A FIXED POINT APPROACH TO THE STABILITY OF GENERAL QUADRATIC EULER-LAGRANGE FUNCTIONAL EQUATIONS IN INTUITIONISTIC FUZZY SPACES

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ABSTRACT

In this paper, we prove the generalized Hyers-Ulam stability of a general k-quadratic Euler-Lagrange functional equation:

$$f(kx + y) + f(kx - y) = 2[f(x + y) + f(x - y)] + 2(k^2 - 2)f(x) - 2f(y)$$

for any fixed positive integer $k \in \mathbb{Z}^+$ in intuitionistic fuzzy normed spaces using a fixed point method.

KEYWORDS

Intuitionistic fuzzy normed spaces; General k-quadratic Euler-Lagrange functional equation; Generalized Hyers-Ulam stability; Fixed point method

SUBJECT CLASSIFICATION

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1. INTRODUCTION

The notion of fuzzy sets was first introduced by Zadeh [30] in 1965 which is a powerful hand set for modeling uncertainty and vagueness in various problems arising in the field of science and engineering. After that, fuzzy theory has become very active area of research in various fields, e.g. population dynamics [5], chaos control [10, 11], computer programming [12], nonlinear dynamical systems [13], nonlinear operators [18], statistical convergence [19] and a lot of developments have been made in the theory of fuzzy sets to find the fuzzy analogues of the classical set theory. The notion of intuitionistic fuzzy norm is also useful one to deal with the inexactness and vagueness arising in modeling. The concept and properties intuitionistic fuzzy metric spaces and normed spaces have been investigated of a number of the authors [8, 14, 17, 20, 25].

The stability problem of functional equations originated from a stability question of Ulam [28] concerning the stability of group homomorphisms. "When is it true that by slightly changing the hypotheses of a theorem one can still assert that thesis of the theorem remains true or approximately true?" In [15], Hyers gave the first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Aoki [2] for additive mappings. In 1978, Rassias [22] generalized Hyers theorem by obtaining a unique linear mapping near an approximately additive mapping. The paper of Rassias has provided a lot of influence in the development of what we call the generalized Hyers-Ulam-Rassias stability of functional equations. In 1996, Issac and Rassias [16] were the first to provide applications of the stability theory of functional equations for the proof new fixed point theorems with applications. Cădariu and Radu [6] used the fixed point method to the investigation of the Jensen functional equation.

Let X be a set. A function $f : X \times X \rightarrow [0, \infty]$ is a called a generalized metric on X if d satisfies

(i) $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(iii) $d(a, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Then (X, d) is a generalized metric space.

We recall the following fixed point theorem which was proved by Diaz and Margolis [9]:

Theorem 1.1. Let (Ω, d) be a complete generalized metric space and $T : \Omega \rightarrow \Omega$ be a strictly contractive mapping with Lipshitz constant L. Then, for any $x \in \Omega$, either $d(T^n x, T^{n+1} x) = \infty$ for all nonnegative integers $n \geq 0$ or other exists a natural number n_0 such that

(i) $d(T^n x, T^{n+1} x) < \infty$ for all $n \geq n_0$;

(ii) the sequence $\{T^n x\}$ is convergent to a fixed point y^* of T;

(iii) y^* is the unique fixed point of T in the set $\Lambda = \{y \in \Omega \mid d(T^{n_0} x, y) < \infty\}$;

(iv) $d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$ for all $y \in \Lambda$.

By using fixed point methods, the stability problems of various functional equations in intuitionistic fuzzy normed spaces have been extensively investigated by a number of authors (see, [1], [7], [21], [23], [24], [26], [27], [29]).

