-dependency of capacitor reactive power, which are compared with the results when these dependencies are neglected.

### A. Transformer Operation with Installed CB

Fig. 7 shows equivalent circuit of a step-down transformer with a CB installed on the secondary (LV) side [2]. Parameters of the transformer are series impedance  $\underline{Z}=R+jX$  and shunt admittance  $\underline{Y}=G+jB$ , both calculated on LV side. The symbol “ID” denotes an ideal transformer, while  $Q_C$  is the reactive power injected by CB. The variables that are recorded in TSs and denoted in Fig. 6 are: complex voltage at LV bus,  $\underline{V}_1$ , and

real and reactive power,  $P_1$  and  $Q_1$ , respectively.

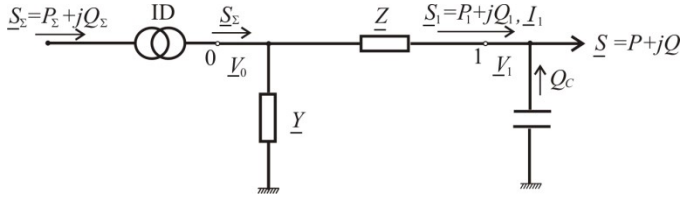


Fig. 7. Equivalent circuit of the transformer with installed capacitor bank.

Under the assumption that the angle of  $\underline{V}_1$  is zero, complex voltage at Bus 0 in Fig. 6, i.e. at the secondary (LV) side of the ideal transformer, can be calculated as:

$$\underline{V}_0 = V_1 + \frac{P_1 R + Q_1 X}{V_1} + j \frac{P_1 X - Q_1 R}{V_1}. \quad (1)$$

The module of current  $I_1$  on the LV side of transformer is:

$$I_1 = \frac{\sqrt{P_1^2 + Q_1^2}}{\sqrt{3}V_1}, \quad (2)$$

while the losses in series and shunt transformer branches are:

$$\Delta S = 3Z I_1^2 \quad (3)$$

and

$$\underline{S}_0 = \underline{Y}^* V_0^2, \quad (4)$$

giving total complex transformer losses, consisting of total real power,  $\Delta P_\Sigma$ , and total reactive power losses,  $\Delta Q_\Sigma$ , of:

$$\Delta \underline{S}_\Sigma = \Delta \underline{S} + \underline{S}_0, \quad (5)$$

and complex power on transformer primary,  $\underline{S}_\Sigma = P_\Sigma + jQ_\Sigma$ , calculated as:

$$\underline{S}_\Sigma = \underline{S}_1 + \Delta \underline{S}_\Sigma. \quad (6)$$

### B. Transformer Operation without CB

For the analysis of operating conditions without CB, transformer primary voltage,  $V_0$ , is assumed to be known, as there were no available measurements before CB installation. It is further assumed that  $V_0$  is the same as in the case with CB installed. This assumption is justified because the changes of  $V_0$  in both cases are relatively small (up to  $\pm 6\%$  of the rated/nominal voltage,  $V_n$ ), although  $V_0$  depends not only on reactive power compensation, but to a much greater extent on load flows in MV network and its supply voltage. Also, the aim of this paper is not precise calculation of particular  $V_0$  voltage value, but the comparison of transformer variables (Section V) for the same loading conditions before and after the installation of CBs.

All the variables calculated without CB installed are denoted by a superscript ' on variable designations used in Subsection IV.A. Under the assumption that the load and capacitor reactive power are not voltage dependent, the power on transformer secondary without a CB installed is equal to the total transformer load. Real power of this load,  $P_1' = P$  is equal to the measured real power on transformer secondary,

$P_1$ , while reactive power  $Q_1' = Q$  is greater than the measured reactive power,  $Q_1$ , for the value of CB-injected reactive power (measurements are done with CB connected):

$$\underline{S}_1' = P_1' + jQ_1' = P_1 + j(Q_1 + Q_C). \quad (7)$$

The value of  $V_0'$  can be obtained from (1) when  $P_1$  and  $Q_1$  are replaced with  $P_1'$  and  $Q_1'$ :

$$(V_0')^2 = \left( V_1' + \frac{P_1' R + Q_1' X}{V_1'} \right)^2 + \left( \frac{P_1' X - Q_1' R}{V_1'} \right)^2. \quad (8)$$

