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Electric Charge transmission through Four Bosons

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ABSTRACT

A non-Maxwellian model based on four bosons is considered. Based on an enlarged $U(1)\times SO(2)$ abelian symmetry and preserving the postulates of light invariance and electric charge conservation its aim is to transfer electric charge $\Delta Q=0$ and $|\Delta Q|=1$. For this, it enlarges Maxwell by associating a fields set $\{A_\mu, U_\mu, V_\mu^{\pm}\}$. An electromagnetic system with four intermediary bosons is proposed. It introduces a new electric charge physics, a new electromagnetic transmission and a new photon physics.

Thus, this work says that, while Maxwell electromagnetism focus on charge distribution and forces, there is still room for an electromagnetism based on charge transmission and gauge bosons. A new interpretation on the meaning of electric charge flux through gauge bosons is taken. Instead of just mediating different nature forces, the gauge bosons physics should be first understood as a mechanism for transferring energy-momentum and quantum numbers as electric charge. At this way, for electric charge transportation four connected gauge bosons are necessary. They are the photon, massive photon and two charged fields.

For this, a whole quantum field theory supported on the set principle is generated. It is based on the fields family meaning associated to a same Lorentz representation (i, j) and tied up by a common gauge symmetry. Our physical case are four bosons from $(\frac{1}{2},\frac{1}{2})$ representation sharing an abelian symmetry. This four set provides the physicity necessary for transmitting any electric charge value.

A new perception for the electromagnetic phenomena is envisaged. It is generalized for a four fields abelian Lagrangian made of with antisymmetric and symmetric fields strengths, producing three and four abelian vertices, and generating whole relativistic equations with seven new features. It holds a dynamics carrying spin-1 and spin-0, based on granular and collective fields strengths, set determinism with directive and chance, quanta network, nonlinearity, neutral charges, photonics.

A new scale for electromagnetic interaction is proposed. New intensities and range are derived through coupled nonlinear equations, massive particles and the presence of coupling constants beyond the electric charge. A light universality is obtained. Light becomes an absolute (light invariance), ubiquous (couplings constants diverse from electric charge), directive (fields set vector), contingencies producer (relativism and chance), inner light (selfinteracting photons).

1 Introduction

Maxwell equations are originated from electric charge and derive light as EM fields propagation. They provided two historical measurements at 19^{th} century. They are the light wave propagation $c=\frac{1}{\mu_0\varepsilon_0}=3.10^8$ m/s and the light invariance. These facts have fixed the physics of our time.

Nevertheless non-Maxwell models have been developed by taken Maxwell equations as starting point for the electromagnetism enlargement. The search is to look for a more complete set of equations describing the EM phenomena. The challenge becomes how through light symmetry to expand Maxwell equations. Symmetry works as the Ariadne's line in physics. However, we should take care how to move through this electromagnetic labyrinth. Not lose its physical context.

Under this challenge, a new fact appears from the electric charge flux. While at 19^{th} century Maxwell introduced by hand the displacement current, at 20^{th} century the elementary particles processes contain a new observation for electric charge conservation law. The electric charge flux moves not only through Maxwell transmission $\Delta Q=0$, but also, through a non-Maxwell transmission $|\Delta Q|=1$. Consequently, given this experimental fact, the EM phenomena should not only be partnered only by the photon. A new EM through four messengers becomes necessary.

Historically, light symmetry opened three phases for physics be developed. The first was Maxwell and the electric charge; the second, relativity and the space-time, matter-energy correlations; the third, Lorentz group and the spin. We

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understand that there is still room for a new physical approach from light symmetry which is the existence of fields families inside of each LG representation. This lead us to introduce for the $\{\frac{1}{2},\frac{1}{2}\}$ representation the fields family $A_{\mu_I} = \{A_\mu, U_\mu, V_\mu^{\pm}\}$. It provides the four messengers required for conducting the electric charge flux at microscopic processes.

Thus, the objective of this work is to develop a non-Maxwellian model able to transmit $\Delta Q = 0, \pm 1$ under four gauge bosons. The paper organizes such new electromagnetic performance as follows. In section 2, the Four Bosons Lagrangian is reintroduced. Based on Lorentz group and gauge invariance the physical structure previously developed is considered. At section 3, historical non-Maxwellian Lagrangians are recapitulated in order to compare their nonlinearities achievements with this work results. At section 4, whole relativistic fields equations are derived. Charges associated to every quanta are obtained diverse from electric charge. Section 5, introduces a new photon. Being the singular quanta where light invariance is expressed the photon is understood as the fields set directive. It yields that its corresponding Euler-Lagrange equation is combined to the symmetry equation associated to representation $\{\frac{1}{2},\frac{1}{2}\}$. At section 6, one explores on the symmetry management. As example, one composes the ratio between the collective and the granular energies. Section 7, separates the spin-1 and spin-0 dynamics inserted in the theory. At section 8, one analyses on the consequences from the four bosons electromagnetism model on the three basic EM elements (electric charge, EM fields, light). At section 9, the corresponding Global Maxwell Equations are studied at vectorial form. The effective photon vectorial equation is expressed at section 10. At section 11, nonlinear EM waves are considered. As conclusion, highlights about this extended EM are taken at section 12.

2 Four Bosons Lagrangian

Electromagnetism is a physics based on light invariance and charge conservation [1]. Its standard model is Maxwell theory [2] which builds up a granular and linear electromagnetism. The challenge is to go beyond. Different efforts occur in literature regarding non-Maxwellian theories. Actually there are 42 models proposing new electromagnetic performances [3]. Our objective is to study on the four bosons electromagnetic model [4]. While Maxwell works with electric charge distribution one extends to the electric charge exchanging. For this, consider four messengers $A_{\mu I} \equiv \{A_\mu, U_\mu, V_\mu^{\pm}\}$ under $\mathrm{U}(1) \times \mathrm{SO}(2)$ abelian symmetry sharing a common gauge parameter

$$
\begin{aligned}
A_\mu' &= A_\mu + k_1 \partial_\mu \alpha, \\
U_\mu' &= U_\mu + k_2 \partial_\mu \alpha, \\
V_\mu^{+}{}' &= e^{iq\alpha} V_\mu^{+} + k_{+} \partial_\mu \alpha, \\
V_\mu^{-}{}' &= e^{-iq\alpha} V_\mu^{-} + k_{-} \partial_\mu \alpha.
\end{aligned}
\tag{2.1}
$$

Eq.(2.1) means an enlarged abelian symmetry [5]. It introduces the presence of different potential fields in the same $\{\frac{1}{2},\frac{1}{2}\}$ Lorentz representation tied up under a common gauge parameter. The Maxwell basic abelian symmetry is preserved and a charged nonlinear abelian gauge theory is generated [6].

The effort of this work is to discover a path for EM physics through the light metric $c^2t'^2 - x'^2 - y'^2 - z'^2 = c^2t^2 - x^2 - y^2 - z^2$. Historically, light symmetry guided Maxwell equation, Einstein relativity, Wigner-Lorentz group and now we would like to understand through a fields set. There is a new EM proposal to be understood where differently from Maxwell, the electric charge will be transported through four bosons.

Eq.(2.1) provides the following whole Lagrangian

$$L = L_K + L_{GF} + L_m + L_I \tag{2.2}$$

where eq (2.2) proposes an electromagnetism which physics foundation is on the energy momentum and electric charge transferring based on four messengers. They are the photon field-A_μ massive photon field-U_μ, massive charged photons fields-$V_\mu^{\pm}$ originated from a fields family derived of light invariance. The task here will be to investigate its performance on the three EM elements which are electric charge, EM fields and photon behavior.

Our study starts with the Lagrangian kinect term. Separating in antisymmetric and symmetric sectors, it gives

$$L_K = L_K^A + L_K^S \tag{2.3}$$

where

$$L_K^A = a_1 F_{\mu\nu} F^{\mu\nu} + a_2 U_{\mu\nu} U^{\mu\nu} + 2a_3 V_{\mu\nu}^{+} V^{\mu\nu-}, \tag{2.4}$$

$$\begin{aligned} L_K^S = {} & b_{(11)} S_{\mu\nu}^{1} S^{\mu\nu 1} + b_{(22)} S_{\mu\nu}^{2} S^{\mu\nu 2} + 2b_{(33)} S_{\mu\nu}^{+} S^{\mu\nu-} + \\ & + c_{(11)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 1} + c_{(22)} S_{\mu}^{\mu 2} S_{\nu}^{\nu 2} + 2c_{(12)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 2} + 2c_{(33)} S_{\mu}^{\mu+} S_{\nu}^{\nu-}, \end{aligned} \tag{2.5}$$

with the following fields strengths definitions

$$\begin{aligned} & F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad U_{\mu\nu} \equiv \partial_\mu U_\nu - \partial_\nu U_\mu, \qquad V_{\mu\nu}^{\pm} \equiv \partial_\mu V_\nu^{\pm} - \partial_\nu V_\mu^{\pm}, \\ & S_{\mu\nu}^{1} \equiv \partial_\mu A_\nu + \partial_\nu A_\mu, \qquad S_{\mu\nu}^{2} \equiv \partial_\mu U_\nu + \partial_\nu U_\mu, \qquad S_{\mu\nu}^{\pm} \equiv \partial_\mu V_\nu^{\pm} + \partial_\nu V_\mu^{\pm}. \end{aligned} \tag{2.6}$$

where separately $S_{\mu\nu}^{I}$ tensors are not gauge invariant, but as a summation it is.

The gauge-fixing term is

$$L_{GF} = \frac{1}{4}\xi_{(11)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 1} + \frac{1}{4}\xi_{(22)} S_{\mu}^{\mu 2} S_{\nu}^{\nu 2} + \frac{1}{2}\xi_{(12)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 2} + \frac{1}{2}\xi_{(33)} S_{\mu}^{\mu+} S_{\nu}^{\nu-}. \tag{2.7}$$

Notice that eq.(2.7) does not necessarily cancel degrees of freedom in order to establish the gauge fixing condition.

The mass term is

$$L_m = -\frac{1}{2}\mu_U^2 U_\mu U^\mu - \mu_V^2 V_\mu^{+} V^{\mu-} \tag{2.8}$$

The corresponding physical masses are that ones associated to the poles of a two point Green functions. Consequently, eq.(2.3) is a model where the spin-1 transverse sector carries a massless photon, a massive photon and two charged photons with the same mass. Similarly for the spin-0 longitudinal sector. Interestingly, the spin-1 and spin-0 families provide a quanta diversity. The associated masses and residues in the transverse and longitudinal sectors are differentiated [7]. As result, this enlarged $\mathrm{U}(1)\times\mathrm{SO}(2)$ gauge invariance produces a model unitary [7], renormalizable [8] and phenomenologically diversified. It provides a healthy phenomenology to be understood through their whole relativistic equations and propagators.

The interaction Lagrangian is composed of a trilinear term and a quadrilinear term

$$L_I = L_3 + L_4 \tag{2.9}$$

Eq.(2.9) is over terms originated from the generic field strength $Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)}$ where $Z_{[\mu\nu]} = F_{\mu\nu} + U_{\mu\nu} + V_{\mu\nu}^{+} + V_{\mu\nu}^{-} + z_{[\mu\nu]}$ and $Z_{(\mu\nu)} = \beta_I S_{\mu\nu}^{I} + g_{\mu\nu}\rho_I S_{\alpha}^{\alpha I} + z_{(\mu\nu)} + g_{\mu\nu}\omega_{\alpha}^{\alpha}$. Physically, aside to granular fields strengths at eq.(2.6), it contains a collective structure from the fields set. It introduces the fields conglomerates $z_{\mu\nu} = \gamma_{IJ} A_\mu^I A_\nu^J$, which antisymmetric expression is given by the following six tensors

$$z_{[\mu\nu]}=\begin{pmatrix}0 & z^{[1,2]} & z^{[1,+]} & z^{[1,-]}\\ -z^{[1,2]} & 0 & z^{[2,+]} & z^{[2,-]}\\ -z^{[+,1]} & -z^{[+,2]} & 0 & z^{[+,-]}\\ -z^{[-,1]} & -z^{[-,2]} & -z^{[-,+]} & 0\end{pmatrix} \tag{2.10}$$

and with more 10 symmetric tensors.

$$z_{(\mu\nu)}=\begin{pmatrix}z^{(1,1)} & z^{(1,2)} & z^{(1,+)} & z^{(1,-)}\\ z^{(1,2)} & z^{(2,2)} & z^{(2,+)} & z^{(2,-)}\\ z^{(1,+)} & z^{(2,+)} & z^{(+,+)} & z^{(+,-)}\\ z^{(1,-)} & z^{(2,-)} & z^{(+,-)} & z^{(-,-)}\end{pmatrix} \tag{2.11}$$

Similarly, for $\omega_{\alpha}^{\alpha}=\tau_{(IJ)}A_{\alpha}^{I}A^{\alpha J}$ coming from the generic symmetric tensor.

Considering the three vertices case, it yields two abelian terms which are separately gauge invariant

$$L_3=L_3^A+L_3^S \tag{2.12}$$

where

$$L_3^A=4(b_1F_{\mu\nu}+b_2U_{\mu\nu})\overset{[12]\mu\nu}{z}+4(b_1F_{\mu\nu}+b_2U_{\mu\nu})\overset{[+-]\mu\nu}{z}+4(\beta_1F_{\mu\nu}+\beta_2U_{\mu\nu})\overset{(+-)\mu\nu}{z}$$
$$+4b_3\{V_{\mu\nu}^{+}(\overset{[-1]\mu\nu}{z}+\overset{[-2]\mu\nu}{z})\}+4b_3\{V_{\mu\nu}^{-}(\overset{[+1]\mu\nu}{z}+\overset{[+2]\mu\nu}{z})\}$$

(2.13)

and

$$L_3^S=2(\beta_1S_{\mu\nu}^1+\beta_2S_{\mu\nu}^2)(\overset{(11)\mu\nu}{z}+2\overset{(12)\mu\nu}{z}+\overset{(22)\mu\nu}{z})$$
$$+2(\rho_1S_{\mu}^{\mu1}+\rho_2S_{\mu}^{\mu2})(\overset{(11)}{z}{}_{\nu}^{\nu}+2\overset{(12)}{z}{}_{\nu}^{\nu}+\overset{(22)}{z}{}_{\nu}^{\nu})$$
$$+2[(\beta_1+4\rho_1)S_{\mu}^{\mu1}+(\beta_2+4\rho_2)S_{\mu}^{\mu2}](\overset{(11)}{\omega}{}_{\nu}^{\nu}+2\overset{(12)}{\omega}{}_{\nu}^{\nu}+\overset{(22)}{\omega}{}_{\nu}^{\nu})$$
$$+8\beta_3\mathrm{Re}\{(\overset{(-1)\mu\nu}{z}+\overset{(-2)\mu\nu}{z})S_{\mu\nu}^{+}\}+4\overset{+-3\mu\nu}{z}(\beta_1S_{\mu\nu}^1+\beta_2S_{\mu\nu}^2)$$
$$+8\rho_3\mathrm{Re}\{(\overset{(-1)}{z}{}_{\nu}^{\nu}+\overset{(-2)}{z}{}_{\nu}^{\nu})S_{\mu}^{\mu+}\}+8(\beta_3+4\rho_3)\mathrm{Re}\{(\overset{(-1)}{\omega}{}_{\nu}^{\nu}+\overset{(-2)}{\omega}{}_{\nu}^{\nu})S_{\mu}^{\mu+}\}$$
$$+4\overset{+-3}{z}{}_{\nu}^{\nu}(\rho_1S_{\mu}^{\mu1}+\rho_2S_{\mu}^{\mu2})+4(\beta_1+4\rho_1)\overset{+-3}{\omega}{}_{\nu}^{\nu}S_{\mu}^{\mu1}+4(\beta_2+4\rho_2)\overset{+-3}{\omega}{}_{\nu}^{\nu}S_{\mu}^{\mu2}.$$

(2.14)

The four interactions are expressed as

$$L_4=L_4^A+L_4^S \tag{2.15}$$

which are separately gauge invariant pieces. It gives,

$$
\begin{aligned}
L_4^A = {} & 2\overset{[12]}{z}_{\mu\nu}\overset{[12]}{z}^{\mu\nu}+2\overset{[12]}{z}_{\mu\nu}\overset{[21]}{z}^{\mu\nu}-4\overset{[13+]}{z}{}^{\mu}_{\mu}\overset{[13-]}{z}{}^{\nu}_{\nu}-4\overset{[23+]}{z}{}^{\mu}_{\mu}\overset{[23-]}{z}{}^{\nu}_{\nu} \\
& +4\overset{[13+]}{z}_{\mu\nu}\overset{[13-]}{z}^{\mu\nu}+4\overset{[23+]}{z}_{\mu\nu}\overset{[23-]}{z}^{\mu\nu}+8\overset{[13+]}{z}_{\mu\nu}\overset{[23-]}{z}^{\mu\nu} \\
& -8\mathrm{Re}\{\overset{[13+]}{z}{}^{\mu}_{\mu}\overset{[23-]}{z}{}^{\nu}_{\nu}\}+8\overset{[12]}{z}_{\mu\nu}\mathrm{Re}\{\overset{[+-]}{z}^{\mu\nu}\}+8\mathrm{Im}\{\overset{[13+]}{z}{}^{\mu}_{\mu}\overset{[24-]}{z}{}^{\nu}_{\nu}\} \\
& +2\overset{[+-]}{z}_{\mu\nu}\overset{[+-]}{z}^{\mu\nu}-2\overset{[+-]}{z}_{\mu\nu}\overset{[-+]}{z}^{\mu\nu}-4\overset{[+-]}{z}{}^{\mu}_{\mu}\overset{[+-]}{z}{}^{\nu}_{\nu}
\end{aligned}
\qquad (2.16)
$$

and

$$
\begin{aligned}
L_4^S = {} & \overset{(11)}{z}_{\mu\nu}\overset{(11)}{z}^{\mu\nu}+\overset{(22)}{z}_{\mu\nu}\overset{(22)}{z}^{\mu\nu}+2\overset{(11)}{z}_{\mu\nu}\overset{(11)}{\omega}^{\mu\nu}+2\overset{(22)}{z}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu} \\
& +4\overset{(11)}{\omega}_{\mu\nu}\overset{(11)}{\omega}^{\mu\nu}+4\overset{(22)}{\omega}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu}+4\overset{(11)}{z}_{\mu\nu}\overset{(12)}{z}^{\mu\nu}+4\overset{(12)}{z}_{\mu\nu}\overset{(22)}{z}^{\mu\nu} \\
& +8\overset{(12)}{z}_{\mu\nu}\overset{(11)}{\omega}^{\mu\nu}+8\overset{(12)}{z}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu}+16\overset{(11)}{\omega}_{\mu\nu}\overset{(12)}{\omega}^{\mu\nu}+16\overset{(12)}{\omega}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu} \\
& +2\overset{(11)}{z}_{\mu\nu}\overset{(22)}{z}^{\mu\nu}+2\overset{(12)}{z}_{\mu\nu}\overset{(12)}{z}^{\mu\nu}+8\overset{(12)}{z}_{\mu\nu}\overset{(12)}{\omega}^{\nu\mu}+16\overset{(12)}{\omega}_{\mu\nu}\overset{(12)}{\omega}^{\nu\mu} \\
& +4\overset{(11)}{z}{}^{\mu}_{\mu}\overset{(22)}{\omega}{}^{\nu}_{\nu}+8\overset{(11)}{\omega}{}^{\mu}_{\mu}\overset{(22)}{\omega}{}^{\nu}_{\nu}+2\overset{(12)}{z}_{\mu\nu}\overset{(12)}{z}^{\nu\mu}+4(\overset{(11)}{z}_{\mu\nu}+\overset{(22)}{z}_{\mu\nu})\overset{+-3}{z}^{\mu\nu} \\
& +4\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(13-)}{z}{}^{\nu}_{\nu}+4\overset{(23+)}{z}{}^{\mu}_{\mu}\overset{(23-)}{z}{}^{\nu}_{\nu}+16\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(13-)}{\omega}{}^{\nu}_{\nu}+16\overset{(23+)}{z}{}^{\mu}_{\mu}\overset{(23-)}{\omega}{}^{\nu}_{\nu} \\
& +32\overset{(13+)}{\omega}{}^{\mu}_{\mu}\overset{(13-)}{\omega}{}^{\nu}_{\nu}+32\overset{(23+)}{\omega}{}^{\mu}_{\mu}\overset{(23-)}{\omega}{}^{\nu}_{\nu}+4\overset{(13+)}{z}_{\mu\nu}\overset{(13-)}{z}^{\mu\nu}+4\overset{(23+)}{z}_{\mu\nu}\overset{(23-)}{z}^{\mu\nu} \\
& +8\overset{(13+)}{z}_{\mu\nu}\overset{(23-)}{z}^{\mu\nu}+8\{\overset{(11)}{z}{}^{\mu}_{\mu}+\overset{(22)}{z}{}^{\mu}_{\mu}+2\overset{(11)}{\omega}{}^{\mu}_{\mu}+2\overset{(22)}{\omega}{}^{\mu}_{\mu}+2\overset{(12)}{z}{}^{\mu}_{\mu}+4\overset{(12)}{\omega}{}^{\mu}_{\mu}\}\overset{+-3}{\omega}{}^{\nu}_{\nu} \\
& +8\overset{(12)}{z}_{\mu\nu}\mathrm{Re}\{\overset{+-3}{z}^{\mu\nu}\}+8\mathrm{Re}\{\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(23-)}{z}{}^{\nu}_{\nu}\}+32\mathrm{Re}\{\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(13-)}{\omega}{}^{\nu}_{\nu}\} \\
& +64\mathrm{Re}\{\overset{(13+)}{\omega}{}^{\mu}_{\mu}\overset{(23-)}{\omega}{}^{\nu}_{\nu}\}-8\mathrm{Im}\{\overset{(13+)}{z}_{\mu\nu}\overset{(24-)}{\omega}^{\nu\mu}\}-8\mathrm{Im}\{\overset{(23+)}{z}_{\mu\nu}\overset{(14-)}{\omega}^{\nu\mu}\} \\
& -32\mathrm{Im}\{\overset{(13+)}{\omega}_{\mu\nu}\overset{(24-)}{\omega}^{\nu\mu}\}+32\mathrm{Im}\{\overset{(14+)}{\omega}_{\mu\nu}\overset{(23-)}{\omega}^{\nu\mu}\}+4\mathrm{Im}\{\overset{(13+)}{z}_{\mu\nu}\overset{(24-)}{z}^{\nu\mu}\} \\
& -4\mathrm{Im}\{\overset{(14+)}{z}_{\mu\nu}\overset{(23-)}{z}^{\nu\mu}\}+8\mathrm{Im}\{\overset{(14+)}{z}_{\mu\nu}\overset{(23-)}{\omega}^{\nu\mu}\}+8\mathrm{Im}\{\overset{(24+)}{z}_{\mu\nu}\overset{(13-)}{\omega}^{\nu\mu}\} \\
& +2\{\overset{+-3}{z}_{\mu\nu}\overset{+-4}{z}^{\mu\nu}+\overset{+-3}{z}_{\mu\nu}\overset{-+4}{z}^{\mu\nu}\}-8\{\overset{(+-)}{z}_{\mu\nu}\overset{(+-)}{\omega}^{\mu\nu}-\overset{(+-)}{z}_{\mu\nu}\overset{(-+)}{\omega}^{\mu\nu}\} \\
& -16\{\overset{(+-)}{\omega}_{\mu\nu}\overset{(+-)}{\omega}^{\mu\nu}-\overset{(+-)}{\omega}_{\mu\nu}\overset{(-+)}{\omega}^{\mu\nu}\}-4\overset{(+-)}{z}{}^{\mu}_{\mu}\overset{(+-)}{z}{}^{\nu}_{\nu}+8\overset{+-3}{z}{}^{\mu}_{\mu}\overset{+-4}{\omega}{}^{\nu}_{\nu} \\
& +16\overset{+-3}{\omega}{}^{\mu}_{\mu}\overset{+-4}{\omega}{}^{\nu}_{\nu}.
\end{aligned}
\qquad (2.17)
$$

Eq.(2.9) was chosen to be described through a formalism which expresses the original Lagrangian at eq.(2.2) in terms of granular and collective fields strengths. At Apendice A it is written in terms of potential fields. Nevertheless, given the dualism individual collective present at this model, we prefer to write eqs.(2.12-2.17) as made of by a composition between individual and collective contributions. For this, it was necessary to define the collective fields strengths expressions at eq.(2.10) and eq.(2.11) as

$$\overset{(11)\mu\nu}{z} \equiv \gamma_{(11)}A^{\mu}A^{\nu}, \quad \overset{(22)\mu\nu}{z} \equiv \gamma_{(22)}U^{\mu}U^{\nu}, \quad \overset{(12)\mu\nu}{z} \equiv \gamma_{(12)}A^{\mu}U^{\nu},$$

$$\overset{(21)\mu\nu}{z} \equiv \gamma_{(21)}U^{\mu}A^{\nu} \quad \overset{(11)}{z}{}_{\mu}^{\mu} = \gamma_{(11)}A_{\mu}A^{\mu}, \quad \overset{(22)}{z}{}_{\mu}^{\mu} = \gamma_{(22)}U_{\mu}U^{\mu},$$

$$\overset{(12)}{z}{}_{\mu}^{\mu} = \gamma_{(12)}A_{\mu}U^{\mu}, \quad \overset{[12]\mu\nu}{z} \equiv \gamma_{[12]}A^{\mu}U^{\nu}, \quad \overset{[21]\mu\nu}{z} \equiv \gamma_{[21]}U^{\mu}A^{\nu},$$

$$\overset{(13+)\mu\nu}{z} \equiv \gamma_{(13)}A^{\mu}V^{\nu+}, \quad \overset{(13-)\mu\nu}{z} \equiv \{\overset{(13+)\mu\nu}{z}\}^{*} = \gamma_{(13)}A^{\mu}V^{\nu-},$$

$$\overset{(14+)\mu\nu}{z} \equiv \gamma_{(14)}A^{\mu}V^{\nu+}, \quad \overset{(14-)\mu\nu}{z} \equiv \{\overset{(14+)\mu\nu}{z}\}^{*} = \gamma_{(14)}A^{\mu}V^{\nu-},$$

$$\overset{[13+]\mu\nu}{z} \equiv \gamma_{[13]}A^{\mu}V^{\nu+}, \quad \overset{[13-]\mu\nu}{z} \equiv \{\overset{[13+]\mu\nu}{z}\}^{*} = \gamma_{[13]}A^{\mu}V^{\nu-},$$

$$\overset{[14+]\mu\nu}{z} \equiv \gamma_{[14]}A^{\mu}V^{\nu+}, \quad \overset{[14-]\mu\nu}{z} \equiv \{\overset{[14+]\mu\nu}{z}\}^{*} = \gamma_{[14]}A^{\mu}V^{\nu-},$$

$$\overset{(23+)\mu\nu}{z} \equiv \gamma_{(23)}U^{\mu}V^{\nu+}, \quad \overset{(23-)\mu\nu}{z} \equiv \{\overset{(23+)\mu\nu}{z}\}^{*} = \gamma_{(23)}U^{\mu}V^{\nu-},$$

$$\overset{(24+)\mu\nu}{z} \equiv \gamma_{(24)}U^{\mu}V^{\nu+}, \quad \overset{(24-)\mu\nu}{z} \equiv \{\overset{(24+)\mu\nu}{z}\}^{*} = \gamma_{(24)}U^{\mu}V^{\nu-},$$

$$\overset{[23+]\mu\nu}{z} \equiv \gamma_{[23]}U^{\mu}V^{\nu+}, \quad \overset{[23-]\mu\nu}{z} \equiv \{\overset{[23+]\mu\nu}{z}\}^{*} = \gamma_{[23]}U^{\mu}V^{\nu-},$$

$$\overset{[24+]\mu\nu}{z} \equiv \gamma_{[24]}U^{\mu}V^{\nu+}, \quad \overset{[24-]\mu\nu}{z} \equiv \{\overset{[24+]\mu\nu}{z}\}^{*} = \gamma_{[24]}U^{\mu}V^{\nu-},$$

$$\overset{+-3\mu\nu}{z} \equiv \gamma_{(33)}V^{\mu+}V^{\nu-}, \quad \overset{-+3\mu\nu}{z} \equiv \{\overset{+-3\mu\nu}{z}\}^{*} = \gamma_{(33)}V^{\mu-}V^{\nu+},$$

$$\overset{+-4\mu\nu}{z} \equiv \gamma_{(44)}V^{\mu+}V^{\nu-}, \quad \overset{-+4\mu\nu}{z} \equiv \{\overset{+-4\mu\nu}{z}\}^{*} = \gamma_{(44)}V^{\mu-}V^{\nu+},$$

$$\overset{(+-)\mu\nu}{z} \equiv -i\gamma_{(34)}V^{\mu+}V^{\nu-}, \quad \overset{(-+)\mu\nu}{z} \equiv \{\overset{(+-)\mu\nu}{z}\}^{*} = i\gamma_{(34)}V^{\mu-}V^{\nu+},$$

$$\overset{[+-]\mu\nu}{z} \equiv -i\gamma_{[34]}V^{\mu+}V^{\nu-}, \quad \overset{[-+]\mu\nu}{z} \equiv \{\overset{[+-]\mu\nu}{z}\}^{*} = i\gamma_{[34]}V^{\mu-}V^{\nu+},$$

$$\overset{(+1)\mu\nu}{z} \equiv (\gamma_{(13)} + i\gamma_{(14)})A^{\mu}V^{\nu+}, \quad \overset{(-1)\mu\nu}{z} \equiv \{\overset{(+1)\mu\nu}{z}\}^{*},$$

$$\overset{(+2)\mu\nu}{z} \equiv (\gamma_{(23)} + i\gamma_{(24)})U^{\mu}V^{\nu+}, \quad \overset{(-2)\mu\nu}{z} \equiv \{\overset{(+2)\mu\nu}{z}\}^{*},$$

$$\overset{[+1]\mu\nu}{z} \equiv (\gamma_{[13]} + i\gamma_{[14]})A^{\mu}V^{\nu+}, \quad \overset{[-1]\mu\nu}{z} \equiv \{\overset{[+1]\mu\nu}{z}\}^{*},$$

$$\overset{[+2]\mu\nu}{z} \equiv (\gamma_{[23]} + i\gamma_{[24]}) U^{\mu} V^{\nu+}, \quad \overset{[-2]\mu\nu}{z} \equiv \{\overset{[+2]\mu\nu}{z}\}^{*}. \tag{2.18}$$

Similarly, we define ω-fields by replacing, in the above definitions, z by ω and γ by τ. For example:

$$\overset{(11)\alpha}{\omega}{}_{\alpha} \equiv \tau_{(11)} A^{\alpha} A_{\alpha}, \quad \overset{(22)\alpha}{\omega}{}_{\alpha} \equiv \tau_{(22)} U^{\alpha} U_{\alpha}, \quad \overset{(12)\alpha}{\omega}{}_{\alpha} \equiv \tau_{(12)} A^{\alpha} U_{\alpha},$$

$$\overset{+-3\alpha}{\omega}{}_{\alpha} \equiv \tau_{(33)} V^{\alpha+} V_{\alpha-}, \quad \overset{(+1)\alpha}{\omega}{}_{\alpha} \equiv (\tau_{(13)} + i\tau_{(14)}) A^{\alpha} V_{\alpha}^{+}, \tag{2.19}$$

and so on.

Thus, eq.(2.2) performs a physics composed by granular and collective terms. However, it should be understood that, although eq.(2.12) and eq.(2.15) are gauge invariant, every collective term expressed in eq.(2.18) does not necessarily preserves the symmetry, unless under some circumstances. The corresponding gauge invariant collective physical terms under eq.(2.1) transformations are:

$$z_{\mu\nu}^{\mathrm{GI}} = z_{[\mu\nu]}^{\mathrm{GI}} + z_{(\mu\nu)}^{\mathrm{GI}} \tag{2.20}$$

where

$$z_{[\mu\nu]}^{\mathrm{GI}} \equiv 2\overset{[12]}{z}{}_{[\mu\nu]} + 2\overset{[+-]}{z}{}_{[\mu\nu]} \tag{2.21}$$

and

$$z_{(\mu\nu)}^{\mathrm{GI}} = \overset{(11)}{z}{}_{\mu\nu} + \overset{(22)}{z}{}_{\mu\nu} + 2\overset{(12)}{z}{}_{(\mu\nu)} + 2\overset{+-3}{z}{}_{(\mu\nu)} \tag{2.22}$$

and

$$\overset{(GI)}{\omega}{}_{\alpha}^{\alpha} = \overset{(11)}{\omega}{}_{\alpha}^{\alpha} + \overset{(12)}{\omega}{}_{\alpha}^{\alpha} + 2\overset{(12)}{\omega}{}_{\alpha}^{\alpha} + 2\overset{+-3}{\omega}{}_{\alpha}^{\alpha} \tag{2.23}$$

Under the conditions

$$\gamma_{[13]} = \gamma_{[14]} = \gamma_{[23]} = \gamma_{[24]} = 0$$

$$\gamma_{(13)} = \gamma_{(14)} = \gamma_{(23)} = \gamma_{(24)} = 0$$

$$\gamma_{(33)} = \gamma_{(44)}$$

$$\tau_{(13)} = \tau_{(14)} = \tau_{(23)} = \tau_{(24)}$$

$$\tau_{(33)} = \tau_{(44)} \tag{2.24}$$

Expressions (2.20-23) are the collective physical fields. They are expressed by the Bianchi identities, eqs.(4.51-2). Notice at Apendice A that the restrictions coming from eq.(2.24) do not change Lagrangian shape at eq.(2.2) and their fields interactions according to [7].

Another aspect derived from symmetry is on the electric charge conservation. It is the basic relationship for describing the electromagnetic phenomena. Maxwell has introduced by hand the displacement current, nowadays gauge symmetry become responsible for its implementation through an abelian gauge symmetry. Following this logic, one introduces through eq.(2.1) different potential fields sharing a common gauge parameter. And so, from such interconnected abelian fields set, one derives the charge conservation law. Given the four fields inserted in the $\{\frac{1}{2},\frac{1}{2}\}$ Lorentz Group representation, eq.(2.1) gives the following global Noether conservation law

$$\partial_\mu J_N^\mu = 0$$

where

$$J_N^\mu \equiv iq\{V_\nu^+[c_1V^{\nu\mu-}+c_3S^{\nu\mu-}+c_2(\overset{[-1]}{z}{}^{[\nu\mu]}+\overset{[-2]}{z}{}^{[\nu\mu]})+c_4(\overset{(-1)}{z}{}^{(\nu\mu)}+\overset{(-2)}{z}{}^{(\nu\mu)})+$$
$$+g^{\nu\mu}(c_5S_\alpha^{\alpha 1}+c_6S_\alpha^{\alpha 2}+c_7S_\alpha^{\alpha -}+c_8(\overset{(-1)}{z}{}_\alpha^\alpha+\overset{(-2)}{z}{}_\alpha^\alpha)+c_9(\overset{(-1)}{\omega}{}_\alpha^\alpha+\overset{(-2)}{\omega}{}_\alpha^\alpha))-\mu_+^2V^{\mu-}]+$$
$$-V_\nu^-[d_1V^{\nu\mu+}+d_3S^{\nu\mu+}+d_2(\overset{[+1]}{z}{}^{[\nu\mu]}+\overset{[+2]}{z}{}^{[\nu\mu]})+d_4(\overset{(+1)}{z}{}^{(\nu\mu)}+\overset{(+2)}{z}{}^{(\nu\mu)})+$$
$$+g^{\nu\mu}(d_5S_\alpha^{\alpha 1}+d_6S_\alpha^{\alpha 2}+d_7S_\alpha^{\alpha +}+d_8(\overset{(+1)}{z}{}_\alpha^\alpha+\overset{(+2)}{z}{}_\alpha^\alpha)+d_9(\overset{(+1)}{\omega}{}_\alpha^\alpha+\overset{(+2)}{\omega}{}_\alpha^\alpha))-\mu_+^2V^{\mu+}]\}$$

Eq.(2.25) means the convective electric charge current associated to the fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$. It is the primary equation from where the electric charge must be observed. It shows how this model expresses the electric charge physicity. First, the electric charge factor is denoted by the parameter $q = eq_v$ where $e = 1.6.10^{-19}C$ and q_v is the charge values associated to fields $V_\mu^\pm$. Then, it shows a J_μ^N dependence not only on charge fields $V_\mu^\pm$, as also, on neutral fields A_μ and U_μ. Third, on collective fields. Adding to that, the gauge invariant term $L_I = eJ_\mu^N(A_\mu + U_\mu)$ can be introduced at eq (2.2).

