ISSN 2347-3487

Approximategeneralized Jensen typemappings in proper Lie *CQ**-algebras

GaYaKim* andSeongSik Kim

Department of Urban Engineering, Dong-eui University

Busan 614-714, Repubic of Korea

kimgya@deu.ac.kr

Department of Mathematics, Dong-eui University

Busan 614-714, Repubic of Korea

sskim@deu.ac.kr

ABSTRACT

In this paper, we investigate the stability problems for proper Lie derivations associated to the generalized Jensen type functional equation

$$f\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) + \sum_{j=2}^{n} f\left(\frac{\sum_{i=1,i\neq j}^{n} x_i - (n-1)x_j}{n}\right) = f(x_1)$$

in a proper Lie CQ*-algebra.

KEYWORDS

proper Lie CQ*-algebras, proper Lie CQ*-derivations, proper Lie JCQ*-derivations, generalized Hyers-Ulam stability, generalized Jensen type mappings

SUBJECT CLASSIFICATION

2010 Mathematics Subject Classification: 39B52, 39B82, 46B03, 47H10

INTRODUCTIONANDPRELIMINARIES

The stability problem of functional equations was originated from a question of Ulam [24]concerning the stability of group homomorphisms as follows:

Let $(G_1,*)$ be a group and $(G_2,\lozenge,d)$ be a metric group with the metric $d(\cdot,\cdot)$. Given $\epsilon > 0$, does there exist a $\delta(\epsilon) > 0$ such that if a mapping $h : G_1 \to G_2$ satisfies theinequality $d\big(h(x * y), h(x) \lozenge h(y)\big) < \delta(\epsilon)$ for all $x, y \in G_1$, then there is a homomorphism $H : G_1 \to G_2$ with $d\big(h(x), H(y)\big) < \epsilon$ for all $x \in G_1$?

If the answer is affirmative, we would say that the equation of a homomorphism $H(x * y) = H(x) \lozenge H(y)$ is stable. The famous Ulam stability problem was partially solved by Hyers [8] for linearfunctional equation of Banach spaces. Let $f: X \to Y$ be a mapping between Banach spaces such that

$$||f(x+y) - f(x) - f(y)|| \leq \epsilon$$

for all $x, y \in X$ and for some $\epsilon > 0$. Then, there exists a unique additive mapping $T: X \to Y$ such that

$$||f(x) - T(x)|| \leq \epsilon$$

for all $x \in X$. Hyers's theorem was generalized by Aoki [1] for additive mappings and by Rassias [21] for linear mappings by considering an unbounded Cauchy difference $||f(x+y) - f(x) - f(y)||$ to be controlled by $\epsilon(||x||^p + ||y||^p)$.Taking into consideration a lot of influence of Ulam, Hyers and Rassias on the development of stability problems of functional equations, the stability phenomenon that was proved by Rassias may be called the generalized Hyers-Ulam-Rassias stability. In 1994, a generalization of Rassias theorem was obtained by Găvruta [5], who replaced $\epsilon(||x||^p + ||y||^p)$ by a general control function $\phi(x, y)$. Cădariu and Radu [3] applied the fixed point method to investigation of the stability of a Jensen functional equation. Since then, the stability problems of many algebraic, differential, integral, operatorial equations have been extensively investigated [9, 10, 13, 14] and the references therein.

The theory of finite dimensional complex Lie algebras is an important part of Lie theory. It has several applications in physics and connections with other parts of mathematics. With an increasing amount of theory and applications concerning Lie algebras of various dimensions, it is becoming necessary to ascertain which tools are applicable for handling them. The miscellaneous characteristics of Lie algebras constitute such tools and have also found applications.Recently, many research papers have been published about the generalized Hyer-Ulam-Rassias stability of homomorphisms and derivations in C*-algebras, Lie C*-algebras, JC*-algebras, CQ*-algebras [4, 7, 11, 15, 16, 18, 19, 20, 22] and the references therein.

We recall some basic facts concerning CQ*-algebras [2, 23].

Definition 1.1. Let A be a linear space and A_0 is a *-algebra contained in A as a subspace. $A$is called a quasi $*$-algebra over A_0 if the following three conditions holds:

(i) the right and left multiplications of an element of A and an element of A_0 are defined and linear;

(ii) $x_1(x_2a) = (x_1x_2)a$, $(ax_1)x_2 = a(x_1x_2)$and$x_1(ax_2) = (x_1a)x_2$ for all $x_1, x_2 \in A$ and all $a \in A_0$;

(iii) an involution $*$, which extends the involution of A_0, is defined in A with the property that $(ab)^* = b^*a^*$ whenever the multiplication is defined.