Now, we consider the functional equation

$$f(kx + y) + f(kx - y) = 2[f(x + y) + f(x - y)] + 2(k^2 - 2)f(x) - 2f(y) \tag{1.1}$$

for any fixed positive integer k. The functional equation (1.1) is said to be a general k-quadratic Euler-Lagrange functional equation. It is easy to see that the mapping $f(x) = x^2$ is a solution of the functional equation (1.1). Every solution of the general k-quadratic Euler-Lagrange functional equation is said to be a quadratic mapping. Note that, if we replace $x = y = 0$ in (1.1), then we get $f(0) = 0$. Letting $x = 0, y = x$ in (1.1), f is even. Letting $y = 0$ in (1.1), we obtain $f(kx) = k^2 f(x)$. In this paper, we study some stability results concerning the functional equation (1.1) with the help of the notion of continuous t-representable in the setting of intuitionistic fuzzy normed spaces by the fixed point alternative.

2. PRELIMINARIES

In this section, we recall some definitions, notations and conventions of theory of intuitionistic fuzzy normed spaces which are needed to prove our main results.

Lemma 2.1 [8] Consider the set L^* and the order relation $\leq_{L^*}$ defined by

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2, x_1 + x_2 \leq 1\}$$

and $(x_1, x_2) \leq (y_1, y_2) \Leftrightarrow x_1 \leq y_1, x_2 \leq y_2$ for all $(x_1, x_2), (y_1, y_2) \in L^*$. Then the pair $(L^*, \leq_{L^*})$ is a complete lattice.

Definition 2.2. [3] An intuitionistic fuzzy set $A_{\xi, \eta}$ in a universal set U is an object

$$A_{\xi, \eta} = \left\{ \left(\xi_A(u), \eta_A(u) \right) : u \in U \right\}$$

where $u \in U$, $\xi_A(u) \in [0,1]$ and $\eta_A(u) \in [0,1]$ are called the membership degree and the non-membership degree, respectively, of $u \in A_{\xi, \eta}$ and they satisfy $\xi_A(u) + \eta_A(u) = 1$.

We denote its units by $0_{L^*} = (0,1)$ and $1_{L^*} = (1,0)$. Classically, a triangular norm $T = *$ on $[0.1]$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \rightarrow [0,1]$ satisfying $T(1, x) = 1 * x = x$ for all $x \in [0,1]$. A triangular conorm $S = \diamond$ is defined as an increasing, commutative, associative mapping $S : [0,1]^2 \rightarrow [0,1]$ satisfying $S(0, x) = 0 \diamond x = x$ for all $x \in [0,1]$.

Definition 2.3. A triangular norm (shortly, t-norm) on L^* is a mapping $T : L^* \times L^* \rightarrow L^*$ satisfying the following conditions:

(i) $T(x, 1_{L^*}) = x$ for all $x \in L^*$ (: boundary condition);

(ii) $T(x, y) = T(y, x)$ for all $x, y \in L^*$ (: commutativity);

(iii) $T(x, T(y, z)) = T(T(x, y), z)$ for all $x, y, z \in L^*$ (: associativity);

(iv) $x \leq_{L^*} x'$ and $y \leq_{L^*} y' \Rightarrow T(x, y) \leq_{L^*} T(x', y')$ for all $x, y, x', y' \in L^*$ (: monotonicity).

Definition 2.4. A continuous t-norm T on L^* is said to be continuous t-representable if there exist a continuous t-norm S_1 and a continuous t-conorm S_2 on [0,1] such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,

$$T(x, y) = (S_1(x_1, y_1), S_2(x_2, y_2)).$$

For example, consider $T(x, y) = (x_1 y_1, \min\{x_2 + y_2, 1\})$ and $M(x, y) = (\min\{x_1, y_1\}, \max\{x_2, y_2\})$ for all $x = (x_1, x_2) \in L^*$ and $y = (y_1, y_2) \in L^*$. Then $T(x, y)$ and $M(x, y)$ are t-representable.

Now, we define a sequence T^n recursively by $T^1 = T$ and

$$T^n(x^{(1)}, \cdots, x^{(n+1)}) = T(T^{n-1}(x^{(1)}, \cdots, x^{(n)}), x^{(n+1)})$$

for all $n \geq 2$ and $x^{(i)} \in L^*$.