Overall, the Lagrangian (2.2) introduces new features for the electromagnetic phenomena. It opens it for antisymmetric and symmetric fields strengths, granular and collective fields, massive particles, nonlinearities, coupling constants beyond the electric charge. It also incorporates free coefficients as $a_1, b_{(11)}$ and so on, which can take any value without breaking symmetry. They are carrying the meaning of chance in nature. It also yields a photon behavior that crosses Maxwell frontier. It contains trilinear and four linear photon vertices with a coupling constant not depending from electric charge. Eq.(2.2) enlarges not only Maxwell, as extensions being proposed in literature by London [9], Born-Infeld [10], Euler-Lagrange [11], Proca [12], Podolsky-Lee-Wick [13], Kalb-Rammond [14], Yang-Lee [15], Salam [16], Standard Model [17]. It modifies the three basic EM elements: electric charge, EM fields and light. A new understanding which will be studied at next sections.

3 Non-Maxwellian models

We should understand where eq.(2.1) makes difference. Historically, different tentatives have happened in order to improve Maxwell equations. Literature has been developing 42 non-Maxwellian models for crossing Maxwell frontier. Therefore before working out on the four bosons electromagnetism we should make a brief analysis on the present status of other non-Maxwellian models.

Recapitulating, non-Maxwellian models started in 1930 decade with Bohr-Infeld, Euler-Heisenberg. At that time the idea was to understand the relationship between spin, charge and mass and the meaning of vacuum polarization. Then, working with the electron-positron pair creation come up the nonlinearity presence as considered by Landau [18], Delbrück [19] and others. Something was pointing out that Maxwell should be surpassed.

Since most of physical systems are intrinsically nonlinear in nature a new EM behavior was expected. A solution was to make Maxwell as a nonlinear expansion. Introduce higher orders terms in a Lagrangian based on two Lorentz and gauge invariant terms F and G. It gives the generic non-Maxwellian model

$$\mathsf{L} \equiv \mathsf{L}(\mathsf{F};\mathsf{G})$$

with

$$\mathsf{F} = F_{\mu,\nu}^2 : (\vec{E}^2 - c^2\vec{B}^2)$$
$$\mathsf{G} = F_{\mu,\nu}\widetilde{F}^{\mu,\nu} : (\vec{E}\cdot\vec{B}) \qquad (3.1)$$

Eq.(3.1) summarizes different nonlinear approaches as Born-Infeld, Euler-Heisenberg, exponential-electrodynamics[20], logarithmic[21], arcsin-ED[22], power-law-ED[23]. Three particular cases are

$$L_M = F + G \tag{3.2}$$

$$L_{EH} = F + \xi F^2 + \rho G^2 \tag{3.3}$$

$$L_{BI} = 1 - \sqrt{1 - \frac{E^2}{\varepsilon_s^2} + \frac{G^2}{\varepsilon_s^4}} \tag{3.4}$$

where ξ, ρ are parameters depending on physical constants as $m_e, e, c, \hbar$ and $\varepsilon_s = \frac{m^2 c^3}{e\hbar}$ is called as the critical field. It is the field strengths at which Dirac sea electrons are expected to tunnel into the continuum and produce electron-positron pairs. In modern terms it is the Schwinger critical field that sets the threshold for non-linear QED to become relevant.

Three fundamental points has motivate Born and Infeld to their theory, eq.(3.4). They are the principle of finiteness: a physical theory must imply only finite measurable entities. Parallel to relativity: the Lagrangian is deduced mimicking what happens in relativity where the kinetic energy must take into account the light speed upper limit for velocities. Unitary principle: it exists only one physical entity, the electromagnetic field; matter particles are considered singularities in the field and mass is a derived notion expressed in terms of electromagnetic energy. (electromagnetic mass)

The corresponding generic equation of motion is [3]

$$\left(\frac{\partial L}{\partial F} F^{\mu\nu} + \frac{\partial L}{\partial G} \tilde{F}^{\mu\nu}\right) = \mu_0 j^\nu \tag{3.5}$$

which express a photonic equation beyond Maxwell with j^ν an external current. Instead a passive photon that just propagates, it introduces a photonic charge expressed through the continuity equation

$$\partial_\mu \partial_\nu \frac{\partial L}{\partial F} F^{\mu\nu} + \partial_\mu \partial_\nu \frac{\partial L}{\partial G} \tilde{F}^{\mu\nu} = \partial . j \tag{3.6}$$

and with the following expression in the energy-momentum tensor

$$\partial_\mu \Theta^\mu_\kappa = \mu_0 j^\nu F_{\nu\kappa} \tag{3.7}$$

where

$$\Theta^\mu_\kappa = \frac{\partial L}{\partial F} F^{\mu\nu} F_{\nu\kappa} + \frac{\partial L}{\partial G} \tilde{F}^{\mu\nu} F_{\nu\kappa} - \delta^\mu_\kappa L \tag{3.8}$$

A light electromagnetism appears. A step-forward with respect to Maxwell through nonlinearity was given. Differently from Maxwell, eq.(3.1) shows an alive photon: a quanta producing its own EM fields. It generates photonic charge and current with selfinterating photons. Nevertheless due to QED renormalization in the forties these models lost interest. They become considered just as effective models. The main consequence on the photon interpretation was lost and it came back to be defined just as in Maxwell and QED[24]. The nonlinearity became underneath until these models came up derived by Tseytlin from superstrings[25], and by Denisov[26] who studied the atom emission through Born-Infeld.

A brief comparison between these classical nonlinear models and the four bosons electromagnetism should be taken. Besides questions with respect to renormalizability and unitarity these old models contains nonlinear restrictions. Their selfinteracting photons are restricted to quadrivertices, dimensional coupling constant and preserving the electric charge dependence. Therefore, eq.(2.2) is more than a nonlinearity revival. It brings two qualitative differences. The first one is on three photon vertice[7]. The second one is the photon coupling without electric charge. These structures are more physical for studying at tree level the experimental scatterings $\gamma\gamma \to e^+e^-$, $e^+e^- \to \gamma\gamma$ and $\gamma\gamma \to \gamma\gamma$.

4 Whole Relativistic Equations

Recapitulating, light symmetry holds laws as Maxwell equations, Relativity, Lorentz Group. Then, keeping in mind its Ariadne's line we should go beyond. The next step is to develop a non-Maxwellian model. For this, we have to take in

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account not only Maxwell and relativity but also the group algebra. The global symmetry $\Lambda^{\mu}_{\nu} = (e^{\frac{1}{2}\omega^{\alpha\beta}\Sigma_{\alpha\beta}})^{\mu}_{\nu}$ goes further than Lorentz transformations and defines a Lie Algebra. It commands the spin quantum number physics through fields spin representations, relativistic equations (Klein-Gordon, Maxwell, Proca, Weyl, Dirac, Rarita-Schwinger, Pauli-Fierz), gauge symmetry (cancellation of spurious spins from LG) and CPT theorem (LG and spin statistics). Now something more is expected to happen.

Then, pulling out the light invariance Ariadne's line, one gets a fourth situation where to each irreducible Lorentz representation is associated a fields family. It introduces the presence of fields grouping or Lorentz specie[27]. They are fields set sharing the same spin representation. A whole quanta physics is encountered. An origin where these fields families are germinating a whole relativistic dynamics[28]. They are systemic equations introducing a physics based on a group of fields under a same Lorentz nature and to be developed under a common gauge parameter.

Thus, one takes the $\{\frac{1}{2},\frac{1}{2}\}$ representation through a common abelian gauge symmetry. Preserving the two basic electromagnetic postulates which are electric charge conservation and light invariance, eq.(2.1) extends the Maxwell minimum set. Based on an enlarged $U(1)X\mathrm{SO}(2)$ abelian symmetry a new electromagnetic performance is expected to be derived with new features for electric charge, EM fields and photon. For this, the corresponding whole equations of motion will be derived at this section.

The fields set abelian gauge symmetry develops a system with coupled differential equations which can be systematized either through primitive equations or by basic equations. The first one, means that ones directly derived from eq.(2.1), as the Euler-Lagrange equations, Bianchi identities, Noether equations. Classically, there are 15 primitive covariant equations. They are 4-Euler-Lagrange equations of motion associated to each field, 3-Noether equations, 4-antisymmetric granular Bianchi identities where each one is associated to a given field $A_{\mu I}$, 1-antisymmetric collective Bianchi identity, 2-symmetric Bianchi identities, 1-kinetic identity [5]. As basic equations means equations which are appropriated combinations from those primitive equations.

Physically, it is required a dynamics which splits the spin-1 and spin-0 sectors. For this, we should avoid symmetric tensors due to they are mixing transverse and longitudinal terms in the equations of motion. The kinetic identity $\partial_{\nu}S^{\nu\mu I} = \partial_{\nu}F^{\nu\mu I} + \eta^{\mu\nu}\partial_{\nu}S^{\alpha I}_{\alpha}$ and the symmetric collective Bianchi identities $2\partial_{\nu}z^{\nu)}_{(\mu} = -\partial_{\mu}z^{\nu)}_{(\nu} + \gamma_{(IJ)}A^{I}_{\mu}S^{\nu J}_{\nu} + 2\gamma_{(IJ)}A^{I}_{\nu}S^{\nu J}_{\mu}$ are combined with the original primitive equations. As result, it separates the transverse and longitudinal dynamics. It obtains the following generic expression corresponding to the basic equations of motion inserted in the representation $\{\frac{1}{2},\frac{1}{2}\}$:

$$\partial_{\nu}(F^{\nu\mu} + z^{[\nu\mu]}) + \partial^{\mu}(S^{\alpha}_{\alpha} + z^{\alpha}_{\alpha}) + M^{\mu} + l^{\mu} + c^{\mu} = j^{\mu} \qquad (4.1)$$

Eq.(4.1) performs the physical dynamics. Three structures appear. First, the space-time evolution shows separate transverse and longitudinal sectors. From L^{A}_{K} and L^{A}_{3}, one gets that $F_{[\mu\nu]}$ and $z_{[\mu\nu]}$ tensors develop a spin-1 dynamics, propagating granular and collective fields; similarly, for a spin-0 dynamics S^{α}_{α} and z^{α}_{α} from L^{S}_{K} and L^{S}_{3}. Second, there is a whole physics where nature works individually and collectively. They become variables correlated by space-time dynamics. The individual-collective dualism is the new significance. By third, it is feed up with own physical sources. They are the massive, London, conglomerate and nonlinear current terms.

Analyzing over the source terms, first appears the mass term. It yields a Proca EM with the difference that here masses are introduced without breaking gauge symmetry[5]. Something different from Maxwell-Higgs[3], Podolsky-Lee-Wick[13], Kalb-Rammond[14], Higgs mechanism[29].

The London term corresponds to the L scalar fields composition

$$l^{\mu}_{I} = gLA^{\mu}_{I} \quad , L \equiv A_{\nu J}A^{\nu}_{J} \qquad (4.2)$$

The conglomerate term means the M_{JK} tensor fields matrix representing the polarization and magnetization

$$c_I^\mu = g_{JK} M_{JK} A_I^\mu \quad , M_{JK} \equiv A_{\nu J} A_K^\nu \tag{4.3}$$

The associate current term involving derivative takes the form

$$j_I^\mu = f_J X_J^{\mu\nu} A_{\nu I} \tag{4.4}$$

where $X_I^{\mu\nu}$ is any field strength as $F^{\mu\nu}$, $S^{\mu\nu}$, S_α^α and so on, with f_J as the corresponding coupling constant. For the particular case where the current is expressed as

$$j_I^\mu = f_J \partial_\nu (A_I^\nu A_J^\mu - A_J^\nu A_I^\mu) \tag{4.5}$$

the current can be rewritten in terms of the spin operator

$$j_J^\mu = i\partial_\nu (A_I^\alpha (\Sigma^{\nu\mu})_{\alpha\beta} A_I^\beta) \tag{4.6}$$

where $(\Sigma_{\alpha\beta})_\nu^\mu = -i(\delta_\alpha^\mu \eta_{\alpha\beta} - \eta_{\alpha\nu}\delta_\rho^\mu)$

The corresponding continuity equation is:

$$\mathsf{W}(S_\alpha^\alpha + z_\alpha^\alpha) = \partial.(j - M - l - c) \tag{4.7}$$

Eq.(4.7) is denoting on conserved charges. It reads up that every gauge boson inserted in the fields set develops dynamically its own charge. It shows on the existence of different sources beyond electric charge. Another sources for the electromagnetic phenomena are provided where electric charge is not anymore the only one origin for the electromagnetic phenomena.

We should now investigate on the equations of motion corresponding to every field. For field- A_μ (photon),

$$\partial_\nu (F_A^{\nu\mu} + z_A^{[\nu\mu]}) + \partial^\mu (S_{\alpha A}^\alpha + z_{\alpha A}^\alpha) + l_A^\mu + c_A^\mu = j_A^\mu$$

where the antisymmetric sector is given by:

$$F_A^{\nu\mu} = \bar{a}_1 F^{\nu\mu} \text{(granular)},$$

$$z_A^{[\nu\mu]} = \bar{a}_2 (\overset{[12]^{[\nu\mu]}}{z} + \overset{[+-]^{[\nu\mu]}}{z}) + \bar{a}_3 \overset{(+-)^{[\nu\mu]}}{z} \quad \text{(collective)}$$

and the longitudinal sector by

$$S_{\alpha A}^\alpha = (\bar{a}_4 S_\alpha^{\alpha 1} + \bar{a}_5 S_\alpha^{\alpha 2}) \text{(granular)}$$

$$z_{\alpha A}^\alpha = \bar{a}_6 (\overset{(11)}{z}{}_\alpha^\alpha + \overset{(22)}{z}{}_\alpha^\alpha + 2 \overset{(12)}{z}{}_\alpha^\alpha + 2 \overset{(+-3)}{z}{}_\alpha^\alpha) + \bar{a}_7 (\overset{(11)}{\omega}{}_\alpha^\alpha + \overset{(22)}{\omega}{}_\alpha^\alpha + 2 \overset{(12)}{\omega}{}_\alpha^\alpha + 2 \overset{+-3}{\omega}{}_\alpha^\alpha) \text{(collective)}$$

The so-called London term is represented by l_A^μ.

$$l_A^\mu = -A_\nu \{ f_2^A \overset{(11)(\nu\mu)}{z} + f_3^A \overset{(12)(\nu\mu)}{z} + f_2^A \overset{(22)(\nu\mu)}{z} + f_4^A \overset{(11)(\nu\mu)}{\omega} + f_5^A \overset{(12)(\nu\mu)}{\omega} +$$

$$g^{\nu\mu} (f_4^A \overset{(22)}{\omega}{}_\alpha^\alpha) \} - U_\nu \{ f_3^U \overset{[12]^{[\nu\mu]}}{z} + f_4^U \overset{(12)^{[\nu\mu]}}{\omega} + f_6^U \overset{(11)(\nu\mu)}{z} + f_7^U \overset{(12)(\nu\mu)}{z} +$$

$$+ f_8^U \overset{(22)(\nu\mu)}{z} + f_9^U \overset{(11)(\nu\mu)}{\omega} + f_4^U \overset{(12)(\nu\mu)}{\omega} + f_{10}^U \overset{(22)(\nu\mu)}{\omega} \}$$

and the conglomerate term c_A^μ is

$$c_A^\mu = -A_\nu\{ f_3^A \overset{+-3(\nu\mu)}{z} + g^{\nu\mu}(f_4^A \overset{+-3}{\omega}{}_\alpha^\alpha)\} - U_\nu\{ f_3^U \overset{[+-][\nu\mu]}{z} + f_7^U \overset{+-3(\nu\mu)}{z} + g^{\nu\mu}(f_{13}^U \overset{+-3}{\omega}{}_\alpha^\alpha)\} +$$

$$-V_\nu^+\{ f_2^+ \overset{[13-][\mu\nu]}{z} + f_3^+ \overset{[23-][\nu\mu]}{z} + f_5^+ \overset{[23-](\nu\mu)}{z} + f_6^+ \overset{[24-]\nu\mu}{z} + f_7^+ \overset{(13-)\mu\nu}{z} +$$

$$+ f_8^+ \overset{(23-)[\nu\mu]}{z} + f_9^+ \overset{(23-)(\nu\mu)}{z} + f_{10}^+ \overset{(24-)(\nu\mu)}{z} + f_{11}^+ \overset{(23-)\nu\mu}{\omega} + f_{12}^+ \overset{(24-)\nu\mu}{\omega} +$$

$$+ g^{\nu\mu}(f_{14}^+ \overset{[13-]}{z}{}_\alpha^\alpha + f_{14}^+ \overset{[23-]}{z}{}_\alpha^\alpha + f_7^+ \overset{(13-)}{z}{}_\alpha^\alpha + f_7^+ \overset{(23-)}{z}{}_\alpha^\alpha + f_{15}^+ \overset{(13-)}{\omega}{}_\alpha^\alpha + f_{15}^+ \overset{(23-)}{\omega}{}_\alpha^\alpha)\} +$$

$$-V_\nu^-\{ f_2^{+*} \overset{[13+][\mu\nu]}{z} + f_3^{+*} \overset{[23+][\nu\mu]}{z} + f_5^{+*} \overset{[23+](\nu\mu)}{z} + f_6^{+*} \overset{[24+]\nu\mu}{z} + f_7^{+*} \overset{(13+)\mu\nu}{z} +$$

$$+ f_8^{+*} \overset{(23+)[\nu\mu]}{z} + f_9^{+*} \overset{(23+)(\nu\mu)}{z} + f_{10}^{+*} \overset{(24+)(\nu\mu)}{z} + f_{11}^{+*} \overset{(23+)\nu\mu}{\omega} + f_{12}^{+*} \overset{(24+)\nu\mu}{\omega} +$$

$$+ g^{\nu\mu}(f_{14}^{+*} \overset{[13+]}{z}{}_\alpha^\alpha + f_{14}^{+*} \overset{[23+]}{z}{}_\alpha^\alpha + f_7^{+*} \overset{(13+)}{z}{}_\alpha^\alpha + f_7^{+*} \overset{(23+)}{z}{}_\alpha^\alpha + f_{15}^{+*} \overset{(13+)}{\omega}{}_\alpha^\alpha + f_{15}^{+*} \overset{(23+)}{\omega}{}_\alpha^\alpha)\}$$

The non-linear current is

$$j_A^\mu = A_\nu\{ f_1^A S^{\nu\mu 2} + g^{\nu\mu}(f_6^A S_\alpha^{\alpha 1} + f_7^A S_\alpha^{\alpha 2})\} +$$

$$+ U_\nu\{ f_1^U F^{\nu\mu} + f_2^U U^{\nu\mu} + f_5^U S^{\nu\mu 2} + g^{\nu\mu}(f_{11}^U S_\alpha^{\alpha 1})\} +$$

$$+ V_\nu^+\{ f_1^+ V^{\nu\mu -} + f_4^+ S^{\nu\mu -} + g^{\nu\mu}(f_{13}^+ S_\alpha^{\alpha -})\} +$$

$$+ V_\nu^-\{ f_1^{+*} V^{\nu\mu +} + f_4^{+*} S^{\nu\mu +} + g^{\nu\mu}(f_{13}^{+*} S_\alpha^{\alpha +})\}$$

where the parameters $\bar{a}_1, \ldots f_{15}^{+*}$ are defined in Apendice B. The same for the equations of motion to be studied below.

The corresponding continuity equation is

$$W(S_{\alpha A}^\alpha + z_{\alpha A}^\alpha) = \partial.(j_A + l_A + c_A)$$

For field-U_μ (massive photon),

$$\partial_\nu (F_U^{\nu\mu} + z_U^{[\nu\mu]}) + \partial^\mu (S_{\alpha U}^\alpha + z_{\alpha U}^\alpha) + M_U^\mu + c_U^\mu + l_U^\mu = j_U^\mu \tag{4.8}$$

Sector-T:

$$F_U^{\nu\mu} = \bar{b}_1 U^{\nu\mu} (\text{granular}), \tag{4.9}$$

$$z_U^{[\nu\mu]} = \bar{b}_2 (\overset{[12][\nu\mu]}{z} + \overset{[+-][\nu\mu]}{z}) + \bar{b}_3 \overset{(+-)[\nu\mu]}{z} \quad (\text{collective}) \tag{4.10}$$

Sector-L:

$$S_U^{\nu\mu} = \bar{b}_4 S_\alpha^{\alpha 1} + \bar{b}_5 S_\alpha^{\alpha 2} (\text{granular}) \tag{4.11}$$

$$z^{\alpha}_{\alpha U} = \bar{b}_6(\overset{(11)}{z}{}^{\alpha}_{\alpha} + \overset{(22)}{z}{}^{\alpha}_{\alpha} + 2\overset{(12)}{z}{}^{\alpha}_{\alpha} + 2\overset{(+-3)}{z}{}^{\alpha}_{\alpha}) +$$

$$+\bar{b}_7(\overset{(11)}{\omega}{}^{\alpha}_{\alpha} + \overset{(22)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{(12)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{+-3}{\omega}{}^{\alpha}_{\alpha})(\text{collective}) \tag{4.12}$$

Sector mass:

$$M^{\mu}_{U} = m^2_U U^{\mu} \tag{4.13}$$

Sector-London:

$$l^{\mu}_U = -A_{\nu}\{g^A_3 \overset{[12][\nu\mu]}{z} + g^A_4 \overset{(12)[\nu\mu]}{\omega} + g^A_6 \overset{(11)(\nu\mu)}{z} + g^A_7 \overset{(12)(\nu\mu)}{z} + g^A_8 \overset{(22)(\nu\mu)}{z} +$$

$$+g^A_9 \overset{(11)(\nu\mu)}{\omega} + g^A_4 \overset{(12)(\nu\mu)}{\omega} + g^A_{10} \overset{(22)(\nu\mu)}{\omega}\} - U_{\nu}\{g^U_2 \overset{(11)(\nu\mu)}{z} + 2g^U_2 \overset{(12)(\nu\mu)}{z} +$$

$$+g^U_2 \overset{(22)(\nu\mu)}{z} + g^U_3 \overset{(12)(\nu\mu)}{\omega} + 2g^U_3 \overset{(22)(\nu\mu)}{\omega} + g^{\nu\mu}(g^U_3 \overset{11}{\omega}{}^{\alpha}_{\alpha})\}$$

Sector-conglomerate:

$$c^{\mu}_U = -A_{\nu}\{g^A_3 \overset{[+-][\nu\mu]}{z} + g^A_7 \overset{+-3(\nu\mu)}{z} + g^{\nu\mu}(g^A_4 \overset{(+-3)}{\omega}{}^{\alpha}_{\alpha})\} - U_{\nu}\{2g^U_2 \overset{+-3(\nu\mu)}{z} + 2g^U_3 \overset{+-3}{\omega}{}^{\alpha}_{\alpha}\} +$$

$$+V^+_{\nu}\{g^+_2 \overset{[13-]^{\mu\nu}}{z} + g^+_3 \overset{[14-]^{\nu\mu}}{z} + g^+_4 \overset{[23-]^{\mu\nu}}{z} + g^+_6 \overset{(13-)^{(\nu\mu)}}{z} + g^+_7 \overset{(14-)^{(\nu\mu)}}{z} + g^+_8 \overset{(23-)^{\mu\nu}}{z} +$$

$$+g^+_9 \overset{(13-)^{\nu\mu}}{\omega} + g^+_{10} \overset{(14-)^{(\nu\mu)}}{\omega} + g^{\nu\mu}(-g^+_4 \overset{[13-]}{z}{}^{\alpha}_{\alpha} - g^+_4 \overset{[23-]}{z}{}^{\alpha}_{\alpha} + g^+_8 \overset{(13-)}{z}{}^{\alpha}_{\alpha} + g^+_8 \overset{(23-)}{z}{}^{\alpha}_{\alpha} +$$

$$+g^+_8 \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + g^+_{13} \overset{(23-)}{\omega}{}^{\alpha}_{\alpha})\} + V^-_{\nu}\{g^{+*}_2 \overset{[13+]^{\mu\nu}}{z} + g^{+*}_3 \overset{[14+]^{\nu\mu}}{z} + g^{+*}_4 \overset{[23+]^{\mu\nu}}{z} +$$

$$+g^{+*}_6 \overset{(13+)^{(\nu\mu)}}{z} + g^{+*}_7 \overset{(14+)^{(\nu\mu)}}{z} + g^{+*}_8 \overset{(23+)^{\mu\nu}}{z} + g^{+*}_9 \overset{(13+)^{\nu\mu}}{\omega} + g^{+*}_{10} \overset{(14+)^{(\nu\mu)}}{\omega} +$$

$$+g^{\nu\mu}(-g^{+*}_4 \overset{[13+]}{z}{}^{\alpha}_{\alpha} - g^{+*}_4 \overset{[23+]}{z}{}^{\alpha}_{\alpha} + g^{+*}_8 \overset{(13+)}{z}{}^{\alpha}_{\alpha} + g^{+*}_8 \overset{(23+)}{z}{}^{\alpha}_{\alpha} + g^{+*}_8 \overset{(13+)}{\omega}{}^{\alpha}_{\alpha} + g^{+*}_{13} \overset{(23+)}{\omega}{}^{\alpha}_{\alpha})\}$$

Sector-current:

$$j^{\mu}_U = A_{\nu}\{g^A_1 F^{\nu\mu} + g^A_2 U^{\nu\mu} + g^A_5 S^{\nu\mu 1} + g^{\nu\mu}(g^A_{11} S^{\alpha 1}_{\alpha} + g^A_{12} S^{\alpha 2}_{\alpha} +)\} +$$

$$+U_{\nu}\{g^U_1 S^{\nu\mu 1} + g^{\nu\mu}(g^U_4 S^{\alpha 1}_{\alpha} + g^U_5 S^{\alpha 2}_{\alpha})\} + V^+_{\nu}\{g^+_1 V^{\nu\mu -} + g^+_5 S^{\nu\mu -} + g^{\nu\mu}(g^+_{11} S^{\alpha -}_{\alpha})\} +$$

$$+V^-_{\nu}\{g^{+*}_1 V^{\nu\mu +} + g^{+*}_5 S^{\nu\mu +} + g^{\nu\mu}(g^{+*}_{11} S^{\alpha +}_{\alpha})\}$$

The corresponding continuity equation is

$$W(S^{\alpha}_{\alpha U} + z^{\alpha}_{\alpha U}) = \partial.(j_U + l_U + c_U + M_U)$$

For field-V^+_{μ} (massive positive charged photon), with coefficients related at Appendix B, one gets

$$\partial_\nu (F_+^{\nu\mu} + z_+^{[\nu\mu]}) + \partial^\mu (S_{\alpha+}^{\alpha} + z_{\alpha+}^{\alpha}) + M_+^{\mu} + l_+^{\mu} = j_{V^+}^{\mu} \qquad (4.14)$$

Sector-T:

$$F_+^{\nu\mu} = \bar{c}_1 V^{\nu\mu-} \text{(granular)} \qquad (4.15)$$

$$z_+^{[\nu\mu]} = \bar{c}_2 (\overset{[-1]}{z}{}^{[\nu\mu]} + \overset{[-2]}{z}{}^{[\nu\mu]})\text{(collective)} \qquad (4.16)$$

Sector-L:

$$S_{\alpha+}^{\alpha} = \bar{c}_3 S_\alpha^{\alpha-}, \text{(granular)} \qquad (4.17)$$

$$z_{\alpha+}^{\alpha} = \bar{c}_4 (\overset{(-1)}{z}{}_\alpha^\alpha + \overset{(-2)}{z}{}_\alpha^\alpha) + \bar{c}_5 (\overset{(-1)}{\omega}{}_\alpha^\alpha + \overset{(-2)}{\omega}{}_\alpha^\alpha)\text{(collective)} \qquad (4.18)$$

Sector-mass:

$$M_+^{\mu} = \mu_V^2 V^{\mu-} \qquad (4.19)$$

Sector-London:

$$l_+^{\mu} = V^{+\mu}\{h_{11}^-(\overset{(11)}{z}{}_\alpha^\alpha + \overset{(22)}{z}{}_\alpha^\alpha + 2\overset{(11)}{\omega}{}_\alpha^\alpha + 2\overset{(22)}{\omega}{}_\alpha^\alpha)\}$$

Sector-conglomerate:

$$c_+^{\mu} = A_\nu\{h_2^A \overset{[13-]}{z}{}^{\nu\mu} + h_3^A \overset{[23-]}{z}{}^{[\nu\mu]} + h_4^A \overset{(23-)}{z}{}^{[\nu\mu]} + h_5^A \overset{[24-]}{z}{}^{[\nu\mu]} + h_7^A \overset{[23-]}{z}{}^{(\nu\mu)} +$$

$$+ h_8^A \overset{(13-)}{z}{}^{(\nu\mu)} + h_4^{*A} \overset{(23-)}{z}{}^{(\nu\mu)} + h_9^A \overset{(24-)}{z}{}^{\mu\nu} + h_{10}^A \overset{(23-)}{\omega}{}^{\mu\nu} + h_{11}^A \overset{(24-)}{\omega}{}^{\mu\nu} +$$

$$+ g^{\nu\mu}(-h_2^A \overset{[13-]}{z}{}_\alpha^\alpha - h_2^A \overset{[23-]}{z}{}_\alpha^\alpha + h_8^A \overset{(23-)}{z}{}_\alpha^\alpha + h_{13}^A \overset{(13-)}{\omega}{}_\alpha^\alpha + h_{13}^A \overset{(23-)}{\omega}{}_\alpha^\alpha)\} +$$

$$+ U_\nu\{h_2^U \overset{[13-]}{z}{}^{\mu\nu} + h_3^U \overset{[14-]}{z}{}^{\mu\nu} + h_4^U \overset{[23-]}{z}{}^{\nu\mu} + h_6^U \overset{(13-)}{z}{}^{\mu\nu} + h_7^U \overset{(14-)}{z}{}^{\mu\nu} +$$

$$+ h_8^U \overset{(23-)}{z}{}^{\mu\nu} + h_9^U \overset{(13-)}{\omega}{}^{\mu\nu} + h_{10}^U \overset{(14-)}{\omega}{}^{\mu\nu} + g^{\nu\mu}(+h_{12}^U \overset{[13-]}{z}{}_\alpha^\alpha - h_4^U \overset{[23-]}{z}{}_\alpha^\alpha +$$

$$+ h_8^U \overset{(13-)}{z}{}_\alpha^\alpha + h_8^U \overset{(23-)}{z}{}_\alpha^\alpha + h_{12}^U \overset{(13-)}{\omega}{}_\alpha^\alpha + h_{12}^U \overset{(23-)}{\omega}{}_\alpha^\alpha)\} + V_\nu^-\{h_3^- \overset{[12]}{z}{}^{[\nu\mu]} +$$

$$+ h_4^- S^{\nu\mu 1} + h_6^- \overset{(11)}{z}{}^{(\nu\mu)} + h_6^- \overset{(22)}{z}{}^{(\nu\mu)} + h_7^- \overset{(12)}{z}{}^{(\nu\mu)} + 2h_6^- \overset{+-4}{z}{}^{(\nu\mu)} +$$

$$+ h_7^- \overset{[+-]}{z}{}^{(\nu\mu)} + h_8^- \overset{(+-)}{\omega}{}^{(\nu\mu)} + g^{\nu\mu}(2\overset{(12)}{z}{}_\alpha^\alpha + 4\overset{(12)}{\omega}{}_\alpha^\alpha + h_{12}^- \overset{[+-]}{z}{}_\alpha^\alpha +$$

$$+ h_{13}^- \overset{(+-)}{z}{}_\alpha^\alpha + h_{14}^- \overset{+-4}{\omega}{}_\alpha^\alpha)\} \qquad (4.20)$$

Sector current:

$$j_{V^+}^{\mu} = A_\nu\{h_1^A V^{\nu\mu-} + h_6^A S^{\nu\mu-} + g^{\nu\mu}(h_{12}^A S_\alpha^{\alpha-})\} +$$

$$+ U_\nu\{h_1^U V^{\nu\mu-} + h_5^U S^{\nu\mu-} + g^{\nu\mu}(h_{11}^U S_\alpha^{\alpha-})\} +$$

$$+V_{\nu}^{-}\{h_1^{-}F^{\nu\mu}+h_2^{-}U^{\nu\mu}+h_4^{-}S^{\nu\mu 1}+h_5^{-}S^{\nu\mu 2}+g^{\nu\mu}(h_9^{-}S_{\alpha}^{\alpha 1}+h_{10}^{-}S_{\alpha}^{\alpha 2})\} \quad (4.21)$$

The corresponding continuity equation is

$$W(S_{\alpha+}^{\alpha}+z_{\alpha+}^{\alpha})=\partial.(j_{+}+l_{+}+c_{+}+M_{+})$$

For field-V_{μ}^{-} (massive negative charged photon),

$$\partial_{\nu}(F_{-}^{\nu\mu}+z_{-}^{[\nu\mu]})+\partial^{\mu}(S_{\alpha-}^{\alpha}+z_{\alpha-}^{\alpha})+M_{-}^{\mu}+c_{-}^{\mu}+l_{-}^{\mu}=j_{V^{-}}^{\mu} \quad (4.22)$$

where Sector-T:

$$F_{-}^{\nu\mu}=\bar{c}_1 V^{\nu\mu+},(\text{granular}) \quad (4.23)$$

$$z_{-}^{[\nu\mu]}=\bar{c}_2(\overset{[+1]^{[\nu\mu]}}{z}+\overset{[+2]^{[\nu\mu]}}{z})(\text{collective}) \quad (4.24)$$

Sector-L:

$$S_{\alpha-}^{\alpha}=\bar{c}_3 S_{\alpha}^{\alpha+},(\text{granular}) \quad (4.25)$$

$$z_{\alpha-}^{\alpha}=\bar{c}_4(\overset{(+1)}{z}{}_{\alpha}^{\alpha}+\overset{(+2)}{z}{}_{\alpha}^{\alpha})+\bar{c}_5(\overset{(+1)}{\omega}{}_{\alpha}^{\alpha}+\overset{(+2)}{\omega}{}_{\alpha}^{\alpha})(\text{collective}) \quad (4.26)$$