Under the assumption that  $V_0'$  is known and almost equal to  $V_0$ , the solution for  $V_1'$  is:

$$V_1' = \sqrt[3]{ \frac{\sqrt{(P_1' R + Q_1' X - 0.5V_0'^2)^2} - (R^2 + X^2)(P_1'^2 + Q_1'^2)}{- (P_1' R + Q_1' X - 0.5V_0'^2)} }. \quad (9)$$

The transformer secondary current without CB is:

$$I_1' = \frac{\sqrt{P_1' + Q_1'}}{\sqrt{3}V_1'}, \quad (10)$$

while the losses in series branch are calculated using (3), in which  $I_1$  is replaced with  $I_1'$ . The losses in shunt branch (constant losses) are equal to the losses in the case with CB, (4), since  $V_0$  is assumed to be the same before and after capacitor installation. The total transformer losses,  $\Delta \underline{S}_\Sigma' = \Delta P_\Sigma' + j\Delta Q_\Sigma'$ , and power flowing on transformer primary,  $\underline{S}_\Sigma' = P_\Sigma' + jQ_\Sigma'$ , respectively, can be calculated from relations analogous to (5) and (6).

### C. Inclusion of Voltage-Dependencies in the Analysis

As it is well known that the real and reactive power demands of the supplied load depend on voltage and frequency. The load models that take into account these dependencies are introduced in calculation. If the frequency dependence is neglected (as frequency variations are rather small in steady state operating conditions), a simplified exponential steady state load model is often used [24]:

$$P = P_n \left( \frac{V}{V_n} \right)^{k_{pv}}, \quad (11)$$

$$Q = Q_n \left( \frac{V}{V_n} \right)^{k_{qv}}, \quad (12)$$

where:  $P$  and  $Q$  are real and reactive power, respectively, at an operating voltage  $V$ ;  $P_n$  and  $Q_n$  are real and reactive power at rated/nominal voltage  $V_n$ , while  $k_{pv}$  and  $k_{qv}$  are real and reactive power voltage exponents. When both voltage exponents are 0, 1 or 2, the load model is termed as constant power, constant current or constant impedance, respectively.

The reactive power demand of shunt capacitor depends on the square of voltage:

$$Q_C = Q_{Cn} \left( \frac{V}{V_{Cn}} \right)^2, \quad (13)$$

where:  $Q_{Cn}$  is rated power of the capacitor and  $V_{Cn}$  is its rated voltage. The value of  $V_{Cn}$  of all CBs considered in this paper is 440 V. The injected reactive power of CB at voltage  $V_1$  (i.e. for the case with CB installed) is obtained by (13) with  $V$  replaced with  $V_1$  and denoted by  $Q_C(V_1)$ . Afterwards, reactive power of the load at voltage  $V_1$ ,  $Q(V_1)$ , is calculated by subtracting  $Q_C(V_1)$  from the measured reactive power on secondary LV side,  $Q_1(V_1)$ . If  $k_{qv}$  is known, reactive power of the load at rated voltage,  $V_n$ , will be obtained as:

$$Q_n = \frac{Q(V_1)}{\left( \frac{V_1}{V_n} \right)^{k_{qv}}}. \quad (14)$$

A similar relation for real power of the load at rated voltage, taking into account that  $P_1(V_1)=P(V_1)$ , can be written as:

$$P_n = \frac{P_1(V_1)}{\left( \frac{V_1}{V_n} \right)^{k_{pv}}}. \quad (15)$$

#### D. Calculation Procedure with Voltage-Dependencies

The values of  $Q_n$  and  $P_n$  calculated by (14) and (15) are used as the inputs in the iterative load flow procedure that takes into consideration voltage dependent load characteristics. This procedure is described in [20] on the example of a radial distribution network and is applied in this paper on distribution transformer equivalent circuit from Fig. 6, in order to obtain transformer operating conditions for the case without CB:

- Step 1. Assume flat voltage profile, i.e. that voltage  $V_1$  at Bus 1 in Fig. 7 is the same as voltage  $V_0$ .
- Step 2. Calculate real and reactive power of the load with (11) and (12), respectively, replacing  $V$  with  $V_0$  in the first iteration, and replacing  $V$  with adjusted voltage at Bus 1 in other iterations. These values are actually real and reactive powers flowing through the series branch 0-1 in Fig. 7, corresponding to the case without installed CB.
- Step 3. Calculate voltage at Bus 1 according to (9).
- Step 4. Compare voltage values calculated in the last two iterations. In the case the mismatch is less than a predefined small value (e.g. 0.1 V, which is the resolution of the recording equipment used in measurements in this paper), the iterative procedure is completed. Otherwise, go to Step 2.

