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Similarity in Analytic Behavior Intermediate Radial States of the Hydrogen Atom & Production of Radionuclides and Gamma Decay

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ABSTRACT

The purpose of this paper is to show that analytic behavior of ground and intermediate states of the Hydrogen atom is similar to Gamma decay in molybdenum-99 (99mTc → 99Tc + γ).

Keywords

Meijer's G-function; Hydrogen atom; Gamma decay; Radionuclide.

SUBJECT CLASSIFICATION

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INTRODUCTION

The Meijer's G-functions (MGFs) are an admirable family of functions which each of them is determined by finitely many indices. Analytic manipulations and numerical computations involving Meijer's G-functions have been provided by the software packages such as Maple and Mathematica [1]. The Meijer's G-function (MGF) is a very general function which reduces to simpler special functions and elementary functions in many common cases such as:

- $\sin z = \sqrt{\pi} G^{1,0}_{0,2}(\frac{z^2}{4}|^{-}_{\frac{1}{2},0}), \qquad -\frac{\pi}{2} < \arg z \leq \frac{\pi}{2}$
- $\cos z = \sqrt{\pi} G^{1,0}_{0,2}(\frac{x^2}{4}|^{-}_{0,\frac{1}{2}}), \qquad \forall z$
- $\ln z = G^{1,2}_{2,2}(x-1|^{1,1}_{1,0}), \qquad \forall z$
- $J_\nu(z) = G^{1,0}_{0,2}(\frac{z^2}{4}|^{-}_{\frac{\nu}{2},\frac{-\nu}{2}}), \qquad -\frac{\pi}{2} < \arg z \leq \frac{\pi}{2}$

It is possible to represent the solutions of many problems in terms of MGFs [2-8]. In [7] we show that how all the excited states of radial functions belong to the Hydrogen atom can be made from ground state by introducing the Bohr operator and two important properties of MGFs such that

Definition 1.1 *The Bohr operator is represented by* $\hat{B}$ *and defined as follows:*

$$\hat{B}[(\frac{1}{r_B})^{\frac{3}{2}} G^{1,0}_{0,1}(^{-}_{0}|\frac{r}{r_B})] = [(\frac{1}{2r_B})^{\frac{3}{2}} G^{1,0}_{0,1}(^{-}_{0}|\frac{r}{2r_B})] \tag{1.1}$$

$$R_{10} = (\frac{1}{r_B})^{\frac{3}{2}} 2e^{-\frac{r}{r_B}} == 2(\frac{1}{r_B})^{\frac{3}{2}} G^{1,0}_{0,1}(^{-}_{0}|\frac{r}{r_B}), \tag{1.2}$$

with $r_B = \frac{\hbar^2}{m}\frac{4\pi\epsilon_0}{e^2}$ where R_{10} and r_B are ground radial state and the Bohr radius, *respectively.*

Affecting $\hat{B}$ on the following expression gives

$$\hat{B}[(\frac{r}{r_B})R_{10}] = \hat{B}[2(\frac{1}{r_B})^{\frac{3}{2}}(\frac{r}{r_B})e^{-\frac{r}{r_B}}] = 2(\frac{1}{2r_B})^{\frac{3}{2}}(\frac{r}{2r_B})e^{-\frac{r}{2r_B}} = \sqrt{3}R_{21}$$

$$R_{21} = (\frac{1}{2r_B})^{\frac{3}{2}}\frac{1}{\sqrt{3}}\frac{r}{r_B}e^{-\frac{r}{2r_B}}.$$

