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Approximate Analytical Solution of Magnetohydrodynamics Compressible Boundary Layer flow with Pressure Gradient and suction/injection

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ABSTRACT

The aim of this work is to obtain exact analytical solution to the two dimensional laminar compressible boundary layer flow with an adverse pressure gradient in the presence of heat and mass transfer with MHD. The method applied is homotopy analysis method. It is shown that this solution agrees very well with numerical solution which is obtained by Runge-Kutta Merson method and results are shown graphically for different magnetic parameters.

Indexing terms/Keywords

Compressible boundary layer flow; Homotopy analysis method; Falkner Skan transformations; Domb Syke Plot.

Academic Discipline And Sub-Disciplines

Mathematics, Fluid Mechanics

SUBJECT CLASSIFICATION

76N20

TYPE (METHOD/APPROACH)

Homotopy analysis method. Runge-Kutta Merson method

INTRODUCTION

The boundary layer theory for incompressible fluids under the effect of an applied magnetic field has been studied extensively by many scientists. But the study of compressible fluid flow problems in boundary layer with an applied magnetic field is studied by very few. In history Rossow [12] seems to be the first person to study this topic. After him recently, these problems have received lot of attention [10]. Compressible boundary layer flow is important in many devices of Mechanical and Aerospace engineering. Flow of compressible fluid in boundary layer region tends to have an unpredictable behavior due to frictional effects and which leads to flow control problems. These problems may become more complicated if there exist an adverse pressure gradient which may lead to reverse flows and flow separation [4] (Adverse pressure gradient means $\frac{\partial y}{\partial x} > 0$ i.e., pressure increases in the direction of flow).

Boundary layer Suction is the best solution for prevention of separation. Control of flow instabilities is studied by Arnal [11]. Kafoussias [1] have studied two dimensional laminar boundary layer numerically for compressible fluids and have shown that suction or cooling of wall will stabilize the boundary layer.

Naidu et al. [6] discussed the growth of the boundary layer due to applied electric and magnetic fields during their study of laminar compressible MHD flow over a flat plate. Martin et al. [7] suggests that the high gradients created by MHD terms will lead to continuum breakdown, during the study of flow in a laminar boundary layer of an electrically conducting gas near the continuum limit, which is computed numerically. Kumari et al.[5] studied the steady laminar compressible boundary layer flow of an electrically conducting fluid in the stagnation region of a sphere with an applied magnetic field taking into account the effects of the induced magnetic field, mass transfer, and viscous dissipation and solved numerically using shooting method. The results obtained can be used in controlling the heat transfer rate. Also They observed that boundary layer solutions break down as the magnetic parameter tends to a certain critical value. Adhikari et al.[8] applied Keller - box method for solving nonlinear ordinary differential equations obtained during the study of steady three-dimensional magnetohydrodynamic (MHD) boundary layer viscous flow and heat transfer due to a permeable stretching sheet with prescribed surface heat flux in presence of a uniform applied magnetic field transverse to the flow.

Xenos et al. [2] studied numerically the problem of magnetohydrodynamic compressible boundary-layer flow over a flat plate, in the presence of an adverse pressure gradient. Xenos et al. [3] numerically examined the steady compressible boundary-layer flow with adverse pressure gradient and heat transfer over a wedge under the effects of blowing and suction.

In this paper, boundary layer flow of compressible fluid with adverse pressure gradient under the effect of heat and mass transfer with MHD is studied [2]. The governing partial differential equations of this model are transformed into ordinary differential equations by Falkner Skan Transforms and the solution is studied by Homotopy Analysis Method (HAM). The same is compared with numerical studies.