Sector-mass:

$$M_{-}^{\mu}=\mu_V^2 V^{\mu-} \quad (4.27)$$

Sector-London:

$$l_{-}^{\mu}=V^{\mu+}\{h_{11}^{-*}(\overset{(11)}{z}{}_{\alpha}^{\alpha}+\overset{(22)}{z}{}_{\alpha}^{\alpha}+2\overset{(11)}{\omega}{}_{\alpha}^{\alpha}+2\overset{(22)}{\omega}{}_{\alpha}^{\alpha})\}$$

Sector-conglomerate:

$$c_{-}^{\mu}=A_{\nu}\{h_2^{A*}\overset{[13+]^{\nu\mu}}{z}+h_3^{A*}\overset{[23+]^{[\nu\mu]}}{z}+h_4^{A*}\overset{(23+)^{[\nu\mu]}}{z}+h_5^{A*}\overset{[24+]^{[\nu\mu]}}{z}+$$

$$+h_7^{A*}\overset{[23+]^{(\nu\mu)}}{z}+h_8^{A*}\overset{(13+)^{(\nu\mu)}}{z}+h_4^{*A}\overset{(23+)^{(\nu\mu)}}{z}+h_9^{A*}\overset{(24+)^{\mu\nu}}{z}+h_{10}^{A*}\overset{(23+)^{\mu\nu}}{\omega}+h_{11}^{A*}\overset{(24+)^{\mu\nu}}{\omega}+$$

$$+g^{\nu\mu}(-h_2^{A*}\overset{[13+]}{z}{}_{\alpha}^{\alpha}-h_2^{A*}\overset{[23+]}{z}{}_{\alpha}^{\alpha}+h_8^{A*}\overset{(13+)}{z}{}_{\alpha}^{\alpha}+h_8^{A*}\overset{(23+)}{z}{}_{\alpha}^{\alpha}+h_{13}^{A*}\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+h_{13}^{A*}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+U_{\nu}\{h_2^{U*}\overset{[13+]^{\mu\nu}}{z}+h_3^{U*}\overset{[14+]^{\mu\nu}}{z}+h_4^{U*}\overset{[23+]^{\nu\mu}}{z}+h_5^{U*}S^{\nu\mu+}+h_6^{U*}\overset{(13+)^{\mu\nu}}{z}+$$

$$+h_7^{U*}\overset{(14+)^{\mu\nu}}{z}+h_8^{U*}\overset{(23+)^{\mu\nu}}{z}+h_9^{U*}\overset{(13+)^{\mu\nu}}{\omega}+h_{10}^{U*}\overset{(14+)^{\mu\nu}}{\omega}+$$

$$+g^{\nu\mu}(+h_{12}^{U}\overset{[13+]}{z}{}_{\alpha}^{\alpha}-h_4^{U*}\overset{[23+]}{z}{}_{\alpha}^{\alpha}+h_8^{U*}\overset{(13+)}{z}{}_{\alpha}^{\alpha}+h_8^{U*}\overset{(23+)}{z}{}_{\alpha}^{\alpha}+h_{12}^{U*}\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+h_{12}^{U*}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+V_{\nu}^{+}\{h_3^{-*}\overset{[12]^{[\nu\mu]}}{z}+h_6^{-*}\overset{(11)^{(\nu\mu)}}{z}+h_6^{-*}\overset{(22)^{(\nu\mu)}}{z}+h_7^{-*}\overset{(12)^{(\nu\mu)}}{z}+2h_6^{-*}\overset{+-4^{(\nu\mu)}}{z}+$$

$$+h_7^{-*}\overset{[+-](\nu\mu)}{z}+h_8^{-*}\overset{(+-)(\nu\mu)}{\omega}+g^{\nu\mu}(2\overset{(12)}{z}{}_{\alpha}^{\alpha}+4\overset{(12)}{\omega}{}_{\alpha}^{\alpha})\}\tag{4.28}$$

Sector-current:

$$j_{V^-}^{\mu}=A_\nu\{h_1^{A*}V^{\nu\mu+}+h_6^{A*}S^{\nu\mu+}+g^{\nu\mu}(h_{12}^{A*}S_{\alpha}^{\alpha+})\}+$$
$$+U_\nu\{h_1^{U*}V^{\nu\mu+}+h_5^{U*}S^{\nu\mu+}+g^{\nu\mu}(h_{11}^{U*}S_{\alpha}^{\alpha+})\}+$$
$$+V_\nu^{+}\{h_1^{-*}F^{\nu\mu}+h_2^{-*}U^{\nu\mu}+h_4^{-*}S^{\nu\mu 1}+h_5^{-*}S^{\nu\mu 2}+g^{\nu\mu}(h_9^{-*}S_{\alpha}^{\alpha 1}+h_{10}^{-*}S_{\alpha}^{\alpha 2})\}$$

The corresponding continuity equation is

$$W(S_{\alpha-}^{\alpha}+z_{\alpha-}^{\alpha})=\partial.(j_-+l_-+M_-+c_-)$$

These whole equations provide five features. They are set space-time evolution, potential fields physicity, fields conglomerates, fields sources, neutral coupling constants. The first one, corresponds to fields strengths evolutions on space-time, with granular and collective, transversal and longitudinal electromagnetic fields propagations. The second fact on these equations is the explicit presence of the $A_{\mu I}$ fields. They are not more the subsidiary terms prescripted by Maxwell and Heaviside in the 19^{th} century. They provide relationships as $A_\mu F^{\mu\nu}$ at eq.(4.11). The third one, is on fields relationships with masses (Proca) and just between fields (London scalars, polarization and magnetization tensors). The fourth one, is on non-linear currents acting as fields own sources, for instance, the photon field interacting with its own electric and magnetic field. The fifth aspect is the presence of couplings constants beyond electric charge.

These five basic aspects are expressing the four bosons electromagnetism. There is no more individual dynamics. These four fields are interlaced providing a whole causality. A set determinism with granular and collective fields. The whole relativistic equations are implementing a new relationship between EM fields and sources. The fields set $\{A_\mu, U_\mu, V_\mu^{\pm}\}$ shows a physics where each field space-time evolution is associated to a source depending on the fields set. Consequently, electric charge will be transported through four messengers, bringing new aspects on its conservation law, conduction, transmission and interaction. New couplings constants appear. As result, the EM fields transmission contains variables and expressions different from Maxwell. A new light physics emerges. Photon becomes an active element in the EM phenomena.

Nevertheless, there is still a fifth equation of motion. It is called as the symmetry equation derived from Noether theorem[4]. It gives,

$$\partial_\nu K^{\nu\mu}+J_N^{\mu}=0,\quad K^{\nu\mu}\equiv\sum_{I=1}^{4}k_I\frac{\partial L}{\partial(\partial_\nu A_{\mu I})}\tag{4.29}$$

Taking

$$K^{\nu\mu}=K^{[\mu\nu]}+\tilde{K}^{(\mu\nu)}+g_{\mu\nu}K_{\alpha}^{\alpha}\tag{4.30}$$

one gets the continuity equation

$$\partial_\mu\partial_\nu\tilde{K}^{(\nu\mu)}+WK_{\alpha}^{\alpha}=0\tag{4.31}$$

which gives,

$$\sum_{I=1}^{4}k_I W(S_{\alpha}^{\alpha I}+z_{\alpha}^{\alpha I})=0\tag{4.32}$$

Substituting eq.(4.47) in eq.(4.1), one gets the whole conservation law

$$k_I\partial\cdot(m^I+l^I+c^I)=k_I\partial\cdot j^I \tag{4.33}$$

where eq.(4.48) means the whole conservation law involving the fields set.

The granular antisymmetric Bianchi identity are

$$\partial_\mu F^I_{\nu\rho}+\partial_\rho F^I_{\mu\nu}+\partial_\nu F^I_{\rho\nu}=0 \tag{4.34}$$

where $F^I_{\mu\nu}=\{F_{\mu\nu},U_{\mu\nu},V^{\pm}_{\mu\nu}\}$. The granular symmetric relationship $\varepsilon_{\mu\nu\rho\sigma}\partial^\nu S^{\rho\sigma}$ does not provide a Bianchi identity.

The collective Bianchi antisymmetric relations are:

$$\partial_\mu z^{GI}_{[\nu\rho]}+\partial_\rho z^{GI}_{[\mu\nu]}+\partial_\nu z^{GI}_{[\rho\mu]}=\gamma_{[12]}\{U_\rho F_{\nu\mu}+U_\mu F_{\nu\rho}+U_\nu F_{\rho\mu}-A_\mu U_{\rho\nu}-A_\nu U_{\rho\mu}-A_\rho U^{\mu\nu}\}$$

$$+2\gamma_{[34]}\mathrm{Im}\{V^-_\mu V^+_{\nu\rho}+V^-_\nu V^+_{\rho\mu}+V^-_\rho V^+_{\mu\nu}\} \tag{4.35}$$

or

$$\partial_\mu\tilde{z}^{(\mu\nu)}=j^\nu_B,\quad \partial_\nu j^\nu_B=0 \tag{4.36}$$

Eq.(4.52) introduces a kind of fields monopole. Instead of the Dirac puntiform magnetic charge[30] its fields composition is more close to the 't Hooft[31] and Polyakov[32] monopoles based on Higgs fields. Monopoles depending on fields contain opportunities in condensed matter as spin ice[33].

The corresponding collective Bianchi symmetric identity is given by

$$\partial_\mu z^{GI}_{(\nu\rho)}+\partial_\rho z^{GI}_{(\mu\nu)}+\partial_\nu z^{GI}_{(\rho\mu)}=\gamma_{(11)}\{A_\mu S^1_{\rho\nu}+A_\nu S^1_{\rho\mu}+A_\rho S^1_{\mu\nu}\}+$$

$$+\gamma_{(22)}\{U_\mu S^2_{\rho\nu}+U_\nu S^2_{\rho\mu}+U_\rho S^2_{\mu\nu}\}+$$

$$+\gamma_{(12)}\{A_\mu S^2_{\rho\nu}+A_\nu S^2_{\rho\mu}+A_\rho S^2_{\mu\nu}+U_\mu S^1_{\rho\nu}+U_\nu S^1_{\rho\mu}+U_\rho S^1_{\mu\nu}\}+$$

$$+2\gamma_{(33)}\mathrm{Re}\{V^-_\mu S^+_{\rho\nu}+V^-_\nu S^+_{\rho\mu}+V^-_\rho S^+_{\mu\nu}\} \tag{4.37}$$

Finally, there is a Bianchi identity similar to eq.(4.52) for ω^α_α.

5 Photon effective equation

A whole system with (4+1) equations of motion is derived from eq.(2.1). However, given that the system which flows in space-time is just with four fields, we should introduce an effective equation in such a way that it reduces the whole EM system to four equations of motion. Given that, after moving from primitive to basic equations, the next step will be to select which field will be responsible for the directive determinism. This means that one that assumes the symmetry equation.

A fields set whole dynamics was obtained and we should now identify how light symmetry acts between them. Recapitulating, the main difficulty from Maxwell equations is on light interpretation. For Maxwell, light is not the beginning charge is the origin. Thus, a light electromagnetism is still expected to be developed. Our interpretation here is to make the photon as the fields set directive. The EM messenger guiding the four fields as the group vector.

The physical choice is on the photon. It must be separated between other fields due to the fact that light symmetry is on it. From the light invariance paradigm a new photon equation must exist. Something where light symmetry choose the photon as a guideline. This differentiation shall incorporate the eq.(4.44) symmetry equation into the eq.(4.8) photon equation. Carrying the singular property of being an absolute one should interpret it as the directive set.

A photon effective equation appears. Being the chosen directive quanta, it associates the Noether symmetry equation with its Euler-Lagrange equation. These combined equations make the photon assuming the directive causality. It conducts an equation where the photon acts as a whole maker. It gives,

$$\partial_\nu(F^{\nu\mu}_w+z^{[\nu\mu]}_w)+\partial^\mu(S^\alpha_{\alpha w}+z^\alpha_{\alpha w})+l^\mu_w+c^\mu_w=j^\mu_w \tag{5.1}$$

where the transversal sector is

$$F_w^{\nu\mu} = \bar{\bar{a}}_1 F^{\nu\mu}(\text{granular}) \tag{5.2}$$

$$z_w^{[\nu\mu]} = \bar{\bar{a}}_2\left(\overset{[12]}{z}{}^{[\nu\mu]} + \overset{[+-]}{z}{}^{[\nu\mu]}\right) + \bar{\bar{a}}_3 \overset{(+-)}{z}{}^{[\nu\mu]} \quad (\text{collective}) \tag{5.3}$$

and the longitudinal sector is

$$S^{\alpha}_{\alpha w} = (\bar{\bar{a}}_4 S^{\alpha 1}_{\alpha} + \bar{\bar{a}}_5 S^{\alpha 2}_{\alpha})(\text{granular}) \tag{5.4}$$

$$z^{\alpha}_{\alpha w} = \bar{\bar{a}}_6\left(\overset{(11)}{z}{}^{\alpha}_{\alpha} + \overset{(22)}{z}{}^{\alpha}_{\alpha} + 2\overset{(12)}{z}{}^{\alpha}_{\alpha} + 2\overset{(+-3)}{z}{}^{\alpha}_{\alpha}\right) + \bar{\bar{a}}_7\left(\overset{(11)}{\omega}{}^{\alpha}_{\alpha} + \overset{(22)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{(12)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{+-3}{\omega}{}^{\alpha}_{\alpha}\right)(\text{collective}) \tag{5.5}$$

The corresponding London term is

$$l^{\mu}_{w} = A^{\mu}\left(r^A_{28}\overset{(22)}{\omega}{}^{\alpha}_{\alpha} + r^A_{29}\overset{+-3}{\omega}{}^{\alpha}_{\alpha}\right) + U^{\mu}\left(r^U_{27}\overset{(11)}{\omega}{}^{\alpha}_{\alpha} + r^U_{28}\overset{+-3}{\omega}{}^{\alpha}_{\alpha}\right) \tag{5.6}$$

The corresponding conglomerate term is

$$\begin{aligned}
c^{\mu}_{w} = A_{\nu}\{ & r^A_4 \overset{[12]}{z}{}^{[\nu\mu]} + r^A_5 \overset{[+-]}{z}{}^{(\nu\mu)} + r^A_6 \overset{(12)}{\omega}{}^{(\nu\mu)} + r^A_7 \overset{[13-]}{z}{}^{[\nu\mu]} + r^A_8 \overset{[23-]}{z}{}^{[\nu\mu]} + r^A_9 \overset{[24-]}{z}{}^{[\nu\mu]} + \\
& + r^A_{10} \overset{(23-)}{z}{}^{[\nu\mu]} - r^A_{11} \overset{(24-)}{z}{}^{[\nu\mu]} - r^A_{12} \overset{(23-)}{\omega}{}^{[\nu\mu]} - r^A_{13} \overset{(24-)}{\omega}{}^{[\nu\mu]} + r^{A*}_7 \overset{[13+]}{z}{}^{[\nu\mu]} + r^{A*}_8 \overset{[23+]}{z}{}^{[\nu\mu]} + \\
& + r^{A*}_9 \overset{[24+]}{z}{}^{[\nu\mu]} + r^{A*}_{10} \overset{(23+)}{z}{}^{[\nu\mu]} - r^{A*}_{11} \overset{(24+)}{z}{}^{[\nu\mu]} - r^{A*}_{12} \overset{(23+)}{\omega}{}^{[\nu\mu]} - r^{A*}_{13} \overset{(24+)}{\omega}{}^{[\nu\mu]} + r^A_{17} \overset{(11)}{z}{}^{(\nu\mu)} + \\
& + r^A_{18} \overset{(22)}{z}{}^{(\nu\mu)} + r^A_{19} \overset{+-3}{z}{}^{(\nu\mu)} + r^A_{20} \overset{(11)}{\omega}{}^{(\nu\mu)} + r^A_{21} \overset{(12)}{\omega}{}^{(\nu\mu)} + r^A_{22} \overset{(22)}{\omega}{}^{(\nu\mu)} + r^A_7 \overset{[13-]}{z}{}^{(\nu\mu)} + r^A_{23} \overset{[23-]}{z}{}^{(\nu\mu)} + \\
& + r^A_{24} \overset{(13-)}{z}{}^{(\nu\mu)} + r^{A*}_9 \overset{(23-)}{z}{}^{(\mu\nu)} + r^A_{12} \overset{(24-)}{z}{}^{(\nu\mu)} + r^A_{13} \overset{(23-)}{\omega}{}^{[\nu\mu]} + r^A_{14} \overset{(24-)}{\omega}{}^{(\nu\mu)} + r^{A*}_7 \overset{[13+]}{z}{}^{(\nu\mu)} + \\
& + r^{A*}_{23} \overset{[23+]}{z}{}^{(\nu\mu)} + r^{A*}_{24} \overset{(13+)}{z}{}^{(\nu\mu)} + r^A_9 \overset{(23+)}{z}{}^{(\mu\nu)} + r^{A*}_{12} \overset{(24+)}{z}{}^{(\nu\mu)} + r^{A*}_{13} \overset{(23+)}{\omega}{}^{[\nu\mu]} + r^{A*}_{14} \overset{(24+)}{\omega}{}^{(\nu\mu)} + \\
& + g^{\nu\mu}\left(-r^A_7 \overset{[13-]}{z}{}^{\alpha}_{\alpha} - r^A_7 \overset{[23-]}{z}{}^{\alpha}_{\alpha} + r^A_{24} \overset{(13-)}{z}{}^{\alpha}_{\alpha} + r^A_{24} \overset{(23-)}{z}{}^{\alpha}_{\alpha} + r^A_{30} \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + r^A_{30} \overset{(23-)}{\omega}{}^{\alpha}_{\alpha} - r^{A+}_7 \overset{[13+]}{z}{}^{\alpha}_{\alpha} + \right. \\
& \left. - r^{A*}_7 \overset{[23+]}{z}{}^{\alpha}_{\alpha} + r^{A*}_{24} \overset{(13+)}{z}{}^{\alpha}_{\alpha} + r^{A*}_{24} \overset{(23+)}{z}{}^{\alpha}_{\alpha} + r^{A*}_{30} \overset{(13+)}{\omega}{}^{\alpha}_{\alpha} + r^{A*}_{30} \overset{(23+)}{\omega}{}^{\alpha}_{\alpha}\right)\} + \\
& + U_{\nu}\{ r^U_4 \overset{[12]}{z}{}^{[\nu\mu]} + r^U_4 \overset{[+-]}{z}{}^{(\nu\mu)} + r^U_5 \overset{(12)}{\omega}{}^{[\nu\mu]} + r^U_6 \overset{[13-]}{z}{}^{[\nu\mu]} + r^U_7 \overset{[14-]}{z}{}^{[\nu\mu]} + r^U_8 \overset{[23-]}{z}{}^{[\nu\mu]} + \\
& + r^U_9 \overset{(13-)}{z}{}^{[\nu\mu]} + r^U_{10} \overset{(14-)}{z}{}^{[\nu\mu]} + r^U_{11} \overset{(23-)}{z}{}^{[\nu\mu]} + r^U_{12} \overset{(13-)}{\omega}{}^{[\nu\mu]} + r^U_{13} \overset{(14-)}{\omega}{}^{[\nu\mu]} + r^{U*}_6 \overset{[13+]}{z}{}^{[\nu\mu]} + \\
& + r^{U*}_7 \overset{[14+]}{z}{}^{[\nu\mu]} + r^{U*}_8 \overset{[23+]}{z}{}^{[\nu\mu]} + r^{U*}_9 \overset{(13+)}{z}{}^{[\nu\mu]} + r^{U*}_{10} \overset{(14+)}{z}{}^{[\nu\mu]} + r^{U*}_{11} \overset{(23+)}{z}{}^{[\nu\mu]} + r^{U*}_{12} \overset{(13+)}{\omega}{}^{[\nu\mu]} + \\
& + r^{U*}_{13} \overset{(14+)}{\omega}{}^{[\nu\mu]} + r^U_{17} \overset{(11)}{z}{}^{(\nu\mu)} + r^U_{18} \overset{(12)}{z}{}^{(\nu\mu)} + r^U_{19} \overset{(22)}{z}{}^{(\nu\mu)} + r^U_{20} \overset{+-3}{z}{}^{(\nu\mu)} + r^U_{21} \overset{(11)}{\omega}{}^{(\nu\mu)} + r^U_{22} \overset{(12)}{\omega}{}^{(\nu\mu)} + \\
& + r^{U*}_{13} \overset{(14+)}{\omega}{}^{[\nu\mu]} + r^U_{17} \overset{(11)}{z}{}^{(\nu\mu)} + r^U_{18} \overset{(12)}{z}{}^{(\nu\mu)} + r^U_8 \overset{[23-]}{z}{}^{(\nu\mu)} - r^U_9 \overset{(13-)}{z}{}^{(\nu\mu)} + - r^U_{10} \overset{(14-)}{z}{}^{(\nu\mu)}
\end{aligned}$$

$$-r_{11}^{U}\overset{(23-)}{z}{}^{(\nu\mu)}-r_{12}^{U}\overset{(13-)}{\omega}{}^{(\nu\mu)}-r_{13}^{U}\overset{(14-)}{\omega}{}^{(\nu\mu)}-r_{6}^{U*}\overset{[13+]}{z}{}^{(\nu\mu)}-r_{7}^{U*}\overset{[14+]}{z}{}^{(\nu\mu)}+r_{8}^{U*}\overset{[23+]}{z}{}^{(\nu\mu)}$$

$$-r_{9}^{U*}\overset{(13+)}{z}{}^{(\nu\mu)}-r_{10}^{U*}\overset{(14+)}{z}{}^{(\nu\mu)}-r_{12}^{U*}\overset{(13+)}{\omega}{}^{(\nu\mu)}-r_{13}^{U*}\overset{(14+)}{\omega}{}^{(\nu\mu)}+$$

$$+g^{\nu\mu}(r_{29}^{U}\overset{[13-]}{z}{}_{\alpha}^{\alpha}+r_{30}^{U}\overset{[23-]}{z}{}_{\alpha}^{\alpha}+r_{31}^{U}\overset{(13-)}{z}{}_{\alpha}^{\alpha}+r_{31}^{U}\overset{(23-)}{z}{}_{\alpha}^{\alpha}+r_{29}^{U}\overset{(13-)}{\omega}{}_{\alpha}^{\alpha}$$

$$+r_{29}^{U}\overset{(23-)}{\omega}{}_{\alpha}^{\alpha}+r_{29}^{U*}\overset{[13+]}{z}{}_{\alpha}^{\alpha}+r_{30}^{U*}\overset{[23+]}{z}{}_{\alpha}^{\alpha}+r_{31}^{U*}\overset{(13+)}{z}{}_{\alpha}^{\alpha}+r_{31}^{U*}\overset{(23+)}{z}{}_{\alpha}^{\alpha}+r_{29}^{U*}\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+r_{29}^{U*}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+V_{\nu}^{+}\{r_{4}^{+}\overset{[12]}{z}{}^{[\nu\mu]}-iqc_{2}(\overset{[-1]}{z}{}^{[\nu\mu]}+\overset{[-2]}{z}{}^{[\nu\mu]})+r_{5}^{+}\overset{[13-]}{z}{}^{[\nu\mu]}+r_{6}^{+}\overset{[14-]}{z}{}^{[\nu\mu]}+r_{7}^{+}\overset{[23-]}{z}{}^{[\nu\mu]}+$$

$$+f_{6}^{+}\overset{[24-]}{z}{}^{[\nu\mu]}-f_{7}^{+}\overset{(13-)}{z}{}^{[\nu\mu]}+r_{8}^{+}\overset{(23-)}{z}{}^{[\nu\mu]}+r_{9}^{+}\overset{(13-)}{\omega}{}^{[\nu\mu]}+f_{12}^{+}\overset{(24-)}{\omega}{}^{[\nu\mu]}+r_{13}^{+}\overset{(11)}{z}{}^{(\nu\mu)}+$$

$$+r_{14}^{+}\overset{(22)}{z}{}^{(\nu\mu)}+r_{15}^{+}\overset{(12)}{z}{}^{(\nu\mu)}+r_{16}^{+}\overset{+-4}{z}{}^{(\nu\mu)}+r_{17}^{+}\overset{[+-]}{z}{}^{(\nu\mu)}+r_{18}^{+}\overset{(+-)}{\omega}{}^{(\nu\mu)}-iqc_{4}(\overset{(-1)}{z}{}^{(\nu\mu)}+\overset{(-2)}{z}{}^{(\nu\mu)})+$$

$$+r_{19}^{+}\overset{[13-]}{z}{}^{(\nu\mu)}+r_{20}^{+}\overset{[14-]}{z}{}^{(\nu\mu)}+r_{21}^{+}\overset{[23-]}{z}{}^{(\nu\mu)}+f_{6}^{+}\overset{[24-]}{z}{}^{(\nu\mu)}+r_{22}^{+}\overset{(13-)}{z}{}^{(\nu\mu)}+r_{23}^{+}\overset{(14-)}{z}{}^{(\nu\mu)}+$$

$$+r_{24}^{+}\overset{(23-)}{\omega}{}^{(\nu\mu)}+f_{10}^{+}\overset{(24-)}{\omega}{}^{(\nu\mu)}+r_{24}^{+}\overset{(13-)}{\omega}{}^{(\nu\mu)}+r_{25}^{+}\overset{(14-)}{\omega}{}^{(\nu\mu)}+f_{11}^{+}\overset{(23-)}{\omega}{}^{(\mu\nu)}+f_{12}^{+}\overset{(24-)}{z}{}^{(\nu\mu)}+$$

$$+g^{\nu\mu}(-iqc_{6}(\overset{(-1)}{z}{}_{\alpha}^{\alpha}+\overset{(-2)}{z}{}_{\alpha}^{\alpha})-iqc_{7}(\overset{(-1)}{\omega}{}_{\alpha}^{\alpha}+\overset{(-2)}{\omega}{}_{\alpha}^{\alpha})+r_{29}^{+}\overset{[13-]}{z}{}_{\alpha}^{\alpha}+$$

$$+r_{29}^{+}\overset{[23-]}{z}{}_{\alpha}^{\alpha}++r_{30}^{+}\overset{(13-)}{z}{}_{\alpha}^{\alpha}+r_{30}^{+}\overset{(23-)}{z}{}_{\alpha}^{\alpha}+r_{31}^{+}\overset{(13-)}{\omega}{}_{\alpha}^{\alpha}+r_{32}^{+}\overset{(23-)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+V_{\nu}^{-}\{r_{4}^{+*}\overset{[12]}{z}{}^{[\nu\mu]}+iqc_{2}(\overset{[+1]}{z}{}^{[\nu\mu]}+\overset{[+2]}{z}{}^{[\nu\mu]})+r_{5}^{+*}\overset{[13+]}{z}{}^{[\nu\mu]}+r_{6}^{+*}\overset{[14+]}{z}{}^{[\nu\mu]}+r_{7}^{+*}\overset{[23+]}{z}{}^{[\nu\mu]}+$$

$$+f_{6}^{+*}\overset{[24+]}{z}{}^{[\nu\mu]}-f_{7}^{+*}\overset{(13+)}{z}{}^{[\nu\mu]}+r_{8}^{+*}\overset{(23+)}{z}{}^{[\nu\mu]}+r_{9}^{+*}\overset{(13+)}{\omega}{}^{[\nu\mu]}+f_{12}^{+}\overset{(24+)}{\omega}{}^{[\nu\mu]}+r_{13}^{+*}\overset{(11)}{z}{}^{(\nu\mu)}+$$

$$+r_{14}^{+*}\overset{(22)}{z}{}^{(\nu\mu)}+r_{15}^{+*}\overset{(12)}{z}{}^{(\nu\mu)}+r_{16}^{+*}\overset{+-4}{z}{}^{(\nu\mu)}+r_{17}^{+*}\overset{[+-]}{z}{}^{(\nu\mu)}+r_{18}^{+*}\overset{(+-)}{\omega}{}^{(\nu\mu)}++r_{19}^{+*}\overset{[13+]}{z}{}^{(\nu\mu)}+$$

$$+iqc_{4}(\overset{(+1)}{z}{}^{(\nu\mu)}+\overset{(+2)}{z}{}^{(\nu\mu)})+r_{20}^{+*}\overset{[14+]}{z}{}^{(\nu\mu)}+r_{21}^{+*}\overset{[23+]}{z}{}^{(\nu\mu)}+f_{6}^{+*}\overset{[24+]}{z}{}^{(\nu\mu)}+r_{22}^{+*}\overset{(13+)}{z}{}^{(\nu\mu)}+$$

$$+r_{23}^{+*}\overset{(14+)}{z}{}^{(\nu\mu)}+r_{24}^{+*}\overset{(23+)}{\omega}{}^{(\nu\mu)}+f_{10}^{+*}\overset{(24+)}{\omega}{}^{(\nu\mu)}+r_{24}^{+*}\overset{(13+)}{\omega}{}^{(\nu\mu)}+r_{25}^{+*}\overset{(14+)}{\omega}{}^{(\nu\mu)}+$$

$$+f_{11}^{+*}\overset{(23+)}{\omega}{}^{(\mu\nu)}+f_{12}^{+}\overset{(24+)}{z}{}^{(\nu\mu)}+g^{\nu\mu}(iqc_{6}(\overset{(+1)}{z}{}_{\alpha}^{\alpha}+\overset{(+2)}{z}{}_{\alpha}^{\alpha})+iqc_{7}(\overset{(+1)}{\omega}{}_{\alpha}^{\alpha}+\overset{(+2)}{\omega}{}_{\alpha}^{\alpha})+$$

$$+r_{29}^{+*}\overset{[13+]}{z}{}_{\alpha}^{\alpha}+r_{29}^{+*}\overset{[23+]}{z}{}_{\alpha}^{\alpha}+r_{30}^{+*}\overset{(13+)}{z}{}_{\alpha}^{\alpha}+r_{30}^{+*}\overset{(23+)}{z}{}_{\alpha}^{\alpha}+r_{31}^{+*}\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+r_{32}^{+*}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha})\} \qquad (5.7)$$

The non-linear systemic current is written as

$$j_{\rm w}^{\mu}=A_{\nu}\{r_{1}^{A}F^{\nu\mu}+r_{2}^{A}U^{\nu\mu}+r_{3}^{A}V^{\nu\mu-}+r_{3}^{A*}V^{\nu\mu+}+r_{14}^{A}S^{\nu\mu1}+$$

$$+r_{15}^{A}S^{\nu\mu2}+r_{16}^{A}S^{\nu\mu-}+r_{16}^{A*}S^{\nu\mu-}+g^{\nu\mu}(r_{25}^{A}S_{\alpha}^{\alpha1}+r_{26}^{A}S_{\alpha}^{\alpha2}+r_{27}^{A}S_{\alpha}^{\alpha-}+r_{27}^{A*}S_{\alpha}^{\alpha+})+$$

$$+U_{\nu}\{r_{1}^{U}F^{\nu\mu}+r_{2}^{U}U^{\nu\mu}+r_{3}^{U}V^{\nu\mu+}+r_{3}^{U*}V^{\nu\mu-}+r_{14}^{U}S^{\nu\mu1}+r_{15}^{U}S^{\nu\mu2}+$$

$$+r_{16}^{U}S^{\nu\mu-}+r_{16}^{U*}S^{\nu\mu+}+g^{\nu\mu}(r_{24}^{U}S_{\alpha}^{\alpha1}+r_{25}^{U}S_{\alpha}^{\alpha2}+r_{26}^{U}S_{\alpha}^{\alpha-}+r_{26}^{U*}S_{\alpha}^{\alpha+})\}+$$

$$+V_{\nu}^{+}\{r_{1}^{+}F^{\nu\mu}+r_{2}^{+}U^{\nu\mu}+(r_{3}^{+}-iqc_{1})V^{\nu\mu-}+r_{10}^{+}S^{\nu\mu1}+$$

$$+r_{11}^{+}S^{\nu\mu2}+(r_{12}^{+}-iqc_{3})S^{\nu\mu-}+g^{\nu\mu}(r_{26}^{+}S_{\alpha}^{\alpha1}+r_{27}^{+}S_{\alpha}^{\alpha2}+(r_{28}^{+}-iqc_{5})S_{\alpha}^{\alpha-})\}+$$

$$+V_{\nu}^{-}\{r_{1}^{+*}F^{\nu\mu}+r_{2}^{+*}U^{\nu\mu}+(r_{3}^{+*}+iqc_{1})V^{\nu\mu+}+r_{10}^{+*}S^{\nu\mu1}+$$

$$+r_{11}^{+}S^{\nu\mu2}+(r_{12}^{+}+iqc_{3})S^{\nu\mu+}++g^{\nu\mu}(r_{26}^{+*}S_{\alpha}^{\alpha1}+r_{27}^{+*}S_{\alpha}^{\alpha2}+(r_{28}^{+*}+iqc_{5})S_{\alpha}^{\alpha+})\} \quad (5.8)$$

The corresponding coefficients in these equations are determined at Appendix C.

The whole continuity equation is

$$\mathsf{W}(S_{\alpha w}^{\alpha}+z_{\alpha w}^{\alpha})=\partial\cdot(j_{w}-l_{w}-c_{w}) \quad (5.9)$$

Eq.(5.9) provides a photonic charge, the difference for (4.13) is on its dependence on electric charge. Notice at Appendix C that for some terms the electric charge (or a multiple of that) is the leading coefficient term. Physically, a photonic charge depending on electric charge is obtained.

Thus, the fields set $\{A_{\mu},U_{\mu},V_{\mu}^{\pm}\}$ flows a whole dynamics based on four equations. They are the photon effective equation plus 3-tributary fields associated to the photon main flux. Together they flow at space-time a whole where Maxwell becomes just a piece of this enlargement. There is a major EM ocean carrying different charges and fields to be explored.

6 Model Management

A new symmetry behavior appears through the fields set physics. Eq.(2.1) anti-reductionist approach provides the directive and circumstantial manifestations. The first one was studied in the previous section by assuming the photon as the fields set vector direction. The second one involves the meaning of chance inside of the set. This section intends to explore the meaning of opportunity that the systemic symmetry produces.

A first result to be understood from this model is on existence of electromagnetic fields being their own sources associated to electric charge. For this, we are going to study a simplified model of the equations of motion studied at section 4. Taking a special combination between the free coefficients one derives the following equation of motion for the field A^{μ} without breaking gauge invariance:

$$\partial_{\nu}F^{\nu\mu}+a_{1}\partial^{\mu}\overset{(11)}{z}{}_{\alpha}^{\alpha}=f_{1}A_{\nu}S^{\nu\mu1} \quad (6.1)$$

This equation raise a system of two coupled equations where we use $\vec{A}=A(r)\hat{r}$, $\phi(\vec{r})=\phi(r)$, $\frac{d}{dt}\rightarrow 0$. It gives $\vec{\nabla}\cdot\vec{\nabla}\phi=f_{1}\vec{A}\cdot\vec{\nabla}\phi$, and $\phi(r)=\xi A(r)$. So we achieve the equation $\phi''+2\frac{\phi'}{r}+\frac{f_{1}\xi}{a_{1}}\phi\phi'=0$. Making the approximation $\frac{f_{1}\xi}{a_{1}}\rightarrow 0$ we get $\phi''+2\frac{\phi'}{r}=0$. Using appropriate boundary conditions, we get $\phi(r)=\frac{\phi_{0}}{r}$ and $\vec{A}=\frac{A_{0}}{r}\hat{r}$. For the field $U^{\mu}:\partial_{\nu}U^{\nu\mu}-m_{U}^{2}U^{\mu}=0$ Leading to: $\vec{\nabla}\cdot\vec{\nabla}\phi_{U}+m_{U}^{2}\phi_{U}=0,\quad \vec{U}=0$. Solving these equations we get: $\phi_{U}(r)=\phi_{0}\frac{e^{-m_{U}r}}{r}$. For the charged fields, we get the same equation $\partial_{\nu}V^{\nu\mu\pm}-\mu_{+}^{2}V^{\mu\pm}=0$.