Once the calculation procedure is completed, transformer secondary current, real and reactive power losses and apparent power on transformer primary side can be obtained by (10), (5) and (6), respectively. To allow for a clear distinction, variables obtained without CB but with voltage dependent characteristics are in further text denoted by a superscript  $^{vd}$ :  $V_1^{vd}$ ,  $I_1^{vd}$ ,  $P_1^{vd}$ ,  $Q_1^{vd}$ ,  $\Delta S_\Sigma^{vd}$  ( $\Delta P_\Sigma^{vd}$  and  $\Delta Q_\Sigma^{vd}$ ) and  $S_\Sigma^{vd}$ , in order

to distinguish them from the previous variables  $V_1'$ ,  $I_1'$ ,  $P_1'$ ,  $Q_1'$ ,  $\Delta S_\Sigma'$  ( $\Delta P_\Sigma'$  and  $\Delta Q_\Sigma'$ ) and  $S_\Sigma'$ , which correspond to the cases without CB and without considered voltage dependent characteristics. For better insight, the flowchart of iterative calculation procedure presented in this subsection is depicted in Fig. 8, where iteration number is denoted with  $k$ .

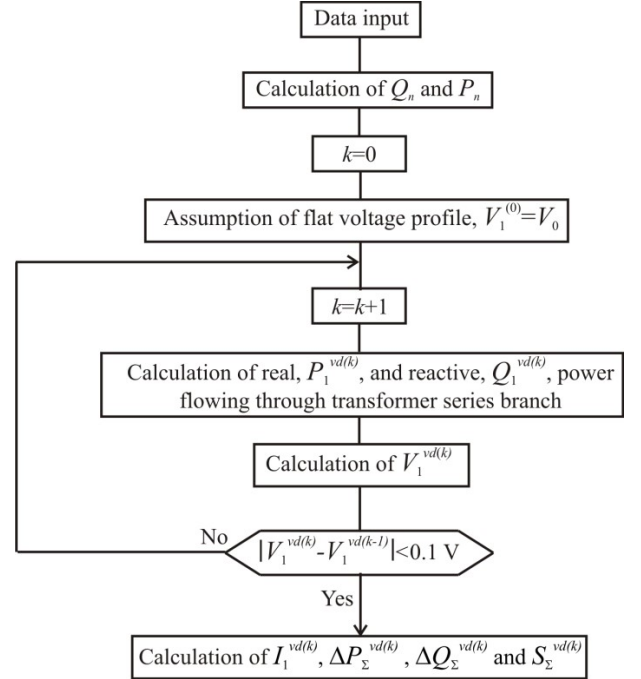


Fig. 8. The flowchart of the presented calculation procedure.

#### V. COMPARISON OF TRANSFORMER OPERATING CONDITIONS

The values of mentioned transformer variables are calculated after and before CB installation, according to the procedures described in Section IV. The differences are expressed in percentage of the corresponding variables obtained in the case before CB installation with included voltage dependent load characteristics. Accordingly, percentage change of transformer secondary current taking into account real and reactive load power and capacitor reactive power dependencies on voltage can be calculated as:

$$\Delta i_1 = \frac{I_1 - I_1^{vd}}{I_1^{vd}} \cdot 100. \quad (16)$$

Percentage error in transformer secondary current calculation made by neglecting real and reactive load power and capacitor reactive power dependencies on voltage is denoted by adding the superscript  $n$ :

$$\Delta i_1^n = \frac{I_1' - I_1^{vd}}{I_1^{vd}} \cdot 100. \quad (17)$$

Using equations analogous to (16) and (17), percentage change and calculation error, respectively, of: real power losses ( $\Delta p_\Sigma$  and  $\Delta p_\Sigma^n$ ), reactive power losses ( $\Delta q_\Sigma$  and  $\Delta q_\Sigma^n$ ) and apparent power ( $\Delta s_\Sigma$  and  $\Delta s_\Sigma^n$ ) can be obtained. Since transformer operating conditions change during the measurement period, the average (mean) values of the

individual percentage changes in all measurement instants ( $\Delta\bar{I}_1$ ,  $\Delta\bar{P}_2$ ,  $\Delta\bar{Q}_1$  and  $\Delta\bar{S}_2$ ) and average (mean) percentage calculation errors for all this instants ( $\Delta\bar{I}_1^e$ ,  $\Delta\bar{P}_2^e$ ,  $\Delta\bar{Q}_1^e$  and  $\Delta\bar{S}_2^e$ ) for the considered set of transformer variables calculated during one week of measurements are summarized in Tables I and III, respectively.