Quasi *-algebras arise in natural way as completions of locally convex *-algebras whose multiplication is not jointly continuous; in this case, one has to deal with topological quasi *-algebras. A quasi *-algebra (A, A_0) is called topological if a locally convex topology τ on A is given such that:

(i) the involution is continuous and the multiplications are separately continuous;

(ii) A_0 is dense in $A[\tau]$.

Throughout this paper, we suppose that a locally convex quasi *-algebra (A, A_0) is complete. Many authors have considered a special class of quasi *-algebras, called proper CQ*-algebras, which arise as completions of C*-algebras.

Definition 1.2. Let A be a Banach module over the C*-algebra A_0 with involution $*$ and C*-norm $||\cdot||_0$such that $A_0 \subset A$. Then the pair (A, A_0) is called a proper CQ*-algebra if

(i) A_0 is dense in A with respect to its norm $||\cdot||$;

(ii) $(ab)^* = b^*a^*$for all $a, b \in A$, whenever the multiplication is defined;

(iii) $||y||_0 = \sup_{a \in A, ||a|| \leq 1} ||ay||$for all $y \in A_0$.

A proper CQ*-algebra $(A, A_0)$is said to have a unit e if there exists an element $e \in A_0$ such that $ae = ea = a$ for all $a \in A$. A proper CQ*-algebra with an identity is called anunital proper CQ*-algebra. In this paper, we will always assume that (A, A_0) is an unital proper CQ*-algebra.

Definition 1.3.A properCQ*-algebra (A, A_0), endowed with a bilinear multiplication $[\cdot,\cdot] : (A \times A_0) \cup (A_0 \times A) \to A$, is called the Lie bracket, which satisfies the following properties:

(i) $[x_1, x_2] = -[x_2, x_1]$for all $(x_1, x_2) \in (A \times A_0) \cup (A_0 \times A)$;

ISSN 2347-3487

(ii) $[x_1,[x_2,x_3]] = [[x_1,x_2],x_3] + [x_1,[x_2,x_3]]$for all $x_1,x_2,x_3 \in A_0$

is called a proper Lie CQ*-algebra.

Now, we consider a mapping $f : A_0 \to A$satisfying the following functional equation:

$$f(\tfrac{1}{n}\textstyle\sum_{i=1}^{n} x_i) + \sum_{j=2}^{n} f\left(\frac{\sum_{i=1,i\neq j}^{n} x_i-(n-1)x_j}{n}\right) = f(x_1) \tag{1.1}$$

for all $x_i, x_j \in A_0$, where n is a fixed positiveinteger with $n \geq 2$. Gordji et al. [7] establish thestability of n-Lie homomorphisms and Jordan n-Lie homomorphisms on n-Lie algebras associated with the functional equation (1.1) by the fixed pointmethod. Kim et al. [12] proved the stabilityand superstability problems of derivations for the functional equation (1.1) on Lie C*-algebras bythe direct method.

Motivated by the above works, we prove the generalized Hyers-Ulamstability problems for proper Lie derivations associated with the generalized Jensen type functional equation (1.1) on proper Lie CQ*-algebras. In section 2, we give Lie CQ*-derivations in proper Lie CQ*-algebras associated with the functional equation (1.1). In section 3, we give Lie JCQ*-derivations in proper Lie Jordan CQ*-algebras associated with the functional equation (1.1).

Throughout this paper, we assume that (A, A_0) is a proper Lie CQ*-algebras associated with the C*-norm $||\cdot||_0$and norm $||\cdot||$. For convenience, we use the following abbreviation for any given mapping $f : A_0 \to A$,

$$D_\mu f(x_1, \dots, x_n) = f(\frac{1}{n}\sum_{i=1}^{n} \mu x_i) - \mu \sum_{j=2}^{n} f\left(\frac{\sum_{i=1,i\neq j}^{n} x_i - (n-1)x_j}{n}\right) - f(\mu x_1)$$

for all $\mu \in T^1 = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $x_1, \dots, x_n \in A_0 (n \geq 2)$.

DERIVATIONS IN PROPER LIE CQ*-ALGEBRAS

In this section, we prove the generalized Hyers-Ulam stability results of proper CQ*-derivations in proper CQ*-triples for the functional equation $D_\mu f(x_1, \dots, x_n) = 0$.

