Dust-Charge Variation Effects on Dust ion Acoustic Shock Waves in Four Component Quantum Plasma

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Abstract

The behavior of nonlinear quantum dust ion acoustic (QDIA) shock waves in a collisionless, unmagnetized plasma consisting of inertialess quantum electrons and positrons, classical cold ions and stationary negatively charged dust grains with dust charge variation is investigated using quantum hydrodynamic (QHD) equations. The propagation of small amplitude QDIA shock waves is governed by Burgers equation. It is shown that the dust charge variation plays an important role in the formation of such QDIA shock structures. The dependence of the shock wave's amplitude and thickness on the chemical potential is investigated. The present theory is applicable to analyze the formation of nonlinear structures at quantum scales in dense astrophysical objects.

Indexing terms/Keywords

Quantum plasma, Dust ion acoustic shock wave, Burgers equation, Dust charge variation, Quantum hydrodynamic

Academic Discipline And Sub-Disciplines

Physics; Plasma Physics; Quantum Plasma

Introduction

In recent years, a huge number of works have been devoted to the investigation of the collective behavior of quantum plasmas [1]. Extremely dense plasmas behave like a quantum ideal gas, due to the exclusion principle. However, also dilute charged particle systems can exhibit quantum features, provided the dimensions of the system are small enough. Small enough here means dimensions comparable to the de Broglie wavelength, $\lambda_B = \frac{h}{\sqrt{2\pi m k_B T}}$, where h is Planck's constant, T is thermodynamic temperature, m is the mass of charge particle, and kB is Boltzmann's constant. For classical regimes, the de Broglie wavelength is so small that particles can be considered as point like; therefore there is no overlapping of the wave functions and no quantum interference. On this basis there is reasonable to postulate that quantum effects start playing a significant role when de Broglie wavelength is similar to or larger than the average interparticle distance, $n^{-1/3}$, i.e., when $n\lambda_B^3 \geq 1$. On the other hand, from the statistical mechanics of ordinary gases [2] that quantum effects become important when the thermal temperature is lower than the so-called Fermi temperature T_F which is related to the equilibrium density of the charged particles (n_0) [3]. The dense quantum plasmas exist in microelectronic devices [4], in intense laser–solid density plasma interaction experiments [5], in laser fusion plasma [6], in quantum diodes [7] and in astrophysical environments [8,9]. Recently, there has been a growing interest in investigating new aspects of dense quantum plasmas by developing the quantum hydrodynamic (QHD) [10-12] and quantum kinetic equations [13,14] by incorporating the quantum force associated with the Bohm potential. The Wigner–Poisson (WP) model has been used to derive a set of QHD equations for a dense quantum plasma. The (QHD) model generalizes the fluid model with the inclusion of quantum statistical pressure and quantum diffraction (also known as Bohm potential) term.