Definition 2.5. A negation on L^* is any strictly decreasing mapping $N : L^* \rightarrow L^*$ satisfying $N(0_{L^*}) = 1_{L^*}$ and $N(1_{L^*}) = 0_{L^*}$. If $N(N(x)) = x$ for all $x \in L^*$, then N is called an involutive negation. A negator N on [0,1] is a decreasing mapping $N : [0,1] \rightarrow [0,1]$ satisfying $N(0) = 1$ and $N(1) = 0$. N_s denotes the standard negator on $([0,1], \leq)$ defined by $N_s(x) = 1 - x$ for all $x \in [0,1]$.

Definition 2.6. [3] Let μ and v be the membership and the non-membership degrees of an intuitionistic fuzzy set from

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$X \times (0, \infty)$ to $[0,1]$ such that $\mu + v \leq 1$ for all $x \in X$ and $t > 0$. The triple $(X, P_{\mu,v}, T)$ is said to be an intuitionistic fuzzy normed space if X is a linear space, T is a continuous and $P_{\mu,v} : X \times (0, \infty) \to L^*$ is a mapping satisfying the following conditions:

(i) $P_{\mu,v}(x, 0) = 0_{L^*}$;

(ii) $P_{\mu,v}(x, t) = 1_{L^*}$ if and only if $x = 0$;

(iii) $P_{\mu,v}(ax, t) = P_{\mu,v}(x, \frac{t}{|a|})$ for all $a \neq 0$;

(iv) $P_{\mu,v}(x + y, t + s) \geq T(P_{\mu,v}(x, t), P_{\mu,v}(y, s))$;

(iv) $P_{\mu,v}(x, \cdot) : (\{0, \infty) \to L^*$ is continuous;

(v) $\lim_{n \to \infty} P_{\mu,v}(x, t) = 1_{L^*}$

for all $x, y \in X$ and $t, s > 0$. In this case, $P_{\mu,v}$ is called an intuitionistic fuzzy norm and we write $P_{\mu,v}(x, t) = (\mu(t), v(t))$ and $\mu(t) + v(t) = 1$ for all $x \in X$ and $t > 0$.

Example 2.7. Let $(X, \| \cdot \|)$ be a normed space and $T(x, y) = (x_1 y_1, \min\{x_2 + y_2, 1\})$ for all $x = (x_1, x_2)$, $y = (y_1, y_2) \in L^*$ and let μ and v be the membership and the non-membership degrees of an intuitionistic fuzzy set from $X \times (0, \infty)$ to $[0,1]$. Let $P_{\mu,v}$ be the intuitionistic fuzzy set on $X \times (0, \infty)$ defined as follows:

$$P_{\mu,v}(x, t) = (\mu(t), v(t)) = \left(\frac{t}{t + \|x\|}, \frac{t}{t + \|y\|} \right)$$

for all $x \in X$ and $t > 0$. Then $(X, P_{\mu,v}, T)$ is an intuitionistic fuzzy normed space.

Definition 2.8. Let $(X, P_{\mu,v}, T)$ be an intuitionistic fuzzy normed space.

(i) A sequence $\{x_n\}$ is said to be intuitionistic fuzzy convergent to $x \in X$ if $P_{\mu,v}(x_n - x, t) \to 1_{L^*}$ as $n \to \infty$ and for all $t > 0$.

(ii) A sequence $\{x_n\}$ is said to be a intuitionistic fuzzy Cauchy sequence in X if, for all $\epsilon > 0$ and $t > 0$, there exists an $n_0 \in \mathbb{Z}^+$ such that $P_{\mu,v}(x_n - x_m, t) >_{L^*} (N_s(\epsilon), \epsilon)$ for all $m, n \geq n_0$, where N_s is the standard negator.

(iii) $(X, P_{\mu,v}, T)$ is complete if every intuitionistic fuzzy Cauchy sequence in $(X, P_{\mu,v}, T)$ is intuitionistic fuzzy convergent in $(X, P_{\mu,v}, T)$. A complete intuitionistic fuzzy normed space is called an intuitionistic fuzzy Banach space.