Let (A, A_0) be a proper Lie *CQ**-algebra with respect to the Lie product

$$[z, x] = \frac{zx - xz}{2}$$

for all $x \in A$ and all $z \in A_0$. A $\mathbb{C}$-linear mapping $\delta: A_0 \to A$ is called a proper Lie CQ-derivation if

$$\delta([z, x]) = [z, \delta(x)] + [\delta(z), x]$$

for all $z, x \in A_0$. In addition, if δ satisfies the additional condition

$$\delta(a^*) = \delta(a)^*$$

for all $a \in A_0$. Then, it is called a proper Lie CQ*-derivation.

Lemma 2.1. [17] Let$f : A_0 \to A$ is an additive mapping such that $f(x)\mu = \mu f(x)$ for all $x \in A_0$ and all $\mu \in T^1$. Then f is $\mathbb{C}$-linear.

Theorem 2.2. Suppose that$f : A_0 \to A$ is a mapping with $f(0) = 0$ for which there exist mappings $\varphi : A_0^n \to [0, \infty)$, $\psi : A_0^2 \to [0, \infty)$ and$\eta : A_0 \to [0, \infty)$ be mappings such that

$$\|D_\mu f(x_1, \cdots, x_n)\| \leq \varphi(x_1, \cdots, x_n) \tag{2.1}$$

$$\|f([w_0, w_1]) - [f(w_0), w_1] - [w_0, f(w_1)]\| \leq \psi(w_0, w_1) \tag{2.2}$$

$$\|f(w_2^*) - f(w_2)^*\| \leq \eta(w_2) \tag{2.3}$$

for all $\mu \in T^1$ and all $x_1, \dots, x_n, w_0, w_1, w_2 \in A_0$ $(n \geq 2)$. Let L (< 1) be a constant such that

$$\varphi\left(\frac{x_1}{n}, \dots, \frac{x_n}{n}\right) \leq \frac{L}{n}\varphi(x_1, \cdots, x_n) \tag{2.4}$$

$$\psi\left(\frac{w_0}{n}, \frac{w_1}{n}\right) \leq \frac{L}{n^2}\psi(w_0, w_1) \tag{2.5}$$

$$\eta\left(\frac{w_2}{n}\right) \leq \frac{L}{n}\eta(w_2) \tag{2.6}$$

for all $x_1, \dots, x_n, w_0, w_1, w_2 \in A_0$. Then there exists a unique Lie CQ*-derivation $\delta : A_0 \to A$ satisfying

$$\|f(x) - \delta(x)\| \leq \frac{L}{1-L}\varphi(x, 0, \dots, 0) \tag{2.7}$$

for all $x \in A_0$.

Proof. Letting $\mu = 1$ and $x_1 = x$, $x_2 = \cdots = x_n = 0$ in (2.1), we get

$$\left\|nf\left(\frac{x}{n}\right) - f(x)\right\| \le \varphi(x, 0, \dots, 0), \quad (2.8)$$

for all $x \in A_0$. Replacing x by $\frac{x}{n^j}$ and multiplying n^j both the sides of (2.8),

$$\left\|n^{j+1} f\left(\frac{x}{n^{j+1}}\right) - n^j f\left(\frac{x}{n^j}\right)\right\| \le n^j \varphi\left(\frac{x}{n^j}, 0, \dots, 0\right),$$

for all $x \in A_0$ and all integer $j \in \mathbb{Z}^+$ with $j = 0,1,2, \dots$. Hence, we obtain

$$\left\|n^m f\left(\frac{x}{n^m}\right) - n^k f\left(\frac{x}{n^k}\right)\right\| \le \sum_{j=k}^{m-1} n^j \varphi\left(\frac{x}{n^j}, 0, \dots, 0\right) \le \sum_{j=k}^{m-1} L^j \varphi(x, 0, \dots, 0) \quad (2.9)$$

for all $x \in A_0$ and all nonnegative integers m, k with $m \ge k$. It follows from (2.9) that the sequence $\{n^m f\left(\frac{x}{n^m}\right)\}$ is Cauchy in A for all $x \in A_0$. Since A is complete, it converges. So, we can define a mapping $\delta : A_0 \to A$ defined by

$$\delta(x) = \lim_{m\to\infty} n^m f\left(\frac{x}{n^m}\right) \quad (2.10)$$

for all $x \in A_0$. Passing the limit $m \to \infty$ with $k = 0$ in (2.9), we have