Table I lists the results for 10 transformers in ten transformer substations (TS-1 to TS-10), specifying cases of CB compensation (AC, OC, TC or CAC). The TS-1 to TS-8 are located in urban area of EDL, while TS-9 and TS-10 are in two rural settlements. Transformers in TS-1, TS-4, TS-5 and TS-7 are discussed in Section III as the examples of AC, OC, TC and CAC cases, respectively. The values of power factors measured in other six TSs, as the indicators of real and reactive power profiles and applied reactive power compensation, are briefly discussed below.

Measurements on transformer in TS-2 (AC case) showed that PF is greater than 0.95 lagging during whole measurement period, similar to the result obtained for TS-1. In TS-3 (OC case) power factor was in the narrower range than in the case of TS-4 and varied in the range  $0.83 \text{ leading} \leq \text{PF} \leq 0.99 \text{ leading}$  (50 % of time  $\text{PF} \geq 0.95 \text{ leading}$ ). In the case of total compensation noted in TS-6, PF is between 0.99 leading and 0.99 lagging, as in the described TS-5 case. PF in TS-9 (also with TC), was between 0.99 leading and 0.92 lagging, while for about 88 % of time it was in the range between 0.99 leading and 0.99 lagging. CAC is recorded in TS-8 and TS-10, where power factor is mostly leading (opposite to the case of TS-7, when 81 % of time PF is lagging). The TS-8 had PF in the range from 0.85 leading and 0.99 lagging, which was 90 % of time leading and 10 % of time lagging. PF of the TS-10 changed from 0.86 leading to 0.99 lagging, and for only about 3 % of time was lagging.

The classes of supplied loads are described below and denoted in Table I by the corresponding integer numbers, from 1 to 8. In load classes Class 1 – Class 5, 100 % of the load is of the given class, while Class 6 – Class 8 correspond to different mixed loads:

- Class 1. Residential load 1 (privately built urban houses without central heating),
- Class 2. Residential load 2 (flats in buildings without central heating, mostly using electrical heating),
- Class 3. Commercial load 1 (administrative buildings),
- Class 4. Commercial load 2 (commercial buildings),
- Class 5. Residential load 3 (rural houses),
- Class 6. Mixed load 1 (35 % of Class 2, 45 % of Class 3 and 20 % of Class 4),
- Class 7. Mixed load 2 (65 % of Class 1, 25 % of Class 3 and 10 % of Class 4),
- Class 8. Mixed load 3 (50 % of Class 1, 10 % of Class 2, 25 % of Class 3 and 15 % of Class 4).

Table I also provides information on seasons in which the measurements were performed, denoted as: S-summer, A-autumn and W-winter, as well as measured average real load power,  $P_{lav}$ , and measured average, maximum and minimum reactive powers on transformer secondary,  $Q_{lav}$ ,  $Q_{lmax}$ , and

$Q_{lmin}$ , respectively. Rated powers of installed CBs,  $Q_{Cn}$ , are also given in Table I. Appendix A lists transformer data required for calculation of their equivalent circuit parameters (either taken from transformer nameplate data, or, when this is not available, from [25]), together with the well-known relations for their calculations (e.g. from [2]).

TABLE I  
AVERAGE PERCENTAGE CHANGES OF TRANSFORMER CHARACTERISTIC VARIABLES AFTER CB INSTALLATION (FOUR COMPENSATION CASES)

TS	Comp. case	Load class, season	$P_{lav}$ [kW]	$Q_{lmax}$ [kvar]	$Q_{lmin}$ [kvar]	$Q_{lav}$ [kvar]	$Q_{Cn}$ [kvar]	$\Delta\bar{I}_1$ [%]	$\Delta\bar{P}_2$ [%]	$\Delta\bar{Q}_1$ [%]	$\Delta\bar{S}_2$ [%]
1	AC	6, W	134.2	41.7	8.3	22.9	50	-9.7	-4.6	-5.6	-10.3
2	AC	2, W	136.4	25.4	5.8	13.1	40	-7.1	-2.7	-2.5	-7.7
3	OC	7, A	150.0	-33.8	-65.9	-50.0	100	3.8	2.0	1.9	2.3
4	OC	3, A	60.7	-14.7	-65.5	-47.3	80	29.3	3.4	3.3	22.5
5	TC	1, W	257.5	23.9	-21.3	-2.1	75	-2.4	-2.6	-2.5	-2.8
6	TC	8, S	130.4	18.2	-36.7	-1.9	75	-9.4	-4.7	-4.7	-10.5
7	CAC	4, W	99.8	65.4	-19.6	20.5	25	-5.1	-3.3	-2.9	-5.7
8	CAC	1, A	154.4	23.3	-36.9	-16.4	75	-3.5	-2.1	-2.0	-4.7
9	TC	5, S	53.2	25.7	-8.2	2.2	20	-6.7	-2.9	-2.1	-7.7
10	CAC	5, A	44.2	6.8	-10.8	-5.9	20	-1.1	-0.5	-0.4	-3.1