One of the interesting quantum plasma model that is investigated by many researchers is electron-positron-ion (e-p-i) quantum plasmas, because it exists in astrophysical environments as well as in laser solid matter interaction plasmas [15,16]. Khan and Haque (2008) studied nonlinear structures in dissipative e-p-i quantum plasmas [17]. Ion acoustic shock waves in quantum e-p-i plasmas have been studied by K. Roy et al. (2008). They found that the KdVB equation has either oscillatory or monotonic shock wave solutions depending on the system parameters proportional to quantum diffraction [18]. On the other hand, most of the astrophysical and laboratory plasmas usually contain highly charged (negative/positive) impurities or dust particles in addition to electrons, positrons and ions; for instance, in different environments of low temperature laboratory, tokamak edges, interplanetary spaces, cometary tails and in the planetary ring systems. When the dust grains are immersed into a plasma, they are usually charged due to the absorption of the charged particles [19,20]. The plasma with the negatively charged dust grains may be regarded as simply multispecies plasma for process with a time scale longer than the characteristic time of grain-charging. Many of interesting investigations pertaining to the dusty plasma fall in this category. Shukla and Ali (2005) used the quantum hydrodynamic model for plasma and derived a new dispersion for the dust acoustic wave in quantum plasma [21]. Misra (2009) studied dust-ion acoustic (DIA) shock waves in quantum dusty pair ion plasmas. He derived the KdV-B equation and showed that both oscillatory and monotonic shocks depending not only on the viscosity parameter but also on the quantum parameter H [22]. The quantum DIA shock waves in an unmagnetized, collisionless four component quantum plasma was studied by M. R. Rouhani (2014) et al.; They investigated the effects of dust grain density as well as quantum diffraction parameter and dissipation parameter on these waves [23]. However, an important distinctive feature of a dusty plasma is the grain-charge variations arising due to the wave motion induced oscillations in the plasma currents that flow to the grains. Clearly, the dust charge is a time-dependent quantity and one has to treat it as a dynamical variable in the plasma. The study of collective effects in dusty plasmas is of significant interest. Duha and Mamun (2009) have investigated the nonlinear propagation of DIA waves in a dusty plasma containing Boltzmann electrons, mobile ions and charge fluctuating stationary Dust, has been investigated. They showed that the dust charge variation is a source of dissipation and is responsible for the formation of the dust-ion acoustic shock waves [24]. Hongyan and kaibiao (2015) studied the nonlinear propagation of DIA shock waves in a dusty multi-ion plasma with negatively dust charge variation. They found that the dust charge variation plays an important role in the formation of such DIA shock structures [25]. Hossain et al. investigated the progation of nonplannar non-linear DIA waves by the reductive perturbation method, in a dusty multi-ion plasma, which leads to the the derivation of the Burgers [Korteweg de-Vries (K-dV)] equation for the shock (solitary) wave propagation. The shock (solitary) waves are found to be formed in a multi-ion dusty plasma due to the balance between nonlinearity and dissipation (dispersion), and that the dissipation (dispersion) arises due to the dust charge fluctuation (deviation of charge neutrality condition) [26].

In the other words the dust charge variation is one of the main effects governing the structure of nonlinear DIA waves. The other publications in this field are: Alinejad (2010) [27], Amour et.al. (2012) [28].

To the best of our knowledge, no investigation for nonlinear structures in four component quantum plasma with consideration of dust charge variation has been made.

On the other hand, the nonlinear wave propagation of any plasma system depends on the velocity distribution of the corresponding plasma species. The Maxwellian velocity distribution is considered as the usual distribution in a collisionless plasma [29]. But it is often noticed that the velocity distribution of plasma particles in space and laboratory are not exactly Maxwellian and may be deviated from that [30,31]. In quantum plasmas, the Fermi-Dirac statistical distribution is usually employed rather than the widely used Boltzman-Maxwell distribution in classical plasmas. Therefore, one would expect a great deal of interest effects to follow from the inclusion of the grain charge evolution and Fermi-distribution function for quantum particles.

in this paper, using the QHD model, we investigate the DIA shock waves in an unmagnetized collisionless four component quantum plasma consisting of inertialess quantum electrons and positrons with Fermi-distribution function, classical cold ions and stationary negative-dust-charge variation. The rate of dust grain charge variation is determined by the current associated with the electrons, positrons and ions in the plasma.

Our manuscript is organized as follows: the basic set of equations for QDIA shock waves in e-p-i dust plasma is presented; the dust charge variation is studied and the currents of particles in model plasma are calculated; Derivation of Burgers equation is given using the reductive perturbation method [32]; the solution of the Burgers equation is investigated and then the main findings of the paper are presented.