3. MAIN RESULTS

Throughout this section, for any mapping $f : X \to Y$ let us define a general k-quadratic Euler-Lagrange difference operator

$$D f(x, y) = f(kx + y) + f(kx - y) - 2[f(x + y) + f(x - y)] - 2(k^2 - 2)f(x) + 2f(y)$$

for any fixed positive integers $k \in \mathbb{Z}^+$.

We prove the intuitionistic fuzzy stability of general k-quadratic

Euler-Lagrange functional equation (1.1) in the setting intuitionistic fuzzy normed space.

Theorem 3.1. Let X be a linear space, $(Z, P'_{\mu,v}, T_M)$ be an intuitionistic fuzzy normed space and $\varphi : X \times X \to Z$ be a function such that for some $0 \leq \alpha < k^2$,

$$P'_{\mu,v}(\varphi(kx, ky), t) \geq_{L^*} P'_{\mu,v}(\alpha \varphi(x, y), t), \tag{3.1}$$

$$\lim_{n \to \infty} P'_{\mu,v}(\varphi(k^n x, k^n y), k^{2n} t) = 1_{L^*} \tag{3.2}$$

for all $x, y \in X$ and $t > 0$. Suppose that $(Y, P_{\mu,v}, T_M)$ is a complete intuitionistic fuzzy normed space. If $f : X \to Y$ is a mapping such that

$$P_{\mu,v}(Df(x, y), t) \geq_{L^*} P'_{\mu,v}(\varphi(x, y), t)$$

(3.3)

for all $x, y \in X$ and $t > 0$ and $f(0) = 0$, then there is a unique k-quadratic Euler-Lagrange mapping $Q : X \to Y$ such that

$$P_{\mu,v}(f(x) - Q(x)) \geq_{L^*} P'_{\mu,v}(\varphi(x, 0), 2(k^2 - \alpha)t) \tag{3.4}$$

for all $x \in X$ and $t > 0$.

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Proof. Putting $y = 0$ in (3.3), we have

$$P_{\mu,v}\left(f(x) - \frac{f(kx)}{k^2}, t\right) \geq_{L^*} P'_{\mu,v}\left(\frac{1}{2k^2}\,\varphi(x,0),t\right)$$

(3.5)

for all $x \in X$ and $t > 0$. Let Ω be a set of all mappings from X into Y and define a generalized metric d on Ω as follows:

$$d(g,h) = \inf\{c \in [0,\infty) \mid P_{\mu,v}(g(x) - h(x),t) \geq_{L^*} P'_{\mu,v}(c\varphi(x,y),t) \text{ for all } x \in X, t > 0\}$$

where, as usual, $\inf \phi = +\infty$. It is easy to show (Ω, d) is a generalized complete metric space. We consider the mapping $T : \Omega \rightarrow \Omega$ is defined by

$$Tg(x) = \frac{g(kx)}{k^2}$$

for all $x \in X$. Let $g, h \in \Omega$ be such that $d(g,h) < c$. Then, it is not difficult to see that

$$d(Tg, Th) \leq \frac{\alpha d(g,h)}{k^2}$$

for all $g, h \in \Omega$. This means that T is a strictly contractive mapping on Ω with Lipschitz constant $L = \frac{\alpha}{k^2} < 1$. Also, it follows from (3.5) that

$$d(f, Tf) \leq \frac{1}{k^2} < \infty. \tag{3.6}$$

It follows from the conditions (2) and (3) of Theorem 1.1 that there is a mapping Q which is a unique fixed point of T in the set $\Omega_1 = \{g \in \Omega \mid d(f,g) < \infty\}$ such that

$$Q(x) = \lim_{n \to \infty} \frac{1}{k^{2n}} f(k^n x)$$

for all $x \in X$, since $\lim_{n \to \infty} d(T^n f, Q) = 0$. Again, it follows from (3.6) and the condition (4) of Theorem 1.1 that

$$d(f, Q) \leq \frac{1}{1-L}\, d(f, Tf) \leq \frac{1}{2(k^2 - \alpha)},$$

which gives $P_{\mu,v}(f(x) - Q(x)) \geq_{L^*} P'_{\mu,v}(\varphi(x,y), 2(k^2 - \alpha)t)$ for all $x \in X$ and $t > 0$. It follows from (3.3) that

