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Specialisation Patterns, GDP Correlations and External Balances*

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Abstract

This paper provides evidence of a link between specialisation patterns − in intermediate inputs or final goods − and business cycle correlations: countries with a similar intermediate-good content of exports tend to have more correlated GDP fluctuations and external balances. We produce a model that replicates these facts. A productivity shock in a large country ("the U.S.") has a smaller effect on the terms of trade of countries that share its specialisation, while being shared fully with countries specialised in the other type of good through a terms-of-trade effect. In the presence of complete asset markets, the trade balance reflects the flow of insurance payments. All countries who benefit little from the shock in the large country will have correlated, negative net exports. The trade balances of all other countries will jointly move in the opposite direction.

KEY WORDS: international business cycles, net exports, intermediate inputs
JEL CLASSIFICATION: F4

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1 Introduction

International trade is widely recognized as an important transmission mechanism of international business cycles (for example, see Baxter and Kouparitsas, 2004). However, as yet there seems to be little understanding of how differences in industrial structures across countries affect the business-cycle transmission via trade linkages, and the determination of trade balances.[1]

This paper considers one key difference between countries – the extent to which they act as intermediate-input suppliers as opposed to final-good producers in international production chains. We gauge the difference in countries' production structures along this dimension from the share of their exports used as production inputs, and highlight that this measure of specialisation seems to be relevant for predicting the strength of business-cycle co-movement: country pairs that display more similar specialisation patterns appear to have more correlated GDP fluctuations and trade balances.

We then offer an explanation for our empirical findings based on a model with a large country ("the U.S.") and many small countries specialised either in intermediate or final-good production. A crucial assumption of the model is that the elasticity of substitution between intermediate and final goods is 1, while the elasticity of substitution between different varieties of intermediate goods, or different varieties of final goods, is larger than 1. In this setting, shocks to the large country affect countries with different specialisation patterns differently through the terms of trade. Countries sharing the large country's specialisation experience a smaller increase in incomes than the rest of the world, because their terms of trade respond less strongly. This gives rise to a positive association between similarity in specialisation patterns and GDP correlations.

When allowing for international asset trade, our model also has implications for the trade balance. The GDPs of countries that share the same specialisation pattern are affected similarly by shocks originating in the large country. Thus, to the extent that the trade balance reflects international insurance payments, the trade balances of similarly specialised countries will move in the same direction.

International production linkages – whereby industries in some countries

[1] Backus et al. (1992) first pointed out a number of anomalies in the canonical open-economy real business cycle model. The most relevant of these to our work is the so-called "trade co-movement puzzle", established by Kose and Yi (2006): the empirically observed relationship between bilateral trade and GDP correlations is far stronger than the standard model would suggest. Subsequently, Kraay and Ventura (2007) pointed out that the strength and synchronisation of business cycles may vary across country pairs in a model of trade due to comparative advantage.

emerge as input suppliers for industries in others − are an increasingly important phenomenon in the global economy: recent estimates suggest that they have been responsible for 30-50% of the expansion of world trade since 1970 (Hummels et al., 2001; Yi, 2003). A growing literature explores the influence of trade in intermediate goods in different areas of macroeconomics: Yi (2003, 2010) first introduced input trade in standard international macro models, showing that this improves the ability of quantitative trade models to account for the growth of trade during the second half the 20th century, and the size of the perceived "home bias" in international trade; Bems et al. (2011) use a global input-output framework to study the decline of international trade during the global recession of 2008-09; Bems (2014) highlights the importance of intermediate inputs trade when assessing the quantitative response of relative prices to external rebalancing; finally, Johnson (2014) shows that introducing intermediate-inputs trade into a standard international business cycle model improves its ability to replicate the relation between bilateral trade and business cycle co-movement.

Our model can be viewed as a stylised version of the many-country, input-output model of international business cycles in Johnson (2014). By adopting stark but tractable assumptions about the nature of international production linkages, production technologies and country sizes we are able to derive analytically several predictions about international business cycle co-movement which seem to accord with the data. In particular, our simple model allows us to cast the spotlight on the cross-country correlation of trade balances, and to illustrate a novel insight: generating a trade surplus over the long run is much less costly for a country (in utility terms) if other countries generating trade surpluses at the same time do not share its specialisation pattern; by contrast, if "similar" countries are attempting to generate a trade surplus at the same time, the resultingly depressed terms of trade imply that lower consumption levels and leisure are required to maintain the same external position in a given country.[2]

The rest of the paper is structured as follows. Section 2 presents some empirical evidence about countries' specialisation in international production networks, and the association between specialisation and business-cycle co-

[2]Casual comments by policy makers suggest that production linkages, specialisation patterns and the cross-country allocation of trade surpluses are understood to play an important role in external adjustment episodes in practice. For example, contrary to the conventional wisdom about German export surpluses in the context of the euro-area crisis, the Spanish government explicitly defended Germany's surpluses in the autumn of 2013 on the basis that *"...Spain benefits from Germany's export success because so many of Spain's own exports to Europe's largest economy come in the form of intermediate goods. Spanish shipments of car parts and chemicals are then used by German companies to create finished products that are in turn sold overseas."* (*Financial Times*, November 19th, 2013)

movement. Section 3 outlines a many-country model with different specialisation patterns and different country sizes which we use to interpret that evidence. We also use the model to assess the determinants of the costs of servicing a given level of external liabilities. Section 4 offers some conclusions.

2 Empirical Evidence

2.1 Data and Variables

In the following, we use data from two main sources: the value of trade in intermediate and final goods from the World Input Output database (WIOD), and seasonally adjusted GDP and net exports statistics from the OECD quarterly national accounts.

The WIOD provides a global input-output table covering 40 economies, detailing the value of goods purchased from 35 industries in each country by the same 35 industries as well as 5 "final" sectors (roughly, consumption, investment and government spending) in each country. The WIOD covers the years 1995-2011, and makes it possible to identify the value of exports of each country-industry, as well as the share of these exports being used as intermediate goods by industries in the importing countries.[3] We use this information to calculate, for each country, the average share of intermediate goods in the value of exports across the 16 goods (i.e. manufacturing) industries in the WIOD, and treat this statistic as a measure of the country's specialisation in international production networks.

GDP and trade-balance correlations are calculated in line with standard practice in the international business cycles literature. We de-trend quarterly data on seasonally adjusted constant-dollar GDP and net exports as a share of GDP using the Hodrick-Prescott filter. Business-cycle correlations are then derived as the simple pairwise correlation of the de-trended GDPs and trade balances. We restrict ourselves to the years 1990-2014 for calculating the correlations, in line with the time coverage of the WIOD. Combining the OECD and WIOD data, we end up with pairwise correlations and specialisation measures for a sample of 30 major economies.

[3] While the construction of international input-output tables generally needs to rely on so-called "import proportionality" assumptions to allocate the observed use of imports by industries and final consumers to their likely countries of origin, the WIOD improves on common practices by using more dis-aggregated trade data in order to ensure that proportionality assumptions are only used within use categories (intermediate or final), rather than across those categories (see Timmer et al., 2015). This makes it especially suitable for our purpose.

2.2 Specialisation Patterns

In this section, we first describe the international specialisation patterns − in intermediate inputs or final goods − which we obtain from the data. To illustrate the importance of international trade in intermediate goods, Figure 1 reports the dollar value of U.S. imports from (left-hand panel) and exports to (right-hand panel) various trade partners for the year 2005, segmented by use category. Trade partners are ranked by the corresponding volume of trade. In both panels, the red segment of each bar represents the value of goods shipped for use as intermediate inputs, whereas the green segment represents the value of final goods shipped. For example, out of a total of $300 billion dollars of U.S. imports from Canada, just over a third were imports of final goods, with the remaining two thirds accounted for by intermediate-good imports.[4] More generally, the right-hand panel shows that most of the value of U.S. exports to its various trading partners in 2005 was derived from intermediate-good shipments.