Mathematical formulation of the problem

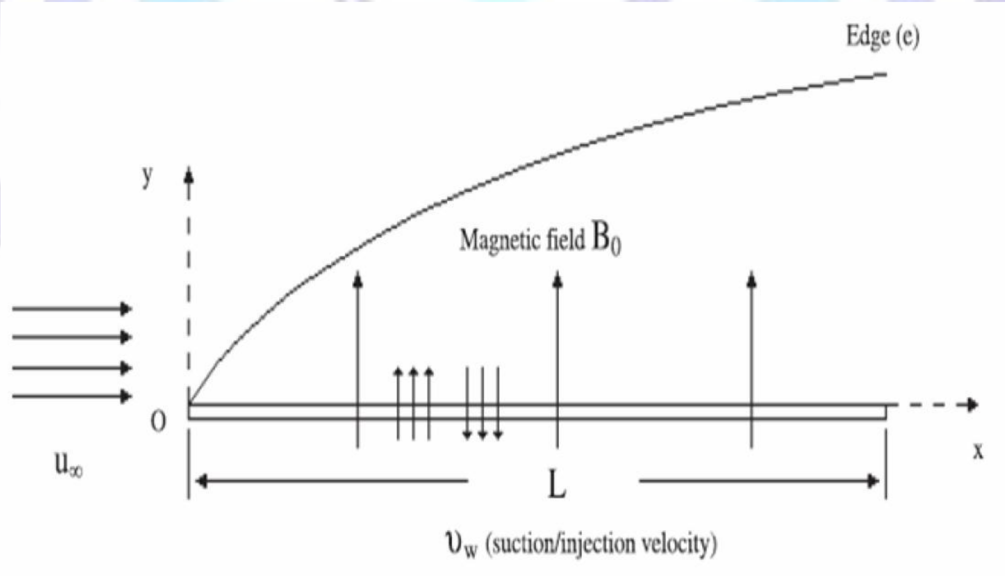

Fig 1: Flow configuration and co-ordinate system

The equations governing the steady, compressible, two-dimensional boundary layer flow, of a heat conducting perfect gas, are the continuity, momentum and energy equations, which, in the absence of body forces, using Prandtl Boundary layer assumptions are

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0, \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma B_0^2\left(u-u_e\right),, \tag{2}$$

$$\rho u\frac{\partial H}{\partial x}+\rho v\frac{\partial H}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial H}{\partial y}+\mu\left(1-\frac{1}{\Pr}\right)u\frac{\partial u}{\partial y}\right),, \tag{3}$$

respectively, where H is the total enthalpy of the fluid defined for a perfect gas by the expression

$H=C_pT+\frac{1}{2}u^2$, $\Pr$ is the Prandtl number defined as $Pr=\frac{\mu C_p}{k}$ and the other quantities have their usual meaning.

The boundary conditions of the problem, including a transpiration velocity v_w at the wall, are

$$y=0:\ u=0;\ v=v_w(x);\ H=H_w(x); \tag{4}$$

$$y\to\delta:\ u=u_e(x);\ H=H_e; \tag{5}$$

where δ is the boundary layer thickness.

The system of equations (1) - (3) constitutes a coupled nonlinear system of partial differential equations with the unknown functions $u=u(x,y)$, $v=v(x,y)$ and $H=H(x,y)$ defined in the rectangular domain $D=(x,y)/0<x<L, 0<y<\infty.$

Introducing the compressible Falkner - Skan transformation, defined by

$$\eta=\int_0^y\left(\frac{u_e(x)}{\nu_e(x)x}\right)^{1/2}\frac{\rho(x,y)}{\rho_e(x)}dy, \psi(x,y)=\left(\rho_e\mu_e u_e x\right)^{1/2} f(x,\eta), \tag{6}$$

where $\nu_e(x)$ is kinematic viscosity at the edge of the boundary layer and $f(x,\eta)$ is the dimensionless stream function.

Using (6), the boundary layer equations reduce to ordinary differential equations for similar flows and defining the stream function ψ, for a compressible flow as

$$\rho u=\frac{\partial\psi}{\partial y},\ \rho v=-\frac{\partial\psi}{\partial x}, \tag{7}$$

which satisfy the continuity equation (1) exactly. The system of equations (2) - (5) reduce to

$$b\frac{\partial^3 f}{\partial\eta^3}+m_1 f\frac{\partial^2 f}{\partial\eta^2}+m_2\left(c-\left(\frac{\partial f}{\partial\eta}\right)^2\right)=x\left(m_3\left(\frac{\partial f}{\partial\eta}-1\right)+\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial x}-\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial x}\right), \tag{8}$$

$$e\frac{\partial^2 S}{\partial\eta^2}+d\frac{\partial f}{\partial\eta}\frac{\partial^3 f}{\partial\eta^3}+d\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2+m_1 f\frac{\partial S}{\partial\eta}=x\left(\frac{\partial f}{\partial\eta}\frac{\partial S}{\partial x}-\frac{\partial S}{\partial\eta}\frac{\partial f}{\partial x}\right), \tag{9}$$

$$\eta = 0; \frac{\partial f}{\partial \eta} = 0,\ f_w = f(0,x) = -\frac{1}{\left(u_e \mu_e \rho_e x\right)^{\frac{1}{2}}} \int_0^x \rho_w v_w(x) dx, S = S_w(x), \tag{10}$$

$$\eta = \eta_e; \frac{\partial f}{\partial \eta} = 1, S = 1, \tag{11}$$

where the quantities b, c, d, e, m_1, m_2 and m_3 are defined as follows

$$\text{b=C}, \text{C} = \frac{\rho\mu}{\rho_e \mu_e}, \text{c} = \frac{\rho_e}{\rho},\ d = \frac{\text{Cu}_e^2}{H_e}\left(1 - \frac{1}{\text{Pr}}\right), e = \frac{b}{\text{Pr}},\ S = \frac{H}{H_e}, \tag{12}$$

$$m_1 = \frac{1}{2}\left[1 + m_2 + \frac{x}{\rho_e \mu_e} \frac{d}{\text{dx}}\left(\rho_e \mu_e\right)\right], m_2 = \frac{x}{u_e} \frac{\text{du}_e}{\text{dx}}, \tag{13}$$

$$R_x = \frac{u_e x}{v_e},\ \text{m}_3 = \frac{m_0 c}{\rho_e u_e},\ m_0 = \sigma B_0^2, \tag{14}$$