Obtaining: $\phi^{\pm}(r) = \phi_0^{\pm} \frac{e^{-\mu_+ r}}{r}, \quad \vec{V}^{\pm} = 0$.

Thus, considering this very particular model, our purpose here is to demonstrate on the presence of different types of electromagnetic fields without having as source the electric charge. Their corresponding mathematical expressions corresponding to eqs(2.6) and (2.11) are:

Granular fields:

$$\vec{E}_A = \frac{E_{0A}}{r^2}\hat{r}$$

$$\vec{E}_U = E_{0U} \frac{e^{-m_U(r-1)}(m_U r+1)}{r^2}\hat{r}$$

$$\vec{E}_{\pm} = E_{0\pm} \frac{e^{-\mu_+(r-1)}(\mu_+ r+1)}{r^2}\hat{r}$$

$$\vec{B}_A = \vec{B}_U = \vec{B}_{\pm} = 0 \quad (6.2)$$

Collective Fields:

$$s^{(11)} = \gamma_{(11)}^2 \frac{\phi_0^2}{r^2}, \qquad \vec{s}^{(12)} = \phi_0 A_0 \frac{e^{-2m_U r}}{r^3}\hat{r},$$

$$\overset{\leftrightarrow}{s}^{(11)} = \frac{A_0^2}{r^2}\hat{r}, \qquad s^{(+-)} = -i\gamma_{(34)}\phi_0^+\phi_0^- \frac{e^{-2\mu_+ r}}{r^2} \quad (6.3)$$

We should investigate now on the meaning of collective energy by comparing with dark matter and energy. Given that in Universe 96% is dark energy-matter we are going to study here the relationship collective/granular energies by taking this proportion. As exercise, making all the parameters $E_{0A} = E_{0U} = E_{0\pm} = 1$, $\gamma_{(11)} = ... = 1$, $\phi_0 = ... = 1$ and the mass parameter $m_+ = m_- = 1$, one gets for the m_U parameter the expression

$$m_U = -78{,}44840715 \quad (6.4)$$

The above result for the mass parameter suggests the possibility of dark matter-energy be interpreted in terms of the collective fields energy. Given that fields predates matter, eq.(6.4) is interpreting the dark energy-matter as consequence from a collective field energy. The whole symmetry management allows the meaning of function. A behavior where m_U value works as a whole function. This dependence introduces a physical possibility where instead of dark-matter energy we should interpret as the presence of collective fields energy.

7 Spin-1 and spin-0 dynamics

A next step for this four bosons electromagnetism is to split the messengers dynamics. Lorentz group introduces not only the space-time correlation but also the spin. Given the representation $(\frac{1}{2}, \frac{1}{2})$, one gets a spin spectroscopy with four particles with spin-1 plus four with spin-0. We should now separate their physics. Classify the corresponding spin dynamics.

The $\{\frac{1}{2},\frac{1}{2}\}$ flavours dynamics must be understood. Eq.(2.2) contains a fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$ propagation. Eq.(4.1) propagates granular fields and collective fields strengths carrying spin-1 and spin-0 families. They contain eight integrated quanta to be systematized. They are carrying a massless scalar and a vectorial photon plus six massive transverse and longitudinal bosons[7].The next step is to dynamically separate the spin-1 and spin-0 sectors.

We should decompose these different sectors. Given the fields set $\{A_{\mu I}\} \equiv \{A_\mu, U_\mu, V_\mu^\pm\}$, there are two families with spin-1 and spin-0 whose dynamics must be identified separately. For this, one has to take the transversal and longitudinal operators $A_{\mu I}^T = \theta_{\mu\nu} A_I^\nu$ and $A_{\mu I}^L = \omega_{\mu\nu} A_I^\nu$ where $\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}$ and $\omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\Box}$. It yields an $A_{\mu I}^T$ dynamics associated to the antisymmetric and symmetric fields strengths while $A_{\mu I}^L$ dynamics is just depending on the symmetric fields strengths.

We should now separate these two spin dynamics multiplying eq.(4.1) by the transverse and longitudinal operators. It gives, the spin-1 dynamics:

$$\partial_\nu (F^{\nu\mu} + z^{[\nu\mu]}) + m^2 X_T^\mu + l_T^\mu + c_T^\mu = j_T^\mu$$

For spin-0 dynamics:

$$\partial^\mu (S_\alpha^\alpha + z_\alpha^\alpha) + m^2 X_L^\mu + l_L^\mu + c_L^\mu = j_L^\mu$$

where $j_T^\mu \equiv \Theta_\nu^\mu j^\nu$ and $j_L^\mu \equiv \omega_\nu^\mu j^\nu$ and so on. Given that, eqs.(7.1) and (7.2) preserves covariance, their splitting from eq.(4.1) shows consistency with the $U(1)XSO(2)$ symmetry. Although the spin operators contain a non-local nature the physicity is not lost. It does not break symmetry. The covariance shows that eq.(2.1) proposal is maintained.

Thus, through a non-local decomposition one derives two different dynamics. Eqs.(7.1) and (7.2) are splitting two covariant spin fluxes, saying that, although the fields are the same, the corresponding quanta propagations are diverse. Covariance shows that, whether this splitting is valid for a given reference system, it will be valid for all others. Their informations do not change in different frames. On other words, the non-locality is preserved for different reference frames. As result, one can assume that the spin dynamics splitting develops different Maxwell equations for spin-1 and spin-0 families. At section 9, one studies their corresponding vectorial and scalar expressions.

The corresponding spin-1 continuity equation is

$$\partial.(m^2 X_T^\mu + l_T^\mu + c_T^\mu - j_T^\mu) = 0 \tag{7.1}$$

and the spin-0 wave equation is

$$\Box(S_\alpha^\alpha + z_\alpha^\alpha) = \partial.(m^2 X_L^\mu + l_L^\mu + c_T^\mu - j_L^\mu) \tag{7.2}$$

Observe that by adding eq.(7.3) with (7.4) one verifies eq.(4.1).

The result of this decomposition is on the appearance of an unity of diversity. A physics based on diversity. The above equations are indicating on the possibility of separating whole quantum field theory into branches. This means that given an (i, j) Lorentz representation is possible to separate through spins operators the diverse quanta inserted. There is a new physical panorama to be explored where every quanta will have its own dynamics. Consequently, given that these type of models carry on the possibility of being renormalizable and unitary, a healthy model involving spin-2 through $(1,1)$ representation, is envisaged. Photons interacting with gravitons become possible.

8 On three EM elements

Eq.(2.1) provokes a new interpretation on the electromagnetic phenomena. It preserves Maxwell symmetry and cross Maxwell frontier. It was tested in previous works as responding some criticisms to Maxwell equations. It answers from first gauge principles on the polarization and magnetization vectors, and on nonlinearity[5]. It also produces a renormalizable and unitary model[7,8] . Now, we should explore on its capacity of opening new phenomenological sectors for the electric charge, EM fields and photonics.

8.1 Electric Charge

Historically, electric charge is the origin of electromagnetism. It is the responsible for the EM fields generation. However, physics should explore beyond to such strict dependence. The step forward is a perception where the EM origin is no more due to electric charge. Let us remember that Maxwell equations in the vacuum already show solutions for the EM fields; also, nonlinear models show EM fields depending on themselves; on this route, section 2 develops another model where the EM fields generation is beyond to the electric charge as source.

The four bosons EM provides a new electric charge physics to be understood. Eq.(2.1) preserves the electric charge conservation, but opens a new panorama for electric charge behavior. The crucial difference occurs through the transmissions. It appears a new phenomena. There is an enlarged electric charge physics associated to eq.(2.2). Instead of the being the universal coupling constant, unique source for EM and light, new relationships between charges and fields are developed. Something to be explored.

There are seven new features on electric charge physics:

The first one is on discovering other charges diverse from electric charge. The fields set physics goes beyond to the e-presence as their continuity equation are saying at section 4 and 5. The four continuity equations, eqs.(4.13),(4.23),(4.33),(4.43), are showing that while Maxwell works with a strict electric charge conservation law, where fields and photon are consequences, now each messenger produces its own charge, EM field and interaction. They will enlarge the EM meaning. Maxwell vision on electric charge EM dependence starts to be surpassed, although the value e= 10^{-19} C remains in the theory basis, it should be understood just as a part of EM phenomena. Electric charge is no more the only one EM source There are more four spin-1 charges, four spin-0 charges, three Bianchi identity sources. In total there are 12 conserved charges playing the EM phenomena where electric charge conservation is just one between others.

The second feature is that while QED considers the electric charge as the universal photon coupling constant through the relationship eA_μ, the term gA_μ appears. The photon field can couple with other fields without the electric charge presence. Eq.(4.1) show that fields and sources as London, conglomerates and current terms express their physicity not necessarily coupled to the electric charge. A photon coupling diversity is generated.

The third aspect is with respect to electric charge conservation. Noether global current, eq.(2.24), establishes its conservation law. However, differently from QED, one notices a dependence not only on charged fields, but also on neutral fields which nature can be granular and collective. They are showing an extension for electric charge conservation law where instead of being restrict to charged fields it depends on the whole system.

The fourth aspect concerns charge conduction. The four bosons model provides two kinds of transportations which are the individual and collective porters. Then, as a new phenomena, one gets neutral A_μ and U_μ equations with collective electric charge transportation. Taking the photon equation (4.8) or (5.1), as example, one notices at lhs a flow conducting a neutral photon plus charges being carried by neutral collective fields; and at rhs, it develops a source $j_A^\mu = a_\mu^1 A^\mu + a_\mu^2 A^\mu + a_\mu^\pm A^\mu$, which although neutral it contains the electric charge presence. Similarly, for U_μ -equation (4.14). $V_\mu^\pm$ charged equations (4.24) and (4.34) are notary charged porters. This conduction can also be either done granularly through $V_\mu^\pm$ or collectively with charged collective fields as $z_{\mu\nu}^{(13+)}$ [34].

The fifth aspect is on charge exchange. Nowadays the exchange $\Delta Q = 0, \pm 1$ is studied in terms of electroweak. Nevertheless, the electric charge transmission must be understood as a more primitive phenomena. Interestingly, electric charge transmission is an old phenomena that should already be treated in early days. Back in time, in 1936, when the particles $e^-, e^+, \mu^+, \mu^-, \gamma$ were already discovered, the question is why they did not think on possible reactions between these particles beyond of being intermediated only by the photon. Why in the famous Warsaw conference in 1938 Oscar Klein did not propose the term Theory of Everything considering transmissions $\Delta Q = 0, \pm 1$? Neutral and charged currents contain a primitive physicity which phenomenology should still be a topic for LHC investigations. Something not restrict to weak interaction.

The sixth feature is on three charges. At 19^{th} century two charges were necessary for revealing the EM phenomena, however, at 20^{th} century with the elementary particle physics development the transmissions processes came aware, and so, the zero charge presence should be taken account for the EM processes. There is an electric charge conversion detected at EM microscopic reactions involving the zero charge into the positive and negative charges. It says that due to their mutual exchange they should be considered as a correlated whole. As consequence, from such mutual conversion, the EM phenomena should be reinterpreted through a three charges set. The positive, negative, zero charges be understood as an unified phenomena.

The seventh feature is on the charge interaction. The new performance for EM interaction is three charges with four messengers. They provoke new EM intensities and range to be explored. It is expected that the fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$ will produce short-range and long-range forces or even the fifth force[35]. There are new physical situations, due to the eight bosons propagators, which solution is not more the inverse square law. The neutral electric charge interaction is not zero, and so, the Yukawa idea of three pions (π^+, π^-, π^0) as responsible for strong interactions can be reinterpreted. Instead of nuclear force they can be components of this extended EM. Be the scalars corresponding to U_μ and $V_\mu^\pm$ fields under a short interaction.

There is a new enterprise for electric charge physics. An unified EM enlarges the electromagnetic processes for positive, negative and zero charges being transmitted by four messengers. It yields an EM model to be understood through eight quanta. It will transport three charges carried by particles with spin-1 and spin-0, through granular and collective fields and able to transmute the charges values. They will be responsible of a new phenomenology for the EM energy-momentum and charges transmissions and with coupling constants not necessarily the electric charge.

8.2 Photonics

The research for a primordial photon is still open. However, the main difficulty of Maxwell equation is on light interpretation. For Maxwell light is not in the beginning, charge is the origin. It considers light as a radiation ocean. Light works as a transmission service where only the electric charge is the source for the electromagnetic fields that propagate on this ocean. A photonic electromagnetism is still on the file to be developed. Something that contains Maxwell and goes beyond to electric charge as the only one source for the electromagnetic phenomena.

There is a photon physics to be understood. We should start by saying that Maxwell equations do not define the photon. There the electron is the active while the photon is passive. The photon behavior is just as a lake transmitting the electromagnetic phenomena. At this interpretation, the left hand side of Maxwell equations works as the transmission lake, while on the right hand side mean the sources being thrown in the lake. Then, we could say that at present moment physics is just throwing charged stones. Besides Dirac stone $\bar{\psi}\gamma^\mu\psi$, another sources as $\bar{\psi}\gamma^\mu\gamma^5\psi$, $\bar{\psi}\Sigma^{\mu\nu}\psi$, and, also with different spins have been thrown in the electromagnetic river. We should now understand on the photonic stone.

Photonics physics is a belief coming from the early universe. Let remind us that in the framework of a Big Bang based on inflationary universe, the radiation era dominated for 10^{13} s. So, matter formation (baryogenesis and matter-antimatter asymmetry) might be suitably treated in our proposal, where light is not only the carrier of the electromagnetic interaction, but it is rather looked upon as an origin for present matter, as pointed out by Born and Infeld.

Physics has been pursuing light as origin. The first published reference on light-light scattering came through the russian S.Vavilov in 1928[35] and the german physicist O.Halpern in 1934[37]. A step forward was given by Euler-Heisenberg, also in 1934, showing in the frame of quantum electrodynamics that photon-photon interaction is possible at tree level [11]. However, this model is limited to an effective theory. Therefore, QED remains until our days as the hegemonic theory even the corresponding Delbrück scattering[19] proposed since 1930 does not happen at tree level. Something more on light at tree level is still expected to be investigated.

Physics is still looking for the primordial photon. Nevertheless, in the actual Zeitgeist, most electromagnetic models understand that the interaction eA_μ is universal, QED does not interact photons between themselves at tree level (radiative corrections are necessary) and there is no photonic charge (just electronic charge). A step forward must be given. Something that preserves light invariance and electric charge conservation, but able to go beyond of the actual light secondary position. A new interpretation is necessary. For this, this work develops a model based on a fourth interpretation of light invariance. After Maxwell equation, relativity, Lorentz Group, the fourth one introduces different fields in the same Lorentz representation. Consequently, from the $\{\frac{1}{2}, \frac{1}{2}\}$ representation one introduces the fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$, which provides new possibilities for light be understood.

Primordial photon means a physical engagement derived directly from the A_μ-field. Then, associated to the fields set, it appears a photon that crosses Maxwell and QED frontiers. After Maxwell radiation field, Planck-Einstein quantum, Quantum Mechanics wave-particle, QED quantum field, emerges an active photon. Instead of a subsidiary field there is an alive photon articulated through four bosons electromagnetism. It brings new aspects for light. They are that photons interactions not necessarily depending on charged particles, selfinteracting photons and photonic charge. Something able to create the photon stone.

Thus, we are under the challenge of throwing this ’photon stone’ in the electromagnetic lake. That should be the origin of a photon physics. As required, it should be defined with A_μ acting directly as a physical field. Therefore, for the photon be associated to a 'stone' it must participate explicitly and as source in the equations of motion. Eq.(4.1) explicit a non subsidiary A_μ-field either through collective fields or as sources for l_μ, c_μ and j_μ. Eq.(2.2) derives a photon stone acting as source, with a nonlinearity through a correct dimensionality, producing a photonic current as a Lorentz vector and associated to a conserved charge which is expressed coupled to EM fields and in terms of porters as London, conglomerates terms, non-linear currents. Notice that while the usual Maxwell approach interprets the photon field as photonic current in terms of Poynting vector flux, the term $f_2^A A_\nu \overset{(11)(\nu\mu)}{z}$ at eq.(4.12) defines more accurately on the photonic current than the Poynting vector $\vec{E}\times\vec{B}$.

There is a new photon physics to be explored. It becomes a possibility from the whole relativistic equations at sections 4 and 5. They introduce a new photon behavior. First, through the relationship gA_μ for the photon field where g is any coupling constant as can be verified through eq.(4.1) prototype. Second, the corresponding photon continuity equation produces a photonic charge. Third, with selfinteracting photons[7]. These three enlargements on photon features introduces the meaning of photonics. A feature is the appearance of an ubiquous photon. For instance, the possibility for the photon interact with neutral particles, as Z^0 and other neutral particles, and neutrinos when matter is included.

There is a photonics physics to be investigated. It is based on a 'photon stone' carrying the A_μ field physicity and with an ubiquous photon interacting with everybody. Sections 4 and 5 encounter new physics without the electric charge presence and with A_μ direct presence at equations of motion. They show an A_μ physicity which precede the electromagnetic fields presence (take $A_\mu = \partial_\mu \phi$ that the equations of motion do not disappear) and proposes new couplings between the photon and EM fields, as $g\vec{A}\times\vec{E}$. The corresponding relativistic whole equations are showing that A_μ generates its own EM fields and propagates at space-time through granular and collective fields. It performs the ubiquous lux manifestation where the photon couples with itself, other fields, $\vec{E}$ and $\vec{B}$ as $g\vec{A}\cdot\vec{B}$, and masses via Global Lorentz force [5].

A new photon age arises. A photonic physical region is generated for a new understanding of the very nature of light itself. New physical aspects appear from selfinteracting photons. A first insight is that photon-photon scattering can be treated at three level and without fermionic matter and electric charge presence. Phenomena such as diffraction, interference, emission of waves will get new contributions. Also effects in non-linear optics, where selfinteracting photons can work as a new argument for exploring the frequencies variations. Up to know a change on frequencies is being treated only as a kinematic topic. Three and four photon vertices can justify why an incident red photon can be transformed into a blue one. Photons nonlinearity may also appear as solitons. Luminosity and vortex effects are expected as the occurrence of light structures like photon clouds, photon-balls and light shots like jets, cones, spots. Another case is the photon conglomerates dynamics and interactions as $z_{\mu\nu}^{(11)}$. It allows the existence of photon clouds. Definitively, gases of photons interacting without electrons and positrons will be showing a physics beyond QED. We expect this fact to be common in the stars formation. There, selfinteracting photons may be responsible for increasing photon's energy. Inelastic light-by-light scattering involving only real photons is another candidate for the photonic physical region.

Nevertheless the main consequence from these photonics properties is that from light one can generate matter. A light big bang becomes possible. Photons contain properties for particles productions. The fundamental aspect coming from eqs.(2.1-2) is that ordinary photons contain reactions to conjure matter from light and vice-versa. The electron-positron offers a first range of energy. It belongs to a region where photons are carrying bundles of energy below 1 MeV as $e^+e^- \to \gamma\gamma$. A next step is to describe reactions as $\gamma\gamma \to e^+e^-$ at tree level. The PVLS experiment to be realized in 2018 in Padova probably goes further to Maxwell and Dirac equations and states a physics were light precedes electric charge. However, from LHC is that one should be taken as the main experiment for photon productions up to 14 TeV.

Concluding, a photonic physics is provided. Eq.(2.1) reveals a photon absolute, ubiquous, set directive, contingencies producer, internal structure and associate to coupling constants diverse from electric charge. These properties are supported respectively by the Michelson-Morley experiment, gA_μ universal interactivity, symmetry equation integrated into the photon Euler-Lagrange equation, physical variables under relativism and chance, selfinteractions. They characterize a photonic physics the following seven features: selfinteracting photons, ubiquous photon, photonic current, non-linear electromagnetic waves, new dispersion relation, photonic Lorentz force, photon matter generation. Three basic consequences appear. They are the photonics big bang, phenomenology and innovation. The first one provokes matter production from photon; follows, new phenomenologies for studying physical properties as light bending, superconductivity, photon balls and so on; and by third, new opportunities for EM innovations where the electron is substituted by the photon.

8.3 Electromagnetic transmission

There is a third element on electromagnetism aside of electric charge and photon behavior which is the EM fields transmission. EM transmission should not be confused with charge conduction. Something is to conduct charge other is to to transmit the electromagnetic energy. In the actual electromagnetism the Poynting vector is responsible for such EM transmission. It shows that, light transmit fields but not conduct charge.

Transmission through EM fields is the physics supporting telecommunications. Three features are necessary for understanding the fields transmissions. They are the energy-momentum tensor conservation laws, EM waves propagations and dispersions relations. We should investigate on these facts at the four bosons model. The fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$ generalize the messengers, Poynting vectors, EM waves, dispersion relations, energy-momentum tensor. They are being described by new neutral and charged messengers, continuity equations, and coupling constants. Opening the electromagnetism for a transmission beyond to electric charge.

There is a new performance when four bosons are considered. While Maxwell has focused on charge distribution, QED on interaction, the four bosons model develops that EM transmission is a phenomena to be understood under three charges (+,-,0) being portered by four messengers and associated to granular and collective fields. A new physicity for transmitting the EM appears. Studying the correspondent energy-momentum tensor there is an EM energy transmission beyond the usual Maxwell Poynting vector[5]. For instance, collective Poynting vectors. Also with nonlinear EM waves as studied at section 9 and new dispersion relations.

Thus, while Maxwell restricts an electromagnetism based on a neutral photon transmission through Poynting vector under a specific dispersion relation, the four bosons electromagnetism extends for a new behavior. It yields a propagation based on a massless photon, massive photon and two charged fields with a spin-0 counterpart. There is a total eight different quanta responsible for the EM transmission through their waves, Poynting vectors, dispersion relations. A new telecommunication to be investigated.

In a first analysis we select three new features. EM transmissions with Poynting vectors as own sources, conservation laws, and nonlinear photons. The four EM flux generalizes the Poynting vector beyond the photon for a fields set with granular and collective transmissions. It appears more three spin-1 and three spin-0 quanta propagations accomplished by collective fields, as eqs.(7.1-4) are showing. And so, new granular and collective Poynting vectors appear[5]. Given their nonlinearities properties these new Poyinting vectors act as their own sources. They are flowing through a whole fields dynamics which generalize the energy-momentum tensor. So the transmission is done under 12 conservation laws aside to the electric charge. Eight are derived from the equations of motion, three from Bianchi identity and one from Noether theorem. Adding to it, one includes two energy-momentum conservation laws. Another fact, is on the photon acting as origin for transmitting nonlinear EM fields. It opens for a nonlinear photon with a new dispersion relation as the next possible telecommunication transmission. An EM transmission to be investigated by astrophysics.

Thus, it appears an EM transmission beyond electric charge. It propagates EM nonlinear fields, new types of granular and collective vectors, new Poynting vectors. A process working as a whole network with an EM flux based on conservation laws which preserve the Maxwell symmetry and opens the electromagnetic phenomena for new transmissions possibilities which scope is for future investigations. A new photonic telecomunication is expected as introducing a new dispersion relation and nonlinear EM waves.

9 Global Maxwell Equations

We will express here the primitive equations in the vectorial form. Considering that a tensor $T_{\mu\nu} = T_{[\mu\nu]} + T_{(\mu\nu)}$, carries 2 spin-1 for $T_{[\mu\nu]}$, eq.(7.1) expresses two vector fields $\vec{E}$ and $\vec{B}$ and two collective fields $\vec{e}$ and $\vec{b}$. Similarly $T_{(\mu\nu)}$ carries 2 spin-0, 1 spin-1 and 1 spin-2. However, dynamically, eq.(7.2) propagates just one granular spin-0 and other collective spin-0.

Defining:

$$F_{0i} = \vec{E}_{Ai}, \quad F_{ij} = \varepsilon_{ijk}\vec{B}_{Ak}$$

$$U_{0i} = \vec{E}_{Ui}, \quad U_{ij} = \varepsilon_{ijk}\vec{B}_{Uk}$$

$$V_{0i}^{\pm} = \vec{E}_{\pm i}, \quad V_{ij}^{\pm} = \varepsilon_{ijk}\vec{B}_{\pm k}$$

$$S_{00}^{1} = S_{A}^{1}, \quad S_{0i}^{1} = \vec{S}_{A}, \quad S_{ij}^{1} = \overset{\leftrightarrow}{S}_{A}$$

$$S_{00}^{2}=S_{U},\quad S_{0i}^{2}=\vec{S}_{U},\quad S_{ij}^{2}=\overset{\leftrightarrow}{S}_{U}$$

$$S_{00}^{\pm}=S_{\pm i},\quad S_{0i}^{\pm}=\vec{S}_{\pm},\quad S_{ij}^{\pm}=\overset{\leftrightarrow}{S}_{\pm}$$

$$z_{[0i]}=\vec{e}_{i},\quad z_{[ij]}=\varepsilon_{ijk}\vec{b}_{k}$$

$$z_{(00)}=s,\quad z_{(0i)}=\vec{s},\quad z_{(ij)}=\overset{\leftrightarrow}{s}$$

$$\omega_{[0i]}=\vec{e}_{\omega i},\quad \omega_{[ij]}=\varepsilon_{ijk}\vec{b}_{\omega k}$$

$$\omega_{(00)}=s_{\omega},\quad \omega_{(0i)}=\vec{s}_{\omega},\quad \omega_{(ij)}=\overset{\leftrightarrow}{s}_{\omega}$$

one derives from eqs.(7.1) the following Maxwell equations set:

9.1 Photon field- A_μ

For spin-1:

$$\vec{\nabla}\cdot(\vec{E}_{A}+\vec{e}_{A})+l_{A}^{T}=\rho_{A}^{T} \tag{9.1}$$

$$\vec{\nabla}\times(\vec{B}_{A}+\vec{b}_{A})-\frac{\partial}{\partial t}(\vec{E}_{A}+\vec{e}_{A})+\vec{l}_{A}^{T}=\vec{J}_{A}^{T} \tag{9.2}$$

$$\vec{\nabla}\cdot\vec{B}_{A}=0 \tag{9.3}$$

$$\vec{\nabla}\times\vec{E}_{A}=-\frac{\partial}{\partial t}\vec{B}_{A} \tag{9.4}$$

with the continuity equation:

$$\frac{\partial}{\partial t}(\rho_{A}^{T}-l_{A}^{T})+\vec{\nabla}\cdot(\vec{J}_{A}^{T}-\vec{l}_{A}^{T})=0 \tag{9.5}$$

For spin-0:

$$\frac{\partial}{\partial t}(S_{A\alpha}^{\alpha}+s_{A\alpha}^{\alpha})+l_{A}^{L}=\rho_{A}^{L} \tag{9.6}$$

$$-\vec{\nabla}(S_{A\alpha}^{\alpha}+s_{A\alpha}^{\alpha})+\vec{l}_{A}^{L}=\vec{J}_{A}^{L} \tag{9.7}$$

with the continuity equation:

$$\frac{\partial}{\partial t}(\rho_{A}^{L}-l_{A}^{L})+\vec{\nabla}\cdot(\vec{J}_{A}^{L}-\vec{l}_{A}^{L})=0 \tag{9.8}$$

9.2 Massive photon field- U_μ

For spin-1:

$$\vec{\nabla}\cdot(\vec{E}_{U}+\vec{e}_{U})+l_{U}^{T}+M_{U}^{T}=\rho_{U}^{T} \tag{9.9}$$

$$\vec{\nabla}\times(\vec{B}_U+\vec{b}_U)-\frac{\partial}{\partial t}(\vec{E}_U+\vec{e}_U)+\vec{l}_U^T+\vec{M}_U^T=\vec{J}_U^T \tag{9.10}$$

$$\vec{\nabla}\cdot\vec{B}_U=0 \tag{9.11}$$

$$\vec{\nabla}\times\vec{E}_U=-\frac{\partial}{\partial t}\vec{B}_U \tag{9.12}$$

with the continuity equation:

$$\frac{\partial}{\partial t}(\rho_U^T-l_U^T-M_U^T)+\vec{\nabla}\cdot(\vec{J}_U^T-\vec{l}_U^T-\vec{M}_U^T)=0 \tag{9.13}$$

For spin-0:

$$\frac{\partial}{\partial t}(S_{U\alpha}^{\alpha}+s_{U\alpha}^{\alpha})+l_U^L+M_U^L=\rho_U^L \tag{9.14}$$

$$-\vec{\nabla}(S_{U\alpha}^{\alpha}+s_{U\alpha}^{\alpha})+\vec{l}_U^L+\vec{M}_U^L=\vec{J}_U^L \tag{9.15}$$

with the continuity equation

$$\frac{\partial}{\partial t}(\rho_U^L-l_U^L-M_U^L)+\vec{\nabla}\cdot(\vec{J}_U^L-\vec{l}_U^L-\vec{M}_U^L)=0 \tag{9.16}$$

9.3 Positive charged photon field-V_μ^+

For spin-1:

$$\vec{\nabla}\cdot(\vec{E}_+ +\vec{e}_+)+l_+^T+M_+^T=\rho_+^T \tag{9.17}$$

$$\vec{\nabla}\times(\vec{B}_+ +\vec{b}_+)-\frac{\partial}{\partial t}(\vec{E}_+ +\vec{e}_+)+\vec{l}_+^T+\vec{M}_+^T=\vec{J}_+^T \tag{9.18}$$

$$\vec{\nabla}\cdot\vec{B}_+=0 \tag{9.19}$$

$$\vec{\nabla}\times\vec{E}_+=-\frac{\partial}{\partial t}\vec{B}_+ \tag{9.20}$$

with the continuity equation

$$\frac{\partial}{\partial t}(\rho_+^T-l_+^T-M_+^T)+\vec{\nabla}\cdot(\vec{J}_+^T-\vec{l}_+^T-\vec{M}_+^T)=0 \tag{9.21}$$

For spin-0:

$$\frac{\partial}{\partial t}(S_{+\alpha}^{\alpha}+s_{+\alpha}^{\alpha})+l_+^L+M_+^L=\rho_+^L \tag{9.22}$$

$$-\vec{\nabla}(S^{\alpha}_{+\alpha}+s^{\alpha}_{+\alpha})+\vec{l}^{L}_{+}+\vec{M}^{L}_{+}=\vec{J}^{L}_{+} \tag{9.23}$$

with the continuity equation

$$\frac{\partial}{\partial t}(\rho^{L}_{+}-l^{L}_{+}-M^{L}_{+})+\vec{\nabla}\cdot(\vec{J}^{L}_{+}-\vec{l}^{L}_{+}-\vec{M}^{L}_{+})=0 \tag{9.24}$$

9.4 Negative charged photon field-V^{-}_{μ}

For spin-1:

$$\vec{\nabla}\cdot(\vec{E}_{-}+\vec{e}_{-})+l^{T}_{-}+M^{T}_{-}=\rho^{T}_{-} \tag{9.25}$$

$$\vec{\nabla}\times(\vec{B}_{-}+\vec{b}_{+})-\frac{\partial}{\partial t}(\vec{E}_{-}+\vec{e}_{+})+\vec{l}^{T}_{-}+\vec{M}^{T}_{-}=\vec{J}^{T}_{-} \tag{9.26}$$

$$\vec{\nabla}\cdot\vec{B}_{-}=0 \tag{9.27}$$

$$\vec{\nabla}\times\vec{E}_{-}=-\frac{\partial}{\partial t}\vec{B}_{-} \tag{9.28}$$

with the continuity equation

$$\frac{\partial}{\partial t}(\rho^{T}_{-}-l^{T}_{-}-M^{T}_{-})+\vec{\nabla}\cdot(\vec{J}^{T}_{-}-\vec{l}^{T}_{-}-\vec{M}^{T}_{-})=0 \tag{9.29}$$

For spin-0:

$$\frac{\partial}{\partial t}(S^{\alpha}_{-\alpha}+s^{\alpha}_{-\alpha})+l^{L}_{-}+M^{L}_{-}=\rho^{L}_{-} \tag{9.30}$$

$$-\vec{\nabla}(S^{\alpha}_{-\alpha}+s^{\alpha}_{-\alpha})+\vec{l}^{L}_{-}+\vec{M}^{L}_{-}=\vec{J}^{L}_{-} \tag{9.31}$$

with the continuity equation

$$\frac{\partial}{\partial t}(\rho^{L}_{-}-l^{L}_{-}-M^{L}_{-})+\vec{\nabla}\cdot(\vec{J}^{L}_{-}-\vec{l}^{L}_{-}-\vec{M}^{L}_{-})=0 \tag{9.32}$$

Notice that the above equations contain term beyond Maxwell. They are the polarization and magnetization vectors, masses, London terms and nonlinear fields as own sources.