In the following, we will use the simple average of the share of intermediates in a country's exports across the 16 WIOD manufacturing industries as a measure of the country's specialisation in international production networks. Figure 2 reports the 2005 value of this statistic for each of the countries in our sample. Countries are ranked from lowest to highest intermediate-good content of exports. In comparison with the rest of the sample, Greece appears to be most specialised in final goods, while Indonesia appears to be primarily an intermediates exporter. Note also that the U.S. has a relatively high intermediate-good content of exports.

Our preferred measure of specialisation is the *simple* average of the intermediates share of exports across industries. Alternatively, we could weight each industry by its share in the country's overall manufacturing exports − which is equivalent to aggregating all industries into one and calculating the intermediate share of exports for the country's manufacturing sector as a whole. Figure 3 plots these two alternative measures of specialisation against each other. The figure shows that, with the exception of Australia, all observations are roughly distributed along a straight line. This suggests that, for most countries, a high aggregate intermediates share of exports is driven by a high intermediate-goods content of exports at the industry level, rather than by composition effects. Not surprisingly in the light of this finding, our results below are robust to the use of either specialisation measure.

Figure 4 shows that our measure of specialisation is relatively persistent

[4]Note that U.S. imports from each of its trading partners exceeded U.S. exports to that trading partner in 2005, reflecting the U.S. trade deficit in that year.

over time. We plot each country's measure of the intermediate-goods content of exports for the year 2011 against the corresponding value for the year 1995 and find that most observations are roughly aligned along the 45-degree line. For the regressions below, we will use the 2005 value of countries' intermediate-goods content of exports, but all results are robust to using other years, or the 1995-2011 average.

2.3 Production Structures and Business-Cycle Correlations

We now turn to the relationship between similarity in specialisation patterns and business-cycle correlations. Table 1 reports some descriptive statistics for the correlations of GDPs and net exports among our sample countries. As is well known, cross-country correlations in GDP fluctuations are generally positive but there is a large degree of variation in the observed pairwise correlations. Cross-country trade-balance correlations have received less attention in the literature. We find that the average net-export correlation is close to zero. However, just as with GDPs, there is significant variation in the correlation of trade balances across country pairs.

Table 2 analyses the relationship between similarity in specialisation − as measured by countries' average intermediates share of industry-level exports − and business-cycle synchronisation. It reports the results from running an OLS regression with the pairwise GDP correlation as the dependent variable and the pairwise absolute difference between specialisation measures as the main independent variable. We also use the log of distance between country pairs as an additional control since GDP correlations are known to decline with distance. In the first five columns, we use simple industry averages to construct our specialisation measure, whereas in the sixth column we use the weighted average. In the first two columns we consider all countries, whereas in columns three and four we exclude non-OECD countries. In columns five and six we exclude Australia only.

In line with the earlier literature, we find a robust and statistically significant negative relationship between distance and GDP correlations. In addition, differences in countries' specialisation patterns appear to reduce business-cycle synchronisation. This effect is statistically and economically significant: our estimates in column (2) suggest that an increase in specialisation differences from the 10th to the 90th percentile of the country-pair distribution should be associated with a reduction in the pairwise GDP correlation by

.07.[5] The effect appears to be weaker for OECD countries, but this may be due to the fact that there is less variation in specialisation patterns in this much smaller, more homogeneous sample.

Table 3 repeats the regression analysis presented in Table 2 with net-export correlations as the dependent variable. We find little evidence of a relationship between distance and net export correlations but, once again, observe that specialisation differences appear to go hand-in-hand with lower trade-balance correlations. This effect is statistically and economically significant, and very robust: again, our estimates in column (2) suggest that an increase in specialisation differences from the 10th to the 90th percentile of the country-pair distribution should be associated with a fall in the pairwise net-export correlation by .07.

If trade balances respond to GDP shocks, it is conceivable that the association between trade-balance correlations and specialisation differences results from the relationship between *GDP correlations* and specialisation differences documented above. To assess this, we regress the correlation of net exports on the part of the GDP correlation explained by specialisation differences. In a first stage, we instrument for the GDP correlation with the similarity proxy (as well as the log of distance). In the second stage, reported in Table 4, we regress the net-export correlation on the instrumented GDP correlation. The results suggest that the intuition set out above may be correct: differences in specialisation patterns result in less correlated business cycles, and these in turn reduce the correlation of trade balances. In the rest of the paper we formalise this intuition and explore some of its implications for the costs of servicing a given level of foreign liabilities through trade surpluses.

3 The Model

Consider a world with one large country and a continuum of small countries.[6] There is an infinitely-lived representative consumer in each country n with instantaneous utility function

$$U\left[C_t(n), L_t(n)\right] = \ln C_t(n) - \frac{L_t(n)^{1+\eta}}{1+\eta}, \tag{1}$$

[5] For comparison, an increase in distance from the 10th to the 90th percentile of the country-pair distribution would reduce the pairwise GDP correlation by .26.

[6] In accordance with some of the evidence presented above, we think of the U.S. as the large country. Figure 5 plots the share of output of each country in our sample over the total sample output for the year 2005. The U.S. stands out as the largest country, with a large advantage over the second-largest country, Japan.

where C denotes final consumption and L denotes labour. Consumers maximise the expected present value of lifetime utility, valued with a common discount factor $\beta \in (0,1)$.

Let there be a total mass $2-n^*$ of small countries. All $n \in [0,1]$ produce a unique variety of F-good. F-goods are used in final consumption. All $n \in [1, 2-n^*]$ produce a unique variety of M-good, whereas the large country produces a mass n^* of varieties of the M-good. M-goods serve as intermediate inputs in the production of F-goods, which are produced by aggregating labour services and a CES aggregator of all available M-goods in a Cobb-Douglas fashion:

$$Q_{Ft}(n) = \left(\frac{A_t(n)L_{Ft}(n)}{1-\gamma}\right)^{1-\gamma}\left[\int_1^2 \left(\frac{X_t(m,n)}{\gamma}\right)^{\frac{\varepsilon-1}{\varepsilon}} dm\right]^{\frac{\gamma\varepsilon}{\varepsilon-1}}. \tag{2}$$

M-goods are produced with labour only:

$$Q_{Mt}(n) = A_t(n)L_{Mt}(n), \tag{3}$$
$$Q_{Mt}(n^*) = A_t^* L_{Mt}^*. \tag{4}$$

$L_{kt}(n)$ is labour used in the production of variety n of good k, $A_t(n)$ is labour productivity in country n, and $X_t(m,n)$ denotes the use of variety m of good M as *intermediate input* in the production of n, with $k \in \{F, M\}$ and asterisks denoting aggregate variables in the large country. The parameter $\varepsilon \geq 1$ captures the elasticity of substitution between varieties of different goods, and the parameter $\gamma \in (0,1)$ reflects the "intermediate-input intensity" of final production.

Final consumption in each country is a CES aggregator of F-goods:

$$C_t(n) = \left[\int_0^1 C_t(f,n)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{5}$$

All goods and labour markets are perfectly competitive. Varieties of F-goods and M-goods can be traded freely between countries. We find it convenient to take the price of the final good as the numéraire.

We consider two different scenarios: one in which consumers cannot trade assets internationally (financial autarky), and one in which consumers can also trade a complete set of state-contingent Arrow-Debreu securities (complete asset markets).