We solve equations (8) and (9) with the boundary conditions (10) and (11) by treating quantities in (12), (13) and (14) as constants. The quantity C in the equation (12) can vary through the boundary layer. However, the constant C assumption is made and is evaluated at the surface conditions by using the Sutherland viscocity law. The flow is assumed to be similar, in other words, $f = f(\eta)$ and $g = g(\eta)$ such that the term $\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta \partial x} - \frac{\partial^2 f}{\partial \eta^2}\frac{\partial f}{\partial x}$ of the momentum equation (8) and right hand side of energy equation (9) become zero. Thus we get

$$b\frac{\partial^3 f}{\partial \eta^3} + m_1 f \frac{\partial^2 f}{\partial \eta^2} + m_2\left(c - \left(\frac{\partial f}{\partial \eta}\right)^2\right) = x m_3\left(\frac{\partial f}{\partial \eta} - 1\right), \tag{15}$$

$$e\frac{\partial^2 S}{\partial \eta^2} + d\frac{\partial f}{\partial \eta}\frac{\partial^3 f}{\partial \eta^3} + d\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2 + m_1 f \frac{\partial S}{\partial \eta} = 0. \tag{16}$$

The equations (15) and (16) are solved by homotopy analysis method.

Homotopy Analysis Method

The given non linear equation (15) is written as

$$N[f(\eta)] = b\frac{\partial^3 F}{\partial \eta^3} + m_1 F \frac{\partial^2 F}{\partial \eta^2} + m_2\left(c - \left(\frac{\partial F}{\partial \eta}\right)^2\right) - x\left(m_3\left(\frac{\partial F}{\partial \eta} - 1\right)\right). \tag{17}$$

Homotopy for this equation is constructed as below

$$(1-p)L\left[F(\eta,p) - f_0(\eta)\right] = \text{hp}\left\{b\frac{\partial^3 F}{\partial \eta^3} + m_1 F \frac{\partial^2 F}{\partial \eta^2} + m_2\left(c - \left(\frac{\partial F}{\partial \eta}\right)^2\right) - x m_3\left(\frac{\partial F}{\partial \eta} - 1\right)\right\}, \tag{18}$$

with boundary conditions

$$F(0,p) = \lambda = f_w, \frac{\partial F}{\partial \eta}(0,p) = 0, \frac{\partial F}{\partial \eta}(\infty, p) = 1, \tag{19}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a non zero parameter. The initial guess approximation $f_0(\eta)$ of $f(\eta)$ chosen in accordance with boundary condition (14).

When p = 0, we have the solution

$$F(\eta,0)=f_0(\eta). \quad (20)$$

When p = 1,

$$F(\eta,1)=f(\eta). \quad (21)$$

Thus as p increases from 0 to 1, the solution varies from the initial guess $f_0(\eta)$ to the exact solution $f(\eta)$. The initial guess approximation $f_0(\eta)$, is chosen as the solution of linear equation

$$Lf=\left(\frac{\partial^3}{\partial\eta^3}+\frac{\partial^2}{\partial\eta^2}\right)f=0, \quad (22)$$

where L must be of same order as N and $f_0(\eta)$ has to satisfy given initial and boundary conditions. Thus we get initial approximation as

$$f_0(\eta)=\phi_0=-1+e^{-\eta}+\eta+\lambda. \quad (23)$$

The final solution consists of a convergence parameter h which has to be selected by drawing a h curve such that the equations (17) and (18) have solutions at each point $p \in [0,1]$. Using Maclaurin series for $F(\eta,p)$ as

$$F(\eta,p)=F(\eta,0)+\sum_{k=1}^{\infty}\frac{p^k}{k!}\frac{\partial^k F(\eta,p)}{\partial p^k}, \quad (24)$$

and defining

$$\phi_0(\eta)=F(\eta,0)=f_0(\eta), \quad (25)$$

Thus we get

$$F(\eta,p)=\phi_0(\eta)+\sum_{k=1}^{\infty}\phi_k(\eta)p^k, \quad (26)$$

Using equations (21) and (26) for p = 1, we get

$$f(\eta)=\phi_0(\eta)+\sum_{m=1}^{\infty}\phi_m(\eta), \quad (27)$$

where $\phi_m(\eta)$ are unknowns to be determined.