$$P_{\mu,v}\left(\frac{1}{k^{2n}}\, Df(k^n x, k^n y), t\right) \geq_{L^*} P'_{\mu,v}(\varphi(k^n x, k^n y), t)$$

for all $x \in X$ and $t > 0$. Taking the limit as $n \to \infty$ in the above inequality, we have

$$P_{\mu,v}\left(\frac{1}{k^{2n}}\, Df(k^n x, k^n y), t\right) \to 1_{L^*}.$$

Thus, a mapping Q is quadratic. To prove uniqueness, let us assume that there exists another Euler-Lagrange quadratic mapping $Q' : X \rightarrow Y$ which satisfies (3.4). Then Q' is a fixed point of T in Ω_1. However, it follows from Theorem 1.1 that T has the unique fixed point in Ω_1. Therefore, we deduce $Q = Q'$. This completes the proof.. □

Corollary 3.2. Let X be a linear space, $(Z, P'_{\mu,v}, T_M)$ be an intuitionistic fuzzy normed space and $(Y, P_{\mu,v}, T_M)$ be a omplete intuitionistic fuzzy normed space. Suppose that $0 < p < 2$ and z_0 is a unit vector in Z. If $f : X \rightarrow Y$ is a mapping such that

$$P_{\mu,v}(Df(x, y), t) \geq_{L^*} P'_{\mu,v}\left(\left(\|x\|^p + \|y\|^2\right)z_0, t\right)$$

and $f(0)=0$ for all $x, y \in X$ and $t > 0$, then there is a unique k-quadratic Euler-Lagrange mapping $Q : X \rightarrow Y$ such that

$$P_{\mu,v}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,v}\left(\|x\|^p z_0, 2(k^2 - k^p)\, t\right)$$

for all $x \in X$ and $t > 0$.

Proof. Let $\varphi(x, y) = \left(\|x\|^p + \|y\|^p\right)z_0$ for all $x, y \in X$. The result follows from Theorem 3.1 with $\alpha = k^p$. □

Now, we present an application of Theorem 3.1 in the classical case:

Example 3.3. Let $(X, \|\cdot\|)$ be a Hilbert space with norm $\|\cdot\|^2 = \langle \cdot, \cdot \rangle$ and let Z be an intuitionistic fuzzy normed space.

Let a function $\varphi : X \rightarrow Z$ be defined by

$$\varphi(x,y) = 2k^2 \left(\|x\| + \|y\| \right) z_o$$

for all $x, y \in X$, where z_o is a unit vector in Z. Let $(Y, P_{\mu,\nu}, T_M)$ be an intuitionistic fuzzy normed space in which

$$T_M(a,b) = (\text{mim}\{a_1, b_1\}, \max\{a_2, b_2\}) \text{ and } P_{\mu,\nu}(a,b) = \left(\frac{t}{t + \|x\|}, \frac{\|x\|}{t + \|x\|} \right)$$

for all $x \in X$ and $t > 0$. Let $(X, P_{\mu,\nu}, T_M)$ be a complete intuitionistic fuzzy normed space. Define $f : X \rightarrow X$ by $f(x) = x^2 + <x, x_o>$ for all $x \in X$ where x_o is a unit vector in X. Then

$$\mu_{Df(x,y)}(t) = \frac{t}{t + \{2(k^2 - k)(|<x, x\_o>| + |<y, x\_o>|)} = \frac{t}{t + 2k^2(\|x\| + \|y\|)} = \mu_{\varphi(x,y)}(t)$$

and

$$\nu_{Df(x,y)}(t) = \frac{2k^2(\|x\| + \|y\|)}{t + 2k^2(\|x\| + \|y\|)} = \nu_{\varphi(x,y)}(t)$$