3.1 Financial Autarky

In the Appendix we discuss the model's equilibrium conditions. Manipulating them we can obtain expressions for each country-type's world income share. In the case of F-good producers,

$$\frac{W_t(n)L_t(n)}{Y_t} = (1-\gamma)\,\frac{[A_t(n)]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}}}{\int_0^1 [A_t(n)]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}}\,dn}, \tag{6}$$

where

$$Y_t \equiv \int_0^{2-n^*} W_t(n)L_t(n)dn + W_t^* L_t^* \tag{7}$$

denotes world income: due to perfect competition, the only source of income in the model is labour income.

Note that the shares in world income of producers of different types of goods are isolated from whatever happens to the other type. This is due to the unitary elasticity of substitution implicit in the Cobb-Douglas production function for F-goods: world expenditure on final output is always split in proportion $(1-\gamma)$ for the producers of final goods and γ for the producers of intermediate inputs used in the production of those final goods, ensuring that the aggregation of income over all producers of a type always yields a constant share of world GDP.

How individual F-goods producers' incomes compare with one another depends on their relative productivity shocks. Since the elasticity of substitution between varieties of final goods is larger than one (by assumption), final-good producers with higher productivity levels command larger market shares in the final-goods market and thus earn a higher income share than low-productivity final-good producers. By the law of large numbers, the income of F-good producers only depends on the realisation of their own productivity level: the average realisation of productivities of F-good producers is always the same.

In the case of M-good producers,

$$\frac{W_t(n)L_t(n)}{Y_t} = \gamma\frac{[A_t(n)]^{\frac{\varepsilon-1}{\varepsilon}}}{\int_1^{2-n^*}[A_t(n)]^{\frac{\varepsilon-1}{\varepsilon}}\,dn + n^*\left(\frac{A_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}}, \tag{8}$$

$$\frac{W_t^* L_t^*}{Y_t} = \gamma\frac{n^*\left(\frac{A_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{\int_1^{2-n^*}[A_t(n)]^{\frac{\varepsilon-1}{\varepsilon}}\,dn + n^*\left(\frac{A_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}}. \tag{9}$$

The presence of a large M-good producer makes a difference for M-producing countries, as the productivity shifter A^* contributes to a positive correlation between the income shares of any two small M-good producers. A shock to A^* also raises world income, Y_t, but alters its distribution because the resulting changes in world prices affect different countries differently: a stronger terms-of-trade response implies that the GDP of F-good producers will rise proportionally with world income, while the GDP of small M-good producers will rise less than proportionally. Thus, the incomes of country pairs with similar specialisation patterns are more positively correlated than the incomes of country pairs with different specialisations.[7]

3.2 Complete Asset Markets[8]

Let us now assume now that representative agents can also trade a complete set of state-contingent Arrow-Debreu securities. In this section we first show that the results we derived above under financial autarky also hold in the presence of asset trade. More importantly, allowing for international asset trade enables us to study the implications of international differences in specialisation patterns for the trade balance (which is, of course, zero under financial autarky), and how changes in asset positions affect different types of countries according to the environment they find themselves in.

With complete asset markets, the competitive-market equilibrium allocations coincide with those of the solution to the following planner problem:

$$\max_{\{C_t(n),L_t(n)\}_{n,t}} E_0 \left\{ \beta^t \left[\int_0^{2-n^*} \theta(n) U \left[C_t(n), L_t(n) \right] dn + \theta^* U \left(C_t^*, L_t^* \right) \right] \right\} \quad (10)$$

subject to

$$\int_0^{2-n^*} C_t(n) dn + C_t^* \leq Y_t. \quad (11)$$

$\theta(n) \geq 0$ represents the planner's weight on country n, which depends on its expected future income and initial foreign asset position, and $\int_0^{2-n^*} \theta(n)dn + \theta^* = 1$.

The above expressions for pricing conditions, goods-market clearing conditions and labour demands still hold under complete asset markets. The solution to the planner's problem above yields final-consumption levels

$$C_t(n) = \theta(n) Y_t, \quad (12)$$

$$C_t^* = \theta^* Y_t, \quad (13)$$

[7]The correlation between the incomes of any small M-country and the large country is lower than the correlation between any two small M-countries, as a positive shock to A^* contributes negatively to the income of the former and positively to the income of the latter.

[8]See the Appendix for the derivation of all the results discussed here.

and labour supplies

$$L_t(n)^\eta = \frac{\partial Y_t / \partial L_t(n)}{\theta(n) Y_t}, \tag{14}$$

$$L_t^{*\eta} = \frac{\partial Y_t / \partial L_t^*}{\theta^* Y_t}. \tag{15}$$

With complete asset markets, a country's consumption level is insulated from any idiosyncratic risk. Its representative consumer simply receives a share of world output (the size of which depends on her share of world wealth in the decentralised equilibrium, as we show in Section 3.2.3). A country's labor supply now increases in the states of nature in which its marginal contribution to world income is high.

3.2.1 Income Shares

Manipulation of the equilibrium conditions yields expressions for each country-type's world income share. In the case of F-good producers,

$$\frac{W_t(n)L_t(n)}{Y_t} = (1-\gamma)\frac{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}}}{\int_0^1 \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}} dn}. \tag{16}$$

In the case of M-good producers:

$$\frac{W_t(n)L_t(n)}{Y_t} = \gamma\frac{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{\int_1^{2-n^*}\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^*\left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}, \tag{17}$$

$$\frac{W_t^* L_t^*}{Y_t} = \gamma\frac{n^*\left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{\int_1^{2-n^*}\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^*\left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}} \tag{18}$$

These expressions are very similar to the ones we obtained under financial autarky and display qualitatively similar results: the incomes of country pairs with similar specialisation patterns are more positively correlated than the incomes of country pairs with different specialisations.

The differences between equations (6), (8) and (9) and (16)-(18) boil down to the presence of parameters θ and η. Parameter η controls the elasticity of labour supply, which now reacts to changes in the marginal product of labour.[9] This modifies the quantitative response of income shares to produc-

[9] In a decentralised equilibrium under financial autarky, the substitution effect from a higher wage is exactly compensated by the corresponding wealth effect, thus leaving labour supply constant. With complete asset markets, the substitution effect continues to be

tivity shocks. Parameter $\theta(n)$ is the corresponding country's planner weight. We will show in Section 3.2.3 that it maps into the country's share of world wealth in the decentralised equilibrium. The higher $\theta(n)$, the less need for country n to generate individual income, as a larger share of world income accrues to it anyway.

3.2.2 Trade Balances

Define $NX_t(n)$ as the trade balance of country n at time t:

$$\frac{NX_t(n)}{Y_t(n)} = 1 - \frac{C_t(n)}{W_t(n)L_t(n)} = 1 - \frac{1}{L_t(n)^{1+\eta}} = 1 - \frac{\theta(n)Y_t}{W_t(n)L_t(n)} \tag{19}$$

Substituting the income share we obtained above yields the following expression for the case of F-good producers:

$$\frac{NX_t(n)}{Y_t(n)} = 1 - \frac{\theta(n)}{1-\gamma} \frac{\int_0^1 \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}} dn}{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}}}. \tag{20}$$

Similarly, for M-good producers we obtain

$$\frac{NX_t(n)}{Y_t(n)} = 1 - \frac{\theta(n)}{\gamma} \frac{\int_1^{2-n^*} \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}, \tag{21}$$

$$\frac{NX_t^*}{Y_t^*} = 1 - \frac{\theta^*}{\gamma} \frac{\int_1^{2-n^*} \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}. \tag{22}$$

In the decentralised equilibrium, a shock to A^* raises Y_t and changes world relative prices. The resulting terms-of-trade changes offer insurance to F-good producers in the sense that their incomes rise proportionally to world income. However, M-producers experience a fall in their share of world GDP.[10] With complete asset markets, goods flow from the large country to the other, small M-good producers as insurance payment in this scenario. The correlation of net exports is therefore also higher for country pairs that share the same specialisation pattern than for country pairs with different specialisations.

present, whereas the wealth effect is not, as a country's consumption is no longer linked to its income.