Differentiating equation (18) m times about the embedding parameter p, using Leibnitz theorem, setting p = 0 and dividing by $m!$, we get

$$L[\phi_m-\chi_m\phi_{m-1}]=hR_m(\eta), \quad (28)$$

where

$$\chi_m=\begin{cases}0 \text{ when } m\le 1\\ 1 \text{ when } m>1\end{cases}, \quad (29)$$

$$R_m[\eta]=b\phi'''_{m-1}+m_1\sum_{k=1}^{m-1}\phi_{m-1-k}\phi''_k-m_2\sum_{k=1}^{m-1}\phi'_{m-1-k}\phi'_k+m_2c(1-\chi_m)-xm_3\phi'_{m-1}, \quad (30)$$

with boundary conditions

$$\phi_m(0)=\phi'_m(0)=\phi'_m(\infty)=0, \quad (31)$$

We solve these linear equations given by (28) for ϕ_m by MATHEMATICA.

We get the solution f as below

$$f = \phi_0 + \sum_{k=1}^{\infty} \phi_k, \tag{32}$$

Equation (9) is also solved by Homotopy analysis method with linear operator L as

$$L_s = \frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}, \tag{33}$$

and nonlinear equation (9) is written as

$$N[S(\eta)] = e\frac{\partial^2 S}{\partial \eta^2} + d\left(\frac{\partial F}{\partial \eta}\frac{\partial^3 F}{\partial \eta^3}\right) + d\left(\frac{\partial^2 F}{\partial \eta^2}\right)^2 + m_1 F\frac{\partial S}{\partial \eta}. \tag{34}$$

The Homotopy for this equation is constructed as below

$$(1-p)L\left[S(\eta,p) - S_0(\eta)\right] = \mathrm{hp}\left\{e\frac{\partial^2 S}{\partial \eta^2} + d\left(\frac{\partial F}{\partial \eta}\frac{\partial^3 F}{\partial \eta^3}\right) + d\left(\frac{\partial^2 F}{\partial \eta^2}\right)^2 + m_1 F\frac{\partial S}{\partial \eta}\right\}, \tag{35}$$

with boundary conditions

$$S(0,p) = \omega = S_w, S(\infty,p) = 1. \tag{36}$$

Initial approximate solution, $S_0(\eta)$ is chosen in accordance with boundary conditions (36) which satisfies the linear equation

$$L(S) = \left(\frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}\right)S = 0, \tag{37}$$

The final solution consists of a convergence parameter h which has to be selected by drawing a h curve such that the equations (21) and (22) have solutions at each point $p \grave{o} [0,1].$

Using Maclaurin series for $S(\eta, p)$ as

$$S(\eta,p) = S(\eta,0) + \sum_{k=1}^{\infty} \frac{p^k}{k!}\frac{\partial^k S(\eta,p)}{\partial p^k}, \tag{38}$$

and defining

$$\psi_0(\eta) = S(\eta,0) = S_0(\eta), \tag{39}$$

Thus we get

$$S(\eta,p) = \psi_0(\eta) + \sum_{k=1}^{\infty} \psi_k(\eta) p^k. \tag{40}$$

Using equations (40) and (41) for p = 1, we get

$$S(\eta) = \psi_0(\eta) + \sum_{k=1}^{\infty} \psi_m(\eta), \tag{41}$$

where $\psi_m(\eta)$ are unknowns to be determined.

Differentiating equation (36) m times about the embedding parameter p, using Leibnitz theorem, setting p = 0 and dividing by $\mathrm{m}!$, we get

$$L\left[\psi_m - \chi_m \psi_{m-1}\right] = hT_m(\eta), \tag{42}$$

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$$\chi_m = \begin{cases} 0 \text{ when } m \le 1 \\ 1 \text{ when } m > 1 \end{cases}, \tag{43}$$

$$T_m[\eta] = e\psi''_{m-1} + d\sum_{k=0}^{m-1}\phi'_{m-1-k}\phi'''_k + d\sum_{k=0}^{m-1}\phi''_{m-1-k}\phi''_k + m_1\sum_{k=0}^{m-1}\phi_{m-1-k}\psi'_k, \tag{44}$$

$$\psi_m(0) = \omega, \psi_m(\infty) = 1, \tag{45}$$

We solve these linear equations given by (44) for ψ_m by MATHEMATICA.