The values of the parameters  $k_{pv}$  and  $k_{qv}$  from the model (11)-(12), for different load classes supplied by the transformers from Table I are obtained using the methodology described in [23]. In order to have more reliable results, parameters  $k_{pv}$  and  $k_{qv}$  are estimated using at least three EDL TSs supplying the same load class in the same season. The average values of  $k_{pv}$  and  $k_{qv}$  for particular season, day of the week (working day, Saturday and Sunday) and four daily time intervals (12 a.m.-6 a.m., 6 a.m.-12 p.m., 12 p.m.-6 p.m. and 6 p.m.-12 a.m.) are used for the calculation of transformer variables without CB installed, as described in Section IV.D.

Parameter values of  $k_{pv}$  and  $k_{qv}$  for the same exponential load model, in which the changes of load with frequency are neglected, for Mixed load 1-3 (again for particular season, day of the week and time interval) are calculated as [26]:

$$k_{pv} = \sum_i c_i k_{pvi}, \quad (18)$$

$$k_{qv} = \sum_i c_i k_{qvi}, \quad (19)$$

where:  $c_i$  is relative participation of load class  $i$  in total bus load, while  $k_{pvi}$  and  $k_{qvi}$  are load parameters of  $i^{\text{th}}$  load class for the corresponding season, day and time interval. In the example of load class denoted as Class 6, i.e. Mixed load 1,  $i=3$ ,  $c_1=0.35$  for Residential load 2,  $c_2=0.45$  for Commercial load 1, and  $c_3=0.2$  for Commercial load 2. Further, for the calculation of  $k_{pv}$  and  $k_{qv}$  of Mixed load 1, e.g. in winter season, during period 12 a.m.-6 a.m. on a working day, one average  $k_{pvi}$  and one average  $k_{qvi}$  value of three mentioned load classes ( $i=3$ ) for the same season, day of the week and time interval should be used. The total number of these average values used in the paper is large, and only characteristic values that correspond to the considered transformers are presented in Appendix B and also denoted as  $k_{pv}$  and  $k_{qv}$ .

The results in Table I demonstrate strong positive effects of adequate compensation (AC) on transformer operation for all considered transformer characteristic variables. Namely,



negative values of  $\Delta \bar{i}_1$ ,  $\Delta \bar{p}_2$ ,  $\Delta \bar{q}_2$  and  $\Delta \bar{s}_2$  denote average percentile decrease of: transformer secondary current, real, reactive and apparent transformer losses due to CB operation, in comparison with the corresponding variables before CB installation. These average percentile decreases are obtained by averaging the individual percentile decreases obtained in all measurement instants. On the other hand, the positive values obtained in the cases of overcompensation denote increase of listed variables and, therefore, negative effects of OC. In the cases of AC, the average values of transformer secondary current,  $\Delta \bar{i}_1$ , and apparent primary power,  $\Delta \bar{s}_2$ , are reduced for no more than approximately 10 %, while total real and reactive power losses,  $\Delta \bar{p}_2$  and  $\Delta \bar{q}_2$ , are reduced up to about 5 %.

Using the specific example of transformer TS-1 on the 27<sup>th</sup> January at 8 a.m., measured real and reactive powers, transformer secondary current and voltage are: 129.6 kW, 31.8 kvar, 187.5 A and 410.7 V, respectively. The secondary current without CB, calculated by the procedure presented in Subsection IV.B is 209.9 A. Thus, according to (16), the difference in transformer currents with and without CB installed is  $-10.7\%$ . The corresponding differences of the total real and reactive power losses, however, are lower:  $-5.6\%$  and  $-6.8\%$ , respectively. The main reason is a notable influence of constant real and reactive power losses, i.e. losses in the shunt branch of transformer equivalent circuit, which do not depend on operating current (these constant losses are the same in the regimes with and without CBs, and form the major parts of the total real and reactive power losses). For the considered operating point with installed CB, these losses are 815.2 W and 4100.2 var, while the total real and reactive power losses are 1083.7 W and 5871.8 var, respectively. In the regime without CB they are only slightly greater: 1147.7 W and 6299.6 var.