[10] See Cuñat and Fons-Rosen (2013) for a related argument.

3.2.3 Decentralisation

A comparison between the planner's problem and the representative agent's optimisation problem in the decentralised equilibrium helps us illustrate the effect of a given level of foreign liabilities on consumption and labour supply. In the decentralised equilibrium, the representative consumer in country n solves

$$\max_{\{C_t(n,s_t),L_t(n,s_t)\}_{s_t}} \sum_{t=0}^{\infty} \beta^t \int_{s_t} \pi_t(s_t) U\left[C_t(n,s_t), L_t(n,s_t)\right] ds_t \tag{23}$$

subject to

$$\sum_{t=0}^{\infty} \int_{s_t} q_t(s_t) C_t(n,s_t) ds_t \leq \sum_{t=0}^{\infty} \int_{s_t} q_t(s_t) W_t(n,s_t) L_t(n,s_t) ds_t - B_0(n), \tag{24}$$

where $\pi_t(s_t)$ denotes the probability of state of nature s at time t, $q_t(s_t)$ is the price of an Arrow-Debreu security yielding one unit of consumption in state s at time t, and $B_0(n)$ represents country n's net foreign liabilities as of period 0.[11] One can show that the solution of this problem implies

$$\theta(n) = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \int_{s_t} \pi_t(s_t) \frac{W_t(n,s_t)L_t(n,s_t)}{Y_t(s_t)} ds_t - \frac{B_0(n)}{Y_0}\right]. \tag{25}$$

A country's planner weight, $\theta(n)$, thus corresponds to a measure of its initial wealth, consisting of two terms: a first term reflecting the discounted expected share of country n in world GDP going forward, and a second term representing n's initial liabilities relative to initial world GDP. Note that, from (16)-(18), the first term depends implicitly on $\{\theta(n)\}_n$. However, as demonstrated in the Appendix, $\theta(n) < \theta(n')$ implies $B_0(n) > B_0(n')$ for all $n \neq n'$, everything else constant.

Consider now a country with a high initial level of indebtedness, $B_0(n)$. This country will need to run trade surpluses in the future in order to service its liabilities. Doing so implies foregoing some final consumption and supplying a larger level of labor for the indebted country: a higher initial level of net foreign liabilities lowers $\theta(n)$, thus prompting a lower consumption level, by equation (12), and a larger supply of labor, by equation (14).

However, if other countries with the same specialisation pattern also have a low planner weight − also due to a high level of foreign liabilities, say −, this will reduce country n's expected share of future incomes, by equations (16) and (17), further lowering $\theta(n)$. Hence, servicing a given amount of initial liabilities under these circumstances will require an increased effort in terms

[11] Note that $\int_0^{2-n^*} B_0(n)dn + B^* = 0$.

of foregone consumption and leisure.

4 Conclusion

This paper draws attention to the importance of countries' specialisation patterns in understanding the international transmission of business cycles as well as the cost of servicing a given level of foreign liabilities. It explores a consequence of the increasing vertical disintegration of production structures into global value added chains in which countries become suppliers of intermediate goods or producers of final goods.

Our empirical evidence distinguishes between intermediate-goods producers and final-good producers using a very crude statistic – the intermediate-goods content of a country's sectoral exports. We document that, despite its simplicity, this statistic appears to capture significant differences between countries which affect the international transmission of business cycles. A very simple model of specialisation reproduces the main stylised facts featured in the data. The model can also be used to assess the determinants of the utility cost to a country of servicing a given level of foreign liabilities. It highlights the importance of the distribution of foreign assets among the country's trading partners: if many countries with the same specialisation pattern are servicing a high level of foreign liabilities, doing so will require all of them to forgo more consumption and leisure.

The evidence and theory presented here suggest a host of questions for future research. For example, it would be important to establish whether the empirical findings presented here are robust to the use of more sophisticated, bilateral measures of vertical integration, and whether there are other key determinants of trade balance correlations. We plan to address these issues in future work.

Finally, our theory highlights as-yet unexploited avenues for empirical research. The GDP correlations implicit in our model suggest that final-good producers have an incentive to invest foremost in other, economically sizeable final-good producers, as productivity shocks in the latter affect the incomes of the former countries in the opposite direction. A symmetric argument applies to intermediate-input-producing countries. In this manner, the theory outlined above could be used to derive, and test, implications for international investment portfolios.