9.5 Collective Bianchi Equations in Vectorial Form

9.5.1 Spin-1

$$\vec{\nabla}\times\vec{e}+\frac{\partial\vec{b}}{\partial t}=\vec{j}_{c} \tag{9.33}$$

where

$$-\vec{j}_c = \gamma_{[12]}\{\phi_U \vec{B}_A - \phi \vec{B}_U + \vec{U}\times\vec{E}_A - \vec{A}\times\vec{E}_U\} + 2\gamma_{[34]}\mathrm{Im}\{\phi^- \vec{B}^+ + \vec{V}^- \times \vec{E}^+\}$$

$$\vec{\nabla}\cdot\vec{b} = \rho_c \qquad (9.34)$$

where

$$\rho_c = \gamma_{[12]}\{\vec{U}\cdot\vec{B}_A - \vec{A}\cdot\vec{B}_U\} + 2\gamma_{[34]}\mathrm{Im}\{\vec{V}^- \cdot \vec{B}^+\} \qquad (9.35)$$

which yields the continuity equation

$$\frac{\partial \rho_c}{\partial t} + \vec{\nabla}\cdot\vec{j}_c = 0 \qquad (9.36)$$

9.5.2 Spin-0

In scalar form

$$\frac{\partial s}{\partial t} = \gamma_{(11)}(\phi S_A) + \gamma_{(22)}(\phi_U S_U) + \gamma_{(12)}(\phi S_U + \phi_U S_A) + 2\gamma_{(33)}\mathrm{Re}(\phi_- S_+) \qquad (9.37)$$

In tensor form

$$\frac{\partial s_{ij}}{\partial t} + \partial_j \vec{s}_i + \partial_i \vec{s}_j = \gamma_{(11)}\{\phi S_{Aij} + \vec{A}_i \vec{S}_{Aj} + \vec{A}_j \vec{S}_{Ai}\} +$$
$$+\gamma_{(22)}\{\phi_U S_{Uij} + \vec{U}_i \vec{S}_{Uj} + \vec{U}_j \vec{S}_{Ui}\} + 2\gamma_{(33)}\mathrm{Re}\{\phi_- S_{+ij} + \vec{V}_i^- \vec{S}_{+j} + \vec{V}_j^- \vec{S}_{+i}\} +$$
$$+\gamma_{(12)}\{\phi S_{Uij} + \vec{A}_i \vec{S}_{Uj} + \vec{A}_j \vec{S}_{Ui} + \phi_U S_{Aij} + \vec{U}_i \vec{S}_{Aj} + \vec{U}_j \vec{S}_{Ai}\}$$

In a second tensor form

$$\partial_i s_{(jk)} + \partial_k s_{(ij)} + \partial_j s_{(ik)} = \gamma_{(11)}\{A_i S_{Ajk} + A_j S_{Aik} + A_k S_{Aij}\} +$$
$$+\gamma_{(22)}\{U_i S_{Ujk} + U_\nu S_{Uik} + U_k S_{Uij}\} + 2\gamma_{(33)}\mathrm{Re}\{V_i^- S_{+jk} + V_j^- S_{+ik} + V_k^- S_{+ij}\} +$$
$$+\gamma_{(12)}\{A_i S_{Ujk} + A_j S_{Uik} + A_j S_{Uij} + U_i S_{Ajk} + U_j S_{Aik} + U_k S_{Aij}\} \qquad (9.38)$$

10 Photon Whole Equation in Vectorial Form

For spin-1:

$$\vec{\nabla}\cdot(\vec{E}_w + \vec{e}_w) + M_w^T + l_w^T = \rho_w^T \qquad (10.1)$$

$$\vec{\nabla}\times(\vec{B}_w + \vec{b}_w) - \frac{\partial}{\partial t}(\vec{E}_w + \vec{e}_w) - + \vec{M}_w^T + \vec{l}_w^T = \vec{J}_w^T \qquad (10.2)$$

For spin-0:

$$\frac{\partial}{\partial t}(S_{w\alpha}^{\alpha}+s_{w\alpha}^{\alpha})+l_{w}^{L}=\rho_{w}^{L} \tag{10.3}$$

$$-\vec{\nabla}(S_{w\alpha}^{\alpha}+s_{w\alpha}^{\alpha})+\vec{M}_{w}^{L}+\vec{l}_{w}^{L}=\vec{J}_{w}^{L} \tag{10.4}$$

11 EM Waves

11.1 Field A^{μ}

Spin-1:

$$\mathsf{W}(F_{\nu\mu}^{A}+z_{[\nu\mu]}^{A})=\partial_{\mu}d_{\nu}^{A}-\partial_{\nu}d_{\mu}^{A}+\partial^{\alpha}f_{\alpha\mu\nu}^{A}$$

where

$$d_{\mu}^{A}=l_{\mu}^{AT}+c_{\mu}^{A}+j_{\mu}^{AT}\text{ and }f_{\alpha\mu\nu}^{A}=\bar{a}_{2}(f_{\alpha\mu\nu}^{[12]}+f_{\alpha\mu\nu}^{[+-]})+\bar{a}_{3}f_{\alpha\mu\nu}^{(+-)} \tag{11.1}$$

Spin-0:

$$\mathsf{W}(S_{\alpha A}^{\alpha}+z_{\alpha A}^{\alpha})=\partial.(j_{A}-l_{A}-c_{A})$$

11.2 Field U^{μ}

Spin-1:

$$\mathsf{W}(F_{\nu\mu}^{U}+z_{[\nu\mu]}^{U})=\partial_{\mu}d_{\nu}^{U}-\partial_{\nu}d_{\mu}^{U}+\partial_{\mu}M_{\nu}^{U}-\partial_{\nu}M_{\mu}^{U}+\partial^{\alpha}f_{\alpha\mu\nu}^{U}$$

where

$$d_{\mu}^{U}=l_{\mu}^{UT}+c_{\mu}^{U}+j_{\mu}^{UT}\text{ and }f_{\alpha\mu\nu}^{U}=\bar{b}_{2}(f_{\alpha\mu\nu}^{[12]}+f_{\alpha\mu\nu}^{[+-]})+\bar{b}_{3}f_{\alpha\mu\nu}^{(+-)} \tag{11.2}$$

Spin-0:

$$\mathsf{W}(S_{\alpha U}^{\alpha}+z_{\alpha U}^{\alpha})=\partial.(j_{U}-l_{U}-c_{U}) \tag{11.3}$$

11.3 Field $V^{+\mu}$

Spin-1:

$$\mathsf{W}(F_{\nu\mu}^{+}+z_{[\nu\mu]}^{+})=\partial_{\mu}d_{\nu}^{+}-\partial_{\nu}d_{\mu}^{+}+\partial_{\mu}M_{\nu}^{+}-\partial_{\nu}M_{\mu}^{+}+\partial^{\alpha}f_{\alpha\mu\nu}^{+}$$

where

$$d_{\mu}^{+}=l_{\mu}^{+T}+c_{\mu}^{+}+j_{\mu V^{+}}^{T}\text{ and }f_{\alpha\mu\nu}^{+}=\bar{c}_{2}(f_{\alpha\mu\nu}^{[-1]}+f_{\alpha\mu\nu}^{[-2]}) \tag{11.4}$$

Spin-0:

$$W(S^{\alpha}_{\alpha+} + z^{\alpha}_{\alpha+}) = \partial.(j_{+} - l_{+} - c_{+}) \quad (11.5)$$

11.4 Field $V^{-\mu}$

Spin-1:

$$W(F^{-}_{\nu\mu} + z^{-}_{[\nu\mu]}) = \partial_{\mu} d^{-}_{\nu} - \partial_{\nu} d^{-}_{\mu} + \partial_{\mu} M^{-}_{\nu} - \partial_{\nu} M^{-}_{\mu} + \partial^{\alpha} f^{-}_{\alpha\mu\nu}$$

$$d^{-}_{\mu} = l^{-T}_{\mu} + c^{-}_{\mu} + j^{T}_{\mu V^{-}} \text{ and } f^{-}_{\alpha\mu\nu} = \bar{c}^{*}_{2}(f^{[+1]}_{\alpha\mu\nu} + f^{[+2]}_{\alpha\mu\nu}) \quad (11.6)$$

Spin-0:

$$W(S^{\alpha}_{\alpha-} + z^{\alpha}_{\alpha-}) = \partial.(j_{-} - l_{-} - c_{-}) \quad (11.7)$$

At Apendice F the corresponding $f_{\alpha\mu\nu}$ expressions are defined in terms of fields.

12 Conclusion

Electromagnetism is being considered as the energy derived from electric charge. Up to now, it still follows the Hendrick Lorentz statement from 1895 at Philosophical Magazine: 'The electromagnetism axiom is that electromagnetic field is an excited state established in the space through the electric charge. Based on that vectors $\vec{E}$ and $\vec{B}$ build up an electromagnetic wave which is identified as light'. Nevertheless, non-Maxwellian models have been moving for an EM dependence beyond electric charge.

The effort of this work is the four bosons electromagnetism model [4]. At these days the challenge is to go beyond Maxwell. Although 150 years of development we still are under the interrogation: what is EM? There are 42 models trying to answer that[3]. There are three elements for the EM phenomena be understood. They are electric charge, EM fields and light. The challenge for any model is to explore on these elements differently from Maxwell.

Science is challenged to surpass Maxwell. There is an enlargement still to be understood through light invariance. The above Lorentz argument just follows a vision based at EM dependence on electric charge. It did not work on the meaning of light symmetry as primary feature of nature. In true most EM models preserve this idea. Section 3 review some nonlinear EM models where they are still based on this Lorentz interpretation. Although nonlinearity be a request they enlarge the Lagrangian but preserving the same Maxwell invariants and electric charge dependence. Eq.(2.1) think different. It argues that the EM testimony should be given from light symmetry. Instead of electric charge as origin, it provides the allowable laws.

Our viewpoint is that there is still a fourth interpretation for light symmetry. Maxwell equations had appeared as its first signal; relativity came after; Lorentz Group introduced the third interpretation. The next one, as fourth, comes from the LG representations. The meaning of quanta family is proposed. It says that, given the $\{\frac{1}{2},\frac{1}{2}\}$ representation, one can associate to it a fields set $\{A_{\mu I}\}$ where I is a flavor index $I = 1,...N$. A new context is opened. Naturalistically, it reads that given a certain LG representation there is the meaning of species of fields [27]. Philosophically, it interprets that the spin content moves from the reductionist to the antireductionist approach. Physically, instead of associating just one relativistic equation to each LG representation, it generates a whole system of relativistic equations. Dynamically, it derives the symmetry equation for the quanta specie, as eq.(4.44) shows, plus N-Euler-Lagrange equations of motion and N-Bianchi identities with respect to each field inserted in the LG representation. It also includes collective Bianchi identities. Phenomenologically, a whole quanta flavor diversity is implemented dynamically. As principle, an unity of diversity is revealed.

A next step is to define the number of flavors for the quanta specie $\{\frac{1}{2},\frac{1}{2}\}$. Given that, EM is the theory for electric charge, it should first be understood through its features for charge transmission like conservation law, conduction, conversion, interaction. Analyzing on the electric charge flux, one notices a qualitative difference between the Maxwell macroscopic approach and the elementary particles microscopic reactions. It is on charges transmutations. While the Maxwell current preserves the charges signal, the subatomic reactions exchange their signs. A new interpretation must be given at microscopic level. The EM must be understood as a phenomena based on three charges positive,negative, neutral interchanging their values. Consequently, an EM extension is requested for these charges exchanging. It is a gauge theory transmitting the electric charge through four intermediate bosons. They will be porters for any electromagnetic process transmitting the physicity $\Delta Q = 0,\pm 1$. As result, there is an EM to be explored through a fields set $\{A_{\mu}, U_{\mu}, V_{\mu}^{\pm}\}$.

A third stage, coming from this fourth light symmetry interpretation, is on which symmetry such fields set should be arranged. The answer is to go beyond Maxwell but preserving the Maxwell abelian symmetry. This makes this work to propose an enlarged $\mathrm{U(1)XSO(2)}$ symmetry, as eq.(2.1). It ensures the principle of electric charge conservation.

A four bosons electromagnetic model is performed. It is based on three postulates:

1. Light invariance symmetry introduces $\{A_{\mu I}\}$;
2. Electric charge transmission establishes $I = 4: \{A_{\mu}, U_{\mu}, V_{\mu}^{\pm}\}$;
3. Electric charge conservation requires an abelian symmetry: $U(1)XSO(2)$.

Thus preserving light invariance and electric charge conservation principles the above postulates redefine what EM is. They say that electromagnetism is more than charges and fields being transmitted by photons. Essentially it is a phenomena derived from light symmetry and electric charge flux. Under this perspective, eq.(2.2) cross Maxwell frontier and moves from Maxwell minimal case (two charges, one messenger) to an extended case (three charges, four messengers). It yields a Lagrangian where, associated through a common gauge parameter the fields set $\{A_{\mu}, U_{\mu}, V_{\mu}^{\pm}\}$ develops a systemic EM with a new structure beyond Maxwell. We should emphasis on their consequences now.

Overall at this conclusion, we would like to highlight on this four bosons electromagnetism. Analysis on new features, scenarios, experiments for electric charge, EM fields and light. First, it introduces three charges physicity due to the fact that they transform into themselves. Second, the photon lives with partners. Besides the usual photon, massive and charged photons are expected, including a spin-0 family. Third, eight gauge bosons become responsible for to the electric charge flux being intermediated at elementary particles reactions. It yields an EM enlargement to be prospected that does not depend on new accelerators. A fact just for observation. It says that, given that the physical processes related to electric charge flux do not require necessarily be associated to fundamental particles, various particles are viable as porters of positive, negative and neutral charges. For instance, J/Ψ can be interpreted as charged spin-1 photon, and similarly, different particles as pions can be accommodated at photon spin-0 family.

The three EM elements are improved as:

A new electric charge physics appears from three charges coming up among together. EM has been discovered through electric charges properties, as Gauss and Biot-Savart laws, where electric charge is the origin and acts as positive and negative. The qualitative difference is that the four bosons model redefines the electric charge performance by introducing the neutral charge. The EM is enlarged from a dyad to a tryad electric charge phenomena. Eq.(2.2) understands that while Maxwell is focused on charge distribution, it is on charge transmission. Differently from Maxwell, the third charge becomes determinant on the transmission processes. It yields new possibilities for electric charge interactions through eq.(2.9), conservation law eq.(2.24), and conduction as at section 4 and 5.

A second EM element is on fields behavior. A new EM fields physics is developed with granular and collective fields as eqs.(2.6),(2.10),(2.11) are showing. They do not require external sources in order to establish their own EM fields. They are nonlinear fields and with news induction laws. A new connectivity between EM fields is produced. While Maxwell four equations have unified electricity and magnetism where Faraday and Ampère law show that EM fields varying in time transform on each other, section 9 shows different possibilities where electric collective fields transform into magnetic collective fields, nonlinear electric fields generate electric fields, and so on. Even without the electric charge presence they are generated and transformed between themselves. They produce a nonlinear interconnected whole.

The third element is light. Although Maxwell has been considered as standard theory there is a new EM sector to be investigated which is the photonics. It is derived from an active photon. Recapitulating, due to light symmetry one should introduce light in the beginning, but in practice Maxwell equations do not do that. The four bosons electromagnetism reinterprets light as an origin. Light is more than just an electric charge consequence, it is a cause for the physical processes. Five features are derived. The photon becomes an absolute from Michelson-Morley experiment, ubiquous through an universality where its coupling constant do not depend on electric charge, directive on the fields set behavior, contingent on physical entities as space-time relativism and particles properties depending on circumstances, selfinteracting as source for particles processes. New phenomenological properties are expected from the photon such as photonic current, selfinteracting photons, nonlinear EM waves, new dispersion relation, forces depending on photon field[5].

Three scenarios are opened:

The first scenario is on three charges exchange. A panorama where EM is no more a physics established just by opposite charges as the 19^{th} century understood. EM energy becomes operated by three charges. Differently from Maxwell and QED there are three charges transmissions working as a whole. The exchange $\Delta Q = 0,\pm 1$ three charges (+,-,0) associated to four interconnected gauge bosons. In these realistic scenario since elementary particles reactions at 1930 decade. Theoretically it is supported from the passage from Quantum Mechanics to Quantum Field Theory where particles transform between themselves. A concept from where in the thirties physics was ready accept the electric charge transmutation. This three charges exchange is a fact with implications on subjects as atomic physics, nuclear physics, neutrino physics and at large scale at LHC.

The second scenario is on nonlinear set dynamics. The three charges exchanges require four bosons messengers. A set performance appears. It yields a whole dynamics, a coupled nonlinear granular and collective dynamics with spin-1 and spin-0 families. A network based on a set determinism with directive and chance, granular and collective variables, nonlinearity, coupling constants diverse from electric charge, and a new range for EM interaction.

The third scenario is on the photon as a particle producer. Photons producing their own EM fields and phenomenology are predating the Maxwell world. They are opening a light electromagnetism with a physicity beyond the electric charge. Deriving the Feynman rules for eq.(2.2), one notices that the Maxwell passive photon becomes active. It appears three and four photons interactions between themselves and with another particles with spin-1 and spin-0 without being necessarily coupled to electric charge[7]. Consequently from radiation field it becomes a particles emitter. Relationships as gA_{μ}, selfinteracting photons and photonic charge are supporting the particles productions. Different implications on photon interaction, conduction and transmission are obtained. Besides the usual photon interaction with charged particles there is the photon interaction without charge. For instance, photon clouds not depending on electric charge are viable.

A third analysis for this new EM concerns on their experimental evidences. Preserving the Maxwell symmetry the four bosons electromagnetism upsides down Maxwell equations. It proposes a Lagrangian from where one derives the polarization and magnetization vectors and a nonlinear EM; new induction laws; dualism individual-collective; renormalizable and unitary model; EM beyond electric charge; photonics. It enlarges the three basic EM elements, opens three EM scenarios and will propose new experiments. They are expected through electric charge, nonlinearity and light. Their measurements will test on the veracities that this model contain in order to enlarge Maxwell.

Three experimental tests are proposed:

A first one is on electric charge exchange through four bosons. New measurements for electric charge are expected beyond Coulomb balance or neutral charge dipole[28]. They are charge conservation beyond granular charged fields, charge conduction through neutral currents, and range of interactions. It also proposes effects on the standard value $e = 10^{-19}C$ due to the presence of electric charge running coupling constant with a dependence on other coupling constants, selfinteracting photons and so on. There is an electric charge dependence on the four bosons whole variables to be studied through the renormalization group equations.

A second test of this model is nonlinearity. Deviations from Maxwell theory have been sought in terrestrial laboratories for a long time. Among these many Euler-Heisenberg theory predictions like the Schwinger effect, vacuum birefringence under a strong magnetic field, photon splitting on lasers. Many consequences have already been studied as light lensing due to the optical property of the vacuum in the presence of a magnetic field [39], polarization phase lag [40], how quantum vacuum friction influences spindown [41].

Nevertheless, for present literature, at the macroscopic level the superposition principle is widely proven with a 0,1 percent precision (EM field generated by a group of charges and currents, transformers, standing waves, diffraction patterns in optic X-ray crystallography, refraction,...). Problems might arise at atomic or nucleon level. Considering charged particles as a localized distribution of charge, then, it is obvious from Maxwell's theory that the field strength and, hence, electromagnetic energy increase as the charge is more and more localized. For atoms, linearity has been tested up to 10^3 -10^5 V/cm fields, the ones one can find in electron orbits. Energy level distances in light atoms allow us to say that the experimental data are in agreement with superposition up to 1 part in 10^6. In nuclei field strength of the order of 10^{19} V/cm can be reached. In this regime Coulomb energies of heavy nuclei have tested and found in agreement with linearity up to 1 part in 10^6. Although at first glance, at classical level, there are no great motivations for nonlinearity be introduced, there are many evidences of the derivation from linearity coming from condensate matter and astrophysics.

The next step is to be understood through classical nonlinearities. Despite of the constraining limits, efforts have been done to create a non-linear classical theory. The first and more relevant try was that of Born and Infield in 1934. It allows the principle of finiteness where a physical theory must imply only finite measurable quantities. The cutting fields $E_c = \frac{m_e^2 c^3}{eh} = 10^{17} N/C$ and $B_c = \frac{m_e^2 c^2}{e\hbar} = 10^9 T$ established by Euler-Heisenberg are bringing nonlinearity shifts for Maxwell equation. Atomic nucleons, neutron stars and magnet-stars reserve a room for nonlinear electromagnetism required by strong fields and high frequencies. Nonlinear atomic spectral lines were analyzed by Denisov at al [26]. From astrophysics, magnetars are stars endowed with an overcritical field estimated $10-10^2 B_c$ at the surface. Similarly the high-ray collisions for frequencies.

There are several evidences of the deviation from linearity in the quantum regime. The Standard Model shows a nonlinear photon. A first test should be on light-light scattering. Actually this phenomena is treated through the uncertainty principle which allows the momentary creation of electron-positron couple and subsequent annihilation with creation of two photons.

A third experimental test is on light new features. Experiments where light predates electric charge will redefine our thought about what EM is. Nonlinear photons and gA_μ are predicted. These properties reveals a nonlinear photon as particles producer. They are transposing the passive photon at Maxwell equations to a creative photon. From radiation field to photon big bang. Particles creation not only as e^+e^- but to various compositions depending on energy range. Therefore for accepting a possible photon big bang we should first investigate how the nonlinearity of theory affects the photon propagation by calculating the red or blue shifts due to this.

Light electromagnetism turns a candidate to be prospected. Historically, since 1934 physics is bringing an investigation beyond the photon derived from Maxwell equations. First, Euler-Heisenberg studied the virtual vacuum polarization, $\gamma \to e^+e^- \to \gamma$ [11]; in the same year Breit and Wheeler calculated on two light quanta collision, $\gamma\gamma \to e^+e^-$ [42]; also in 1934 Delbrück studied a pure quanta effect through the reaction $\gamma\gamma \to \gamma\gamma$; in the sixties SLAC measured the reaction $e^+e^- \to \gamma\gamma$; and in 1971 Adler studied the $\gamma\gamma$ collision [43]. Nevertheless all this context was performed under QED approach. Although selfinteractions photons simplify all these processes with a tree level contribution they were not considered. Something is missing. The suggestion here is to understand these processes under gA_μ and selfinteracting photons. Eq.(2.2) advocates a photon phenomenology based on nonlinearity and with interactions not depending on electric charge. A challenge to be understood through the Photon Collider being prepared in Germany for 2018, the PVLAS' Padova experiment with laser, LSW German experiment. They should work as test for selfinteracting photons and for a photon behavior beyond electric charge.

The next question is what would happen when the photon is running at high energies as LHC 14 TeV. The question is: which new photon performance is expected? While Maxwell horizon was eV and keV and produced a linear light with a strict dependence on electric charge something different is expected at MeV and so on range. New light behavior is expected with the increasing of the energy[45]. It propitiates reactions as $\gamma\gamma \to e^+e^-$ which will be able to inform on selfinteracting photons. At GeV, we expect light interact with neutral particles as Z_0; and at TeV, we will start to understand light as a source for producing particles. Our expectation is on the existence of neutral photons realized by gA_μ and selfinteracting. 'Neutral photons' can be responsible for photon balls, photon moment magnetic anomalous, photon-neutrino interaction, dark matter without electric charge presence. Another investigation is on the photon behavior at high frequencies. Given a spectrometer study the relationships between frequency and wavelength. Laboratories at present days have being able to produce photons at high frequencies. From Maxwell regions, visible at 10^{15} Hz (eV) and X-rays at 10^{18} Hz (keV), one moves to γ-rays at MeV region with 10^{21} Hz, and for accelerators at GeV region (Standard Model) and TeV region (LHC). The question is, the dispersion relations will remain the same?

Finally, we would select three consequences for philosophy of science. Based on the set principle which says that nature works as a group, physics moves from the reductionist to the antireductionist approach. Under this new context, it introduces three topics. They are the evolution physics, a new unification approach and the primordial photon.

The whole quantum field theories changes the meaning of parts. Quanta depending on the whole brings a new perspective to be understood. It changes our actual common sense of elementary particle. The meaning of elementary particles (parts) moves from building blocks into parts inserted in the whole context. Their physicity is no more on their properties but in which grouping they are working. Their quantum numbers will be functions of the whole structure being described by the corresponding Lagrangian. They will be depending on set, diversity and chance.

On diversity, Lorentz fields families and gauge symmetry provides a dynamics that differentiates particles. It generates a flavor diversity on masses, charges and spin. At this work we have studied on $\{\frac{1}{2},\frac{1}{2}\}$ representation. Future papers will study on $\{\Psi_{LI}\}$, $\{\Psi_{RI}\}$ and $\{h_{\mu\nu I}\}$ families through $\{(0,\frac{1}{2})+(\frac{1}{2},0)\}$ and $\{1,1\}$ representations. These families will introduce diversity as a first principle. They are saying that instead of searching for fundamental particles nature should be understood based on varieties. It provides the meaning of unity diversity. It is the paradigm for complexity.

On chance, there is a new relationship between absolute and contingency for physics be realized. The absolute coming from light invariance derives not only the relativism studied by Einstein relativity; it also introduces the meaning of chance in the physical variables. Physical processes being analyzed under relativism and chance are two basic consequences derived from light symmetry. Free coefficients in eq.(2.2) can take any value without breaking gauge symmetry. They introduce a volume of circumstances in theory[5]. Equations with chance go beyond the Quantum Mechanics probability and uncertainly. It introduces the meaning of opportunity in nature. It says that nature can be created and organized.

Set, diversity and chance basis the meaning of evolution for physics. A relationship to be explored through a whole quanta field theory where the parts do not follow more the reductionist determinism. The new origin is the whole environment It brings a possible comparison to be done between the Lorentz group quanta tree and the Darwin tree of life. Although under different perspectives, one related to spin content and other to species content, they share a common horizon where the environment precedes the parts. So, while Darwin introduces a branch where mankind and monkeys types are inserted together, eq.(2.2) proposes another branch with spin-1 and spin-0 families. Both express a dynamics of evolution through the unity of diversity freedom.

The second topic is about unification. The standard viewpoint is that charge exchange physics is related through the usual unification between electromagnetism and weak interaction. Nevertheless there is a more primitive mechanism to be understood for modeling the physical processes which is based on three charges and four gauge bosons. It provides a charge exchange mechanism which also shows how the neutron could be transformed into proton, but with a new understanding on electric charge, EM fields and light.

The third topic is about primordial photon. A recent tridimensional Via Lactea map from ESA (2016) catalogs 1.142 billions of stars. It shows the Universe based on an industry of stars. So the question is whether the light being produced by these stars is Maxwell like or there is a primordial light to be understood. This work understand that a creative light is still a challenge for physical theories.

13 Interaction Lagrangian on Potential Fields

13.1 L_3

Expressing eq.(2.12) in terms of fields, one gets

$$L_3 = \alpha_{111}(\partial_\mu A_\nu)A^\mu A^\nu + \alpha_{112}(\partial_\mu A_\nu)A^\mu U^\nu + \alpha_{121}(\partial_\mu A_\nu)U^\mu A^\nu + \alpha_{122}(\partial_\mu A_\nu)U^\mu U^\nu +$$

$$+\alpha_{211}(\partial_\mu U_\nu)A^\mu A^\nu + \alpha_{212}(\partial_\mu U_\nu)A^\mu U^\nu + \alpha_{221}(\partial_\mu U_\nu)U^\mu A^\nu + \alpha_{222}(\partial_\mu U_\nu)U^\mu U^\nu +$$

$$+(\alpha_{331}A^\nu + \alpha_{332}U^\nu)[(\partial_\mu V_\nu^+)V^{\nu-} + (\partial_\mu V_\nu^-)V^{\nu+}] +$$

$$-i(\alpha_{341}A^\nu + \alpha_{342}U^\nu)[(\partial_\mu V_\nu^+)V^{\nu-} - (\partial_\mu V_\nu^-)V^{\nu+}] +$$

$$+(\alpha_{313}A^\mu + \alpha_{323}U^\mu)[(\partial_\mu V_\nu^+)V^{\nu-} + (\partial_\mu V_\nu^-)V^{\nu+}] +$$

$$-i(\alpha_{314}A^\mu + \alpha_{324}U^\mu)[(\partial_\mu V_\nu^+)V^{\nu-} - (\partial_\mu V_\nu^-)V^{\nu+}] +$$

$$+[\alpha_{133}(\partial_\mu A_\nu) + \alpha_{233}(\partial_\mu U_\nu)][V^{\mu+}V^{\nu-} + V^{\mu-}V^{\nu+}] +$$

$$-i[\alpha_{134}(\partial_\mu A_\nu) + \alpha_{234}(\partial_\mu U_\nu)][V^{\mu+}V^{\nu-} - V^{\mu-}V^{\nu+}]$$

$$+\beta_{1(11)}(\partial_\mu A^\mu)A_\nu A^\nu + 2\beta_{1(12)}(\partial_\mu A^\mu)A_\nu U^\nu +$$

$$+\beta_{1(22)}(\partial_\mu A^\mu)U_\nu U^\nu + \beta_{2(11)}(\partial_\mu U^\mu)A_\nu A^\nu +$$

$$+2\beta_{2(12)}(\partial_\mu U^\mu)A_\nu U^\nu + \beta_{2(22)}(\partial_\mu U^\mu)U_\nu U^\nu +$$

$$+2(\beta_{3(13)}A^\nu + \beta_{3(23)}U^\nu)[(\partial_\mu V^{\mu+})V_\nu^- + (\partial_\mu V^{\mu-})V_\nu^+] +$$

$$-2i(\beta_{3(14)}A^\nu + \beta_{3(24)}U^\nu)[(\partial_\mu V^{\mu+})V_\nu^- + (\partial_\mu V^{\mu-})V_\nu^+] +$$

$$+2[\beta_{1(33)}(\partial_\mu A^\mu) + \beta_{2(33)}(\partial_\mu U^\mu)]V_\nu^+ V_\nu^-$$

where coefficients

$$\alpha_{IJK} = 2a_{I(JK)} + 2b_{I(JK)}$$

$$\alpha_{IJK} = 2c_{I(JK)} \tag{13.1}$$

are derived from the original $U(1)$-Lagrangian

$$L_3 = a_{I[JK]} G^I_{\mu\nu} G^{\mu J} G^{\nu K} + b_{I(JK)} S^I_{\mu\nu} G^{\mu J} G^{\nu K} + c_{I(JK)} S^{\mu I}_{\mu} G^{\mu J} G^{\nu K}$$

with

$$G^I_{\mu\nu} = \partial_\mu G^I_\nu - \partial_\nu G^I_\mu \, \text{and} \, S^I_{\mu\nu} = \partial_\mu G^I_\nu + \partial_\nu G^I_\mu \tag{13.2}$$

where $G^1_\mu \equiv A_\mu; \quad G^2_\mu \equiv U_\mu; \quad G^3_\mu \equiv \frac{1}{\sqrt{2}}(V^+_\mu + V^-_\mu); \quad G^4_\mu \equiv \frac{i}{\sqrt{2}}(V^+_\mu - V^-_\mu)$

13.2 L_4

Rewriting eq.(2.15),

$$\begin{aligned}
L_4 = &\{\gamma_{(11)}\gamma_{(11)} + 2\gamma_{(11)}\tau_{(11)} + \tau_{(11)}\tau_{(11)}\} A_\mu A_\nu A^\mu A^\nu + \\
&+\{\gamma_{(22)}\gamma_{(22)} + 2\gamma_{(22)}\tau_{(22)} + 4\tau_{(22)}\tau_{(22)}\} U_\mu U_\nu U^\mu U^\nu + \\
&+4\{\gamma_{(11)}\gamma_{(12)} + 2\gamma_{(11)}\tau_{(12)} + 4\tau_{(11)}\tau_{(12)}\} A_\mu A_\nu A^\mu U^\nu + \\
&+4\{\gamma_{(12)}\gamma_{(22)} + 2\gamma_{(12)}\tau_{(22)} + 4\tau_{(12)}\tau_{(22)}\} A_\mu U_\nu U^\mu U^\nu + \\
&+2\{\gamma_{(11)}\gamma_{(22)} + 2\gamma_{(12)}\tau_{(12)} + 4\tau_{(12)}\tau_{(12)}\} A_\mu A_\nu U^\mu U^\nu + \\
&+2\{\gamma_{[12]}\gamma_{[12]} + \gamma_{(12)}\gamma_{(12)} + 2\gamma_{(11)}\tau_{(22)} + 4\tau_{(11)}\tau_{(22)}\} A_\mu U_\nu A^\mu U^\nu + \\
&+2\{\gamma_{[12]}\gamma_{[21]} + \gamma_{(12)}\gamma_{(21)} + 2\gamma_{(11)}\tau_{(22)} + 4\tau_{(11)}\tau_{(22)}\} A_\mu U_\nu U^\mu A^\nu + \\
&+2\{\gamma_{(11)}\gamma_{(33)} + 2\gamma_{(13)}\tau_{(13)} + 4\tau_{(13)}\tau_{(13)}\} A_\mu A_\nu (V^{\mu+}V^{\nu-} + V^{\mu-}V^{\nu+}) + \\
&+2\{\gamma_{[13]}\gamma_{[31]} + \gamma_{(13)}\gamma_{(31)} + 2\gamma_{(13)}\tau_{(31)} + 4\tau_{(13)}\tau_{(31)}\} A_\mu A_\nu (V^{\mu+}V^{\nu-} + V^{\mu-}V^{\nu+}) + \\
&+4\{\gamma_{[13]}\gamma_{[13]} + \gamma_{(13)}\gamma_{(13)} + 2\gamma_{(11)}\tau_{(33)} + 4\tau_{(11)}\tau_{(33)}\} A_\mu A^\mu V^+_\nu V^{\nu-} + \\
&+2\{\gamma_{(22)}\gamma_{(33)} + 2\gamma_{(23)}\tau_{(23)} + 4\tau_{(23)}\tau_{(23)}\} U_\mu U_\nu (V^{\mu+}V^{\nu-} + V^{\mu-}V^{\nu+}) + \\
&+2\{\gamma_{[23]}\gamma_{[32]} + \gamma_{(23)}\gamma_{(32)} + 2\gamma_{(23)}\tau_{(32)} + 4\tau_{(23)}\tau_{(32)}\} U_\mu U_\nu (V^{\mu+}V^{\nu-} + V^{\mu-}V^{\nu+}) + \\
&+4\{\gamma_{[23]}\gamma_{[32]} + \gamma_{(23)}\gamma_{(23)} + 2\gamma_{(22)}\tau_{(33)} + 4\tau_{(22)}\tau_{(33)}\} U_\mu U^\mu V^+_\nu V^{\nu-} + \\
&+4\{\gamma_{(12)}\gamma_{(33)} + 2\gamma_{(13)}\tau_{(23)} + 4\tau_{(13)}\tau_{(23)}\} A_\mu U^\mu (V^{\mu+}V^{\nu-} + V^{\mu-}V^{\nu+}) + \\
&+8\{\gamma_{[13]}\gamma_{[23]} + \gamma_{(13)}\gamma_{(23)} + 2\gamma_{(12)}\tau_{(33)} + 4\tau_{(12)}\tau_{(33)}\} A_\mu U^\mu V^+_\nu V^{\nu-} \\
&-4i\{\gamma_{[12]}\gamma_{[34]} + \gamma_{(12)}\gamma_{(34)} + 2\gamma_{(13)}\tau_{(24)} + 4\tau_{(13)}\tau_{(24)}\} A_\mu U_\nu (V^{\mu+}V^{\nu-} - V^{\mu-}V^{\nu+}) + \\
&+4i\{\gamma_{[13]}\gamma_{[42]} + \gamma_{(13)}\gamma_{(42)} + 2\gamma_{(14)}\tau_{(32)} + 4\tau_{(14)}\tau_{(32)}\} A_\mu U^\nu (V^{\mu+}V^{\nu-} - V^{\mu-}V^{\nu+}) +
\end{aligned}$$

$$+\{\gamma_{(33)}\gamma_{(24)}+2\gamma_{(34)}\tau_{(34)}+4\tau_{(34)}\tau_{(34)}\}(V^{\mu+}V^{\nu-}+V^{\mu-}V^{\nu+})(V^{\mu+}V^{\nu-}+V^{\mu-}V^{\nu+})+$$

$$+4\{\gamma_{[34]}\gamma_{[34]}+\gamma_{(34)}\gamma_{(34)}+2\gamma_{(33)}\tau_{(44)}+4\tau_{(33)}\tau_{(44)}\}(V_{\mu}^{+}V^{\mu-})^{2}$$

14 Equations of motion free coefficients

The equations of motion develop the following parameters written in terms of the original free Lagrangian coefficients, eq(2.2):

14.1 Field A_μ

14.1.1 Left Side

$$\bar{a}_1=4(a_1+b_{(11)}),\quad \bar{a}_2=8b_1,\quad \bar{a}_3=8\beta_1,$$

$$\bar{a}_4=4(b_{(11)}+c_{(11)})+\xi_{(11)}$$

$$\bar{a}_5=4c_{(12)}+\xi_{(22)}$$

$$\bar{a}_6=2(2\rho_1-\beta_1),\quad \bar{a}_7=4(\beta_1+4\rho_1) \tag{14.1}$$

14.1.2 Field A_μ Right Side : Field A_μ

$$f_1^A=4\gamma_{(11)}\beta_2-4\beta_1\gamma_{(12)}\quad f_2^A=4\gamma_{(11)}\quad f_3^A=2f_3^A,$$

$$f_4^A=8(\gamma_{(11)}+2\tau_{(11)}),\quad f_5^A=2f_4,$$

$$f_6^A=2\gamma_{(11)}(2\rho_1-\beta_1)+8\tau_{(11)}(\beta_1+4\rho_1),$$

$$f_7^A=4\gamma_{(11)}\rho_2-2\beta_1\gamma_{(12)}+8\tau_{(11)}(\beta_2+4\rho_2)$$