Basic equations:

Let us consider a homogeneous system of an unmagnetized, collisionless and dissipative dusty plasma composed of inertialess quantum electrons and positrons, classical cold ions, and stationary negatively-charged dust grains with dust charge variations. The effect of the kinematic viscosity of the ion fluid and the thermal effect for ions are also taken to be account. Therefore, assume that the electrons and positrons agree the equation of state for a one dimensional zero-temperature Fermi gas $P_j = 2k_B T_{Fj} n_j^3 / 3n_{j0}^2$, Where $T_{Fj} = \hbar^2 (3\pi^2 n_j)^{2/3} / (2m_j k_B)$ is the Fermi temperature, k_B is the Boltzmann constant, and nj0 is the equilibrium number density of the jth species (j=e, p). The quasi-neutrality condition in the equilibrium state can be written as

$$n_{i0} + n_{p0} - n_{e0} - n_{d0}\frac{Q_{d0}}{e} = 0, \tag{1}$$

Where n_{d0} and Q_{d0} are, respectively, equilibrium number density and equilibrium charge of dust grain.

The nonlinear dynamics of the dust ion acoustic waves in the quantum plasma system are governed by the following normalized QHD equations:

$$\partial_t n_i + \partial_x (n_i u_i) = 0 \tag{2}$$

$$(\partial_t - \eta_i \partial_x^2) u_i + u\partial_x u_i = -\partial_x \varphi \tag{3}$$

$$\partial_x \varphi - n_e \partial_x n_e + \kappa \frac{H^2}{2} \partial_x \left[\frac{\partial_x^2 \sqrt{n_e}}{\sqrt{n_e}} \right] = 0 \tag{4}$$

$$-\partial_x\varphi-\sigma n_p\partial_x n_p+\kappa\frac{H^2}{2}\partial_x\left[\frac{\partial_x^2\sqrt{n_p}}{\sqrt{n_p}}\right]=0 \tag{5}$$

$$\partial_x^2\varphi=(1-d)\alpha_d n_e+(1-d)(1-\alpha_d)n_p-n_i+d\frac{Q_d}{Q_{d0}} \tag{6}$$

Where $d=n_{d0}Q_{d0}/en_{i0}$, $\alpha_d=e_i/(1-d)=1+(p_i/(1-d))$, $e_i=n_{e0}/n_{i0}$, $p_i=n_{p0}/n_{i0}$, $\sigma=T_{Fp}/T_{Fe}$ and $m_e=m_p=m$, $\kappa=n_{i0}/n_{e0}$, $\eta_i=\beta_i\omega_{pi}/c_i^2$, β_i is kinematic viscosity of ions. The nondimensional quantum diffraction parameter H is defined as $H=\hbar\omega_{pe}/k_BT_{Fe}$ that $\omega_{pe}=(4\pi n_{e0}e^2/m_e)^{1/2}$.

The number density, n_j (j=e, i, p, d), of the jth species is normalized by their unperturbed density n_{j0}, the electrostatic wave potential φ is normalized by k_BT_{Fe}/e ,the fluid ion velocity u by the quantum ion-acoustic speed $c_i=(k_BT_{Fe}/m_i)^{1/2}$, the space and time coordinates x and t are normalized, respectively, by the quantum Debye length $\lambda_D=(k_BT_{Fe}/4\pi e^2 n_{i0})^{1/2}$ and the ion plasma period $\omega_{pi}^{-1}=(4\pi n_{i0}e^2/m_i)^{-1/2}$.

Dust charge variations:

The elementary processes that lead to the charging of dust grains are quite complex and depend mainly on the environment around the dust grains. When dust grains are immersed in a gaseous plasma, the plasma particles (electrons, positrons and ions) are collected by the dust grains which act as probes. The dust grains, therefore, charged by the collection of the plasma particles flowing onto their surfaces [19].

The charge of dust grains depends on the number density and thermal speed of charged particles. The dust grains can acquire the negative charge when the number density of the electrons is larger than that of the positrons.