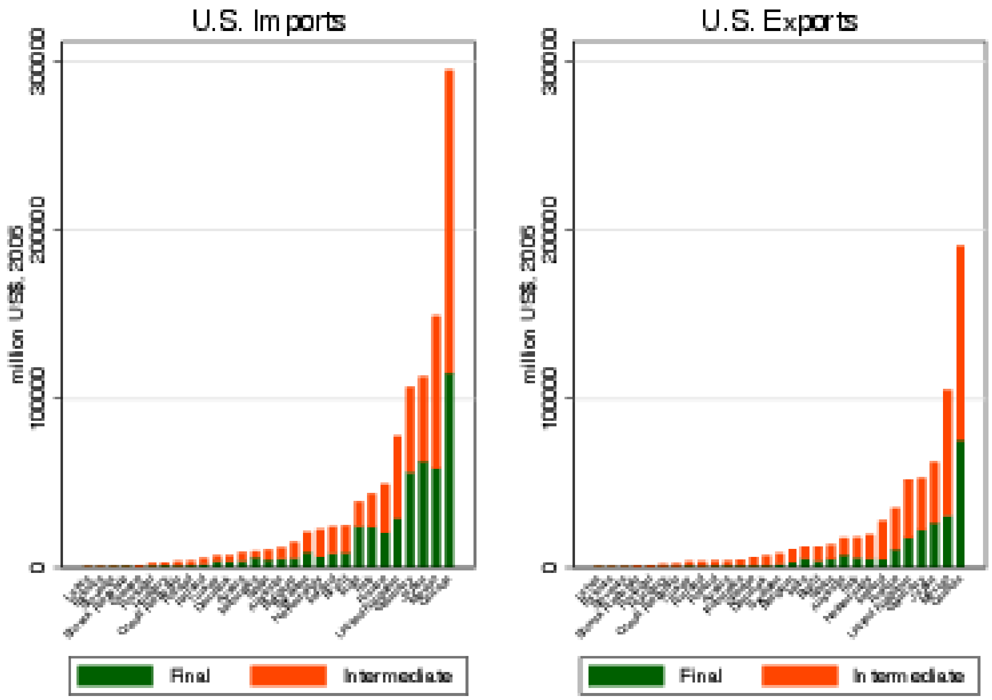

Figure 1: U.S. Trade in Intermediate and Final Goods

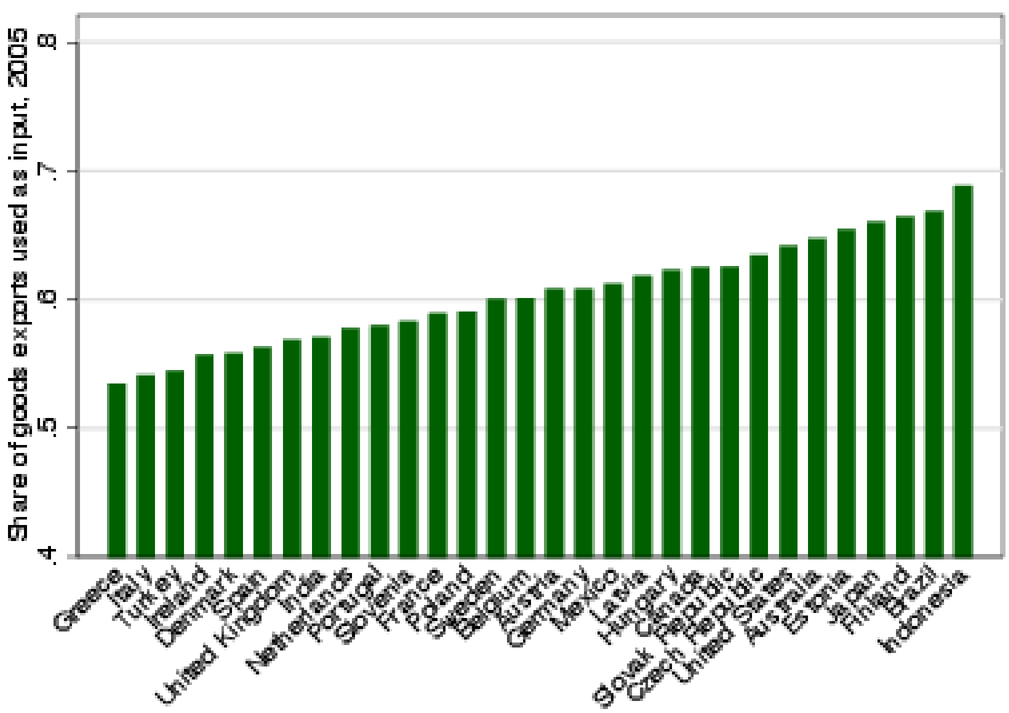

Figure 2: Intermediate-Good Content of Exports

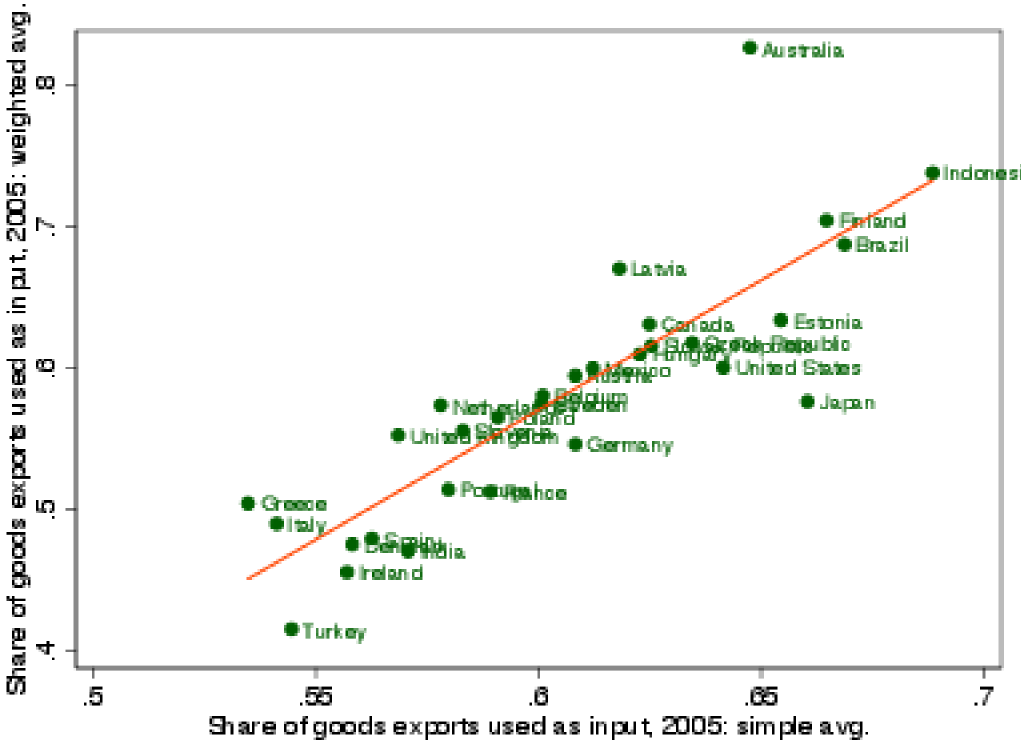

Figure 3: Alternative Measures of Specialisation

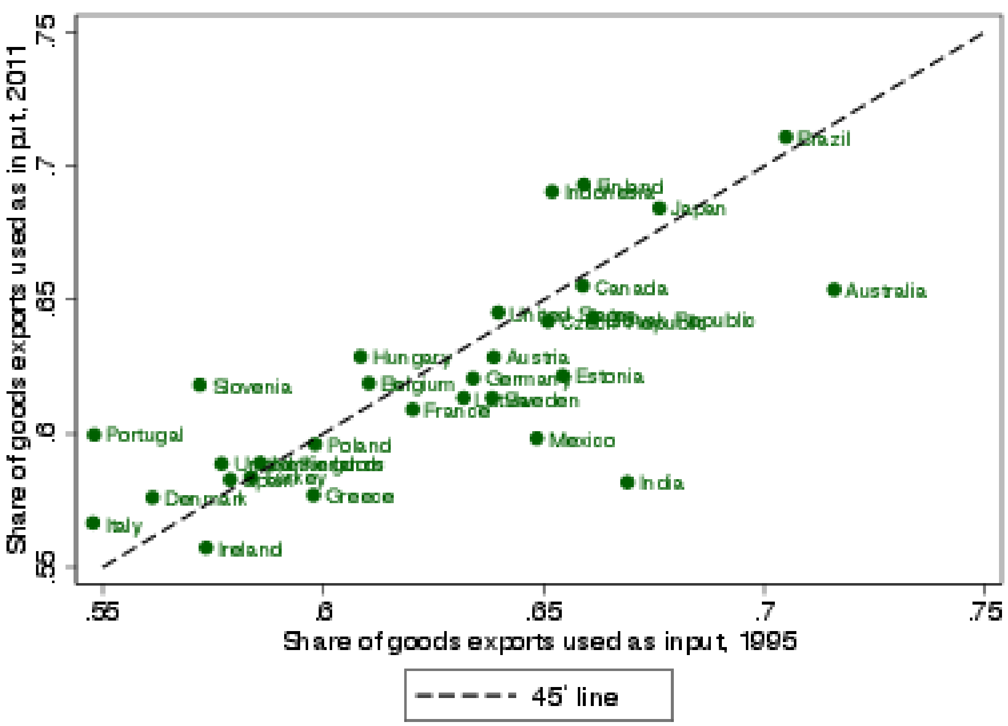

Figure 4: Intermediate-Good Content of Exports Over Time

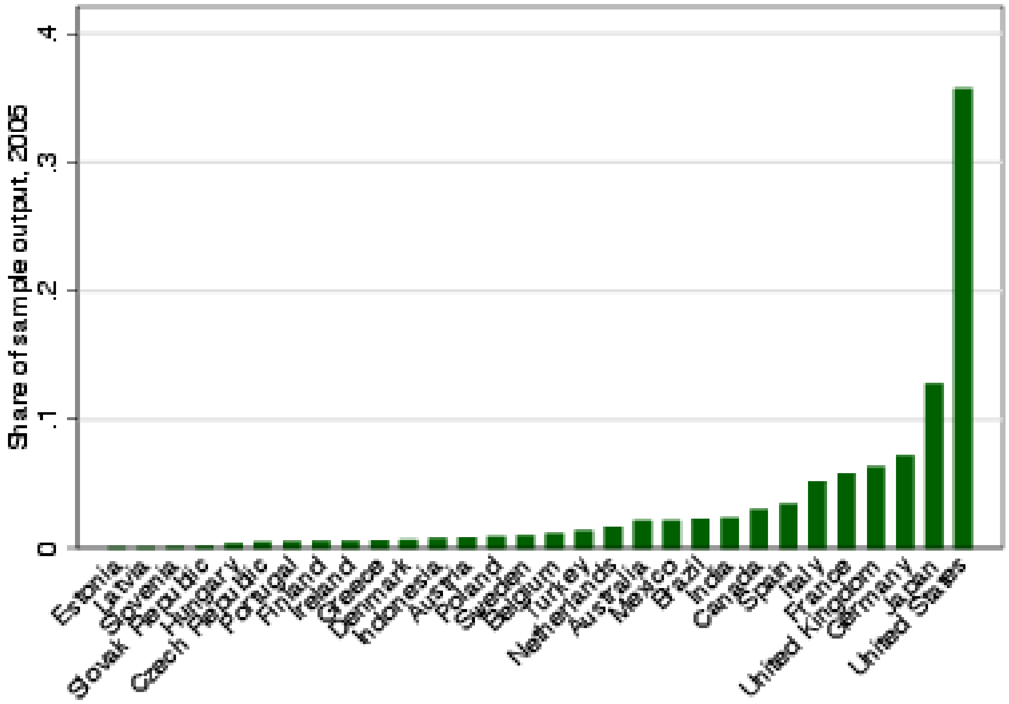

Figure 5: Country Shares of Sample Output

Variable	Obs.	Mean	Min.	Max.
Corr(Y1,Y2)	435	.553	-.212	.935
Corr(NX1/Y1,NX2/Y2)	435	.024	-.563	.625

Note: 30 countries, 1990-2014

Table 1: Business-Cycle Correlations – Summary Statistics

Dep. variable: Corr(Y1,Y2)	goods average	goods average	goods average, OECD countries	goods average, OECD countries	goods average, w/o Australia	wgt. average, w/o Australia
Difference inputs/exports	-2.231	-0.878	-1.200	-0.188	-0.895	-0.647
	(0.389)***	(0.365)**	(0.460)***	(0.417)	(0.381)**	(0.174)***
Log distance (capitals, km)		-0.094		-0.071	-0.089	-0.086
		(0.008)***		(0.009)***	(0.009)***	(0.009)***
Adj. R2	0.08	0.27	0.03	0.17	0.24	0.25
Observations	435	435	276	276	406	406

* p<0.1; ** p<0.05; *** p<0.01
Note: Robust standard errors in parentheses.

Table 2: GDP Correlations

Dep. variable: Corr(NX1/Y1,NX2/Y2)	goods average	goods average	goods average, OECD countries	goods average, OECD countries	goods average, w/o Australia	wgt. average, w/o Australia
Difference inputs/exports	-0.942	-0.837	-1.175	-1.035	-1.146	-0.297
	(0.330)***	(0.348)**	(0.431)***	(0.458)**	(0.359)***	(0.166)*
Log distance (capitals, km)		-0.007		-0.010	-0.011	-0.017
		(0.009)		(0.012)	(0.010)	(0.010)*
Adj. R2	0.02	0.01	0.02	0.02	0.03	0.02
Observations	435	435	276	276	406	406

* p<0.1; ** p<0.05; *** p<0.01
Note: Robust standard errors in parentheses.

Table 3: NX Correlations

Dep. variable: Corr(NX1/Y1,NX2/Y2)	goods avg.	goods avg.	goods avg., OECD	goods avg., OECD	goods avg., w/o Australia	wgt. avg., w/o Australia
Corr(Y1,Y2)	0.422	0.953	0.979	5.497	1.280	1.280
	(0.155)***	(0.527)*	(0.513)*	(12.338)	(0.638)**	(0.638)**
Log distance		0.083		0.383	0.102	0.102
		(0.055)		(0.892)	(0.062)*	(0.062)*
F statistic	7.40	2.58	3.64	0.15	3.01	3.01
Observations	435	435	276	276	406	406