Furthermore, in [9] we introduce the fractional Bohr operator to deduce intermediate states of the Hydrogen atom. We obtain that the operator $\hat{B}^f(\frac{r}{r_B})^f$ maps the ground radial state R_{10} into multiple of intermediate radial states such that

$$\hat{B}^{\mathfrak{f}}\{[\frac{1}{r_B}]^{\frac{3}{2}} G^{1,0}_{0,1}(^{-}_{0}|\frac{r}{r_B})\} = \{[\frac{1}{(1+\mathfrak{f})r_B}]^{\frac{3}{2}} G^{1,0}_{0,1}(^{-}_{0}|\frac{r}{[(1+\mathfrak{f})r_B}]\}$$

$$\hat{B}^{\mathfrak{f}}(\frac{r}{r_B})^{\mathfrak{f}}R_{10} = 2[\frac{1}{(1+\mathfrak{f})r_B}]^{\frac{3}{2}}[\frac{r}{(1+\mathfrak{f})r_B}]^{\mathfrak{f}}e^{-\frac{r}{(1+\mathfrak{f})r_B}} \tag{1.3}$$

In this paper we show that by choosing convenient values of *f* there exist similarity in analytic behavior of Technetium-99m (99mTc)in generator and intermediate states of Hydrogen atom (between R_{21} and R_{10}).

GAMMA DECAY IN (99mTc → 99Tc + γ)

In gamma decay a nucleus changes from a higher energy state to a lower energy state through the emission of electromagnetic radiation (photons). The number of protons (and neutrons) in the nucleus does not change in this process, so the parent and daughter atoms are the same chemical element. Gamma decays happen most often after a alpha or beta decay. Gamma radiation is the most useful type of radiation for medical purposes, but at the same time it is the most dangerous because of its ability to penetrate large thicknesses of material [11-12].

A quantity is subject to exponential decay if it decreases at a rate proportional to its current value. Symbolically, this process can be expressed by the following differential equation, where N is the quantity and λ is a positive rate called the exponential decay constant:

$$\frac{dN}{dt} = -\lambda N. \qquad (2.1)$$

The solution to this equation is

$$N(t) = N_0 e^{-\lambda t} \qquad (2.2)$$

Here N(t) is the quantity at time t, and $N_0 = N(0)$ is the initial quantity, i.e. the quantity at time t = 0 [11].

Mean lifetime

If the decaying quantity, N(t), is the number of discrete elements in a certain set, it is possible to compute the average length of time that an element remains in the set. This is called the mean lifetime (or simply the lifetime or the exponential time constant), τ, and it can be shown that it relates to the decay rate, λ, in the following way [11-12]:

$$\tau = \frac{1}{\lambda}. \qquad (2.3)$$

The mean lifetime can be looked at as a "scaling time", because we can write the exponential decay equation in terms of the mean lifetime, , instead of the decay constant, λ:

$$N(t) = N_0 e^{-\frac{t}{\tau}}. \qquad (2.4)$$

Technetium-99m was discovered as a product of cyclotron bombardment of molybdenum. This procedure produced molybdenum-99, a radionuclide with a longer half-life (2.75 days), which decays to Tc-99m. At present, molybdenum-99 (Mo-99) is used commercially as the easily transportable source of medically used Tc-99m [10].

Technetium-99m is a metastable nuclear isomer, as indicated by the "m" after its mass number 99. This means it is a decay product whose nucleus remains in an excited state that lasts much longer than is typical. The nucleus will eventually relax (i.e., de-excite) to its ground state through the emission of gamma rays or internal conversion electrons. Both of these decay modes rearrange the nucleons without transmuting the technetium into another element.

Tc-99m decays mainly by gamma emission, slightly less than 88% of the time. (99mTc → 99Tc +γ) About 98.6% of these gamma decays result in 140.5 keV gamma rays and the remaining 1.4% are to gammas of a slightly higher energy at 142.6 keV. These are the radiations that are picked up by a gamma camera when 99mTc is used as a radioactive tracer for medical imaging. The remaining approximately 12% of 99mTc decays are by means of internal conversion, resulting in ejection of high speed internal conversion electrons in several sharp peaks (as is typical of electrons from this type of decay) also at about 140 keV (99mTc → 99Tc+ + e−). These conversion electrons will ionize the surrounding matter like beta radiation electrons would do, contributing along with the 140.5 keV and 142.6 keV gammas to the total deposited dose [10-12].