Initial approximate solution, $S_0(\eta)$ is chosen in accordance with boundary conditions (38) as

$$S_0(\eta) = \psi_0(\eta) = 1 + (\omega - 1)e^{-\eta}. \tag{46}$$

Thus we get the solution as

$$S = \psi_0 + \sum_{k=0}^{\infty}\psi_k, \tag{47}$$

The HAM solution of equations (15) and (16) are depicted graphically in next section for different parameters and are compared with the solution obtained by Xenos [2].

Graphs of Homotopy Analysis Method

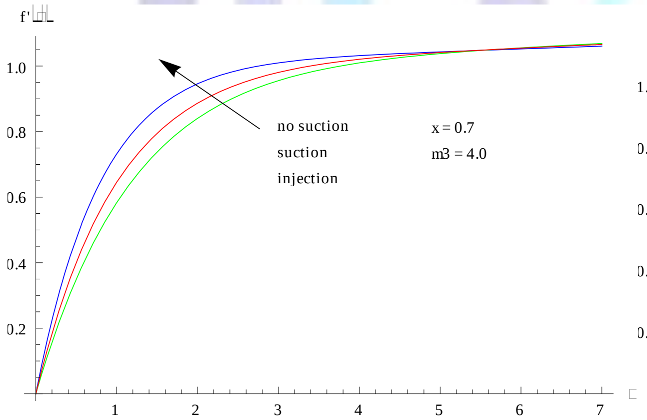

Figure 2: Velocity curves for m_3 = 4.0

(1) no suction , (2) with suction,(3) with injection

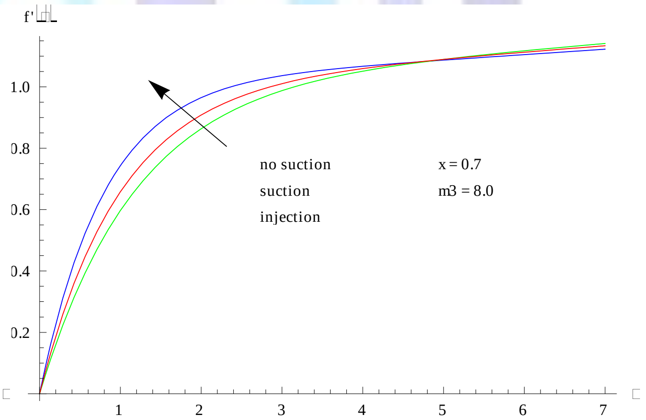

Figure 3: Velocity curves for m_3 = 8.0

(1) no suction , (2) with suction,(3) with injection

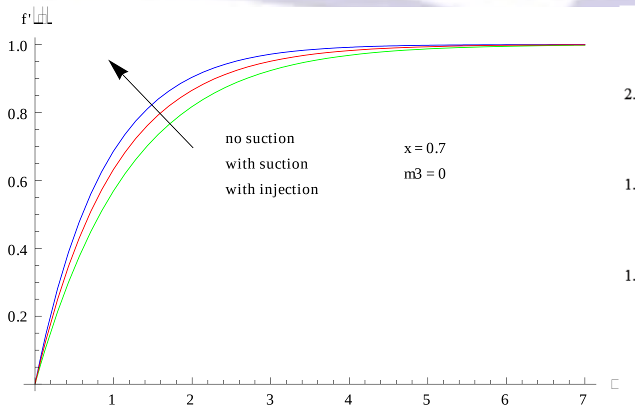

Figure 4: Velocity curves for m_3 = 0

(1) no suction , (2) with suction,(3) with injection

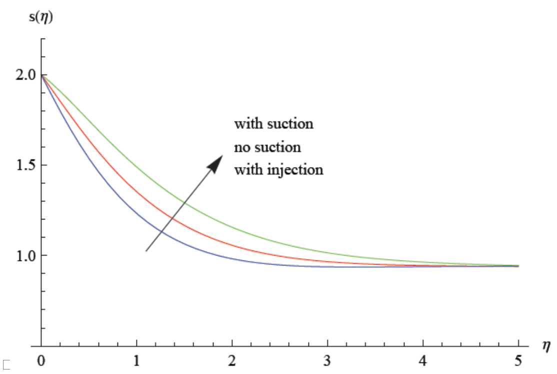

Figure 5: Temperature Profiles $\lambda = -2, 0, 2$

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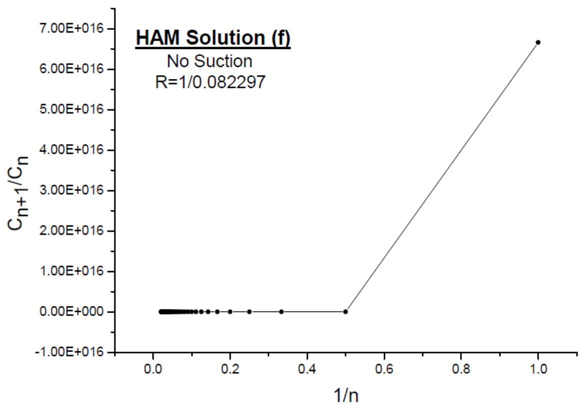