For the two examined OC cases in TS-3 and TS-4, the values of transformer variables increase after CBs are installed, suggesting that reactive power of CBs should be reduced, or existing CBs should be uninstalled. The change of reactive power flows through the transformers in OC cases results in the increase of current, apparent power and total real and reactive power losses, compared with the regimes without CBs. The average increases of current and apparent power in TS-4 are especially large (29.3 % and 22.5 %, respectively, as presented in Table I), resulting in a notable increase of transformer heating losses and reduction of transformer capacity for supplying additional real power load.

For instance, measured real and reactive powers in TS-4 on 27<sup>th</sup> October at 8:15 p.m with CBs installed are: 53.4 kW and  $-50.0$  kvar, respectively, as presented in Table II, while secondary voltage is 402.9 V. Calculated transformer secondary current, and total real and total reactive power losses in this regime, and in the regime without CBs are also presented in this table. Due to overcompensation, transformer current is 30.8 % greater with CBs than in the case without CBs (80.1 A). Total real and total reactive power losses with CBs are 866.2 W and 4338.4 var, i.e. only 3.9 % and 3.8 % greater than those without CBs, because of low load and

therefore significant contributions of constant real (787.1 W) and reactive power (3959.1 var) losses to the total transformer losses.

TABLE II  
EXAMPLE OF TRANSFORMER CHARACTERISTIC VARIABLES WITH AND WITHOUT CBs (OC CASE)

$P_1$ [kW]	$Q_1$ [kvar]	$I_1$ [A]	$\Delta P_\Sigma$ [kW]	$\Delta Q_\Sigma$ [kvar]	$I_1^{yd}$ [A]	$\Delta P_\Sigma^{yd}$ [kW]	$\Delta Q_\Sigma^{yd}$ [kvar]	$\Delta i_1$ [%]	$\Delta p_\Sigma$ [%]	$\Delta q_\Sigma$ [%]
53.4	-50.0	104.8	866.2	4338.4	80.1	833.4	4180.9	30.8	3.9	3.8

Table I shows that total compensation results in a decrease of transformer current and apparent power, with the average reduction of up to around 10 %. The average decreases of real and reactive power losses are up to around 5 %. For CAC cases, somewhat lower reductions of current and apparent power, up to around 5 %, and reduction of total real and reactive power losses, up to about 3 %, are found. The reduction of transformer current and apparent power enables additional load to be connected and defers related investments in additional transformers. Thus, capacities of approximately 79 % of TSs with CBs, with AC, TC and CAC (140 TSs) are released for about 5 % for connecting the new load. This should be taken into account when the procedures for sizing TSs are applied. In terms of the overall network effects, total real and reactive power losses in 140 TSs with AC, TC, and CAC are decreased for approximately 3%, while further economic effects of these compensation cases are the result of the released capacities of supplying MV lines, again manifested in the deferral of the investments in line upgrading and in reduced line losses.

Table III presents results for the calculation errors of transformer variables before the CB installation (see Section IV.B.), which are caused by neglecting the voltage dependencies of the TS-supplied loads and CBs' reactive powers. The average percentage error for calculated secondary current values,  $\Delta \bar{i}_1^{\%}$ , is obtained by averaging the results obtained by (17) at every measuring instant. Similar equations are used for averaging other characteristics, giving average percentage errors for primary apparent power,  $\Delta \bar{s}_2^{\%}$ , and total real,  $\Delta \bar{p}_2^{\%}$ , and total reactive,  $\Delta \bar{q}_2^{\%}$ , power losses. In TS-4 (OC case) and TS-6 (TC case), the average errors for secondary current and average primary apparent power are greater than 5 %, which is generally considered inadequate in engineering calculations. These results confirm that the voltage dependencies of loads and CB reactive powers should be taken into account for the accurate analysis of transformer operation.

Finally, the results in Table III show that the average errors for the total real and reactive power losses due to neglected voltage dependencies might be up to 3 %, as obtained in TS-6 (TC case). The errors are positive, indicating that the greater values for transformer characteristics are obtained when voltage dependencies are not taken into account, i.e. when the load and CB reactive power are assumed to be constant. Accordingly, transformer variables calculated without considering actual voltage dependencies will result in underestimation of transformers' capacity and stricter than necessary operating conditions.