Operation of a 99m-Tc generator

Suppose we have a sample of 99Mo and suppose that at time t=0 there are N_0 nuclei in our sample and nothing else. The number N(t) of 99Mo nuclei decreases with time according to radioactive decay law:

$$N(t) = N_0 e^{-\lambda_{\mathrm{Mo}} t},$$

where λ_{Mo} is the decay constant for 99Mo.

Thus the number of 99Mo nuclei that decay during a small time interval dt is given by

$$dN(t) = -\lambda_{Mo} N_0 e^{-\lambda_{Mo} t} dt$$

Since 99Mo decays into 99mTc, the same number of 99mTc nuclei are formed during the time period dt. At a time t', only a fraction dn(t') of these nuclei will still be around since the 99mTc is also decaying. The time for 99mTc to decay is given by *t'-t*. Plugging this into radioactive the decay law we arrive at:

$$dn(\acute{t}) = -dN(t) e^{-\lambda_{Tc}(t-\acute{t})} = \lambda_{Mo} N_0 e^{-\lambda_{Mo} t} e^{-\lambda_{Tc}(t-\acute{t})} dt$$

If we sum up the little contributions dn(t') and integrate over t in order to find the number n(t'), that is the number of all 99mTc nuclei present at the time t', we find:

$$n(\acute{t}) = \frac{\lambda_{Mo}}{\lambda_{Tc} - \lambda_{Mo}} N_0 e^{-\lambda_{Tc} \acute{t}} [e^{(\lambda_{Tc} - \lambda_{Mo}) \acute{t}} - 1] \qquad (2.5)$$

The Fig. 1 illustrates the outcome of this calculation. The horizontal axis represents time (in days), while the vertical one represents the number of nuclei present (in arbitrary units). The green curve illustrates the exponential decay of a sample of pure 99mTc. The red curve shows the number of 99mTc nuclei present in a 99mTc generator that is never eluted. Finally, the blue curve shows the situation for a 99mTc generator that is eluted every 12 hours.

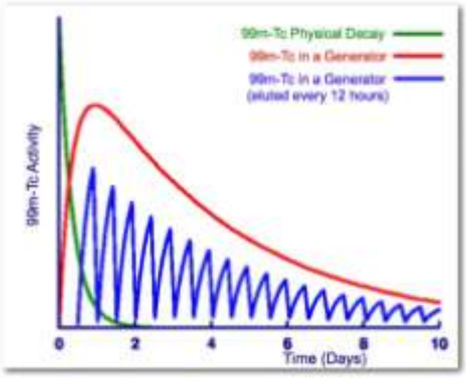

Figure 1: Comparison of the physical decay of 99m-Tc with its activity arising from 99Mo decay in a radioisotope generator with and without elution at 12 hour intervals [see 12].

MAIN RESULTS

Now we compare 99mTc physical decay and 99mTc in generator with ground radial state and intermediate state of the Hydrogen atom, respectively. Regarding Eq. (1.3) and assigning six different values for $f = 0, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ gives the following functions in Table 1.

Table 1 : Analytic radial states of the Hydrogen atom (both stationary and intermediate)

f	Function
0	$g_1 = e^{-r}$
$\frac{1}{8}$	$g_2 = (\frac{8r}{9})^{\frac{1}{8}} e^{-\frac{8r}{9}}$
$\frac{1}{4}$	$g_3 = (\frac{4r}{5})^{\frac{1}{4}} e^{-\frac{4r}{5}}$
$\frac{1}{2}$	$g_4 = (\frac{2r}{3})^{\frac{1}{2}} e^{-\frac{2r}{3}}$
$\frac{3}{4}$	$g_5 = (\frac{4r}{7})^{\frac{3}{4}} e^{-\frac{4r}{7}}$
1	$g_6 = (\frac{r}{2}) e^{-\frac{r}{2}}$