The charged particles in a plasma respond to perturbations that are applied externally and can fluctuate. The dust grain charge Q_d is determined by $dQ_d/dt=\sum_j I_j$ where j represents the plasma species (i.e., j=e, p and i) and I_j is the current associated with the species j. The charging current Ij to the dust grain carried by the plasma particle j has been calculated with the orbit limited motion approach [22]

$$I_j(\vec{r},t,q_d)=q_j\int v\sigma_j(q_d,v)f_j(\vec{r},v,t)d^3p \tag{7}$$

Where f_j and qj are, respectively, the velocity distribution and charge of the plasma species j and σj is the cross section for charging collisions between the dust and the plasma particle species j. distribution function for classical particles (i.e., ions) is Maxwell-Boltzmann distribution and for quantum particles (i.e., electrons and positrons) is Fermi-Dirac distribution as defined as:

$$\begin{aligned} &f_i(\varepsilon)\propto\frac{1}{e^{\beta(\varepsilon-\mu_i)}},\\ &f_j(\varepsilon)\propto\frac{1}{e^{\beta(\varepsilon-\mu_j)}+1}\qquad j=e,p \end{aligned} \tag{8}$$

Therefore, the charging currents of electrons, positrons and ions to be obtained as follows:

$$I_e=-er_d^2n_eV_{Te}\left((\mu_e+\varphi_d)^2+\frac{\pi^2}{3}\right) \tag{9}$$

$$I_p=er_d^2n_pV_{Tp}\left(\mu_p^2-2\mu_p\varphi_d+\frac{\pi^2}{3}\right) \tag{10}$$

$$I_i=4\pi r_d^2n_ieV_{Ti}\left(1-\varphi_d\left(\frac{T_e}{T_i}\right)\right) \tag{11}$$

Where r_d is the radius of the dust grain, $T_e=T_p=T$ is the thermal temperature of electrons and positrons. μ_e *and* μ_p are, respectively, chemical potential of electrons and positrons that are normalized by k_BT and $\varphi_d=\frac{Q_d}{r_d}$ is the electrostatic potential of dust particles and is normalized by k_BT/e.

Note that, to obtain Ie and Ip in relations (9) and (10), the integral in (7) has been solved for large values of μ.

Summation of equilibrium currents; say, $I_{j0}=I_j(Q_d=Q_{d0})$, are zero

$$I_{e0}+I_{p0}+I_{i0}=0\qquad ,$$

Then the equation of dust charge variation in terms of perturbed currents is:

$$\frac{dq_d}{dt} = I_{e1} + I_{p1} + I_{i1} \quad . \tag{12}$$

Where, I_{j1} are the perturbed currents that normalized by I_{i0}:

$$\begin{aligned}
&I_{j1} = I_j - I_{j0} \\
&I_{e1} = I_{e0}\left[\delta n_e + n_e(Aq_d^2 + Bq_d)\right] \\
&I_{p1} = I_{p0}\left[\delta n_p - n_p C q_d\right] \\
&I_{i1} = \left[\delta n_i - n_i D q_d\right] \\
&\text{with } \delta n_j = n_j - 1, \text{where } j = e, p, i
\end{aligned} \quad . \tag{13}$$

The coefficients A, B, C and D are the functions of plasma parameters as follow:

$$A = \frac{\varphi_{d0}^2}{(\mu_e + \varphi_d)^2 + \frac{\pi^2}{3}} \tag{14}$$

$$B = \frac{2\left(\varphi_{d0}^2 + \mu_e \varphi_{d0}\right)}{(\mu_e + \varphi_{d0}) + \frac{\pi^2}{3}} \tag{15}$$

$$C = \frac{2\mu_p \varphi_{d0}}{\mu_p^2 - 2\mu_p \varphi_{d0} + \frac{\pi^2}{3}} \tag{16}$$

$$D = \frac{\varphi_{d0}}{1 - \varphi_{d0}\frac{T_e}{T_i}} \quad . \tag{17}$$

Derivation of Burgers Equation:

To study the dynamics of small amplitude quantum dust ion acoustic shock waves, we use two stretched coordinates, $\xi = \varepsilon(x - \lambda t)$ and $\tau = \varepsilon^2 t$ **.** Where λ is the phase velocity normalized to the fixed acoustic speed ci and the slow varying time scale is incorporated in the ε^2 dependence of τ **.**The perturbed variables are expended about their equilibrium values as power series:

$$\begin{aligned}
&n_j = 1 + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + ..., \\
&u_i = 0 + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + ..., \\
&\varphi = 0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + ..., \\
&q_d = \varepsilon q_{d1} + \varepsilon^2 q_{d2} + ...
\end{aligned} \tag{18}$$

Note that, q_d is the perturbed charge of dust particle that normalized by Q_{d0}. Substituting the expansions (18) into the one dimensional Eqs. (2)-(6) and (12), we have a set of equations in the form of lowest-order power series on ε **,**

$$\begin{aligned}
&u_{i1} = \frac{\varphi_1}{\lambda}, \\
&n_{i1} = (1-d)\left[\alpha_d - \frac{(1-\alpha_d)}{\sigma}\right]\varphi_1, \\
&n_{p1} = \frac{-\varphi_1}{\sigma}, \\
&n_{e1} = \varphi_1. \\
&q_{d1} = \frac{I_{e0} n_{e1} + I_{p0} n_{p1} + n_{i1}}{\gamma}
\end{aligned} \tag{19}$$

Where,

$$\begin{aligned}
&\gamma = -BI_{e0} + CI_{p0} + D, \\
&\lambda = \pm\left[\frac{1 - d/\gamma}{(1-d)\left\{\alpha_d - \frac{(1-\alpha_d)}{\sigma}\right\} + d\left(\frac{I_{e0} - I_{p0}/\sigma}{\gamma}\right)}\right]^{1/2} .
\end{aligned} \tag{20}$$

This shows that the QDIA wave can propagate left or right depending on the negative or positive sign of λ.

The following equations are the next higher order equations in ε

$$-\lambda\partial_\xi n_{i2} + \partial_\xi u_{i2} = -\partial_\tau n_{i1} - \partial_\xi \left(n_{i1}u_{i1}\right) \tag{21}$$

$$\partial_\xi \varphi_2 - \lambda\partial_\xi u_{i2} = -u_1\partial_\xi u_{i1} - \partial_\tau u_{i1} + \eta_i \partial_\xi^2 u_{i1} \tag{22}$$

$$\partial_\xi \varphi_2 - \partial_\xi n_{e2} = n_{e1}\partial_\xi n_{e1} \tag{23}$$

$$\partial_\xi \varphi_2 + \sigma\partial_\xi n_{p2} = -\sigma n_{p1}\partial_\xi n_{p1} \tag{24}$$

$$(1-d)\alpha_d n_{e2} + (1-d)(1-\alpha_d)n_{p2} - n_{i2} + dq_{d2} = 0 \tag{25}$$

$$\begin{aligned} &-\lambda\partial_\xi q_{d1} = I_{e0}\left[n_{e2} + Aq_{d1}^2 + Bn_{e1}q_{d1} + Bq_{d2}\right] + \\ &I_{p0}\left[n_{p2} - Cn_{p1}q_{d1} - Cq_{d2}\right] + \left[n_{i2} - Dn_{i1}q_{d1} - Dq_{d2}\right] = 0 \end{aligned} \tag{26}$$

Eliminating the second order quantities from Eqs. (21)- (26) by using the first order relations, one can derive the Burgers equation for dust-ion acoustic waves as described in following:

$$\partial_\tau \varphi_1 + P\varphi_1\partial_\xi\varphi_1 + Q\partial_\xi^2\varphi_1 = 0 \tag{27}$$