Instruments	Diff. inp./exp.	Diff. inp./exp., Log dist.	Diff. inp./exp.	Diff. inp./exp., Log dist.	Diff. inp./exp., Log dist.	Diff. inp./exp., Log dist.

* p<0.1; ** p<0.05; *** p<0.01
Note: Robust standard errors in parentheses.

Table 4: NX Correlations (Instrumented)

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A Appendix

A.1 Competitive Pricing, Goods Market Clearing, and Labor Demand

Competitive pricing conditions yield

$$P_{Mt}(n) = \frac{W_t(n)}{A_t(n)}, \tag{26}$$

$$P^*_{Mt} = \frac{W^*_t}{A^*_t}, \tag{27}$$

$$P_{Ft}(n) = \left[\frac{W_t(n)}{A_t(n)}\right]^{1-\gamma} \left[\int_1^2 P_{Mt}(n)^{1-\varepsilon} dn\right]^{\frac{\gamma}{1-\varepsilon}}, \tag{28}$$

$$1 = \left[\int_0^1 P_{Ft}(n)^{1-\varepsilon} dn\right]^{\frac{1}{1-\varepsilon}}. \tag{29}$$

Goods market-clearing requires

$$Q_{Ft}(f) = \int_0^{2-n^*} C_t(f,n)dn + C^*_t(f), \tag{30}$$

$$Q_{Mt}(m) = \int_1^2 X_t(m,n)dn. \tag{31}$$

Labour demand is given by the following expressions:

$$L_t(n) = \begin{cases} L_{tF}(n) = Q_{Ft}(n)/A_t(n) & \forall : n \in [0,1] \\ L_{tM}(n) = Q_{Mt}(n)/A_t(n) & \forall : n \in [1, 2-n^*] \end{cases}, \tag{32}$$

$$L^*_t = n^* L^*_{Mt} = Q_{Mt}(n^*)/A^*_t. \tag{33}$$

Combining (26)-(29),

$$1 = \left\{\int_0^1 \left[\frac{W_t(n)}{A_t(n)}\right]^{(1-\gamma)(1-\varepsilon)} dn \left[\int_1^{2-n^*} \left[\frac{W_t(n)}{A_t(n)}\right]^{1-\varepsilon} dn + n^* \left(\frac{W^*_t}{A^*_t}\right)^{1-\varepsilon}\right]^{\gamma}\right\}^{\frac{1}{1-\varepsilon}}. \tag{34}$$

From utility/profit maximisation and goods market clearing, we find that

$$W_t(n)L_t(n) = (1-\gamma)\frac{\left[\frac{W_t(n)}{A_t(n)}\right]^{(1-\gamma)(1-\varepsilon)}}{\int_0^1\left[\frac{W_t(n)}{A_t(n)}\right]^{(1-\gamma)(1-\varepsilon)}dn}Y_t \quad \forall : n \in [0,1], \tag{35}$$

$$W_t(n)L_t(n) = \gamma\frac{\left[\frac{W_t(n)}{A_t(n)}\right]^{1-\varepsilon}}{\int_1^{2-n^*}\left[\frac{W_t(n)}{A_t(n)}\right]^{1-\varepsilon}dn + n^*\left(\frac{W_t^*}{A_t^*}\right)^{1-\varepsilon}}Y_t \quad \forall : n \in [1, 2-n^*], \tag{36}$$

$$W_t^*L_t^* = \gamma\frac{n^*\left(\frac{W_t^*}{A_t^*}\right)^{1-\varepsilon}}{\int_1^{2-n^*}\left[\frac{W_t(n)}{A_t(n)}\right]^{1-\varepsilon}dn + n^*\left(\frac{W_t^*}{A_t^*}\right)^{1-\varepsilon}}Y_t, \tag{37}$$

where $Y_t \equiv \int_0^{2-n^*} W_t(n)L_t(n)dn + W_t^*L_t^*$.