TABLE III  
AVERAGE PERCENTAGE ERRORS OF TRANSFORMER VARIABLES DUE TO  
NEGLECTED VOLTAGE DEPENDENCIES (FOUR COMPENSATION CASES)

TS	Comp. case	$\Delta I_1^n$ [%]	$\Delta p_F^n$ [%]	$\Delta q_F^n$ [%]	$\Delta s_F^n$ [%]
1	AC	2.9	1.6	1.9	3.0
2	AC	1.8	0.8	0.7	1.8
3	OC	4.1	2.6	2.5	4.3
4	OC	9.1	1.5	1.4	10.0
5	TC	2.1	2.3	2.2	2.1
6	TC	5.6	3.0	3.0	5.6
7	CAC	1.7	1.1	1.0	1.8
8	CAC	3.4	1.9	1.8	3.5
9	TC	2.8	1.4	1.1	2.8
10	CATC	3.5	1.3	1.0	3.8

A more detailed overview of the potential impact of voltage dependencies is presented as box/whisker plots in Fig. 9, where the ranges of  $\Delta I_1^n$ ,  $\Delta p_F^n$ ,  $\Delta q_F^n$  and  $\Delta s_F^n$  from Table III are represented as the extent of whiskers, the extent of boxes show the 25<sup>th</sup> and 75<sup>th</sup> percentiles and the lines in boxes indicate the median (50<sup>th</sup> percentile) values of the calculation errors of transformer variables. It can be concluded that the errors in calculation if voltage dependencies are neglected will vary in the wide ranges:  $\Delta I_1^n \in (1.7; 9.1)$ ,  $\Delta p_F^n \in (0.8; 3.0)$ ,  $\Delta q_F^n \in (0.7; 3.0)$  and  $\Delta s_F^n \in (1.8; 10.0)$ . The median values of  $\Delta I_1^n$  and  $\Delta s_F^n$  (around 3.2 %) and median values of  $\Delta p_F^n$  and  $\Delta q_F^n$  (about 1.6 %) can be adopted as typical errors introduced by ignoring voltage dependencies in the analyzed operation conditions of considered transformers. As presented in [20], the influence of load voltage dependencies on load flow calculation of medium voltage distribution network strongly depends on both load parameters and actual supply voltage. Thus, the quantification of the overall influence of voltage dependencies on the whole distribution network performance requires additional research on typical network voltage profiles.

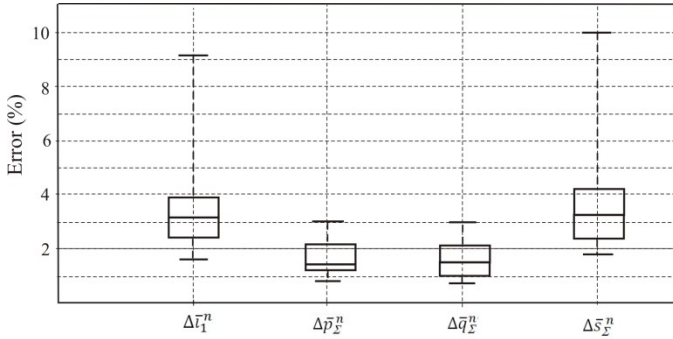


Fig. 9. Box/Whisker plots for the errors in calculation of transformer variables if voltage dependencies are neglected.

## VI. CONCLUSIONS

This paper presents the analysis of an extensive field measurement campaign of transformer substations (TSs) with dedicated capacitor banks (CBs) for reactive power compensation. In particular, the paper evaluates and quantifies the effects of CBs on the operation of distribution transformers, which are not considered in any of the previous work. Furthermore, the paper compares the results for main

operational transformer characteristics with and without taking into account the actual voltage dependencies of supplied load and CB reactive power. These results quantify the average effects of reactive power compensation identified during the measurement period. Positive effects, manifested as reduced operating currents and apparent powers are found in eight of ten in-detail examined TSs (up to about 10 % is found for TS-1, AC case, and TS-6, TC case), as well as the reduction of total real and reactive power losses (up to around 5 % for the same two TSs). The analysis also allows for a clear identification of overcompensation cases, when existing CBs should be uninstalled or modified.

The results for voltage dependent load characteristics, obtained in the same network for different load classes, seasons, days of the week and time periods of the day, clearly indicate that voltage dependencies should be applied for the evaluation of effects of shunt capacitors on the operation of distribution transformers. If they are neglected, average errors for calculated current and apparent powers might be greater than 5 %, as found for some of considered TSs, with typical values around 3 %. Since distribution networks might have very large number of secondary TSs, the aggregate-level impact of installed shunt capacitors and considered voltage dependencies can be significant.