Where the coefficients of the nonlinear (P) and dissipative (Q) are modified by dust charge variation as follows:

$$\begin{aligned} P &= P_0 + P_1 \\ Q &= Q_0 + Q_1 \end{aligned} \tag{28}$$

Where, P_0 and Q_0 are, respectively, the nonlinear and dissipative coefficients in absence of dust charge variations while P_1 and Q_1 are, respectively, the correction of nonlinear and dissipative coefficients due to dust charge variations

$$\begin{aligned} P_0 &= \frac{3}{2\lambda} + \frac{\lambda^3}{2}(1-d)\left(\mu_d + \frac{(1-\mu_d)}{\sigma^2}\right) \\ P_1 &= \frac{1}{N}\left[\begin{array}{l} \frac{2\delta}{\gamma}\left(BI_{e0} + C\frac{I_{p0}}{\sigma} - D\frac{I_{i0}}{\lambda^2}\right) - \left(I_{e0} + \frac{I_{p0}}{\sigma^2}\right) \\ + \frac{2A\delta^2}{\gamma^2}I_{e0} - I_{i0}(1-d)\left(\mu_d + \frac{(1-\mu_d)}{\sigma^2}\right) \end{array}\right], \\ Q_0 &= -\frac{\eta_i}{2} \\ Q_1 &= \frac{\lambda\delta}{\gamma N}. \end{aligned} \tag{29}$$

With $N = \frac{2}{d\lambda^3}\left(BI_{e0} - CI_{p0} + (d-D)\right)$ and $\delta = \left(I_{e0} - \frac{I_{p0}}{\sigma} + \frac{1}{\lambda^2}\right)$.

Since the sign of Q_0, due to the viscosity, is opposite to Q_1, due to dust charge variation, One can find that, the thickness of shock wave decreases with increase in dust charge variation and in the other words the sharper shock waves is obtained.

Solution of Burgers equation:

In this section, we will find the analytical solution of Burgers equation described in Eq. (27). By using the transformation $\rho = \xi - u_0\tau$, where u_0 is the normalized constant speed of the wave frame, the stationary solution of the Burgers equation is obtained from:

$$-u_0\frac{d\phi}{d\rho} + \frac{A}{2}\frac{d\phi^2}{d\rho} + Q\frac{d^2\phi}{d\rho^2} = 0 \tag{30}$$

Integrating Eq. (30) with boundary conditions; $\phi = 0, \partial_\rho\phi = 0, \partial_\rho^2\phi = 0$ for $\rho \to +\infty$; we have:

$$\left(\frac{A}{2}\phi - u_0\right)\phi = -Q\frac{d\phi}{d\rho}, \tag{31}$$

Therefore,

$$\int_{\phi_m}^{\phi}\frac{d\phi}{\phi(1-\frac{A}{2u_0}\phi)} = \int_0^\rho \frac{u_0}{Q}d\rho\,. \tag{32}$$

Where $\phi_m = \frac{2u_0}{A}$. One can obtain the monotonic shock structure solution of Eq. (32) as follow:

$$\phi(\rho) = \frac{u_0}{P}\left\{1 + \tanh\left(\frac{u_0}{2Q}\right)\rho\right\} \tag{33}$$

Where, $\frac{2Q}{u_0}$ and $\frac{u_0}{P}$ are, respectively, thickness and amplitude of the shock wave. It is clear that dust charge variation effect on the thickness and amplitude of shock wave by coefficients P and Q. The effect of chemical potential on shock profile of QDIA shock waves in model plasma is shown in Fig. 1. The amplitude of the shock increases and it's thickness decreases with increase in μ, as shown in Figs. 2 and 3.