Figure 6: Domb Syke Plot for f HAM (no suction)

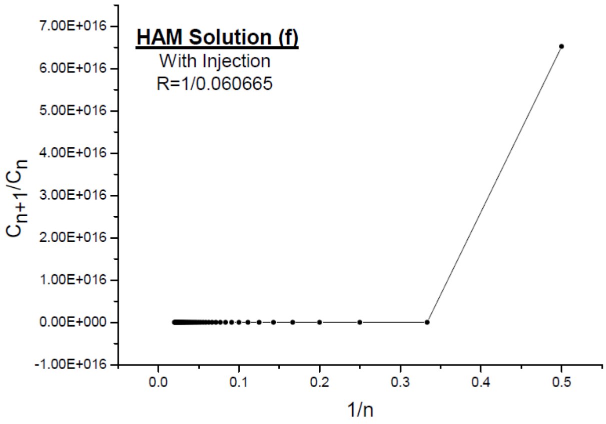


Figure 7: Domb Syke Plot for f HAM (injection)

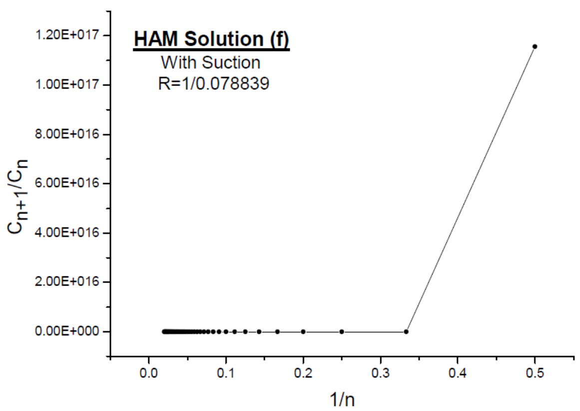


Figure 8: Domb Syke Plot for f HAM (with suction)

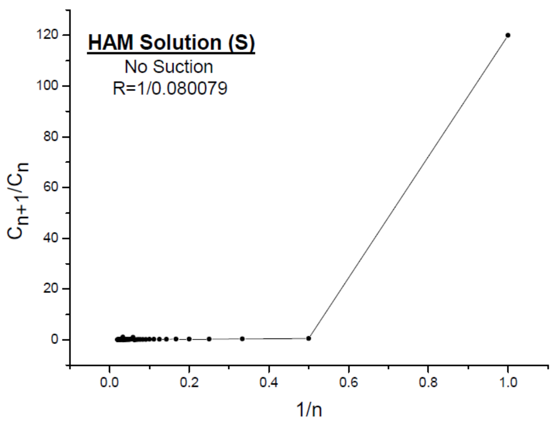


Figure 9: Domb Syke Plot for S HAM (no suction)

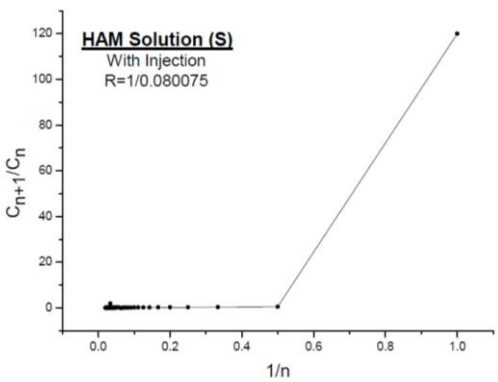


Figure 10: Domb Syke Plot for S HAM (with injection)

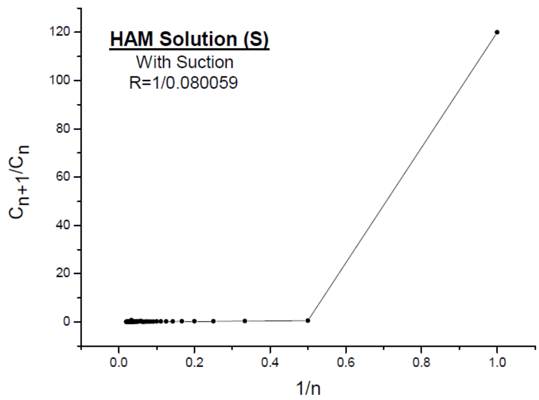
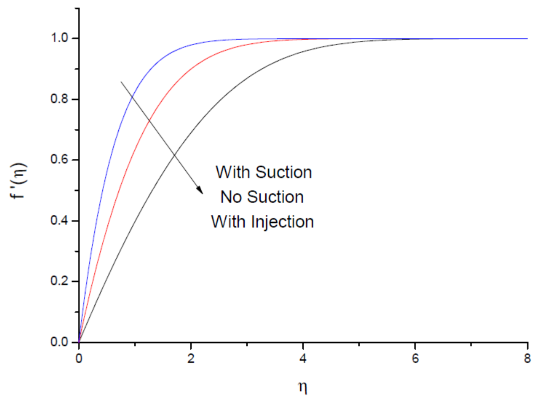