for all $x, y \in X$ and all $t > 0$. Thus, we get

$$P_{\mu,\nu}(Df(x,y),t) = \left(\mu_{Df(x,y)}(t), \nu_{Df(x,y)}(t) \right) \geq_{L^*} \left(\mu_{\varphi(x,y)}(t), \nu_{\varphi(x,y)}(t) \right) = P_{\mu,\nu}(\varphi(x,y),t)$$

and

$$P_{\mu,\nu}(\varphi(kx, ky), t) \geq_{L^*} P_{\mu,\nu}(k\varphi(x,y),t)$$

for all $x, y \in X$ and $t > 0$. It is from $\lim_{n \to \infty} \mu_{\varphi(k^n x, k^n y)}(k^{2n}t) = 1$ and $\lim_{n \to \infty} \nu_{\varphi(k^n x, k^n y)}(k^{2n}t) = 0$ that

$$\lim_{n \to \infty} P_{\mu,\nu}(\varphi(k^n x, k^n y), k^{2n}t) = (1,0) = 1_{L^*}$$

for all $x, y \in X$ and $t > 0$. Hence, all the conditions of Theorem 3.1 for $\alpha = k$ are fulfilled. So, there exists a unique k-quadratic Euler-Lagrange mapping $Q : X \rightarrow Y$ such that

$$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P_{\mu,\nu}(\varphi(x,0), 2(k^2 - k)t)$$

for all $x \in X$ and $t > 0$. $\square$

Modifying the proof of Theorem 3.1, we can easily prove the following:

Theorem 3.4. Let X be a linear space, $(Z, P'_{\mu,\nu}, T_M)$ be an intuitionistic fuzzy normed space and $\varphi : X \times X \rightarrow Z$ be a function such that for some $k^2 < \alpha$,

$$P'_{\mu,\nu}(\varphi(x,y),t) \geq_{L^*} P'_{\mu,\nu}(\varphi(kx, ky), \alpha t),$$

$$\lim_{n \to \infty} P'_{\mu,\nu}\left(\left(\varphi(\tfrac{x}{k^n}, \tfrac{y}{k^n}), \tfrac{t}{k^{2n}} \right) \right) = 1_{L^*}$$

for all $x, y \in X$ and $t > 0$. Suppose that $(Y, P_{\mu,\nu}, T_M)$ is a complete intuitionistic fuzzy normed space. If $f : X \rightarrow Y$ is a mapping such that

$$P_{\mu,\nu}(Df(x,y),t) \geq_{L^*} P'_{\mu,\nu}(\varphi(x,y),t).$$

for all $x, y \in X$ and $t > 0$ and $f(0) = 0$, then there is a unique k-quadratic Euler-Lagrange mapping $Q : X \rightarrow Y$ such that

$$P_{\mu,\nu}(f(x) - Q(x)) \geq_{L^*} P'_{\mu,\nu}(\varphi(x,0), 2(\alpha - k^2)t) \tag{3.7}$$

for all $x \in X$ and $t > 0$.

Proof. Let Ω and d be as in the proof of Theorem 3.1. Then (Ω, d) becomes a generalized complete metric space and the mapping $T : \Omega \rightarrow \Omega$ defined by $Tg(x) = k^2 g(\tfrac{x}{k})$ for all $g \in \Omega$. Let $g, h \in \Omega$ be such that $d(g, h) < c$. Then, it is not difficult to see that

$$d(Tg, Th) \leq \frac{k^2 d(g,h)}{\alpha}$$

for all $g, h \in \Omega$. This means that T is a strictly contractive mapping on Ω with Lipschitz constant $L = \frac{k^2}{\alpha} < 1$. Also, it follows

from (3.5) that $d(f, Tf) \leq \frac{1}{2\alpha} < \infty$. Using Theorem 1.1, we obtain

$$d(f, Q) \leq \frac{1}{1-L} \; d(f, Tf) \; \leq \; \frac{1}{2(\alpha-k^2)}$$

which implies the inequality (3.7) holds for all $x \in X$ and $t > 0$. The remaining assertion goes through in a similar method to the corresponding part of Theorem 3.1. This complete the proof. □

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