In Fig.1 the green curve illustrates the exponential physical decay of a sample of pure 99mTc and horizontal axis represents time, while in Fig 2. the blue curve illustrates the ground radial state of the Hydrogen atom R_{10} and horizontal axis represents radial distance. There is an interesting fact. In spite of this difference in type of variables there exists an interesting fact which analytical behavior both of them is equal. In the same way, in Fig.1 the red curve illustrates 99m-Tc in a radioisotope generator and horizontal axis represents time, while in Fig.4 the red curve the radial state R_{21} and horizontal axis represents radial distance. As we can see from these figures (1-4) analytical behavior of these two phenomena is similar.

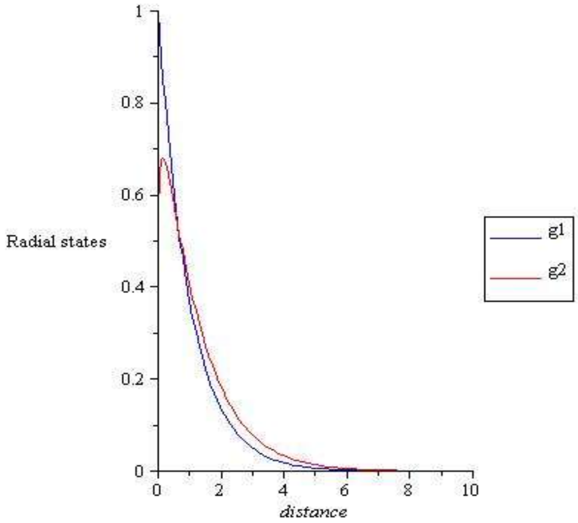

Figure 2: Curves of Ground radial state (f=0) and intermediate state of the Hydrogen atom (f=1/8)

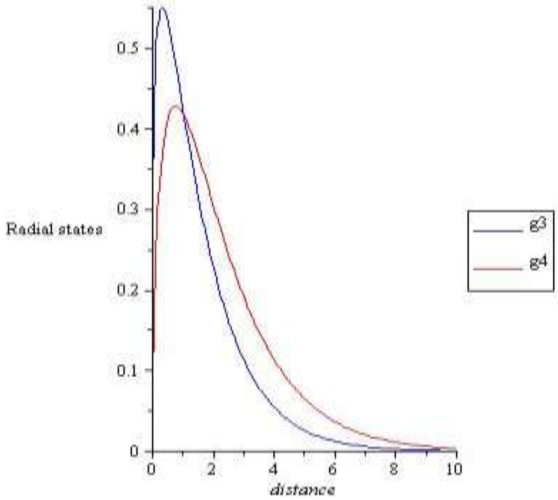

Figure 3: Curves of two intermediate states of the Hydrogen atom (f=1/4, 1/2)

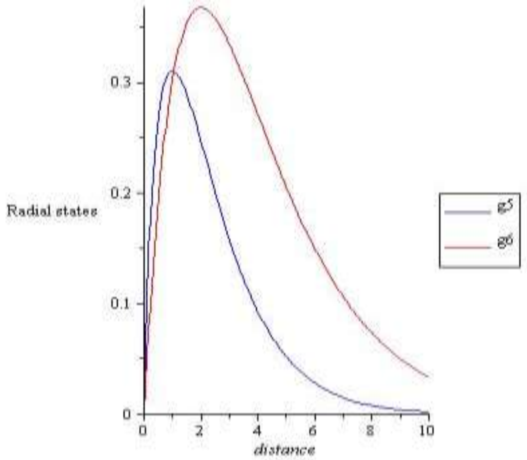

Figure 4: Curves of intermediate state of the Hydrogen atom (f=3/4) and Radial state R_{21}

CONCLUSIONS

This paper shows that two phenomena in scale of atomic and nuclear physics gives similarity in analytic behavior which is obtained from figures.

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