Figure 11: Domb Syke Plot for S HAM (with suction)



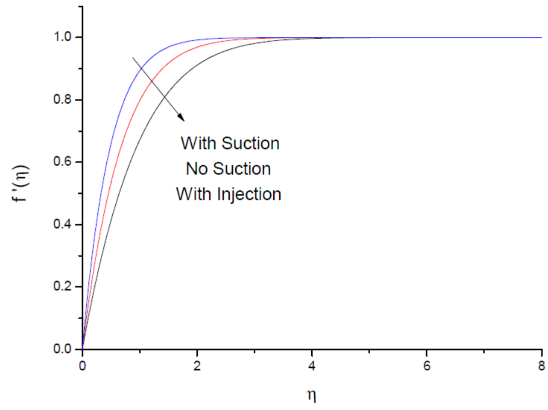
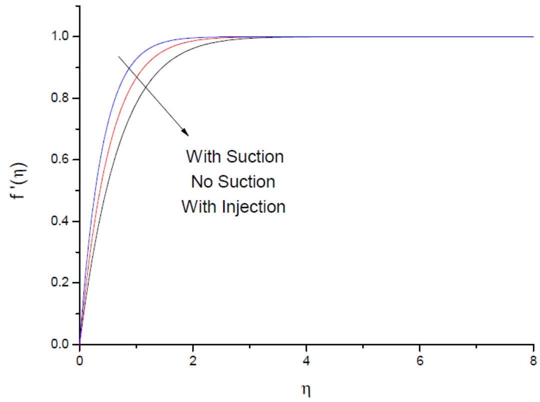
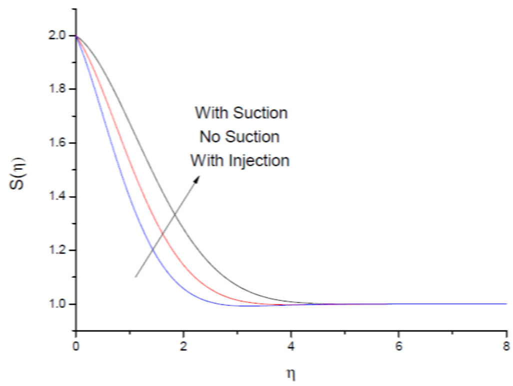


Figure 12: Velocity Profile for $m_3 = 0$

Figure 13: Velocity Profile for

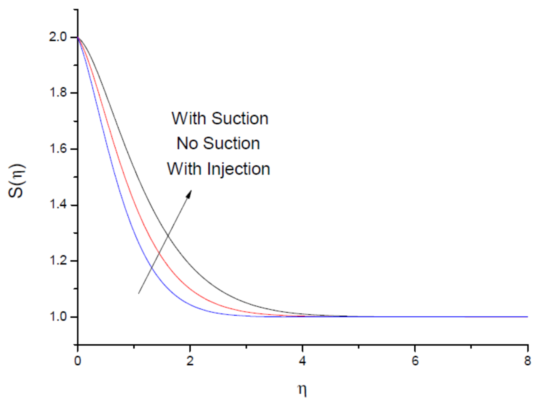




$m_3 = 4$

Figure 14: Velocity Profile for $m_3 = 8$

Figure 15: Temperature Profile for $m_3 = 0$



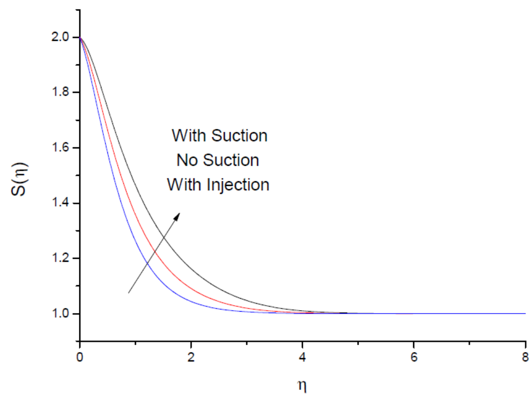


Figure 16: Temperature Profile for $m_3 = 4$

Figure 17: Temperature Profile for $m_3 = 8$

Result and Discussions

In this paper, the problem of MHD compressible boundary layer flow over a flat plate, in presence of adverse pressure gradient is studied by HAM and Ruge-Kutta Merson methods. The fluid is Newtonian, the transverse magnetic field applied is constant, the flow is subjected to a constant velocity of suction or injection and there is no heat transfer between the plate and fluid. The governing equations are a system of nonlinear partial differential equations and are transformed into nonlinear ordinary differential equations by Falkner-Skan transforms.