$$\|f(x) - \delta(x)\| \le \lim_{m\to\infty} \|n^m f\left(\frac{x}{n^m}\right) - f(x)\| \le \sum_{j=0}^{\infty} n^j \varphi\left(\frac{x}{n^j}, 0, \dots, 0\right) \le \sum_{j=0}^{\infty} L^j \varphi(x, 0, \dots, 0),$$

which implies the inequality (2.7) holds for all $x \in A_0$. On the other hand, it follows from (2.4), (2.5), (2.6) and $L < 1$ that

$$\lim_{m\to\infty} n^m \varphi\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right) = 0, \qquad \lim_{m\to\infty} n^{2m} \psi\left(\frac{w_0}{n^m}, \frac{w_1}{n^m}\right) = 0, \qquad \lim_{m\to\infty} n^m \eta\left(\frac{w_2}{n^m}\right) = 0.$$

Substituting $\mu = 1$ and $(x_1, \dots, x_n)$ by $\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right)$ in (2.1), we have

$$\|D_1\delta(x_1, \cdots, x_n)\| = \lim_{m\to\infty} n^m \|D_1 f\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right)\|$$
$$\le \lim_{m\to\infty} n^m \varphi\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right) = 0$$

for all $x_1, \dots, x_n \in A_0$. Then $D_1\delta(x_1, \cdots, x_n) = 0$, and so the mapping δ is additive [12]. Letting $x_1 = x$, $x_2 = \cdots = x_n = 0$ in (2.1), we get $\|\mu f(x) - f(\mu x)\| \le \varphi(x, 0, \dots, 0)$ for all $x \in A_0$. Thus,

$$\|\mu f(x) - f(\mu x)\| = \lim_{m\to\infty} n^m \|\mu f\left(\frac{x}{n^m}\right) - f\left(\mu\frac{x}{n^m}\right)\|$$
$$\le \lim_{m\to\infty} n^m \varphi\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right) = 0,$$

which implies $\mu f(x) = f(\mu x)$ for all $\mu \in T^1$ and all $x \in A_0$. It follows from Lemma 2.1 that the mapping δ is $\mathbb{C}$-linear.

Now, it follows from (2.2), (2.5) and (2.10) that

$$\|\delta([w_0, w_1]) - [\delta(w_0), w_1] - [w_0, \delta(w_1)]\|$$
$$= \lim_{m\to\infty} n^{2m} \left\|f\left(\frac{[w_0, w_1]}{n^{2m}}\right) - \left[f\left(\frac{w_0}{n^m}\right), \frac{w_1}{n^m}\right] - \left[\frac{w_0}{n^m}, f\left(\frac{w_1}{n^m}\right)\right]\right\|$$
$$\le \lim_{m\to\infty} n^{2m} \psi\left(\frac{w_0}{n^m}, \frac{w_1}{n^m}\right) = 0,$$

which proves

$$\delta([w_0, w_1]) = [\delta(w_0), w_1] + [w_0, \delta(w_1)]$$

for all $w_0, w_1 \in A_0$. And it follows from (2.3), (2.6) and (2.10) that

$$\|\delta(w_2^*) - \delta(w_2)^*\| = \lim_{m\to\infty} n^m \left\|f\left(\frac{w_2^*}{n^m}\right) - f\left(\frac{w_2}{n^m}\right)^*\right\| \le \lim_{m\to\infty} n^m \eta\left(\frac{w_2}{n^m}\right) = 0$$

for all $w_2 \in A_0$. Thus, we obtain that the mapping $\delta : A_0 \to A$ satisfies

$$\delta(w_2^*) = \delta(w_2)^*$$

for all $w_2 \in A_0$.

Finally, let $\delta' : A_0 \to A$ be another proper Lie CQ*-derivation on A_0 satisfying (2.7). Then, we have

$$\|\delta(x) - \delta'(x)\| \le \lim_{m\to\infty} n^m \|f\left(\frac{x}{n^m}\right) - \delta'\left(\frac{x}{n^m}\right)\|$$
$$\le \lim_{m\to\infty} \frac{n^m L}{1 - L} \varphi\left(\frac{x}{n^m}, 0, \dots, 0\right) = 0$$

for all $x \in A_0$. So, we can conclude that $\delta(x) = \delta'(x)$ for all $x \in A_0$. Therefore, the mapping δ is a unique proper Lie CQ*-derivation on A_0satisfying (2.7). This completes the proof.