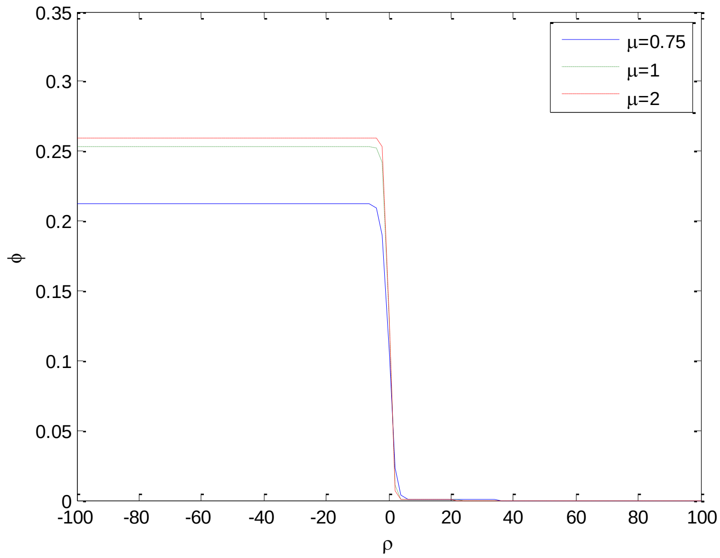

Fig.1. shock solution for different values of μe with:

P=1.3, d=0.8, u0=0.1

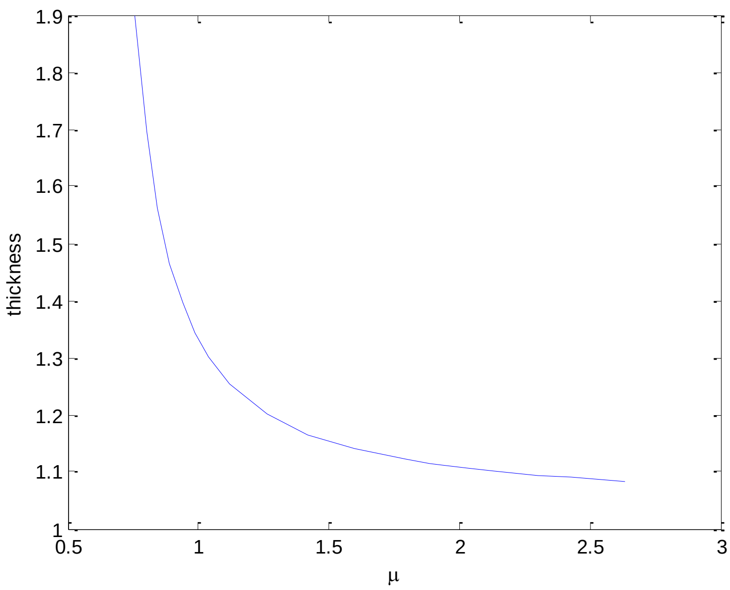

Fig. 2 Thickness of the shock wave versus μe with:

p=1.3, d=0.8, η=0.3, u0=0.1

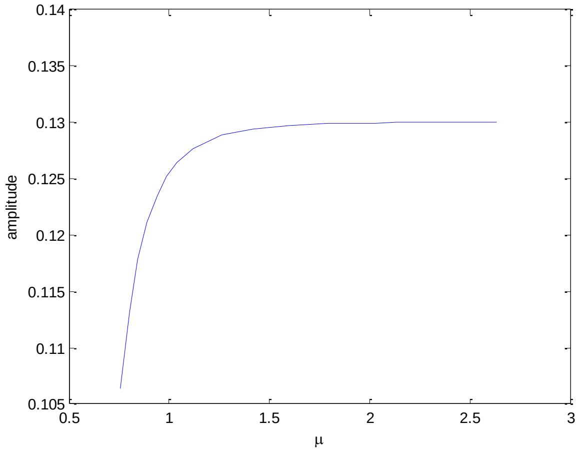

Fig. 3 Amplitude of the shock wave versus μe with:

p=1.3, d=0.8, η=0.3, u0=0.1

Conclusions

In this paper, the dust charge variation effects on nonlinear quantum dust ion acoustic shock waves in a plasma composed of inertialess quantum electrons and positrons, classical cold ions and stationary dust grains have been investigated by employing QHD model. Using the reductive perturbation method, the Burgers equation is obtained to study small amplitude shock waves. It is found that, in addition to the viscosity of ions, the dust charge variation is a sourse of dissipation and is responsible for the formation of the QDIA shock waves. In this condition, the shock wave parameters such as velocity, amplitude and thickness are modified.

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