In considered EDL network, the current and apparent power of transformers in about 140 TSs decrease on average by 5% due to installed CBs, helping to release the transformer capacity for the additional load and to defer investments in the new transformers. On the other hand, in about 37 TSs with overcompensation, it is found that CBs should be retrofitted, since they reduce available transformer capacities and increase their losses. Thus, the presented analysis can be also used as a part of the standard procedures for sizing distribution transformers with installed dedicated CBs for reactive power compensation.

## VII. APPENDIX A

This appendix provides data from transformer nameplates required for the calculation of equivalent circuit parameters: rated power,  $S_n$ , rated voltage,  $V_{Tn}$  and relative short-circuit voltage,  $u_k$ . Other transformer data in EDL are adopted from [25]: core losses,  $P_0$ , full load copper losses,  $P_{Cum}$  and no-load current,  $i_0$ . Their values for all considered transformers in TSs are given in Table A.I and are used for the calculation of transformer characteristics on secondary/LV side, assuming rated transformer secondary voltage,  $V_{Tn}=0.42$  kV:

$$Z = \frac{u_k}{100} \frac{V_{Tn}^2}{S_n/1000}, \quad (20)$$

$$R = P_{Cum} 1000 \left( \frac{V_{Tn}}{S_n} \right)^2, \quad (21)$$

$$X = \sqrt{Z^2 - R^2}, \quad (22)$$

$$Y = \frac{i_0}{100} \frac{S_n/1000}{V_{Tn}^2}, \quad (23)$$

$$G = \frac{P_0 / 1000}{V_{Tn}^2}, \quad (24)$$

$$B = -\sqrt{Y^2 - G^2}. \quad (25)$$

TABLE A.I  
TRANSFORMER DATA

TS	$S_n$ [kVA]	$P_0$ [kW]	$P_{Cun}$ [kW]	$i_0$ [%]	$u_k$ [%]
1.	630	0.86	5.4	0.7	5.8
2.	630	0.86	5.4	0.7	4.05
3, 4.	630	0.86	5.4	0.7	4.2
5, 8.	630	0.86	5.4	0.7	4.1
6.	630	0.86	5.4	0.7	4.4
7.	400	0.61	3.85	0.8	4.1
9.	250	0.425	2.75	0.9	3.9
10.	250	0.425	2.75	0.9	3.9

### VIII. APPENDIX B

The total number of  $k_{pv}$  and  $k_{qv}$  values used in the calculation is 120. This number is obtained by multiplying the number of the considered transformers (i.e. 10) by the number of characteristic days of the week (i.e. three, for working days, Saturday and Sunday) and the number of time intervals (i.e. four, for 12 a.m.-6 a.m., 6 a.m.-12 p.m., 12 p.m.-6 p.m. and 6 p.m.-12 a.m.). Due to space limitation, Table A.II lists the maximum and minimum values, sorted by the corresponding TSs, as well as the average (“Aavg”) values and standard deviations (“StD”) obtained for a one week period (five working days, Saturday and Sunday). According to Table A.II, it can be concluded that the ranges of  $k_{pv}$  and  $k_{qv}$  values of all considered transformer loads are wide, but standard deviations are relatively small.

TABLE A.II  
THE MINIMUM AND MAXIMUM VALUES OF LOAD PARAMETERS APPLIED TO  
CALCULATIONS FOR CONSIDERED TSs

TS	Load class, season	$k_{pv}$				$k_{qv}$			
		min	max	Avg	StD	min	max	Avg	StD
1.	6, W	1.21	1.77	1.49	0.13	2.54	3.79	3.05	0.32
2.	2, W	0.77	2.04	1.72	0.24	1.24	3.21	2.49	0.43
3.	7, A	1.06	2.35	1.57	0.28	2.25	3.83	2.87	0.34
4.	3, A	0.53	2.11	1.31	0.34	1.76	4.84	3.64	0.60
5.	1, W	1.44	2.26	1.80	0.20	0.93	4.10	2.54	0.91
6.	8, S	0.84	1.93	1.65	0.22	2.00	3.08	2.52	0.27
7.	4, W	0.54	1.98	1.59	0.32	1.63	4.97	2.86	0.69
8.	1, A	1.09	3.18	1.71	0.46	2.08	3.63	2.61	0.38
9.	5, S	1.08	2.16	1.46	0.22	2.15	3.57	2.86	0.35
10.	5, A	1.45	2.18	1.78	0.16	2.20	4.32	3.11	0.58

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