14.1.3 Field A_μ Right Side : Field U_μ

$$f_1^U=-4\gamma_{[12]}b_1,\quad f_2^U=-4\gamma_{[12]}b_1,\quad f_3^U=-8\gamma_{[12]},$$

$$f_4^U=16(\gamma_{(12)}+2\tau_{(12)}),\quad f_5^U=-4\gamma_{(12)}\beta_2-4\beta_1\gamma_{(22)},$$

$$f_6^U=4(\gamma_{(12)}+2\tau_{(12)}),\quad f_7^U=8\gamma_{(12)},\quad f_8^U=4\gamma_{(12)},$$

$$f_9^U=16\tau_{(12)},\quad f_{10}^U=8(\gamma_{(12)}+2\tau_{(12)}),$$

$$f_{11}^U=4\gamma_{(12)}\rho_1-2\beta_1\gamma_{(12)}+8\tau_{(12)}(\beta_1+4\rho_1),$$

$$f_{12}^U=4\gamma_{(12)}\rho_2-2\beta_1\gamma_{(22)}+8\tau_{(12)}(\beta_2+4\rho_2)$$

$$f_{13}^U=4(\gamma_{(12)}+2\tau_{(12)}) \tag{14.2}$$

14.1.4 Field A_μ Right Side : Fields V_μ^+ and V_μ^-

$$f_1^+ = -8b_3(\gamma_{[13]} + i\gamma_{[14]}), \quad f_2^+ = -4\gamma_{[13]}, \quad f_3^+ = -8\gamma_{[13]} - 2i\Delta\gamma_{[14]}$$

$$f_4^+ = 8\beta_3(\gamma_{(13)} + i\gamma_{(14)}), \quad f_5^+ = -(f_3^+)^*, \quad f_6^+ = 2i\gamma_{[13]}$$

$$f_7^+ = 4\gamma_{[13]}, \quad f_8^+ = -8\gamma_{(13)} + 2i(\gamma_{(14)} + 2\tau_{(14)}), \quad f_9^+ = -(f_8^+)^*$$

$$f_{10}^+ = 2i(\gamma_{(13)} + 2\tau_{(13)}), \quad f_{11}^+ = 4i(\gamma_{(14)} + 4\tau_{(14)}), \quad f_{12}^+ = 4i(\gamma_{(13)} + 4\tau_{(13)}),$$

$$f_{13}^+ = 8\rho_3(\gamma_{(13)} + I\gamma_{(14)}) + 8(\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)}) - 2\beta_1\gamma_{(33)}$$

$$f_{14}^+ = -f_2^+, \quad f_{15}^+ = 16(\gamma_{(13)} + 2\tau_{(13)})$$

14.2 Field U_μ

14.2.1 Left Side

$$\bar{b}_1 = 4(a_2 + b_{(22)}), \quad \bar{b}_2 = 8b_2, \quad \bar{b}_3 = 8\beta_2,$$

$$\bar{b}_4 = 4c_{(12)} + \xi_{(12)}$$

$$\bar{b}_5 = 4(b_{(22)} + c_{(22)}) + \xi_{(22)}$$

$$\bar{b}_6 = 2(2\rho_2 - \beta_2), \quad \bar{b}_7 = 4(\beta_2 + 4\rho_2)$$

14.2.2 Field U_μ Right Side : Field A_μ

$$g_1^A = 4\gamma_{[12]}b_1, \quad g_2^A = 4\gamma_{[12]}b_2, \quad g_3^A = 8\gamma_{[12]}, \quad g_4^A = 16(\gamma_{(12)} + \tau_{(12)})$$

$$g_5^A = 4(\beta_1\gamma_{(12)} - \beta_2\gamma_{(11)}), \quad g_6^A = 4\gamma_{(12)}, \quad g_7^A = 8\gamma_{(12)} \quad g_8^A = 4(\gamma_{(12)} + 2\tau_{(12)}),$$

$$g_9^A = 8(\gamma_{(12)} + 2\tau_{(12)}), \quad g_{10}^A = 16\tau_{(12)}, \quad g_{11}^A = 4(\rho_1\gamma_{(12)} + 2\tau_{(12)}(\beta_1 + 4\rho_1)) - 2\beta_2\gamma_{(11)}$$

$$g_{12}^A = 4(\rho_2\gamma_{(12)} + 2\tau_{(12)}(\beta_2 + 4\rho_2)) - 2\beta_2\gamma_{(12)}$$

14.2.3 Field U_μ Right Side : Field U_μ

$$g_1^U = 4(\gamma_{(22)}\beta_1 - \gamma_{(12)}\beta_2), \quad g_2^U = 4\gamma_{(22)}, \quad g_3^U = 8(\gamma_{(22)} + 2\tau_{(22)})$$

$$g_4^U = 4(\gamma_{(22)}\rho_1 + 2\tau_{(22)}(\beta_1 + 4\rho_1)) - 2\beta_2\gamma_{(12)}$$

$$g_5^U = 4(\gamma_{(22)}\rho_1 + 2\tau_{(22)}(\beta_2 + 4\rho_2)) - 2\beta_2\gamma_{(22)}$$

14.2.4 Field U_μ Right Side : Fields V_μ^+ and V_μ^-

$$g_1^+ = -8b_3(\gamma_{[23]} + i\gamma_{[24]}), \quad g_2^+ = 2i\gamma_{[24]}, \quad g_3^+ = 2i\gamma_{[23]}$$

$$g_4^+ = 4\gamma_{[23]}, \quad g_5^+ = 8\beta_3(\gamma_{(23)} + i\gamma_{(24)}) - 4\beta_2\gamma_{(33)}, \quad g_6^+ = 2i(\gamma_{(24)} + 2\tau_{(24)})$$

$$g_7^+ = -2i(\gamma_{(23)} - 2\tau_{(23)}), \quad g_8^+ = 4\gamma_{(23)}$$

$$g_9^+ = -4i(\gamma_{(24)} + 2\tau_{(24)}), \quad g_{10}^+ = 4i(\gamma_{(23)} + 4\tau_{(23)})$$

$$g_{11}^+ = 8\rho_3(\gamma_{(23)} + i\gamma_{(24)}) + 8(\beta_3 + 4\rho_3)(\tau_{(23)} + i\tau_{(24)}) - 2\beta_2\gamma_{(33)}$$

$$g_{12}^+ = 16(\gamma_{(23)} + 2\tau_{(23)}) \tag{14.3}$$

14.3 Field V_μ^+

14.3.1 Left Side

$$\bar{c}_1 = 4(a_3 + b_{(33)}), \quad \bar{c}_2 = 8b_3, \quad \bar{c}_3 = 4(b_{(33)} + c_{(33)}) + \frac{1}{2}\xi_{(33)}$$

$$\bar{c}_4 = 4(2\rho_3 - \beta_3), \quad \bar{c}_5 = 8(\beta_3 + 4\rho_3)$$

14.3.2 Field V_μ^+ Right Side : Field A_μ

$$h_1^A = 4b_3(\gamma_{[13]} + i\gamma_{[14]}), \quad h_2^A = 4\gamma_{[13]}, \quad h_3^A = 8\gamma_{[13]} + 2i\gamma_{[14]},$$

$$h_4^A = 8\gamma_{(13)} - 2i(\gamma_{(14)} - 2\tau_{(14)}), \quad h_5^A = 2i\gamma_{[13]}$$

$$h_6^A = 8\beta_3 i\gamma_{(14)}, \quad h_7^A = 8\gamma_{[13]} - 2i\gamma_{[14]}$$

$$h_8^A = 4\gamma_{(13)}, \quad h_9^A = 2i(\gamma_{(13)} - 2\tau_{(13)})$$

$$h_{10}^A = -4i(\gamma_{(14)} + 2\tau_{(14)}), \quad h_{11}^A = 4i(\gamma_{(13)} + 4\tau_{(13)})$$

$$h_{12}^A = 4(\rho_3(\gamma_{(13)} + i\gamma_{(14)}) + (\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)})) - 2\beta_3(\gamma_{(13)} - i\gamma_{(14)})$$

$$h_{13}^A = 16(\gamma_{(13)} + 2\tau_{(13)}) \tag{14.4}$$

14.3.3 Field V_μ^+ Right Side : Field U_μ

$$h_1^U = 4b_3(\gamma_{[13]} + i\gamma_{[14]}), \quad h_2^U = 2i\gamma_{[24]}, \quad h_3^U = -2i\gamma_{[23]}, \quad h_4^U = 4\gamma_{[23]}$$

$$h_5^U = 8\beta_3 i\gamma_{(24)}, \quad h_6^U = 2i(\gamma_{(24)} - 2\tau_{(24)}), \quad h_7^U = 2i(\gamma_{(23)} + 2\tau_{(23)}),$$

$$h_8^U = 4\gamma_{(23)}, \quad h_9^U = -4i(\gamma_{(24)} + 4\tau_{(24)}), \quad h_{10}^U = 4i(\gamma_{(23)} + 4\tau_{(23)})$$

$$h_{11}^U = 4(\rho_3(\gamma_{(13)} + i\gamma_{(14)}) + (\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)})) - 2\beta_3(\gamma_{(23)} - i\gamma_{(24)})$$

$$h_{12}^U = 16(\gamma_{(23)} + 2\tau_{(23)}) \qquad (14.5)$$

14.3.4 Field V_μ^+ Right Side : Field V_μ^-

$$h_1^- = -4i(b_1\gamma_{[34]} + \beta_1\gamma_{(34)}), \quad h_2^- = -4i(b_2\gamma_{[34]} + \beta_2\gamma_{(34)})$$

$$h_3^- = 8i\gamma_{[34]}, \quad h_4^- = 4\gamma_{(33)}\beta_1 - 4\beta_3(\gamma_{(13)} - i\gamma_{(14)}),$$

$$h_5^- = 4\gamma_{(33)}\beta_2 - 4\beta_3(\gamma_{(23)} - i\gamma_{(24)})$$

$$h_6^- = 4\gamma_{(33)}, \quad h_7^- = -8i\gamma_{[34]}, \quad h_8^- = 32i(\gamma_{[34]} + 2\tau_{(34)}),$$

$$h_9^- = 8\gamma_{(33)}\rho_1 + 4(\beta_1 + 4\rho_1)\tau_{(33)} - 2\beta_3(\gamma_{(13)} - i\gamma_{(14)}),$$

$$h_{10}^- = 8\gamma_{(33)}\rho_1 + 4(\beta_2 + 4\rho_2)\tau_{(33)} - 2\beta_3(\gamma_{(23)} - i\gamma_{(24)})$$

$$h_{11}^- = 8\tau_{(33)}, \quad h_{12}^- = 8i\gamma_{[34]}, \quad h_{13}^- = 8i\gamma_{(34)},$$

$$h_{14}^- = 16(\gamma_{(33)} + 2\tau_{(33)}),$$

15 Photon Equation Coefficients

15.1 Field A_μ

$$r_1^A = -k_2 g_1^A, \quad r_2^A = -k_2 g_2^A, \quad r_3^A = -k_+ h_1^A, \quad r_4^A = -k_2 g_3^A$$

$$r_5^A = r_4^A, \quad r_6^A = -k_2 g_4^A, \quad r_7^A = -k_+ h_2^A, \quad r_8^A = -k_+ h_3^A$$

$$r_9^A = -k_+ h_5^A, \quad r_{10}^A = -k_+ h_4^A, \quad r_{11}^A = -k_+ h_9^A, \quad r_{12}^A = -k_+ h_{10}^A,$$

$$r_{13}^A = -k_+ h_{11}^A, \quad r_{14}^A = -k_2 g_5^A, \quad r_{15}^A = f_1^A - k_2 g_6^A, \quad r_{16}^A = -k_+ h_6^A,$$

$$r_{17}^A = f_2^A - k_2 g_6^A, \quad r_{18}^A = f_3^A - k_2 g_7^A, \quad r_{19}^A = f_2^A - k_2 g_4^A, \quad r_{20}^A = -k_2 g_7^A,$$

$$r_{21}^A = f_4^A - k_2 g_9^A, \quad r_{22}^A = f_5^A - k_2 g_4^A, \quad r_{23}^A = -k_2 g_{10}^A, \quad r_{24}^A = -k_+ h_7^A,$$

$$r_{25}^A = -k_+ h_8^A, \quad r_{26}^A = f_6^A - k_2 g_{11}^A, \quad r_{27}^A = f_7^A - k_2 g_{12}^A, \quad r_{28}^A = -k_+ h_{12}^A,$$

$$r_{29}^A = -k_2 g_{10}^A, \quad r_{30}^A = -k_2 g_4^A, \quad r_{31}^A = -k_+ h_{13}^A, \qquad (15.1)$$

15.2 Field U_μ

$$r_1^U = f_1^U, \quad r_2^U = f_2^U, \quad r_3^U = -k_+ h_1^U, \quad r_4^U = f_3^U$$
$$r_5^U = f_4^U, \quad r_6^U = k_+ h_2^U, \quad r_7^U = k_+ h_3^U, \quad r_8^U = -k_+ h_4^U$$
$$r_9^U = k_+ h_6^U, \quad r_{10}^U = k_+ h_7^U, \quad r_{11}^U = +k_+ h_8^U, \quad r_{12}^U = k_+ h_9^U,$$
$$r_{13}^U = k_+ h_{10}^U, \quad r_{14}^U = -k_2 g_1^U, \quad r_{15}^U = f_5^U - k_2 g_2^U, \quad r_{16}^U = -k_+ h_5^U,$$
$$r_{17}^U = f_6^U - k_2 g_2^U, \quad r_{18}^U = f_7^U - 2k_2 g_2^U, \quad r_{19}^U = f_8^U, \quad r_{20}^U = f_7^U,$$
$$r_{21}^U = f_9^U, \quad r_{22}^U = f_4^U - 2k_2 g_3^U, \quad r_{23}^U = f_{10}^A - 2k_2 g_3^U, \quad r_{24}^U = f_{11}^U - k_2 g_4^U,$$
$$r_{25}^U = f_{12}^U - k_2 g_5^U, \quad r_{26}^U = -k_+ h_{11}^U, \quad r_{27}^U = -k_2 g_3^U, \quad r_{28}^U = f_{13}^U - 2k_2 g_3^U,$$
$$r_{29}^U = -k_+ h_{12}^U, \quad r_{30}^U = k_2 h_4^U, \quad r_{31}^U = -k_+ h_8^U, \qquad (15.2)$$

15.3 Field V_μ^+

$$r_1^+ = -k_- h_1^{-*}, \quad r_2^+ = -k_- h_2^{-*}, \quad r_3^+ = f_1^+ - k_2 g_1^+, \quad r_4^+ = -k_- h_3^{-*},$$
$$r_5^+ = f_2^+ - k_2 g_2^+, \quad r_6^+ = -k_2 g_3^+, \quad r_7^+ = f_3^+ - k_2 g_4^{-*}, \quad r_8^+ = f_8^+ - k_2 g_8^+,$$
$$r_9^+ = f_{11}^+ - k_2 g_9^+, \quad r_{10}^+ = -k_- h_4^{-*}, \quad r_{11}^+ = -k_- h_5^{-*}, \quad r_{12}^+ = f_4^+ - k_2 g_5^+,$$
$$r_{13}^+ = -k_- h_6^{-*}, \quad r_{14}^+ = -k_- h_6^{-*}, \quad r_{15}^+ = -k_- h_7^{-*}, \quad r_{16}^+ = -2k_- h_6^{-*},$$
$$r_{17}^+ = -k_- h_7^{-*}, \quad r_{18}^+ = -k_- h_8^{-*}, \quad r_{19}^+ = -k_2 g_2^+, \quad r_{20}^+ = -k_2 g_3^+,$$
$$r_{21}^+ = f_5^+ - k_2 g_4^+, \quad r_{22}^+ = f_7^+ - k_2 g_1^+, \quad r_{23}^+ = -k_2 g_7^+, \quad r_{24}^+ = f_9^+ - k_2 g_8^+,$$
$$r_{25}^+ = -k_2 g_{10}^+, \quad r_{26}^+ = -k_- h_9^{-*}, \quad r_{27}^+ = -k_- h_{10}^{-*}, r_{28}^+ = f_{13} - k_2 g_{11}^+,$$
$$r_{29}^+ = f_{14} - k_2 g_4^+, \quad r_{30}^+ = f_7^+ - k_2 g_8^+, \quad r_{31}^+ = f_{15}^+ - k_2 g_{13}^+,$$

16 Vectorial Equations

16.1 Photon-field A_μ

Transversal Sector

$$\vec{E}_A = \bar{a}_1 \vec{E}_A \qquad (16.1)$$

$$\vec{B}_A = \bar{a}_1 \vec{B}_A \qquad (16.2)$$

$$\vec{e}_A = \bar{a}_2 (\vec{e}^{\,[12]} + \vec{e}^{\,[+-]}) + a_3 \vec{e}^{\,(+-)} \qquad (16.3)$$

$$\vec{b}_A = \bar{a}_2 (\vec{b}^{\,[12]} + \vec{b}^{\,[+-]}) + \bar{a}_3 \vec{b}^{\,(+-)} \qquad (16.4)$$

Longitudinal Sector

$$\vec{S}_A = \bar{a}_4 S_\alpha^{\alpha 1} + \bar{a}_5 S_\alpha^{\alpha 2} \qquad (16.5)$$

$$\vec{s}_A = \bar{a}_6(\overset{(11)}{z}{}^{\alpha}_{\alpha} + \overset{(22)}{z}{}^{\alpha}_{\alpha} + 2\overset{(12)}{z}{}^{\alpha}_{\alpha} + 2\overset{+-3}{z}{}^{\alpha}_{\alpha}) + \bar{a}_7(\overset{(11)}{\omega}{}^{\alpha}_{\alpha} + \overset{(22)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{(12)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{+-3}{\omega}{}^{\alpha}_{\alpha}) \quad (16.6)$$

London Sector

$$l_A = \phi(f_2^A s^{(11)} + f_3^A s^{(12)} + f_2^A s^{(22)} + f_4^A s_\omega^{(11)} + f_5^A s_\omega^{(12)} + f_3^A s^{+-3} + f_4^A \overset{+-3}{\omega}{}^{\alpha}_{\alpha} + f_4^A \overset{(22)}{\omega}{}^{\alpha}_{\alpha}) +$$
$$+\phi_U(f_6^U s^{(11)} + f_7^U s^{(12)} + f_8^U s^{(12)} + f_7^U s^{+-3} + f_9^U s_\omega^{(11)} + f_4^U s_\omega^{(12)} + f_{10}^U s_\omega^{(22)} + f_{13}^U \overset{+-3}{\omega}{}^{\alpha}_{\alpha}) +$$
$$+\phi_+(f_5^+ s^{[23-]} + f_6^+ s^{[24-]} + f_7^+ s^{(13-)} + f_9^+ s^{(23-)} + f_{10}^+ s^{(24-)} + f_{11}^+ s_\omega^{(23-)} + f_{12}^+ s_\omega^{(24-)} +$$
$$+ f_{14}^+ \overset{[13-]}{z}{}^{\alpha}_{\alpha} + f_{14}^+ \overset{[23-]}{z}{}^{\alpha}_{\alpha} + f_7^+ \overset{(13-)}{z}{}^{\alpha}_{\alpha} + f_7^+ \overset{(23-)}{z}{}^{\alpha}_{\alpha} + f_{15}^+ \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + f_{16}^+ \overset{(23-)}{\omega}{}^{\alpha}_{\alpha}) +$$
$$+\phi_-(f_5^{*+} s^{[23+]} + f_6^{*+} s^{[24+]} + f_7^{*+} s^{(13+)} + f_9^{*+} s^{(23+)} + f_{10}^{*+} s^{(24+)} + f_{11}^{*+} s_\omega^{(23+)} + f_{12}^{*+} s_\omega^{(24+)} +$$
$$+ f_{14}^{*+} \overset{[13+]}{z}{}^{\alpha}_{\alpha} + f_{14}^{*+} \overset{[23+]}{z}{}^{\alpha}_{\alpha} + f_7^{*+} \overset{(13+)}{z}{}^{\alpha}_{\alpha} + f_7^{*+} \overset{(23+)}{z}{}^{\alpha}_{\alpha} + f_{15}^{*+} \overset{(13+)}{\omega}{}^{\alpha}_{\alpha} + f_{16}^{*+} \overset{(23+)}{\omega}{}^{\alpha}_{\alpha})$$
$$+\vec{A}\cdot(f_2^A \vec{s}^{(11)} + f_3^A \vec{s}^{(12)} + f_2^A \vec{s}^{(22)} + f_4^A \vec{s}_\omega^{(11)} + f_4^A \vec{s}_\omega^{(12)}) +$$
$$+\vec{U}\cdot(f_3^U \vec{e}^{[12]} + f_3^U \vec{e}^{[+-]} + f_4^U \vec{e}_\omega^{(12)} + f_6^U \vec{s}^{(11)} + f_7^U \vec{s}^{(12)} + f_8^U \vec{s}^{(22)} + f_7^U \vec{s}^{+-3} +$$
$$+ f_9^U \vec{s}_\omega^{(11)} + f_4^U \vec{s}_\omega^{(12)} + f_{10}^U \vec{s}_\omega^{(22)}) +$$
$$+\vec{V}^+\cdot(f_2^+ \vec{e}^{[13-]} + f_3^+ \vec{e}^{[23-]} + f_6^+ \vec{e}^{[24-]} - f_7^+ \vec{e}^{(13-)} + f_8^+ \vec{e}^{(23-)} + f_{11}^+ \vec{e}_\omega^{(23-)} + f_{12}^+ \vec{e}_\omega^{(24-)} +$$
$$+ f_5^+ \vec{s}^{[23-]} + f_6^+ \vec{s}^{[24-]} + f_7^+ \vec{s}^{(13-)} + f_9^+ \vec{s}^{(23-)} + f_{10}^+ \vec{s}^{(24-)} + f_{11}^+ \vec{s}_\omega^{(23-)} + f_{12}^+ \vec{s}_\omega^{(24-)}) +$$
$$+\vec{V}^-\cdot(f_2^{*+} \vec{e}^{[13+]} + f_3^{*+} \vec{e}^{[23+]} + f_6^{*+} \vec{e}^{[24+]} - f_7^{*+} \vec{e}^{(13+)} + f_8^{*+} \vec{e}^{(23+)} + f_{11}^{*+} \vec{e}_\omega^{(23+)} + f_{12}^{*+} \vec{e}_\omega^{(24+)} +$$
$$+ f_5^{*+} \vec{s}^{[23+]} + f_6^{*+} \vec{s}^{[24+]} + f_7^{*+} \vec{s}^{(13+)} + f_9^{*+} \vec{s}^{(23+)} + f_{10}^{*+} \vec{s}^{(24+)} + f_{11}^{*+} \vec{s}_\omega^{(23+)} + f_{12}^{*+} \vec{s}_\omega^{(24+)})$$

$$\vec{l}_A = -\vec{A}\cdot(f_2^A \overset{\leftrightarrow(11)}{s} + f_3^A \overset{\leftrightarrow(12)}{s} + f_3^A \overset{\leftrightarrow(22)}{s} + f_4^A \overset{\leftrightarrow(11)}{s}_\omega + f_5^A \overset{\leftrightarrow(12)}{s}_\omega) +$$
$$-\phi(f_2^A \vec{s}^{(11)} + f_3^A \vec{s}^{(12)} + f_2^A \vec{s}^{(22)} + f_4^A \vec{s}_\omega^{(11)} + f_5^A \vec{s}_\omega^{(12)} + f_3^A \overset{\leftrightarrow+-3}{s}) +$$
$$+\vec{A}(f_4^A \overset{(22)}{\omega}{}^{\alpha}_{\alpha} + f_3^A s^{+-3} + f_4^A \overset{+-3}{\omega}{}^{\alpha}_{\alpha}) - \vec{U}\times(f_3^U \vec{b}^{[12]} + f_3^U \vec{b}^{[+-]} + f_4^U \vec{b}_\omega^{(12)}) +$$
$$-\vec{U}\cdot(f_6^U \overset{\leftrightarrow(11)}{s} + f_7^U \overset{\leftrightarrow(12)}{s} + f_8^U \overset{\leftrightarrow(22)}{s} + f_7^U \overset{\leftrightarrow+-3}{s} + f_9^U \overset{\leftrightarrow(11)}{s}_\omega + f_4^U \overset{\leftrightarrow(12)}{s}_\omega + f_{10}^U \overset{\leftrightarrow(22)}{s}_\omega) +$$
$$-\phi_U(f_3^U \vec{e}^{[12]} + f_3^U \vec{e}^{[+-]} + f_4^U \vec{e}_\omega^{(12)} + f_6^U \vec{s}^{(11)} + f_7^U \vec{s}^{(12)} + f_8^U \vec{s}^{(22)} +$$
$$+ f_7^U \vec{s}^{+-3} + f_9^U \vec{s}_\omega^{(11)} + f_4^U \vec{s}_\omega^{(12)} + f_{10}^U \vec{s}_\omega^{(22)}) + \vec{U}(f_{13}^U \overset{+-3}{\omega}{}^{\alpha}_{\alpha}) +$$
$$-\vec{V}^+\times(-f_2^+ \vec{b}^{[13-]} + f_3^+ \vec{b}^{[23-]} + f_6^+ \vec{b}^{[24-]} - f_7^+ \vec{b}^{(13-)} + f_8^+ \vec{b}^{(23-)} + f_{11}^+ \vec{b}_\omega^{(23-)} + f_{12}^+ \vec{b}_\omega^{(24-)}) +$$
$$-\vec{V}^+\cdot(f_5^+ \overset{\leftrightarrow[23-]}{s} + f_6^+ \overset{\leftrightarrow[24-]}{s} + f_7^+ \overset{\leftrightarrow(13-)}{s} + f_9^+ \overset{\leftrightarrow(23-)}{s} + f_{10}^+ \overset{\leftrightarrow(24-)}{s} + f_{11}^+ \overset{\leftrightarrow(23-)}{s}_\omega + f_{12}^+ \overset{\leftrightarrow(24-)}{s}_\omega) +$$
$$-\phi_+(-f_2^+ \vec{e}^{[13-]} + f_3^+ \vec{e}^{[23-]} + f_6^+ \vec{e}^{[24-]} - f_7^+ \vec{e}^{(13-)} + f_8^+ \vec{e}^{(23-)} + f_{11}^+ \vec{e}_\omega^{(23-)} + f_{12}^+ \vec{e}_\omega^{(24-)}$$

$$+f_5^+\vec{s}^{\,[23-]}+f_6^+\vec{s}^{\,[24-]}+f_7^+\vec{s}^{\,(13-)}+f_9^+\vec{s}^{\,(23-)}+f_{10}^+\vec{s}^{\,(24-)}+f_{11}^+\vec{s}_\omega^{(23-)}+f_{12}^+\vec{s}_\omega^{(24-)})+$$

$$+\vec{V}^+(f_{14}^+\overset{[13-]}{z}{}^{\alpha}_{\alpha}+f_{14}^+\overset{[23-]}{z}{}^{\alpha}_{\alpha}+f_7^+\overset{(13-)}{z}{}^{\alpha}_{\alpha}+f_7^+\overset{(23-)}{z}{}^{\alpha}_{\alpha}+f_{15}^+\overset{(13-)}{\omega}{}^{\alpha}_{\alpha}+f_{16}^+\overset{(23-)}{\omega}{}^{\alpha}_{\alpha})$$

$$-\vec{V}^-\times(-f_2^{*+}\vec{b}^{\,[13+]}+f_3^{*+}\vec{b}^{\,[23+]}+f_6^{*+}\vec{b}^{\,[24+]}-f_7^{*+}\vec{b}^{\,(13+)}+f_8^{*+}\vec{b}^{\,(23+)}+f_{11}^{*+}\vec{b}_\omega^{(23+)}+f_{12}^{*+}\vec{b}_\omega^{(24+)})+$$

$$-\vec{V}^-\cdot(f_5^{*+}\overset{\leftrightarrow}{s}{}^{[23+]}+f_6^{*+}\overset{\leftrightarrow}{s}{}^{[24+]}+f_7^{*+}\overset{\leftrightarrow}{s}{}^{(13+)}+f_9^{*+}\overset{\leftrightarrow}{s}{}^{(23+)}+f_{10}^{*+}\overset{\leftrightarrow}{s}{}^{(24+)}+f_{11}^{*+}\overset{\leftrightarrow}{s}{}_\omega^{(23+)}+f_{12}^{*+}\overset{\leftrightarrow}{s}{}_\omega^{(24+)})+$$

$$-\phi_-(-f_2^{*+}\vec{e}^{\,[13+]}+f_3^{*+}\vec{e}^{\,[23+]}+f_6^{*+}\vec{e}^{\,[24+]}+f_7^{*+}\vec{e}^{\,(13+)}+f_5^{*+}\vec{s}^{\,[23+]}+f_6^{*+}\vec{s}^{\,[24+]}+f_8^{*+}\vec{e}^{\,(23+)}+$$

$$+f_{11}^{*+}\vec{e}_\omega^{(23+)}+f_{12}^{*+}\vec{e}_\omega^{(24+)}+f_7^{*+}\vec{s}^{\,(13+)}+f_9^{*+}\vec{s}^{\,(23+)}+f_{10}^{*+}\vec{s}^{\,(24+)}+f_{11}^{*+}\vec{s}_\omega^{(23+)}+f_{12}^{*+}\vec{s}_\omega^{(24+)})+$$

$$-\vec{V}^-(f_{14}^{*+}\overset{[13+]}{z}{}^{\alpha}_{\alpha}+f_{14}^{*+}\overset{[23+]}{z}{}^{\alpha}_{\alpha}+f_7^{*+}\overset{(13+)}{z}{}^{\alpha}_{\alpha}+f_7^{*+}\overset{(23+)}{z}{}^{\alpha}_{\alpha}+f_{15}^{*+}\overset{(13+)}{\omega}{}^{\alpha}_{\alpha}+f_{16}^{*+}\overset{(23+)}{\omega}{}^{\alpha}_{\alpha})$$

$$\rho_A=\phi(f_1^U S_A+f_6^A S_{A\alpha}^\alpha+f_7^A S_{U\alpha}^\alpha)+\vec{A}\cdot(f_1^A\vec{S}_U)+\phi_U(f_5^U S_U+f_{11}^U S_\alpha^{\alpha 1}+f_{12}^U S_\alpha^{\alpha 2})+$$

$$+\vec{U}\cdot(f_1^U\vec{E}_A+f_2^U\vec{E}_U)+\phi_+(f_4^+S_-+f_{13}^+S_\alpha^{\alpha-})+\vec{V}^+\cdot(f_1^+\vec{E}_-+f_4^+\vec{S}_-)+$$

$$+\phi_-(f_4^{*+}S_++f_{13}^{*+}S_\alpha^{\alpha+})+\vec{V}^-\cdot(f_1^{*+}\vec{E}_++f_4^{*+}\vec{S}_+)$$

$$\vec{J}_A=\vec{A}\cdot(f_1^A\overset{\leftrightarrow}{S}_U)+\phi(f_1^A\vec{S}_U)-\vec{A}(f_6^A S_{A\alpha}^\alpha+f_7^A S_{U\alpha}^\alpha)+$$

$$+\vec{U}\times(f_1^U\vec{B}_A+f_2^U\vec{B}_U)+\vec{U}\cdot(f_5^U\overset{\leftrightarrow}{S}_U)+\phi_U(f_1^U\vec{E}_A+f_2^U\vec{E}_U)+$$

$$-\vec{U}(f_{11}^U S_\alpha^{\alpha 1}+f_{12}^U S_\alpha^{\alpha 2})+\vec{V}^+\times(f_1^+\vec{B}_-)+\vec{V}^+\cdot(f_4^+\overset{\leftrightarrow}{S}_-)+$$

$$+\phi_+(f_1^+\vec{E}_-+f_4^+\vec{S}_-)-\vec{V}^+(f_{13}^+S_\alpha^{\alpha-})+\vec{V}^-\times(f_1^{*+}\vec{B}_+)+$$

$$+\vec{V}^{*+}\cdot(f_4^{*+}\overset{\leftrightarrow}{S}_+)+\phi_-(f_1^{*+}\vec{E}_++f_4^{*+}\vec{S}_+)+\vec{V}^-(f_{13}^{*+}S_\alpha^{\alpha+})$$