A.2 Financial Autarky: Income Shares

Under financial autarky, and given the preferences we assumed, it is easy to show that labour supply equals 1 in every country-period. This result, combined with equations (35)-(37), yields equations (6)-(9).

A.3 Complete Asset Markets

A.3.1 Consumption and Labour Supply

With complete asset markets and competitive product markets the market equilibrium of the world economy coincides with the solution of the planner problem

$$\max_{\{C_t(n),L_t(n)\}_{n,t}} E_0\left\{\beta^t\int_0^{2-n^*}\theta(n)U\left[C_t(n),L_t(n)\right]dn + \theta^*U\left(C_t^*,L_t^*\right)\right\} \tag{38}$$

subject to

$$\int_0^{2-n^*} C_t(n)dn + C_t^* \leq \int_0^{2-n^*} W_t(n)L_t(n)dn + W_t^*L_t^* \equiv Y_t, \tag{39}$$

where $\theta(n) \geq 0$ represents the planner weight on country n (which depends on its expected future income and initial foreign asset position), $\int_0^{2-n^*}\theta(n)dn + \theta^* = 1$, and $\{W_{nt}\}_{n,t}$ are as pinned down by (26)-(29).

The first-order conditions of the planner problem are

$$
\begin{aligned}
\beta^t \theta(n) \frac{1}{C_t(n)} - \lambda_t &= 0, && (40)\\
\beta^t \theta^* \frac{1}{C_t^*} - \lambda_t &= 0, && (41)\\
-\beta^t \theta(n) L_t(n)^{\eta} + \lambda_t \frac{\partial Y_t}{\partial L_t(n)} &= 0, && (42)\\
-\beta^t \theta^* L_t^{*\eta} + \lambda_t \frac{\partial Y_t}{\partial L_t^*} &= 0, && (43)
\end{aligned}
$$

which yield (12)-(15).

A.3.2 Income Shares

From (36) and (37),

$$
\begin{aligned}
\frac{W_t(n) L_t(n)}{W_t^* L_t^*} &= \frac{1}{n^*} \left[\frac{A_t^*}{A_t(n)} \frac{W_t(n)}{W_t^*} \right]^{1-\varepsilon} \quad \forall : n \in [1, 2-n^*], && (44)\\
\frac{W_t(n)}{W_t^*} &= \left\{ \frac{1}{n^*} \left[\frac{A_t^*}{A_t(n)} \right]^{1-\varepsilon} \frac{L_t^*}{L_t(n)} \right\}^{\frac{1}{\varepsilon}} \quad \forall : n \in [1, 2-n^*], && (45)\\
\frac{W_t(n)}{A_t(n)} &= \left[\frac{A_t^* L_t^*}{n^* A_t(n) L_t(n)} \right]^{\frac{1}{\varepsilon}} \frac{W_t^*}{A_t^*} \quad \forall : n \in [1, 2-n^*]. && (46)
\end{aligned}
$$

Similarly, from (35),

$$
\frac{W_t(n)}{A_t(n)} = \left[\frac{A_t(1) L_t(1)}{A_t(n) L_t(n)} \right]^{\frac{1}{1+(1-\gamma)(\varepsilon-1)}} \frac{W_t(1)}{A_t(1)} \quad \forall : n \in [0, 1]. \quad (47)
$$

Substituting (47) into (37),

$$
\begin{aligned}
\frac{W_t^*}{A_t^*} &= \gamma \frac{n^* (A_t^* L_t^*)^{-1}}{\int_1^{2-n^*} \left[\frac{A_t^* L_t^*}{n^* A_t(n) L_t(n)} \right]^{\frac{1-\varepsilon}{\varepsilon}} dn + n^*} Y_t, && (48)\\
\frac{W_t^*}{A_t^*} &= \gamma \frac{\left(\frac{A_t^* L_t^*}{n^*} \right)^{-\frac{1}{\varepsilon}}}{\int_1^{2-n^*} \left[A_t(n) L_t(n) \right]^{\frac{\varepsilon-1}{\varepsilon}} dn + n^* \left(\frac{A_t^* L_t^*}{n^*} \right)^{\frac{\varepsilon-1}{\varepsilon}}} Y_t. && (49)
\end{aligned}
$$

Similarly, substituting (47) into (35),

$$
\frac{W_t(1)}{A_t(1)} = (1-\gamma) \frac{\left[A_t(1) L_t(1) \right]^{-\frac{1}{1+(1-\gamma)(\varepsilon-1)}}}{\int_0^1 \left[A_t(n) L_t(n) \right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}} dn} Y_t. \quad (50)
$$

From the normalisation (29),

$$Y_t = \left\{ \frac{\left[\int_0^1 \left[A_t(n)\, L_t(n)\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}} dn\right]^{\frac{1+(1-\gamma)(\varepsilon-1)}{(1-\gamma)(\varepsilon-1)}}}{1-\gamma} \right\}^{(1-\gamma)} \times \tag{51}$$

$$\times \left\{ \frac{\left[\int_1^{2-n^*} \left[A_t(n)\, L_t(n)\right]^{\frac{\varepsilon-1}{\varepsilon}} dn + n^* \left(\frac{A_t^* L_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}{\gamma} \right\}^{\gamma}.$$

This gives us

$$\frac{\partial Y_t/\partial L_t(n)}{Y_t} = \frac{A_t(n)^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}}\, L_t(n)^{-\frac{1}{1+(1-\gamma)(\varepsilon-1)}}}{\int_0^1 \left[A_t(n)\, L_t(n)\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}} dn} (1-\gamma) \quad \forall : n \in [0,1], \tag{52}$$

$$\frac{\partial Y_t/\partial L_t(n)}{Y_t} = \frac{A_t(n)^{\frac{\varepsilon-1}{\varepsilon}}\, L_t(n)^{-\frac{1}{\varepsilon}}}{\int_1^{2-n^*} \left[A_t(n)\, L_t(n)\right]^{\frac{\varepsilon-1}{\varepsilon}} dn + n^* \left(\frac{A_t^* L_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}} \gamma \quad \forall : n \in [1, 2-n^*], \tag{53}$$

$$\frac{\partial Y_t/\partial L_t^*}{Y_t} = \frac{n^* \left(\frac{A_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}} L_t^{*-\frac{1}{\varepsilon}}}{\int_1^{2-n^*} \left[A_t(n)\, L_t(n)\right]^{\frac{\varepsilon-1}{\varepsilon}} dn + n^* \left(\frac{A_t^* L_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}} \gamma \quad \forall : n \in [1, 2-n^*]. \tag{54}$$

Using (14) and (52) implies, after some manipulation,

$$L_t(n) = \left\{ \frac{1-\gamma}{\theta(n)} \frac{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}}}{\int_0^1 \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}} dn} \right\}^{\frac{1}{1+\eta}} \quad \forall : n \in [0,1]. \tag{55}$$

Combining (14) with (53) and (54),

$$L_t(n) = \left[\frac{\theta^*}{n^*\theta(n)}\right]^{\frac{\varepsilon}{1+\eta\varepsilon}} \left[\frac{n^* A_t(n)}{A_t^*}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} L_t^* \quad \forall : n \in [1, 2-n^*]. \tag{56}$$