Corollary 2.3. Let $r(> 1)$ and θ be positive real numbers. Suppose that a mapping $f : A_0 \to A$ satisfies

$$\|D_1 f(x_1, \cdots, x_n)\| \le \theta(||x_1||_0^r + \cdots + ||x_n|_0^r)$$

$$\|f([w_0, w_1]) - [f(w_0), w_1] - [w_0, f(w_1)]\| \le \theta(||w_0||_0^{2r} + ||w_1|_0^{2r}) \quad (2.11)$$

$$\|f(w_2^*) - f(w_2)^*\| \le \theta||w_2||_0^r$$

for all $x_1, \dots, x_n,\ w_0, w_1, w_2 \in A_0$. Then there exists a unique proper Lie CQ*-derivation $\delta : A_0 \to A$ such that

$$\|f(x) - \delta(x)\| \le \frac{n^r \theta}{n^r - n} ||x||_0^r \quad (2.12)$$

for all $x \in A_0$.

Proof. Letting $\varphi(x_1, \cdots, x_n) = \theta(||x_1||_0^r + \cdots + ||x_n|_0^r)$for all $x_1, \dots, x_n \in A_0$ and $L = n^{1-r}$, we obtain the claimed stability result (2.12). This completes the proof.

Corollary 2.4. Let $r, r_j (j = 1,2, \dots, n)$ and θ be positive real numbers such that $0 < \sum_{j=1}^{n} r_j < 1$. Suppose that a mapping $f : A_0 \to A$ satisfies

$$\|D_\mu f(x_1, \cdots, x_n)\| \le \theta\left(||x_1||_0^{r_1} \times \cdots \times ||x_n||_0^{r_n}\right) \quad (2.13)$$

$$\|f([w_0, w_1]) - [f(w_0), w_1] - [w_0, f(w_1)]\| \le \theta(||w_0||_0^r \cdot ||w_1||_0^r)$$

$$\|f(w_2^*) - f(w_2)^*\| \le \theta||w_2||_0^r$$

for all $x_1, \dots, x_n,\ w_0, w_1, w_2 \in A_0$ and all $\mu \in T^1$. Then f is a proper Lie CQ*-derivation on A_0.

Proof.Letting $\varphi(x_1, \cdots, x_n) = \theta\left(||x_1||_0^{r_1} \times \cdots \times ||x_n||_0^{r_n}\right)$, $\psi(w_0, w_1) = \theta(||w_0||_0^r \cdot ||w_1||_0^r)$, $\eta(w_2) = \theta||w_2||_0^r$ in Theorem 2.2, we have the conditions (2.4), (2.5), (2.6). Putting $x_1 = \cdots = x_n = 0$and $\mu = 1$ in (2.13), we obtain $f(0) = 0$. Furthermore, if we put $x_1 = x,\ x_2 = \cdots = x_n = 0$ and $\mu = 1$ in (2.13), then we have $f(x) = nf\left(\frac{x}{n}\right)$ for all $x \in A_0$. It is easy to see that by induction, we get$f(x) = n^m f\left(\frac{x}{n^m}\right)$for all $x \in A_0$ and all $m \in \mathbb{Z}^+$. Now, it follows from Theorem 2.2 that f is a proper Lie CQ*-derivation on A_0. This completes the proof.

DERIVATIONS IN PROPER LIE JCQ*-ALGEBRAS

In this section, we give the generalized Hyers-Ulam stability of Lie JCQ*-derivations in proper CQ*- algebras associated with the functional equation $D_\mu f(x_1, \dots, x_n) = 0$.

A proper Lie CQ*-algebra (A, A_0), endowed with Jordan product

$$z \circ x = \frac{zx + xz}{2}$$

for all $x \in A_0$and all $z \in A_0$ is called a proper Lie JCQ*-algebra.

Definition 3.1. Let (A, A_0) be a proper Lie JCQ*-algebra. A $\mathbb{C}$-linear mapping $\delta : A_0 \to A$is called a proper Lie JCQ*-derivation if δsatisfies a proper Lie CQ*-derivation and

$$\delta(x \circ y) = x \circ \delta(y) + \delta(x) \circ y \quad (3.1)$$

for all $x,\ y \in A_0$.