Homotopy analysis method is applied to these transformed ODEs for some limiting cases. The solution obtained matches exactly with numerical solution. HAM solution curves for velocity are drawn for different values of magnetic parameter m_3 in fig 2, 3 and 4 for $\lambda < 0,$ $\lambda = 0$ and $\lambda > 0$.

Temperature profile for for $\lambda < 0,$ $\lambda = 0$ and $\lambda > 0$ are drawn in fig 5.

The HAM solution obtained for velocity and temperature is also tested for convergence by drawing Domb-Syke plots (fig 6, 7, 8, 9) where we estimate the radius of convergence for suction, no suction and injection which has values R=12.68 for $\lambda < 0$ (fig 8), R=12.15 for $\lambda = 0$ (fig 6) and R=16.48 for $\lambda > 0$ (fig 7) for velocity, and R=12.49 for $\lambda < 0$ (fig 11), R=12.49 for $\lambda = 0$ (fig 9) and R=12.49 for $\lambda > 0$ (fig 10) for temperature. In fig 12, 13, 14, the effect of magnetic parameter m_3 on velocity profile is drawn by Runge-Kutta Merson method for suction, no suction and injection. In fig 15, 16, 17, the effect of m_3 on temperature profile is shown by Runge-Kutta Merson method for $\lambda < 0,$ $\lambda = 0$ and $\lambda > 0$.

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REFERENCES

[1] N. Kafoussias, A. Karabis, M. Xenos,Numerical study of two dimensional laminar boundary layer compressible flow with pressure gradient and heat and mass transfer,International Journal of Engineering Science,37,1999,1795 -1812.

[2] M. Xenos, N. Kafoussias, and G. Karahalios,Magnetohydrodynamic compressible laminar boundary layer adiabatic flow with adverse pressure gradient and continuous or localized mass transfer, Can. J. Phys. ,79,2001,1247-1263.

[3] M. Xenos, E. Tzirtzilakis and N. Kafoussias,Compressible turbulent boundary-layer ow control over a wedge. 2nd International Conference, Scientfic Computing to Computational Engineering,2nd IC-SCCE Athens , 5-8 July, 2006.

[4] F. Hirt and H. Thomann,Measurement of wall shear stress in turbulent boundary layers subject to strong pressure gradients, J . Fluid Mech,171, 1986,547-562.

[5] M. Kumari, H. S. Takhar And G. Nath, Compressible MHD Boundary Layer In The Stagnation Region Of A Sphere, ht. J. Engg Sci., 28, No. 5, 1990, 357-366.

[6] V. G. Naidu, K. S. Rao, M. L. Mittal, B. Nageswara Rao, Laminar compressible MHD (magneto hydro dynamic) boundary layers , ht. J. Engg Sci.,55, Issue 6,1989, 196-198.

[7] Michael James Martin, Chunpei Cai, and Iain D. Boyd, Slip Flow in a Magnetohydrodynamic Boundary Layer , 43rd AIAA Plasmadynamics and Lasers Conference , AIAA 2012-3295 ,June 2012 New Orleans, Louisiana, 25 - 28 .

[8] A. Adhikari and D.C. Sanyal, Heat Transfer On MHD Viscous Flow Over A Stretching Sheet With Prescribed Heat Flux, Bulletin Of International Mathematical Virtual Institute , www.imvibl.org, Former Bulletin Of Society Of Mathematicians Banja Luka, ISSN 0354-5792 (o), ISSN 1986-521,3 ISSN 1840-4367 , 2013,35-47.

[9] J.H. Davidson and F.A. Kulacki and P.F. Dunn, Handbook of single-phase convective heat transfer. Edited by S. Kakac, R.K. Shah, and W. Aung. John Wiley \& Sons , Springer-Verlag ,1987, 9.1-9.49.

[10] J.M.G. Chanty, Magnetized plasma flow over an insulator at high magnetic reynolds number, J. Thermophy and Heat Transfer, 1994, 3-10.

[11] D. Arnal, In Proceedings of the CISM Courses and Lectures on Control of Flow Instabilities and Unsteady Flows, Udine, Italy. No. 369. September 18-22, 1995. Edited by G.E.A. Meier and G.H.Schnerr, Springer-Verlag , New York, 1996 , 119-153.

[12] V.J. Rossow , Impulsive motion of an infinite flat plate in hydromagnetic viscous incompressible fluid, NACA TN , 3971 , May 1957.

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