Substituting (56) into (54),

$$L_t^* = \left\{ \frac{\gamma}{\theta^*} \frac{n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{\int_1^{2-n^*} \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}} \right\}^{\frac{1}{1+\eta}}, \tag{57}$$

which can be used with (56) to obtain

$$L_t(n) = \left\{ \frac{\gamma}{\theta(n)} \frac{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{\int_1^{2-n^*} \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}} \right\}^{\frac{1}{1+\eta}} \quad \forall : n \in [1, 2-n^*] \,. \tag{58}$$

Substituting (47) in (35),

$$\frac{W_t(n)L_t(n)}{Y_t} = (1-\gamma) \frac{[A_t(n) L_t(n)]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}}}{\int_0^1 [A_t(n) L_t(n)]^{\frac{(1-\gamma)(\varepsilon-1)}{1+(1-\gamma)(\varepsilon-1)}} dn} \quad \forall : n \in [0,1] \,, \tag{59}$$

and (46) in (36) and (37),

$$\frac{W_t(n)L_t(n)}{Y_t} = \gamma \frac{[A_t(n) L_t(n)]^{\frac{\varepsilon-1}{\varepsilon}}}{\int_1^{2-n^*} [A_t(n) L_t(n)]^{\frac{\varepsilon-1}{\varepsilon}} dn + n^* \left(\frac{A_t^* L_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}} \quad \forall : n \in [1, 2-n^*] \,, \tag{60}$$

$$\frac{W_t^* L_t^*}{Y_t} = \gamma \frac{n^* \left(\frac{A_t^* L_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{\int_1^{2-n^*} [A_t(n) L_t(n)]^{\frac{\varepsilon-1}{\varepsilon}} dn + n^* \left(\frac{A_t^* L_t^*}{n^*}\right)^{\frac{\varepsilon-1}{\varepsilon}}}. \tag{61}$$

Now, substituting (55) into (59), and (57) and (58) into (60) and (61) yields equations (16)-(18).

A.3.3 Trade Balances

Define $NX_t(n)$ as the trade balance of country n at time t. Then,

$$\frac{NX_t(n)}{Y_t(n)} = 1 - \frac{C_t(n)}{W_t(n)L_t(n)} = 1 - \frac{1}{L_t(n)^{1+\eta}}, \tag{62}$$

$$\frac{NX_t^*}{Y_t^*} = 1 - \frac{C_t^*}{W_t^* L_t^*} = 1 - \frac{1}{L_t{}^{*1+\eta}}, \tag{63}$$

where the second equality follows from (12)-(15). Substituting in (55), (57) and (58),

$$\frac{NX_t(n)}{Y_t(n)} = 1 - \frac{\theta(n)}{1-\gamma} \frac{\int_0^1 \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}} dn}{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{(1-\gamma)(\varepsilon-1)}{1+\eta+\eta(1-\gamma)(\varepsilon-1)}}} = 1 - \theta(n) \frac{Y_t}{W_t(n)L_t(n)} \quad \forall : n \in [0,1] \,, \tag{64}$$

$$\frac{NX_t(n)}{Y_t(n)} = 1 - \frac{\theta(n)}{\gamma} \frac{\int_1^{2-n^*} \left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*}\right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{\left[\frac{A_t(n)^{1+\eta}}{\theta(n)}\right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}}} =$$

$$= 1 - \theta(n) \frac{Y_t}{W_t(n) L_t(n)} \quad \forall : n \in [1, 2 - n^*] \tag{65}$$

$$\frac{NX_t^*}{Y_t^*} = 1 - \frac{\theta^*}{\gamma} \frac{\int_1^{2-n^*} \left[\frac{A_t(n)^{1+\eta}}{\theta(n)} \right]^{\frac{\varepsilon-1}{1+\eta\varepsilon}} dn + n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*} \right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}}{n^* \left(\frac{A_t^{*1+\eta}}{n^{*\eta}\theta^*} \right)^{\frac{\varepsilon-1}{1+\eta\varepsilon}}} = 1 - \theta^* \frac{Y_t}{W_t^* L_t^*}. \tag{66}$$

Note that this implies that

$$Cov \left[\frac{NX_t(n)}{Y_t(n)}, \frac{NX_t(d)}{Y_t(d)} \right] = \theta(n)\theta(d) Cov \left[\frac{Y_t}{W_t(n) L_t(n)}, \frac{Y_t}{W_t(d) L_t(d)} \right]. \tag{67}$$

A.3.4 Decentralised Equilibrium

Suppose agents can trade a full set of state-contingent Arrow-Debreu securities, with $q_t(s_t)$ denoting the price of security which delivers one unit of final consumption in period t and state s_t. Then the representative consumer in country n solves

$$\max_{\{C_t(n,s_t), L_t(n,s_t)\}_{s_t}} \sum_{t=0}^{\infty} \beta^t \int_{s_t} \pi_t(s_t) U \left[C_t(n, s_t), L_t(n, s_t) \right] ds_t \tag{68}$$

subject to

$$\sum_{t=0}^{\infty} \int_{s_t} q_t(s_t) C_t(n, s_t) ds_t \leq \sum_{t=0}^{\infty} \int_{s_t} q_t(s_t) W_t(n, s_t) L_t(n, s_t) ds_t - B_0(n), \tag{69}$$

where $U[\cdot, \cdot]$ is still defined as in (1). Note that

$$\int_0^{2-n^*} B_0(n) dn + B^* = 0. \tag{70}$$

The first-order conditions of this problem are

$$\beta^t \pi_t(s_t) \frac{1}{C_t(n, s_t)} - \lambda(n) q_t(s_t) = 0, \tag{71}$$

$$-\beta^t \pi_t(s_t) L_t(n, s_t)^\eta + \lambda(n) q_t(s_t) W_t(n, s_t) = 0, \tag{72}$$

and market clearing requires

$$\int_0^{2-n^*} C_t(n, s_t) dn + C_t^*(s_t) = \int_0^{2-n^*} W_t(n, s_t) L_t(n, s_t) dn + W_t^*(s_t) L_t^*(s_t) \equiv Y_t(s_t). \tag{73}$$

From (71) and (73),

$$\frac{\beta^t \pi_t(s_t)}{q_t(s_t)} \left[\int_0^{2-n^*} \frac{1}{\lambda(n)} dn + \frac{1}{\lambda^*} \right] = Y_t(s_t), \tag{74}$$

so

$$C_t(n,s_t) = \frac{\frac{1}{\lambda(n)}}{\left[\int_0^{2-n^*} \frac{1}{\lambda(n)} dn + \frac{1}{\lambda^*}\right]} Y_t(s_t), \tag{75}$$

and $\frac{1}{\lambda(n)} / \left[\int_0^{2-n^*} \frac{1}{\lambda(n)} dn + \frac{1}{\lambda^*}\right]$ thus corresponds to the "planner weight" $\theta(n)$ from the planner problem.

Substituting (71) into (69),

$$\frac{1}{\lambda(n)} = (1-\beta)\left[\sum_{t=0}^{\infty} \int_{s_t} q_t(s_t) W_t(n,s_t) L_t(n,s_t) ds_t - B_0(n)\right]. \tag{76}$$

Now substituting (74) into (76),

$$\theta(n) = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \int_{s_t} \pi_t(s_t) \frac{W_t(n,s_t) L_t(n,s_t)}{Y_t(s_t)} ds_t - \frac{B_0(n)}{Y_0}\right]. \tag{77}$$

Consider two countries n and n' such that $\theta(n) < \theta(n')$. From equation (17), country n's income share will be higher than that of country n', everything else constant. From equation (77), $\theta(n) < \theta(n')$ must therefore imply that $B_0(n) > B_0(n')$.