16.2 Photon-field U_μ

$$\vec{E}_U=\bar{b}_1\vec{E}_U \tag{16.7}$$

$$\vec{B}_U=\bar{b}_1\vec{B}_U \tag{16.8}$$

$$\vec{e}_U=\bar{b}_2(\vec{e}^{\,[12]}+\vec{e}^{\,[+-]})+\bar{b}_3\vec{e}^{\,(+-)} \tag{16.9}$$

$$\vec{b}_U=\bar{b}_2(\vec{b}^{\,[12]}+\vec{b}^{\,[+-]})+\bar{b}_3\vec{b}^{\,(+-)} \tag{16.10}$$

$$\vec{S}_U=\bar{b}_4 S_\alpha^{\alpha 1}+\bar{b}_5 S_\alpha^{\alpha 2} \tag{16.11}$$

$$\vec{s}_U = \bar{b}_6(\overset{(11)}{z}{}^{\alpha}_{\alpha} + \overset{(22)}{z}{}^{\alpha}_{\alpha} + 2\overset{(12)}{z}{}^{\alpha}_{\alpha} + 2\overset{(+-3)}{z}{}^{\alpha}_{\alpha}) + \bar{b}_7(\overset{(11)}{\omega}{}^{\alpha}_{\alpha} + \overset{(22)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{(12)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{+-3}{\omega}{}^{\alpha}_{\alpha}) \quad (16.12)$$

$$M_U = 2\mu_U^2\phi_U \quad (16.13)$$

$$\begin{aligned}
l_U = {} & \phi(g_6^A s^{(11)} + g_7^A s^{(12)} + g_8^A s^{(22)} + g_7^A s^{+-3} + g_9^A s_\omega^{(11)} + g_4^A s_\omega^{(12)} + g_{10}^A s_\omega^{(22)} + g_4^A \overset{+-3}{\omega}{}^{\alpha}_{\alpha}) + \\
& + \vec{A}\cdot(g_3^A \vec{e}^{\,[12]} + g_3^A \vec{e}^{\,[+-]} + g_4^A \vec{e}_\omega^{\,(12)} + g_6^A \vec{s}^{\,(11)} + g_7^A \vec{s}^{\,(12)} + g_8^A \vec{s}^{\,(22)} + g_7^A \vec{s}^{\,+-3} + \\
& + g_9^A \vec{s}_\omega^{\,(11)} + g_4^A \vec{s}_\omega^{\,(12)} + g_{10}^A \vec{s}_\omega^{\,(22)}) + \\
& + \phi_U(g_2^U s^{(11)} + 2g_2^U s^{(12)} + g_2^U s^{(22)} + 2g_2^U s^{+-3} + g_4^U s^{+-3} + g_3^U s_\omega^{(12)} + 2g_3^U s_\omega^{(22)} + \\
& + g_3^U \overset{(11)}{\omega}{}^{\alpha}_{\alpha} + 2g_3^U \overset{+-3}{\omega}{}^{\alpha}_{\alpha} + g_6^A \overset{+-3}{\omega}{}^{\alpha}_{\alpha}) + \\
& + \vec{U}\cdot(g_2^U \vec{s}^{\,(11)} + 2g_2^U \vec{s}^{\,(12)} + g_2^U \vec{s}^{\,(22)} + g_3^U \vec{s}_\omega^{\,(12)} + 2g_3^U \vec{s}_\omega^{\,(22)} + g_4^U \vec{s}^{\,+-3}) + \\
& + \phi_+(g_2^+ s^{[13-]} + g_3^+ s^{[14-]} + g_4^+ s^{[23-]} + g_6^+ s^{(13-)} + g_7^+ s^{(14-)} + g_8^+ s^{(23-)} + g_9^+ s_\omega^{(13-)} + \\
& + g_{10}^+ s_\omega^{(14-)} - g_4^+ \overset{[13-]}{z}{}^{\alpha}_{\alpha} - g_4^+ \overset{[23-]}{z}{}^{\alpha}_{\alpha} + g_8^+ \overset{(13-)}{z}{}^{\alpha}_{\alpha} + g_8^+ \overset{(23-)}{z}{}^{\alpha}_{\alpha} + g_8^+ \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + g_{13}^+ \overset{(23-)}{\omega}{}^{\alpha}_{\alpha}) + \\
& + \vec{V}^+\cdot(-g_2^+ \vec{e}^{\,[13-]} + g_3^+ \vec{e}^{\,[14-]} - g_4^+ \vec{e}^{\,[23-]} - g_8^+ \vec{e}^{\,(23-)} + g_9^+ \vec{e}_\omega^{\,(13-)} + \\
& + g_3^+ \vec{s}^{\,[14-]} + g_4^+ \vec{s}^{\,[23-]} + g_6^+ \vec{s}^{\,(13-)} + g_3^+ \vec{s}^{\,[13-]} + g_7^+ \vec{s}^{\,(14-)} + g_8^+ \vec{s}^{\,(23-)} + g_9^+ \vec{s}_\omega^{\,(13-)} + g_{10}^+ \vec{s}_\omega^{\,(14-)}) + \\
& + \phi_-(g_2^{*+} s^{[13+]} + g_3^{*+} s^{[14+]} + g_4^{*+} s^{[23+]} + g_6^{*+} s^{(13+)} + g_7^{*+} s^{(14+)} + g_8^{*+} s^{(23+)} + \\
& + g_9^{*+} s_\omega^{(13+)} + g_{10}^{*+} s_\omega^{(14+)} - g_4^{*+} \overset{[13+]}{z}{}^{\alpha}_{\alpha} - g_4^{*+} \overset{[23+]}{z}{}^{\alpha}_{\alpha} + g_8^{*+} \overset{(13+)}{z}{}^{\alpha}_{\alpha} + g_8^{*+} \overset{(23+)}{z}{}^{\alpha}_{\alpha} + \\
& + g_8^{*+} \overset{(13+)}{\omega}{}^{\alpha}_{\alpha} + g_{13}^{*+} \overset{(23+)}{\omega}{}^{\alpha}_{\alpha}) + \\
& + \vec{V}^-\cdot(-g_2^{*+} \vec{e}^{\,[13+]} + g_3^{*+} \vec{e}^{\,[14+]} - g_4^{*+} \vec{e}^{\,[23+]} - g_8^{*+} \vec{e}^{\,(23+)} + g_9^{*+} \vec{e}_\omega^{\,(13+)} \\
& + g_3^{*+} \vec{s}^{\,[13+]} + g_3^{*+} \vec{s}^{\,[14+]} + g_4^{*+} \vec{s}^{\,[23+]} + g_6^{*+} \vec{s}^{\,(13+)} + g_3^{*+} \vec{s}^{\,[13+]} + g_7^{*+} \vec{s}^{\,(14+)} + \\
& + g_8^{*+} \vec{s}^{\,(23+)} + g_9^{*+} \vec{s}_\omega^{\,(13+)} + g_{10}^{*+} \vec{s}_\omega^{\,(14+)})
\end{aligned} \quad (16.14)$$

$$\begin{aligned}
\vec{l}_U = {} & \vec{A}\times(g_3^A \vec{b}^{\,[12]} + g_3^A \vec{b}^{\,[+-]} + g_4^A \vec{b}_\omega^{\,(12)}) + \\
& + \vec{A}\cdot(g_6^A \overset{\leftrightarrow(11)}{s} + g_7^A \overset{\leftrightarrow(12)}{s} + g_8^A \overset{\leftrightarrow(22)}{s} + g_7^A \overset{\leftrightarrow +-3}{s} + g_9^A \overset{\leftrightarrow(11)}{s}_\omega + g_4^A \overset{\leftrightarrow(12)}{s}_\omega + g_{10}^A \overset{\leftrightarrow(22)}{s}_\omega) + \\
& + \phi(g_3^A \vec{e}^{\,[12]} + g_3^A \vec{e}^{\,[+-]} + g_4^A \vec{e}_\omega^{\,(12)} + g_6^A \vec{s}^{\,(11)} + \\
& + g_7^A \vec{s}^{\,(12)} + g_8^A \vec{s}^{\,(22)} + g_7^A \vec{s}^{\,+-3} + g_9^A \vec{s}_\omega^{\,(11)} + g_4^A \vec{s}_\omega^{\,(12)} + g_{10}^A \vec{s}_\omega^{\,(22)}) + \\
& - \vec{A}(g_4^A \overset{+-3}{\omega}{}^{\alpha}_{\alpha}) + \vec{U}\cdot(g_2^U \overset{\leftrightarrow(11)}{s} + 2g_2^U \overset{\leftrightarrow(12)}{s} + g_2^U \overset{\leftrightarrow(22)}{s} + 2g_2^U \overset{\leftrightarrow(+-3)}{s} + g_3^U \overset{\leftrightarrow(12)}{s}_\omega + 2g_3^U \overset{\leftrightarrow(22)}{s}_\omega) + \\
& + \phi_U(g_2^U \vec{s}^{\,(11)} + 2g_2^U \vec{s}^{\,(12)} + g_2^U \vec{s}^{\,(22)} + 2g_2^U \vec{s}^{\,+-3} + g_3^U \vec{s}_\omega^{\,(12)} + 2g_3^U \vec{s}_\omega^{\,(22)}) +
\end{aligned}$$

$$-\vec{U}(g_3^U \overset{(11)}{\omega}{}_\alpha^\alpha + 2g_3^U \overset{+-3}{\omega}{}_\alpha^\alpha) + \vec{V}^+ \times (-g_2^+\vec{b}^{[13-]} - g_3^+\vec{b}^{[14-]} - g_4^+\vec{b}^{[23-]} - g_8^+\vec{b}^{(23-)} + g_9^+\vec{b}_\omega^{(13-)}) +$$

$$+\vec{V}^+ \cdot (g_2^+ \overset{\leftrightarrow}{s}{}^{[13-]} + g_3^+ \overset{\leftrightarrow}{s}{}^{[14-]} + g_4^+ \overset{\leftrightarrow}{s}{}^{[23-]} + g_6^+ \overset{\leftrightarrow}{s}{}^{(13-)} + g_7^+ \overset{\leftrightarrow}{s}{}^{(14-)} + g_8^+ \overset{\leftrightarrow}{s}{}^{(23-)} + g_9^+ \overset{\leftrightarrow}{s}{}_\omega^{(13-)} + g_{10}^+ \overset{\leftrightarrow}{s}{}_\omega^{(14-)}) +$$

$$+\phi_+(-g_2^+\vec{e}^{[13-]} + g_3^+\vec{e}^{[14-]} - g_4^+\vec{e}^{[23-]} - g_8^+\vec{e}^{(23-)} + g_9^+\vec{e}_\omega^{(13-)} +$$

$$+g_2^+\vec{s}^{[13-]} + g_3^+\vec{s}^{[14-]} + g_4^+\vec{s}^{[23-]} + g_7^+\vec{s}^{(14-)} + g_8^+\vec{s}^{(23-)} + g_9^+\vec{s}_\omega^{(13-)} + g_{10}^+\vec{s}_\omega^{(14-)}) +$$

$$-\vec{V}^+(-g_4^+ \overset{[13-]}{z}{}_\alpha^\alpha - g_4^+ \overset{[23-]}{z}{}_\alpha^\alpha + g_8^+ \overset{(13-)}{z}{}_\alpha^\alpha + g_8^+ \overset{(23-)}{z}{}_\alpha^\alpha + g_8^+ \overset{(13-)}{\omega}{}_\alpha^\alpha + g_{13}^+ \overset{(23-)}{\omega}{}_\alpha^\alpha) +$$

$$+\vec{V}^- \times (-g_2^{*+}\vec{b}^{[13+]} - g_3^{*+}\vec{b}^{[14+]} - g_4^{*+}\vec{b}^{[23+]} - g_8^{*+}\vec{b}^{(23+)} + g_9^{*+}\vec{b}_\omega^{(13+)}) +$$

$$+\vec{V}^- \cdot (g_2^{*+} \overset{\leftrightarrow}{s}{}^{[13+]} + g_3^{*+} \overset{\leftrightarrow}{s}{}^{[14+]} + g_4^{*+} \overset{\leftrightarrow}{s}{}^{[23+]} + g_6^{*+} \overset{\leftrightarrow}{s}{}^{(13+)} +$$

$$+g_7^{*+} \overset{\leftrightarrow}{s}{}^{(14+)} + g_8^{*+} \overset{\leftrightarrow}{s}{}^{(23+)} + g_9^{*+} \overset{\leftrightarrow}{s}{}_\omega^{(13+)} + g_{10}^{*+} \overset{\leftrightarrow}{s}{}_\omega^{(14+)}) +$$

$$+\phi_-(-g_2^{*+}\vec{e}^{[13+]} + g_3^{*+}\vec{e}^{[14+]} - g_4^{*+}\vec{e}^{[23+]} - g_8^{*+}\vec{e}^{(23+)} + g_9^{*+}\vec{e}_\omega^{(13+)} +$$

$$+g_2^{*+}\vec{s}^{[13+]} + g_3^{*+}\vec{s}^{[14+]} + g_4^{*+}\vec{s}^{[23+]} + g_7^{*+}\vec{s}^{(14+)} + g_8^{*+}\vec{s}^{(23+)} + g_9^{*+}\vec{s}_\omega^{(13+)} + g_{10}^{*+}\vec{s}_\omega^{(14+)}) +$$

$$-\vec{V}^-(g_{11}^{*+}S_\alpha^{\alpha+} - g_4^{*+} \overset{[13+]}{z}{}_\alpha^\alpha - g_4^{*+} \overset{[23+]}{z}{}_\alpha^\alpha + g_8^{*+} \overset{(13+)}{z}{}_\alpha^\alpha + g_8^{*+} \overset{(23+)}{z}{}_\alpha^\alpha + g_8^{*+} \overset{(13+)}{\omega}{}_\alpha^\alpha + g_{13}^{*+} \overset{(23+)}{\omega}{}_\alpha^\alpha)$$

$$\rho_U = \phi(g_5^A S_A + g_{12}^A S_\alpha^{\alpha 2}) + \vec{A} \cdot (g_1^A \vec{E}_A + g_2^A \vec{E}_U + g_5^A \vec{S}_A) +$$

$$+\phi_U(g_1^U S_A + g_4^U S_\alpha^{\alpha 1} + g_5^U S_\alpha^{\alpha 2}) + \vec{U} \cdot (g_1^U \vec{S}_A +) + \phi_+(g_5^+ S_- + g_{11}^+ S_\alpha^{\alpha -}) +$$

$$+\vec{V}^+ \cdot (g_1^+ \vec{E}_-) + \phi_-(g_5^{*+} S_+ + g_{11}^{*+} S_\alpha^{\alpha +}) + \vec{V}^- \cdot (g_1^{*+} \vec{E}_+ + g_5^{*+} \vec{S}_+) \tag{16.15}$$

$$\vec{J}_U = \vec{A} \times (g_1^A \vec{B}_A + g_2^A \vec{B}_U) + \vec{A} \cdot (g_5^A \overset{\leftrightarrow}{S}_A +) + \phi(g_1^A \vec{E}_A + g_2^A \vec{E}_U) +$$

$$-\vec{A}(g_{11}^A S_\alpha^{\alpha 1} + g_{12}^A S_\alpha^{\alpha 2}) + \vec{U} \cdot (g_1^U \overset{\leftrightarrow}{S}_A) + \phi_U(g_1^U \vec{S}_A) +$$

$$-\vec{U}(g_4^U S_\alpha^{\alpha 1} + g_5^U S_\alpha^{\alpha 2}) + \vec{V}^+ \times (g_1^+ \vec{B}_-) + \vec{V}^+ \cdot (g_5^+ \overset{\leftrightarrow}{S}_-) +$$

$$+\phi_+(g_1^+ \vec{E}_- + g_5^+ \vec{S}_-) - \vec{V}^+(g_{11}^+ S_\alpha^{\alpha -}) + \vec{V}^- \times (g_1^{*+} \vec{B}_+) + \vec{V}^- \cdot (g_5^{*+} \overset{\leftrightarrow}{S}_+) +$$

$$+\phi_-(g_1^{*+} \vec{E}_+ + g_5^{*+} \vec{S}_- +) - \vec{V}^-(g_{11}^{*+} S_\alpha^{\alpha +})$$

16.3 Positive charged photon field-V_μ^+

$$\vec{E}_{+} = \bar{c}_1 \vec{E}_{-} \quad (16.16)$$

$$\vec{B}_{+} = \bar{c}_1 \vec{B}_{-} \quad (16.17)$$

$$\vec{e}_{+} = \bar{c}_2 (\vec{e}^{\,[-1]} + \vec{e}^{\,[-2]}) \quad (16.18)$$

$$\vec{b}_{+} = \bar{b}_2 (\vec{b}^{\,[12]} + \vec{b}^{\,[+-]}) + \bar{b}_3 \vec{b}^{\,(+-)} \quad (16.19)$$

$$\vec{S}_{+} = \bar{c}_3 S_{\alpha}^{\alpha -} \quad (16.20)$$

$$\vec{s}_{+} = \bar{c}_4 (\overset{(-1)}{z}{}^{\alpha}_{\alpha} + \overset{(-2)}{z}{}^{\alpha}_{\alpha}) + \bar{c}_5 (\overset{(-1)}{\omega}{}^{\alpha}_{\alpha} + \overset{(-2)}{\omega}{}^{\alpha}_{\alpha}) \quad (16.21)$$

$$M_{+} = -\mu_{+}^{2} \phi_{-} \quad (16.22)$$

$$\begin{aligned}
l_{+} = {} & \phi(h_2^A s^{[13-]} + h_7^A s^{[23-]} + h_8^A s^{(13-)} + h_4^{A*} s^{(23-)} + h_9^A s^{(24-)} + h_{10}^A s_{\omega}^{(23-)} + \\
& + h_{11}^A s_{\omega}^{(24-)} - h_2^A \overset{[13-]}{z}{}^{\alpha}_{\alpha} - h_2^A \overset{[23-]}{z}{}^{\alpha}_{\alpha} + h_8^A \overset{(13-)}{z}{}^{\alpha}_{\alpha} + h_8^A \overset{(23-)}{z}{}^{\alpha}_{\alpha} + h_{13}^A \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + h_{13}^A \overset{(23-)}{\omega}{}^{\alpha}_{\alpha}) + \\
& + \vec{A} \cdot (-h_2^A \vec{e}^{\,[13-]} - h_3^A \vec{e}^{\,[23-]} + h_5^A \vec{e}^{\,[24-]} + h_4^A \vec{e}^{\,(23-)} - h_9^A \vec{e}^{\,(24-)} + h_{10}^A \vec{e}_{\omega}^{\,(23-)} + h_{11}^A \vec{e}\,\omega^{(24-)} \\
& + h_2^A \vec{s}^{\,[13-]} + h_7^A \vec{s}^{\,[23-]} + h_8^A \vec{s}^{\,(13-)} + h_9^A \vec{s}^{\,(23-)} + h_{10}^A \vec{s}^{\,(24-)} + h_{11}^A \vec{s}_{\omega}^{\,(23-)} + h_{12}^A \vec{s}_{\omega}^{\,(24-)}) + \\
& + \phi_U (+h_2^U s^{[13-]} + h_3^U s^{[14-]} + h_4^U s^{[23-]} + h_6^U s^{(13-)} + h_7^U s^{(14-)} + h_8^U s^{(23-)} + h_9^U s_{\omega}^{(13-)} + \\
& + h_{10}^U s_{\omega}^{(23-)} + h_{11}^U S_{\alpha}^{\alpha -} + h_{12}^U \overset{[13-]}{z}{}^{\alpha}_{\alpha} - h_4^U \overset{[23-]}{z}{}^{\alpha}_{\alpha} + h_8^U \overset{(13-)}{z}{}^{\alpha}_{\alpha} + h_8^U \overset{(23-)}{z}{}^{\alpha}_{\alpha} + h_{12}^U \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + h_{12}^U \overset{(23-)}{\omega}{}^{\alpha}_{\alpha}) + \\
& + \vec{U} \cdot (-h_2^U \vec{e}^{\,[13-]} - h_3^U \vec{e}^{\,[14-]} + h_4^U \vec{e}^{\,[23-]} - h_6^U \vec{e}^{\,(13-)} - h_7^U \vec{e}^{\,(14-)} - h_8^U \vec{e}^{\,(23-)} + h_9^U \vec{e}_{\omega}^{\,(13-)} + \\
& + h_{10}^U \vec{e}_{\omega}^{\,(23-)} + h_2^U \vec{s}^{\,[13-]} + h_3^U \vec{s}^{\,[14-]} + h_4^U \vec{s}^{\,[23-]} + h_6^U \vec{s}^{\,(13-)} + h_7^U \vec{s}^{\,(14-)} + \\
& + h_8^U \vec{s}^{\,(23-)} + h_9^U \vec{s}_{\omega}^{\,(13-)} + h_{10}^U \vec{s}_{\omega}^{\,(23-)}) + \\
& + \phi_{-} (h_6^- s^{(11)} + h_6^- s^{(22)} + h_7^- s^{(12)} + 2h_6^- s^{+-4} + h_7^- s^{[+-]} + h_8^- s_{\omega}^{(+-)} + h_{12}^- \overset{[+-]}{z}{}^{\alpha}_{\alpha} + \\
& + h_{13}^- \overset{(+-)}{z}{}^{\alpha}_{\alpha} + h_{14}^- \overset{+-4}{z}{}^{\alpha}_{\alpha} + h_{11}^- (\overset{(11)}{z}{}^{\alpha}_{\alpha} + \overset{(22)}{z}{}^{\alpha}_{\alpha} + 2\overset{(12)}{z}{}^{\alpha}_{\alpha} + 2\overset{(11)}{\omega}{}^{\alpha}_{\alpha} + 2\overset{(22)}{\omega}{}^{\alpha}_{\alpha} + 4\overset{(12)}{\omega}{}^{\alpha}_{\alpha})) + \\
& + \vec{V}_{-} \cdot (h_3^- \vec{e}^{\,[12]} + h_4^- \vec{S}_A + h_6^- \vec{s}^{\,(11)} + h_6^- \vec{s}^{\,(22)} + h_7^- \vec{s}^{\,(12)} + 2h_6^- \vec{s}^{\,+-4} + h_7^- \vec{s}^{\,[+-]} + h_8^- \vec{s}_{\omega}^{\,(+-)}) \quad (16.23)
\end{aligned}$$

$$\begin{aligned}
\vec{l}_{+} = {} & \vec{A} \times (h_2^A \vec{b}^{\,[13-]} + h_3^A \vec{b}^{\,[23-]} + h_5^A \vec{b}^{\,[24-]} + h_4^A \vec{b}^{\,(23-)} - h_9^A \vec{b}^{\,(24-)} - h_{10}^A \vec{b}_{\omega}^{\,(23-)} - h_{11}^A \vec{b}_{\omega}^{\,(24-)}) + \\
& + \vec{A} \cdot (h_2^A \overset{\leftrightarrow}{s}{}^{[13-]} + h_7^A \overset{\leftrightarrow}{s}{}^{[23-]} + h_8^A \overset{\leftrightarrow}{s}{}^{(13-)} + h_4^{A*} \overset{\leftrightarrow}{s}{}^{(23-)} + h_9^A \overset{\leftrightarrow}{s}{}^{(24-)} + h_{10}^A \overset{\leftrightarrow}{s}{}_{\omega}^{(23-)} + h_{11}^A \overset{\leftrightarrow}{s}{}_{\omega}^{(24-)}) + \\
& + \phi(h_2^A \vec{e}^{\,[13-]} + h_3^A \vec{e}^{\,[23-]} + h_5^A \vec{e}^{\,[24-]} + h_4^A \vec{e}^{\,(23-)} - h_{10}^A \vec{e}^{\,(24-)} + h_{11}^A \vec{e}_{\omega}^{\,(23-)} + h_{12}^A \vec{e}\,\omega^{(24-)} \\
& + h_2^A \vec{s}^{\,[13-]} + h_7^A \vec{s}^{\,[23-]} + h_8^A \vec{s}^{\,(13-)} + h_4^{A*} \vec{s}^{\,(23-)} + h_9^A \vec{s}^{\,(24-)} + h_{10}^A \vec{s}_{\omega}^{\,(23-)} + h_{11}^A \vec{s}_{\omega}^{\,(24-)}) + \\
& - \vec{A}(-h_2^A \overset{[13-]}{z}{}^{\alpha}_{\alpha} - h_2^A \overset{[23-]}{z}{}^{\alpha}_{\alpha} + h_8^A \overset{(13-)}{z}{}^{\alpha}_{\alpha} + h_8^A \overset{(23-)}{z}{}^{\alpha}_{\alpha} + h_{13}^A \overset{(13-)}{\omega}{}^{\alpha}_{\alpha} + h_{13}^A \overset{(23-)}{\omega}{}^{\alpha}_{\alpha}) +
\end{aligned}$$

$$+\vec{U}\times(-h_2^U\vec{e}^{\,[13-]}-h_3^U\vec{e}^{\,[14-]}+h_4^U\vec{e}^{\,[23-]}-h_6^U\vec{e}^{\,(13-)}-h_7^U\vec{e}^{\,(14-)}-h_8^U\vec{e}^{\,(23-)}+$$

$$+h_9^U\vec{e}_\omega^{\,(13-)}+h_{10}^U\vec{e}_\omega^{\,(23-)})$$

$$+\vec{U}\cdot(h_2^U\overset{\leftrightarrow}{s}^{[13-]}+h_3^U\overset{\leftrightarrow}{s}^{[14-]}+h_4^U\overset{\leftrightarrow}{s}^{[23-]}+h_6^U\overset{\leftrightarrow}{s}^{(13-)}+h_7^U\overset{\leftrightarrow}{s}^{(14-)}+h_8^U\overset{\leftrightarrow}{s}^{(23-)}+h_9^U\overset{\leftrightarrow}{s}_\omega^{(13-)}+h_{10}^U\overset{\leftrightarrow}{s}_\omega^{(14-)})+$$

$$+\phi_U(-h_2^U\vec{e}^{\,[13-]}-h_3^U\vec{e}^{\,[14-]}+h_4^U\vec{e}^{\,[23-]}-h_6^U\vec{e}^{\,(13-)}-h_7^U\vec{e}^{\,(14-)}-h_8^U\vec{e}^{\,(23-)}+h_9^U\vec{e}_\omega^{\,(13-)}+$$

$$+h_{10}^U\vec{e}_\omega^{\,(23-)}+h_2^U\vec{s}^{\,[13-]}+h_3^U\vec{s}^{\,[14-]}+h_4^U\vec{s}^{\,[23-]}+h_6^U\vec{s}^{\,(13-)}+h_7^U\vec{s}^{\,(14-)}+$$

$$+h_8^U\vec{s}^{\,(23-)}+h_9^U\vec{s}_\omega^{\,(13-)}+h_{10}^U\vec{s}_\omega^{\,(23-)})+$$

$$-\vec{U}(h_{12}^U\overset{[13-]}{z}{}_\alpha^\alpha-h_4^U\overset{[23-]}{z}{}_\alpha^\alpha+h_8^U\overset{(13-)}{z}{}_\alpha^\alpha+h_8^U\overset{(23-)}{z}{}_\alpha^\alpha+h_{12}^U\overset{(13-)}{\omega}{}_\alpha^\alpha+h_{12}^U\overset{(23-)}{\omega}{}_\alpha^\alpha)+$$

$$+\vec{V}^-\times(h_3^-\vec{b}^{\,[12]})+\vec{V}^-\cdot(h_6^-\overset{\leftrightarrow}{s}^{(11)}+h_6^-\overset{\leftrightarrow}{s}^{(22)}+h_7^-\overset{\leftrightarrow}{s}^{(12)}+2h_6^-\overset{\leftrightarrow}{s}^{+-4}+h_7^-\overset{\leftrightarrow}{s}^{[+-]}+h_8^-\overset{\leftrightarrow}{s}_\omega^{(+-)})+$$

$$+\phi_-(h_3^-\vec{e}^{\,[12]}+h_6^-\vec{s}^{\,(11)}+h_6^-\vec{s}^{\,(22)}+h_7^-\vec{s}^{\,(12)}+2h_6^-\vec{s}^{\,+-4}+h_7^-\vec{s}^{\,[+-]}+h_8^-\vec{s}_\omega^{\,(+-)})+$$

$$-\vec{V}^-(h_{11}^-(\overset{(11)}{z}{}_\alpha^\alpha+\overset{(22)}{z}{}_\alpha^\alpha+2\overset{(12)}{z}{}_\alpha^\alpha+2\overset{(11)}{\omega}{}_\alpha^\alpha+2\overset{(22)}{\omega}{}_\alpha^\alpha+4\overset{(12)}{\omega}{}_\alpha^\alpha)+$$

$$+h_{12}^-\overset{[+-]}{z}{}_\alpha^\alpha+h_{13}^-\overset{(+-)}{z}{}_\alpha^\alpha+h_{14}^-\overset{+-4}{z}{}_\alpha^\alpha)$$

$$\rho_+=\phi(h_6^A S_-+h_{12}^A S_\alpha^{\alpha-})+\vec{A}\cdot(h_1^A\vec{E}_-+h_6^A\vec{S}_-)+$$

$$+\phi_U(h_5^U S_-+h_{11}^U S_\alpha^{\alpha-})+\vec{U}\cdot(h_1^U\vec{E}_-+h_5^U\vec{S}_-)+$$

$$+\phi_-(h_4^- S_A+h_5^- S_U+h_9^- S_\alpha^{\alpha 1}+h_{10}^- S_\alpha^{\alpha 2})+$$

$$+\vec{V}_-\cdot(h_1^-\vec{E}_A+h_2^-\vec{E}_U+h_4^-\vec{S}_A+h_5^-\vec{S}_U) \tag{16.24}$$

$$\vec{J}_+=\vec{A}\times(h_1^A\vec{B}_-)+\vec{A}\cdot(h_6^A\overset{\leftrightarrow}{S}_-)+$$

$$+\phi(h_1^A\vec{E}_-+h_6^A\vec{S}_-)-\vec{A}(h_{12}^A S_\alpha^{\alpha-})+\vec{U}\times(h_1^U\vec{E}_-)$$

$$+\vec{U}\cdot(h_5^U\overset{\leftrightarrow}{S}_-)+\phi_U(h_1^U\vec{E}_-+h_5^U\vec{S}_-)+$$

$$-\vec{U}(+h_{11}^U S_\alpha^{\alpha-})+\vec{V}^-\times(h_1^-\vec{B}_A+h_2^-\vec{B}_U)+$$

$$+\vec{V}^-\cdot(h_4^-\overset{\leftrightarrow}{S}_A+h_5^-\overset{\leftrightarrow}{S}_U)+\phi_-(h_1^-\vec{E}_A+h_2^-\vec{E}_U+h_4^-\vec{S}_A+h_5^-\vec{S}_U+)+$$

$$-\vec{V}^-(h_9^- S_\alpha^{\alpha 1}+h_{10}^- S_\alpha^{\alpha 2})$$

16.4 Negative charged photon field- V_μ^-

$$\vec{E}_- = \bar{c}_1^* \vec{E}_+ \tag{16.25}$$

$$\vec{B}_- = \bar{c}_1^* \vec{B}_+ \tag{16.26}$$

$$\vec{e}_- = \bar{c}_2^* (\vec{e}^{\,[+1]} + \vec{e}^{\,[+2]}) \tag{16.27}$$

$$\vec{b}_- = \bar{b}_2 (\vec{b}^{\,[12]} + \vec{b}^{\,[+-]}) + \bar{b}_3 \vec{b}^{\,(+-)} \tag{16.28}$$

$$\vec{S}_- = \bar{c}_3^* S_\alpha^{\alpha+} \tag{16.29}$$

$$\vec{s}_- = \bar{c}_4^* (\overset{(+1)}{z}{}_\alpha^\alpha + \overset{(+2)}{z}{}_\alpha^\alpha) + \bar{c}_5^* (\overset{(+1)}{\omega}{}_\alpha^\alpha + \overset{(+2)}{\omega}{}_\alpha^\alpha) \tag{16.30}$$

$$M_- = -\mu_V^2 \phi_+ \tag{16.31}$$

$$l_- = \phi(h_2^{A*} s^{[13+]} + h_7^{A*} s^{[23+]} + h_8^{A*} s^{(13+)} + h_4^{A*} s^{(23+)} + h_9^{A*} s^{(24+)} + h_{10}^{A*} s_\omega^{(23+)} +$$

$$+ h_{11}^{A*} s_\omega^{(24+)} - h_2^{A*} \overset{[13+]}{z}{}_\alpha^\alpha - h_2^{A*} \overset{[23+]}{z}{}_\alpha^\alpha + h_8^{A*} \overset{(13+)}{z}{}_\alpha^\alpha + h_8^{A*} \overset{(23+)}{z}{}_\alpha^\alpha + h_{13}^{A*} \overset{(13+)}{\omega}{}_\alpha^\alpha + h_{13}^{A*} \overset{(23+)}{\omega}{}_\alpha^\alpha) +$$

$$+ \vec{A} \cdot (-h_2^{A*} \vec{e}^{\,[13+]} - h_3^{A*} \vec{e}^{\,[23+]} + h_5^{A*} \vec{e}^{\,[24+]} + h_4^{A*} \vec{e}^{\,(23+)} - h_9^{A*} \vec{e}^{\,(24+)} + h_{10}^{A*} \vec{e}_\omega^{\,(23+)} + h_{11}^{A*} \vec{e}\,\omega^{(24+)}$$

$$+ h_2^{A*} \vec{s}^{\,[13+]} + h_7^{A*} \vec{s}^{\,[23+]} + h_8^{A*} \vec{s}^{\,(13+)} + h_9^{A*} \vec{s}^{\,(23+)} + h_{10}^{A*} \vec{s}^{\,(24+)} + h_{11}^{A*} \vec{s}_\omega^{\,(23+)} + h_{12}^{A*} \vec{s}_\omega^{\,(24+)}) +$$

$$+ \phi_U (+h_2^{U*} s^{[13+]} + h_3^{U*} s^{[14+]} + h_4^{U*} s^{[23+]} + h_6^{U*} s^{(13+)} + h_7^{U*} s^{(14+)} + h_8^{U*} s^{(23+)} + h_9^{U*} s_\omega^{(13+)} +$$

$$+ h_{10}^{U*} s_\omega^{(23+)} + h_{11}^{U*} S_\alpha^{\alpha-} + h_{12}^{U*} \overset{[13+]}{z}{}_\alpha^\alpha - h_4^{U*} \overset{[23+]}{z}{}_\alpha^\alpha + h_8^{U*} \overset{(13+)}{z}{}_\alpha^\alpha + h_8^{U*} \overset{(23+)}{z}{}_\alpha^\alpha + h_{12}^{U*} \overset{(13+)}{\omega}{}_\alpha^\alpha + h_{12}^{U*} \overset{(23+)}{\omega}{}_\alpha^\alpha) +$$

$$+ \vec{U} \cdot (-h_2^{U*} \vec{e}^{\,[13+]} - h_3^{U*} \vec{e}^{\,[14+]} + h_4^{U*} \vec{e}^{\,[23+]} - h_6^{U*} \vec{e}^{\,(13+)} - h_7^{U*} \vec{e}^{\,(14+)} - h_8^{U*} \vec{e}^{\,(23+)} + h_9^{U*} \vec{e}_\omega^{\,(13+)} +$$

$$+ h_{10}^{U*} \vec{e}_\omega^{\,(23+)} + h_2^{U*} \vec{s}^{\,[13+]} + h_3^{U*} \vec{s}^{\,[14+]} + h_4^{U*} \vec{s}^{\,[23+]} + h_6^{U*} \vec{s}^{\,(13+)} + h_7^{U*} \vec{s}^{\,(14+)} +$$

$$+ h_8^{U*} \vec{s}^{\,(23+)} + h_9^{U*} \vec{s}_\omega^{\,(13+)} + h_{10}^{U*} \vec{s}_\omega^{\,(23+)}) +$$

$$+ \phi_- (h_6^{-*} s^{(11)} + h_6^{-*} s^{(22)} + h_7^{-*} s^{(12)} + 2h_6^{-*} s^{+-4} + h_7^{-*} s^{[+-]} + h_8^{-*} s_\omega^{(+-)} + h_{12}^{-*} \overset{[+-]}{z}{}_\alpha^\alpha +$$

$$+ h_{13}^{-*} \overset{(+-)}{z}{}_\alpha^\alpha + h_{14}^{-*} \overset{+-4}{z}{}_\alpha^\alpha + h_{11}^{-*} (\overset{(11)}{z}{}_\alpha^\alpha + \overset{(22)}{z}{}_\alpha^\alpha + 2\overset{(12)}{z}{}_\alpha^\alpha + 2\overset{(11)}{\omega}{}_\alpha^\alpha + 2\overset{(22)}{\omega}{}_\alpha^\alpha + 4\overset{(12)}{\omega}{}_\alpha^\alpha)) +$$

$$+ \vec{V}_- \cdot (h_3^{-*} \vec{e}^{\,[12]} + h_4^{-*} \vec{S}_A + h_6^{-*} \vec{s}^{\,(11)} + h_6^{-*} \vec{s}^{\,(22)} + h_7^{-*} \vec{s}^{\,(12)} + 2h_6^{-*} \vec{s}^{\,+-4} + h_7^{-*} \vec{s}^{\,[+-]} + h_8^{-*} \vec{s}_\omega^{\,(+-)})$$

(16.32)

$$\vec{l}_- = \vec{A} \times (h_2^{A*} \vec{b}^{\,[13+]} + h_3^{A*} \vec{b}^{\,[23+]} + h_5^{A*} \vec{b}^{\,[24+]} + h_4^{A*} \vec{b}^{\,(23+)} - h_9^{A*} \vec{b}^{\,(24+)} - h_{10}^{A*} \vec{b}_\omega^{\,(23+)} - h_{11}^{A*} \vec{b}_\omega^{\,(24+)}) +$$

$$+ \vec{A} \cdot (h_2^{A*} \overset{\leftrightarrow}{s}{}^{[13+]} + h_7^{A*} \overset{\leftrightarrow}{s}{}^{[23+]} + h_8^{A*} \overset{\leftrightarrow}{s}{}^{(13+)} + h_4^{A*} \overset{\leftrightarrow}{s}{}^{(23+)} + h_9^{A*} \overset{\leftrightarrow}{s}{}^{(24+)} + h_{10}^{A*} \overset{\leftrightarrow}{s}{}_\omega^{(23+)} + h_{11}^{A*} \overset{\leftrightarrow}{s}{}_\omega^{(24+)}) +$$

$$+ \phi(h_2^{A*} \vec{e}^{\,[13+]} + h_3^{A*} \vec{e}^{\,[23+]} + h_5^{A*} \vec{e}^{\,[24+]} + h_4^{A*} \vec{e}^{\,(23+)} - h_{10}^{A*} \vec{e}^{\,(24+)} + h_{11}^{A*} \vec{e}_\omega^{\,(23+)} + h_{12}^{A*} \vec{e}\,\omega^{(24+)}$$

$$+h_2^{A*}\vec{s}^{\,[13+]}+h_7^{A*}\vec{s}^{\,[23+]}+h_8^{A*}\vec{s}^{\,(13+)}+h_4^{A*}\vec{s}^{\,(23+)}+h_9^{A*}\vec{s}^{\,(24+)}+h_{10}^{A*}\vec{s}_{\omega}^{\,(23+)}+h_{11}^{A*}\vec{s}_{\omega}^{\,(24+)})+$$

$$-\vec{A}(-h_2^{A*}\overset{[13+]}{z}{}_{\alpha}^{\alpha}-h_2^{A*}\overset{[23+]}{z}{}_{\alpha}^{\alpha}+h_8^{A*}\overset{(13+)}{z}{}_{\alpha}^{\alpha}+h_8^{A*}\overset{(23+)}{z}{}_{\alpha}^{\alpha}+h_{13}^{A*}\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+h_{13}^{A*}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha})+$$

$$+\vec{U}\times(-h_2^{U*}\vec{e}^{\,[13+]}-h_3^{U*}\vec{e}^{\,[14+]}+h_4^{U*}\vec{e}^{\,[23+]}-h_6^{U*}\vec{e}^{\,(13+)}-h_7^{U*}\vec{e}^{\,(14+)}-h_8^{U*}\vec{e}^{\,(23+)}+$$

$$+h_9^{U*}\vec{e}_{\omega}^{\,(13+)}+h_{10}^{U*}\vec{e}_{\omega}^{\,(23+)})$$

$$+\vec{U}\cdot(h_2^{U*}\overleftrightarrow{s}^{[13+]}+h_3^{U*}\overleftrightarrow{s}^{[14+]}+h_4^{U*}\overleftrightarrow{s}^{[23+]}+h_6^{U*}\overleftrightarrow{s}^{(13+)}+h_7^{U*}\overleftrightarrow{s}^{(14+)}+h_8^{U*}\overleftrightarrow{s}^{(23+)}+h_9^{U*}\overleftrightarrow{s}_{\omega}^{(13+)}+h_{10}^{U*}\overleftrightarrow{s}_{\omega}^{(14+)})+$$

$$+\phi_U(-h_2^{U*}\vec{e}^{\,[13+]}-h_3^{U*}\vec{e}^{\,[14+]}+h_4^{U*}\vec{e}^{\,[23+]}-h_6^{U*}\vec{e}^{\,(13+)}-h_7^{U*}\vec{e}^{\,(14+)}-h_8^{U*}\vec{e}^{\,(23+)}+h_9^{U*}\vec{e}_{\omega}^{\,(13+)}+$$

$$+h_{10}^{U*}\vec{e}_{\omega}^{\,(23+)}+h_2^{U*}\vec{s}^{\,[13+]}+h_3^{U*}\vec{s}^{\,[14+]}+h_4^{U*}\vec{s}^{\,[23+]}+h_6^{U*}\vec{s}^{\,(13+)}+h_7^{U*}\vec{s}^{\,(14+)}+$$

$$+h_8^{U*}\vec{s}^{\,(23+)}+h_9^{U*}\vec{s}_{\omega}^{\,(13+)}+h_{10}^{U*}\vec{s}_{\omega}^{\,(23+)})+$$

$$-\vec{U}(h_{12}^{U*}\overset{[13+]}{z}{}_{\alpha}^{\alpha}-h_4^{U*}\overset{[23+]}{z}{}_{\alpha}^{\alpha}+h_8^{U*}\overset{(13+)}{z}{}_{\alpha}^{\alpha}+h_8^{U*}\overset{(23+)}{z}{}_{\alpha}^{\alpha}+h_{12}^{U*}\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+h_{12}^{U*}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha})+$$

$$+\vec{V}^{-}\times(h_3^{-*}\vec{b}^{\,[12]})+\vec{V}^{-}\cdot(h_6^{-*}\overleftrightarrow{s}^{(11)}+h_6^{-*}\overleftrightarrow{s}^{(22)}+h_7^{-*}\overleftrightarrow{s}^{(12)}+2h_6^{-*}\overleftrightarrow{s}^{+-4}+h_7^{-*}\overleftrightarrow{s}^{[+-]}+h_8^{-*}\overleftrightarrow{s}_{\omega}^{(+-)})+$$

$$+\phi_{-}(h_3^{-*}\vec{e}^{\,[12]}+h_6^{-*}\vec{s}^{\,(11)}+h_6^{-*}\vec{s}^{\,(22)}+h_7^{-*}\vec{s}^{\,(12)}+2h_6^{-*}\vec{s}^{\,+-4}+h_7^{-*}\vec{s}^{\,[+-]}+h_8^{-*}\vec{s}_{\omega}^{\,(+-)})+$$

$$-\vec{V}^{-}(h_{11}^{-*}(\overset{(11)}{z}{}_{\alpha}^{\alpha}+\overset{(22)}{z}{}_{\alpha}^{\alpha}+2\overset{(12)}{z}{}_{\alpha}^{\alpha}+2\overset{(11)}{\omega}{}_{\alpha}^{\alpha}+2\overset{(22)}{\omega}{}_{\alpha}^{\alpha}+4\overset{(12)}{\omega}{}_{\alpha}^{\alpha})+$$

$$+h_{12}^{-*}\overset{[+-]}{z}{}_{\alpha}^{\alpha}+h_{13}^{-*}\overset{(+-)}{z}{}_{\alpha}^{\alpha}+h_{14}^{-*}\overset{+-4}{z}{}_{\alpha}^{\alpha})$$

$$\rho_{-}=\phi(h_6^{A*}S_{-}+h_{12}^{A*}S_{\alpha}^{\alpha-})+\vec{A}\cdot(h_1^{A*}\vec{E}_{-}+h_6^{A*}\vec{S}_{-})+$$

$$+\phi_U(h_5^{U*}S_{-}+h_{11}^{U*}S_{\alpha}^{\alpha-})+\vec{U}\cdot(h_1^{U*}\vec{E}_{-}+h_5^{U*}\vec{S}_{-})+$$

$$+\phi_{-}(h_4^{-*}S_A+h_5^{-*}S_U+h_9^{-*}S_{\alpha}^{\alpha1}+h_{10}^{-*}S_{\alpha}^{\alpha2})+$$

$$+\vec{V}_{-}\cdot(h_1^{-*}\vec{E}_A+h_2^{-*}\vec{E}_U+h_4^{-*}\vec{S}_A+h_5^{-*}\vec{S}_U) \qquad (16.33)$$

$$\vec{J}_{-}=\vec{A}\times(h_1^{A*}\vec{B}_{-})+\vec{A}\cdot(h_6^{A*}\overleftrightarrow{S}_{-})+$$

$$+\phi(h_1^{A*}\vec{E}_{-}+h_6^{A*}\vec{S}_{-})-\vec{A}(h_{12}^{A*}S_{\alpha}^{\alpha-})+\vec{U}\times(h_1^{U*}\vec{E}_{-})$$

$$+\vec{U}\cdot(h_5^{U*}\overleftrightarrow{S}_{-})+\phi_U(h_1^{U*}\vec{E}_{-}+h_5^{U*}\vec{S}_{-})+$$

$$-\vec{U}(+h_{11}^{U*}S_{\alpha}^{\alpha-})+\vec{V}^{-}\times(h_1^{-*}\vec{B}_A+h_2^{-*}\vec{B}_U)+$$

$$+\vec{V}^{-}\cdot(h_4^{-*}\overleftrightarrow{S}_A+h_5^{-*}\overleftrightarrow{S}_U)+\phi_{-}(h_1^{-*}\vec{E}_A+h_2^{-*}\vec{E}_U+h_4^{-*}\vec{S}_A+h_5^{-*}\vec{S}_U+)+$$

$$-\vec{V}^{-}(h_9^{-*}S_{\alpha}^{\alpha1}+h_{10}^{-*}S_{\alpha}^{\alpha2})$$

17 Bianchi relations

17.1 The Collective Symmetric Bianchi Relations

17.1.1 Tensorial Form

$$\partial_\mu z^{(11)}_{\nu\rho} + \partial_\nu z^{(11)}_{\rho\mu} + \partial_\rho z^{(11)}_{\mu\nu} = \gamma_{(11)}\{A_\mu S^1_{\nu\rho} + A_\nu S^1_{\rho\mu} + A_\rho S^1_{\mu\nu}\} \tag{17.1}$$

$$\partial_\mu z^{(12)}_{\nu\rho} + \partial_\nu z^{(12)}_{\rho\mu} + \partial_\rho z^{(12)}_{\mu\nu} = \frac{1}{2}\gamma_{(12)}\{U_\mu S^1_{\nu\rho} + U_\nu S^1_{\rho\mu} + U_\rho S^1_{\mu\nu} +$$

$$+ A_\mu S^2_{\nu\rho} + A_\nu S^2_{\rho\mu} + A_\rho S^2_{\mu\nu}\} \tag{17.2}$$

$$\partial_\mu z^{(22)}_{\nu\rho} + \partial_\nu z^{(22)}_{\rho\mu} + \partial_\rho z^{(22)}_{\mu\nu} = \gamma_{(22)}\{U_\mu S^2_{\nu\rho} + U_\nu S^2_{\rho\mu} + U_\rho S^2_{\mu\nu}\} \tag{17.3}$$

$$\partial_\mu z^{+-3}_{(\nu\rho)} + \partial_\nu z^{+-3}_{(\rho\mu)} + \partial_\rho z^{+-3}_{(\mu\nu)} = \frac{1}{2}\gamma_{(33)}\{V^-_\mu S^+_{\nu\rho} + V^-_\nu S^+_{\rho\mu} + V^-_\rho S^+_{\mu\nu} +$$

$$+ V^+_\mu S^-_{\nu\rho} + V^+_\nu S^-_{\rho\mu} + V^+_\rho S^-_{\mu\nu}\} \tag{17.4}$$

$$\partial_\mu z^{(+1)}_{(\nu\rho)} + \partial_\nu z^{(+1)}_{(\rho\mu)} + \partial_\rho z^{(+1)}_{(\mu\nu)} = \frac{1}{2}(\gamma_{(13)} + i\gamma_{(14)})\{V^+_\mu S^1_{\nu\rho} + V^+_\nu S^1_{\rho\mu} + V^+_\rho S^1_{\mu\nu} +$$

$$+ A_\mu S^+_{\nu\rho} + A_\nu S^+_{\rho\mu} + A_\rho S^+_{\mu\nu}\} \tag{17.5}$$

$$\partial_\mu z^{(-1)}_{(\nu\rho)} + \partial_\nu z^{(-1)}_{(\rho\mu)} + \partial_\rho z^{(-1)}_{(\mu\nu)} = \frac{1}{2}(\gamma_{(13)} - i\gamma_{(14)})\{V^-_\mu S^1_{\nu\rho} + V^-_\nu S^1_{\rho\mu} + V^-_\rho S^1_{\mu\nu} +$$

$$+ A_\mu S^-_{\nu\rho} + A_\nu S^-_{\rho\mu} + A_\rho S^-_{\mu\nu}\} \tag{17.6}$$

$$\partial_\mu z^{(+2)}_{(\nu\rho)} + \partial_\nu z^{(+2)}_{(\rho\mu)} + \partial_\rho z^{(+2)}_{(\mu\nu)} = \frac{1}{2}(\gamma_{(13)} + i\gamma_{(14)})\{V^+_\mu S^2_{\nu\rho} + V^+_\nu S^2_{\rho\mu} + V^+_\rho S^2_{\mu\nu} +$$

$$+ U_\mu S^+_{\nu\rho} + U_\nu S^+_{\rho\mu} + U_\rho S^+_{\mu\nu}\} \tag{17.7}$$

$$\partial_\mu z^{(-2)}_{(\nu\rho)} + \partial_\nu z^{(-2)}_{(\rho\mu)} + \partial_\rho z^{(-2)}_{(\mu\nu)} = \frac{1}{2}(\gamma_{(23)} - i\gamma_{(24)})\{V^-_\mu S^2_{\nu\rho} + V^-_\nu S^2_{\rho\mu} + V^-_\rho S^2_{\mu\nu} +$$

$$+ U_\mu S^-_{\nu\rho} + U_\nu S^-_{\rho\mu} + U_\rho S^-_{\mu\nu}\} \tag{17.8}$$

17.1.2 Vectorial Form

$$\frac{\partial}{\partial t} s^{(11)} + \vec{\nabla}\cdot\vec{s}^{\,(11)} = \frac{1}{2}\gamma_{(11)}(\phi S_{A\alpha}^{\ \alpha} + 2\phi S_A + 2\vec{A}\cdot\vec{S}_A) - \frac{1}{2}\frac{\partial}{\partial t} \overset{(11)}{z}{}_{\alpha}^{\ \alpha} \tag{17.9}$$

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(11)} - \frac{\partial}{\partial t}\vec{s}^{\,(11)} = \frac{1}{2}\gamma_{(11)}(\vec{A}S_{A\alpha}^{\ \alpha} - 2\phi\vec{S}_A + 2\vec{A}\cdot\overset{\leftrightarrow}{S}_A) - \frac{1}{2}\vec{\nabla}\overset{(11)}{z}{}_{\alpha}^{\ \alpha} \tag{17.10}$$

$$\frac{\partial}{\partial t} s^{(12)} + \vec{\nabla}\cdot\vec{s}^{\,(12)} = \frac{1}{4}\gamma_{12}(\phi S_{U\alpha}^{\ \alpha} + \phi_U S_{A\alpha}^{\ \alpha} + 2\phi S_U + 2\phi S_U +$$

$$+ 2\vec{A}\cdot\vec{S}_U + 2\vec{U}\cdot\vec{S}_A) - \frac{1}{2}\frac{\partial}{\partial t}\overset{(12)}{z}{}_{\alpha}^{\ \alpha} \tag{17.11}$$

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(12)} - \frac{\partial}{\partial t}\vec{s}^{\,(12)} = \frac{1}{4}\gamma_{(12)}(\vec{A}S_{U\alpha}^{\ \alpha} + \vec{U}S_{A\alpha}^{\ \alpha} - 2\phi\vec{S}_U - 2\phi_U\vec{S}_A + \tag{17.12}$$

$$+ 2\vec{A}\cdot\overset{\leftrightarrow}{S}_U + 2\vec{U}\cdot\overset{\leftrightarrow}{S}_A) - \frac{1}{2}\vec{\nabla}\overset{(12)}{z}{}_{\alpha}^{\ \alpha}$$

$$\frac{\partial}{\partial t} s^{(22)} + \vec{\nabla}\cdot\vec{s}^{\,(22)} = \frac{1}{2}\gamma_{(22)}(\phi S_{U\alpha}^{\ \alpha} + 2\phi_U S_U + 2\vec{U}\cdot\vec{S}_U) - \frac{1}{2}\frac{\partial}{\partial t}\overset{(22)}{z}{}_{\alpha}^{\ \alpha} \tag{17.13}$$

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(22)} - \frac{\partial}{\partial t}\vec{s}^{\,(22)} = \frac{1}{2}\gamma_{(22)}(\vec{U}S_{U\alpha}^{\ \alpha} - 2\phi_U\vec{S}_U + 2\vec{U}\cdot\overset{\leftrightarrow}{S}_U) - \frac{1}{2}\vec{\nabla}\overset{(22)}{z}{}_{\alpha}^{\ \alpha} \tag{17.14}$$

$$\frac{\partial}{\partial t} s^{+-3} + \vec{\nabla}\cdot\vec{s}^{\,+-3} = \frac{1}{4}\gamma_{(33)}(\phi_- S_{+\alpha}^{\ \alpha} + \phi_+ S_{-\alpha}^{\ \alpha} + 2\phi_- S_+ + 2\phi_+ S_- +$$

$$+ 2\vec{V}^-\cdot\vec{S}_+ + 2\vec{V}^+\cdot\vec{S}_-) - \frac{1}{2}\frac{\partial}{\partial t}\overset{+-3}{z}{}_{\alpha}^{\ \alpha}$$

(17.15)

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{+-3} - \frac{\partial}{\partial t}\vec{s}^{\,+-3} = \frac{1}{4}\gamma_{(33)}(\vec{V}^- S_{+\alpha}^{\ \alpha} + \vec{V}^+ S_{-\alpha}^{\ \alpha} +$$

$$- 2\phi_-\vec{S}_+ - 2\phi_+\vec{S}_- + 2\vec{V}^-\cdot\overset{\leftrightarrow}{S}_+ + 2\vec{V}^+\cdot\overset{\leftrightarrow}{S}_-) - \frac{1}{2}\vec{\nabla}\overset{+-3}{z}{}_{\alpha}^{\ \alpha}$$

$$\frac{\partial}{\partial t}s^{(+1)}+\vec{\nabla}\cdot\vec{s}^{\,(+1)}=\frac{1}{4}(\gamma_{(13)}+i\gamma_{(14)})(\phi_{+}S_{A\alpha}^{\ \alpha}+\phi S_{+\alpha}^{\ \alpha}+2\phi_{+}S_{A}+2\phi S_{+}+2\vec{V}^{+}\cdot\vec{S}_{A}+$$
$$+2\vec{A}\cdot\vec{S}_{+})-\frac{1}{2}\frac{\partial}{\partial t}\overset{(+1)}{z}{}_{\alpha}^{\alpha}$$

(17.16)

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(+1)}-\frac{\partial}{\partial t}\vec{s}^{\,(+1)}=\frac{1}{4}(\gamma_{(13)}+i\gamma_{(14)})(\vec{V}^{+}S_{A\alpha}^{\ \alpha}+\vec{A}S_{+\alpha}^{\ \alpha}-2\phi_{+}\vec{S}_{A}-2\phi\vec{S}_{+}+$$
$$+2\vec{V}^{+}\cdot\overset{\leftrightarrow}{S}_{A}+2\vec{A}\cdot\overset{\leftrightarrow}{S}_{+})-\frac{1}{2}\vec{\nabla}\overset{(+1)}{z}{}_{\alpha}^{\alpha}$$

$$\frac{\partial}{\partial t}s^{(-1)}+\vec{\nabla}\cdot\vec{s}^{\,(-1)}=\frac{1}{4}(\gamma_{(13)}-i\gamma_{(14)})(\phi_{-}S_{A\alpha}^{\ \alpha}+\phi S_{-\alpha}^{\ \alpha}+2\phi_{-}S_{A}+2\phi S_{-}+2\vec{V}^{-}\cdot\vec{S}_{A}+$$
$$+2\vec{A}\cdot\vec{S}_{-})-\frac{1}{2}\frac{\partial}{\partial t}\overset{(-1)}{z}{}_{\alpha}^{\alpha}$$

(17.17)

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(-1)}-\frac{\partial}{\partial t}\vec{s}^{\,(-1)}=\frac{1}{4}(\gamma_{(13)}-i\gamma_{(14)})(\vec{V}^{-}S_{A\alpha}^{\ \alpha}+\vec{A}S_{-\alpha}^{\ \alpha}-2\phi_{-}\vec{S}_{A}-2\phi\vec{S}_{-}+$$
$$+2\vec{V}^{-}\cdot\overset{\leftrightarrow}{S}_{A}+2\vec{A}\cdot\overset{\leftrightarrow}{S}_{-})-\frac{1}{2}\vec{\nabla}\overset{(-1)}{z}{}_{\alpha}^{\alpha}$$

$$\frac{\partial}{\partial t}s^{(+2)}+\vec{\nabla}\cdot\vec{s}^{\,(+2)}=\frac{1}{4}(\gamma_{(23)}+i\gamma_{(24)})(\phi_{+}S_{U\alpha}^{\ \alpha}+\phi_{U}S_{+\alpha}^{\ \alpha}+2\phi_{+}S_{U}+2\phi_{U}S_{+}+2\vec{V}^{+}\cdot\vec{S}_{U}+$$
$$+2\vec{U}\cdot\vec{S}_{+})-\frac{1}{2}\frac{\partial}{\partial t}\overset{(+2)}{z}{}_{\alpha}^{\alpha}$$

(17.18)

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(+2)}-\frac{\partial}{\partial t}\vec{s}^{\,(+2)}=\frac{1}{4}(\gamma_{(23)}+i\gamma_{(24)})(\vec{V}^{+}S_{U\alpha}^{\ \alpha}+\vec{U}S_{+\alpha}^{\ \alpha}-2\phi_{+}\vec{S}_{U}-2\phi_{U}\vec{S}_{+}+$$
$$+2\vec{V}^{+}\cdot\overset{\leftrightarrow}{S}_{U}+2\vec{U}\cdot\overset{\leftrightarrow}{S}_{+})-\frac{1}{2}\vec{\nabla}\overset{(+2)}{z}{}_{\alpha}^{\alpha}$$

$$\frac{\partial}{\partial t}s^{(-2)}+\vec{\nabla}\cdot\vec{s}^{\,(-2)}=\frac{1}{4}(\gamma_{(23)}-i\gamma_{(24)})(\phi_{-}S_{U\alpha}^{\ \alpha}+\phi_{U}S_{-\alpha}^{\ \alpha}+2\phi_{-}S_{U}+2\phi_{U}S_{-}+2\vec{V}^{-}\cdot\vec{S}_{U}+$$

$$+2\vec{U}\cdot\vec{S}_{-})-\frac{1}{2}\frac{\partial}{\partial t}\overset{(-2)}{z}{}_{\alpha}^{\alpha}$$

(17.19)

$$\vec{\nabla}\cdot\overset{\leftrightarrow}{s}{}^{(-2)}-\frac{\partial}{\partial t}\vec{s}^{\,(-2)}=\frac{1}{4}(\gamma_{(23)}-i\gamma_{(24)})(\vec{V}^{-}S_{U\alpha}^{\ \alpha}+\vec{U}S_{-\alpha}^{\ \alpha}-2\phi_{-}\vec{S}_{U}-2\phi_{U}\vec{S}_{-}+$$

$$+2\vec{V}^{-}\cdot\overset{\leftrightarrow}{S}_{U}+2\vec{U}\cdot\overset{\leftrightarrow}{S}_{-})-\frac{1}{2}\vec{\nabla}\overset{(-2)}{z}{}_{\alpha}^{\alpha}$$

18 Bianchi Collective relations

18.1 Antisymmetric Case

$$\partial_{\mu}\overset{+-3}{z}{}_{[\nu\rho]}+\partial_{\nu}\overset{+-3}{z}{}_{[\rho\mu]}+\partial_{\rho}\overset{+-3}{z}{}_{[\mu\nu]}=f_{\mu\nu\rho}^{+-3}$$

where

$$f_{\mu\nu\rho}^{+-3}=\frac{1}{2}\gamma_{(33)}\{V_{\mu}^{-}V_{\nu\rho}^{+}+V_{\nu}^{-}V_{\rho\mu}^{+}+V_{\rho}^{-}V_{\mu\nu}^{+}-V_{\mu}^{+}V_{\nu\rho}^{-}+V_{\nu}^{+}V_{\rho\mu}^{-}-V_{\rho}^{+}V_{\mu\nu}^{+}\} \quad (18.1)$$

$$\partial_{\mu}\overset{(+-)}{z}{}_{[\nu\rho]}+\partial_{\nu}\overset{(+-)}{z}{}_{[\rho\mu]}+\partial_{\rho}\overset{(+-)}{z}{}_{[\mu\nu]}=f_{\mu\nu\rho}^{(+-)}$$

where

$$f_{\mu\nu\rho}^{(+-)}=-\frac{i}{2}\gamma_{(34)}\{V_{\mu}^{-}V_{\nu\rho}^{+}+V_{\nu}^{-}V_{\rho\mu}^{+}+V_{\rho}^{-}V_{\mu\nu}^{+}-V_{\mu}^{+}V_{\nu\rho}^{-}+V_{\nu}^{+}V_{\rho\mu}^{-}-V_{\rho}^{+}V_{\mu\nu}^{+}\} \quad (18.2)$$

$$\partial_{\mu}\overset{[+-]}{z}{}_{[\nu\rho]}+\partial_{\nu}\overset{[+-]}{z}{}_{[\rho\mu]}+\partial_{\rho}\overset{[+-]}{z}{}_{[\mu\nu]}=f_{\mu\nu\rho}^{[+-]}$$

where

$$f_{\mu\nu\rho}^{[+-]}=-\frac{i}{2}\gamma_{[34]}\{V_{\mu}^{-}V_{\nu\rho}^{+}+V_{\nu}^{-}V_{\rho\mu}^{+}+V_{\rho}^{-}V_{\mu\nu}^{+}-V_{\mu}^{+}V_{\nu\rho}^{-}+V_{\nu}^{+}V_{\rho\mu}^{-}-V_{\rho}^{+}V_{\mu\nu}^{+}\} \quad (18.3)$$

$$\partial_{\mu}\overset{(+1)}{z}{}_{[\nu\rho]}+\partial_{\nu}\overset{(+1)}{z}{}_{[\rho\mu]}+\partial_{\rho}\overset{(+1)}{z}{}_{[\mu\nu]}=f_{\mu\nu\rho}^{(+1)}$$

where

$$f_{\mu\nu\rho}^{(+1)}=\frac{1}{2}(\gamma_{(13)}+i\gamma_{(14)})\{V_{\mu}^{+}F_{\nu\rho}+V_{\nu}^{+}F_{\rho\mu}+V_{\rho}^{+}F_{\mu\nu}-A_{\mu}V_{\nu\rho}^{+}+A_{\nu}V_{\rho\mu}^{+}-A_{\rho}V_{\mu\nu}^{+}\}$$

$$\partial_\mu \overset{(+2)}{z}_{[\nu\rho]} + \partial_\nu \overset{(+2)}{z}_{[\rho\mu]} + \partial_\rho \overset{(+2)}{z}_{[\mu\nu]} = f^{(+2)}_{\mu\nu\rho}$$

where

$$f^{(+2)}_{\mu\nu\rho} = \frac{1}{2}(\gamma_{(23)} + i\gamma_{(24)})\{V^+_\mu F_{\nu\rho} + V^+_\nu F_{\rho\mu} + V^+_\rho F_{\mu\nu} - A_\mu V^+_{\nu\rho} + A_\nu V^+_{\rho\mu} - A_\rho V^+_{\mu\nu}\}$$

$$\partial_\mu \overset{[12]}{z}_{[\nu\rho]} + \partial_\nu \overset{[12]}{z}_{[\rho\mu]} + \partial_\rho \overset{[12]}{z}_{[\mu\nu]} = f^{[12]}_{\mu\nu\rho}$$

where

$$f^{[12]}_{\mu\nu\rho} = \frac{1}{2}\gamma_{[12]}\{U_\mu F_{\nu\rho} + U_\nu F_{\rho\mu} + U_\rho F_{\mu\nu} - A_\mu U_{\nu\rho} + A_\nu U_{\rho\mu} - A_\rho U_{\mu\nu}\}$$

$$\partial_\mu \overset{(12)}{z}_{[\nu\rho]} + \partial_\nu \overset{(12)}{z}_{[\rho\mu]} + \partial_\rho \overset{(12)}{z}_{[\mu\nu]} = f^{(12)}_{\mu\nu\rho}$$

where

$$f^{(12)}_{\mu\nu\rho} = \frac{1}{2}\gamma_{(12)}\{U_\mu F_{\nu\rho} + U_\nu F_{\rho\mu} + U_\rho F_{\mu\nu} - A_\mu U_{\nu\rho} + A_\nu U_{\rho\mu} - A_\rho U_{\mu\nu}\}$$

$$\partial_\mu \overset{+-4}{z}_{[\nu\rho]} + \partial_\nu \overset{+-4}{z}_{[\rho\mu]} + \partial_\rho \overset{+-4}{z}_{[\mu\nu]} = g^{+-4}_{\mu\nu\rho}$$

where

$$g^{+-4}_{\mu\nu\rho} = \frac{1}{2}\gamma_{(44)}\{V^-_\mu V^+_{\nu\rho} + V^-_\nu V^+_{\rho\mu} + V^-_\rho V^+_{\mu\nu} - V^+_\mu V^-_{\nu\rho} - V^+_\nu V^-_{\rho\mu} - V^+_\rho V^-_{\mu\nu}\} \quad (18.4)$$

18.2 Symmetric Case

$$\partial_\mu \overset{+-3}{z}_{(\nu\rho)} + \partial_\nu \overset{+-3}{z}_{(\rho\mu)} + \partial_\rho \overset{+-3}{z}_{(\mu\nu)} = g^{+-3}_{\mu\nu\rho}$$

where

$$g^{+-3}_{\mu\nu\rho} = \frac{1}{2}\gamma_{(33)}\{V^-_\mu S^+_{\nu\rho} + V^-_\nu S^+_{\rho\mu} + V^-_\rho S^+_{\mu\nu} + V^+_\mu S^-_{\nu\rho} + V^+_\nu S^-_{\rho\mu} + V^+_\rho S^-_{\mu\nu}\} \quad (18.5)$$

$$\partial_\mu \overset{(+-)}{z}_{(\nu\rho)} + \partial_\nu \overset{(+-)}{z}_{(\rho\mu)} + \partial_\rho \overset{(+-)}{z}_{(\mu\nu)} = g^{(+-)}_{\mu\nu\rho}$$

where

$$g^{(+-)}_{\mu\nu\rho} = -\frac{i}{2}\gamma_{(34)}\{V^-_\mu S^+_{\nu\rho} + V^-_\nu S^+_{\rho\mu} + V^-_\rho S^+_{\mu\nu} + V^+_\mu S^-_{\nu\rho} + V^+_\nu S^-_{\rho\mu} + V^+_\rho S^-_{\mu\nu}\} \quad (18.6)$$

$$\partial_\mu \overset{[+-]}{z}_{(\nu\rho)} + \partial_\nu \overset{[+-]}{z}_{(\rho\mu)} + \partial_\rho \overset{[+-]}{z}_{(\mu\nu)} = g^{[+-]}_{\mu\nu\rho}$$

where

$$g^{[+-]}_{\mu\nu\rho} = -\frac{i}{2}\gamma_{[34]}\{V^-_\mu S^+_{\nu\rho} + V^-_\nu S^+_{\rho\mu} + V^-_\rho S^+_{\mu\nu} + V^+_\mu S^-_{\nu\rho} + V^+_\nu S^-_{\rho\mu} + V^+_\rho S^-_{\mu\nu}\} \quad (18.7)$$

$$\partial_\mu \overset{(+1)}{z}_{(\nu\rho)} + \partial_\nu \overset{(+1)}{z}_{(\rho\mu)} + \partial_\rho \overset{(+1)}{z}_{(\mu\nu)} = g^{(+1)}_{\mu\nu\rho}$$

where

$$g^{(+1)}_{\mu\nu\rho} = \frac{1}{2}(\gamma_{(13)} + i\gamma_{(14)})\{V^+_\mu S^1_{\nu\rho} + V^+_\nu S^1_{\rho\mu} + V^+_\rho S^1_{\mu\nu} + A_\mu S^+_{\nu\rho} + A_\nu S^+_{\rho\mu} + A_\rho S^+_{\mu\nu}\}$$

$$\partial_\mu \overset{(+2)}{z}_{(\nu\rho)} + \partial_\nu \overset{(+2)}{z}_{(\rho\mu)} + \partial_\rho \overset{(+2)}{z}_{(\mu\nu)} = g^{(+2)}_{\mu\nu\rho}$$

where

$$g^{(+2)}_{\mu\nu\rho} = \frac{1}{2}(\gamma_{(23)} + i\gamma_{(24)})\{V^+_\mu S^2_{\nu\rho} + V^+_\nu S^2_{\rho\mu} + V^+_\rho S^2_{\mu\nu} + U_\mu S^+_{\nu\rho} + U_\nu S^+_{\rho\mu} + U_\rho S^+_{\mu\nu}\}$$

$$\partial_\mu \overset{[12]}{z}_{(\nu\rho)} + \partial_\nu \overset{[12]}{z}_{(\rho\mu)} + \partial_\rho \overset{[12]}{z}_{(\mu\nu)} = g^{[12]}_{\mu\nu\rho}$$

where

$$g^{[12]}_{\mu\nu\rho} = \frac{1}{2}\gamma_{[12]}\{U_\mu S^1_{\nu\rho} + U_\nu S^1_{\rho\mu} + U_\rho S^1_{\mu\nu} + A_\mu S^2_{\nu\rho} + A_\nu S^2_{\rho\mu} + A_\rho S^2_{\mu\nu}\}$$

$$\partial_\mu \overset{(12)}{z}_{(\nu\rho)} + \partial_\nu \overset{(12)}{z}_{(\rho\mu)} + \partial_\rho \overset{(12)}{z}_{(\mu\nu)} = g^{(12)}_{\mu\nu\rho}$$

where

$$g^{(12)}_{\mu\nu\rho} = \frac{1}{2}\gamma_{(12)}\{U_\mu S^1_{\nu\rho} + U_\nu S^1_{\rho\mu} + U_\rho S^1_{\mu\nu} + A_\mu S^2_{\nu\rho} + A_\nu S^2_{\rho\mu} + A_\rho S^2_{\mu\nu}\}$$

$$\partial_\mu \overset{11}{z}_{\nu\rho} + \partial_\nu \overset{11}{z}_{\rho\mu} + \partial_\rho \overset{11}{z}_{\mu\nu} = g^{11}_{\mu\nu\rho}$$

where

$$g^{11}_{\mu\nu\rho} = \gamma_{11}\{A_\mu S^1_{\nu\rho} + A_\nu S^1_{\rho\mu} + A_\rho S^1_{\mu\nu}\}$$

$$\partial_\mu \overset{22}{z}_{\nu\rho} + \partial_\nu \overset{22}{z}_{\rho\mu} + \partial_\rho \overset{22}{z}_{\mu\nu} = g^{12}_{\mu\nu\rho}$$

where

$$g^{22}_{\mu\nu\rho} = \gamma_{22}\{U_\mu S^2_{\nu\rho} + U_\nu S^2_{\rho\mu} + U_\rho S^2_{\mu\nu}\}$$

$$\partial_\mu \overset{+-4}{z}_{(\nu\rho)} + \partial_\nu \overset{+-4}{z}_{(\rho\mu)} + \partial_\rho \overset{+-4}{z}_{(\mu\nu)} = g^{+-4}_{\mu\nu\rho}$$

where

$$g^{+-4}_{\mu\nu\rho} = \frac{1}{2}\gamma_{(44)}\{V^-_\mu S^+_{\nu\rho} + V^-_\nu S^+_{\rho\mu} + V^-_\rho S^+_{\mu\nu} + V^+_\mu S^-_{\nu\rho} + V^+_\nu S^-_{\rho\mu} + V^+_\rho S^-_{\mu\nu}\} \qquad (18.8)$$

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