Theorem 3.2. Let $\varphi,\ \psi,\ \eta$ be as Theorem 2.2. Suppose that a mapping $f : A_0 \to A$ with $f(0) = 0$ satisfies (2.1), (2.2), (2.3) and

$$||f(w_0 \circ w_1) - w_0 \circ f(w_1) - f(w_0) \circ w_1|| \le \psi(w_0, w_1) \quad (3.2)$$

for all $w_0, w_1 \in A_0$. Then there exists a unique proper Lie JCQ*-derivation $\delta : A_0 \to A$ such that

$$||f(x) - \delta(x)|| \le \frac{L}{1-L} \varphi(x, 0, \dots, 0) \quad (3.3)$$

for all $x \in A_0$.

Proof. By the same reasoning as in the proof of Theorem 2.2, there exists a unique $\mathbb{C}$-linear mapping δsatisfying (2.7). The mapping $\delta : A_0 \to A$is defined by

$$\delta(x) = \lim_{m\to\infty} n^m f\left(\frac{x}{n^m}\right)$$

for all $x \in A_0$. It is sufficient to show that $f$satisfies the condition (3.1) of Definition 3.1.It follows from (2.5) and (3.2) that

$$\|\delta(w_0 \circ w_1) - \delta(w_0) \circ w_1 - w_0 \circ \delta(w_1)\|$$
$$= \lim_{m\to\infty} n^{2m} \left\| f\left(\frac{w_0 \circ w_1}{n^{2m}}\right) - f\left(\frac{w_0}{n^m}\right) \circ \frac{w_1}{n^m} - \frac{w_0}{n^m} \circ f\left(\frac{w_1}{n^m}\right) \right\|$$
$$\leq \lim_{m\to\infty} n^{2m} \psi\left(\frac{w_0}{n^m}, \frac{w_1}{n^m}\right) = 0,$$

which gives

$$\delta(w_0 \circ w_1) = \delta(w_0) \circ w_1 + w_0 \circ \delta(w_1)$$

for all $w_0, w_1 \in A_0$. Therefore, we conclude the mapping $\delta$is a proper Lie JCQ*-derivation on A_0.This completes the proof.

Corollary 3.3. Let $r(> 1), \theta$and ϵ be positive real numbers. Suppose that a mapping $f: A_0 \to A$satisfies

$$\|D_1 f(x_1, \cdots, x_n)\| \leq \epsilon,$$
$$\|f([w_0, w_1]) - [f(w_0), w_1] - [w_0, f(w_1)]\| \leq \theta(||w_0||_0^{2r} + ||w_1|_0^{2r}),$$
$$\|f(w_2^*) - f(w_2)^*\| \leq \theta||w_2||_0^r,$$
$$||f(w_0 \circ w_1) - w_0 \circ f(w_1) - f(w_0) \circ w_1|| \leq \theta(||w_0||_0^{2r} + ||w_1|_0^{2r})$$

for all $x_1, \ldots, x_n,\ w_0, w_1, w_2 \in A_0$. Then there exists a unique proper Lie JCQ*-derivation $\delta : A_0 \to A$such that

$$||f(x) - \delta(x)|| \leq \frac{n\epsilon}{n^r - n} \tag{3.4}$$

for all $x \in A_0$.

Corollary 3.4. Let r_i, r, θ be as Corollary 2.4. Suppose that a mapping $f: A_0 \to A$ satisfies

$$\|D_\mu f(x_1, \cdots, x_n)\| \leq \theta(||x_1||_0^{r_1} \times \cdots \times ||x_n||_0^{r_n})$$
$$\|f([w_0, w_1]) - [f(w_0), w_1] - [w_0, f(w_1)]\| \leq \theta(||w_0||_0^r \cdot ||w_1||_0^r)$$
$$\|f(w_2^*) - f(w_2)^*\| \leq \theta||w_2||_0^r$$
$$||f(w_0 \circ w_1) - w_0 \circ f(w_1) - f(w_0) \circ w_1|| \leq \theta(||w_0||_0^r \cdot ||w_1||_0^r)$$

for all $x_1, \ldots, x_n,\ w_0, w_1, w_2 \in A_0$. Then $f$is a proper Lie JCQ*-derivation on A_0.

ACKNOWLEDGMENTS

The authors would like to thank the editors for their comments and suggestions that helped them to improve this article.*This work was supported by Dong-eui Foundation Grant (2014).

REFERENCES

[1] T. Aoki,On the stability of the linear transformation in Banach spaces, J. Math. Soc. Japan. 2 (1950), 64-66.

[2] F. Bagarello, A. Inoue and C. Trapani, *-Derivations of quasi-*-algebras, Internat. J. Math. Math. Sci. 21 (2004), 1077-1096.

[3] L. Cădariu and V. Radu, Fixed points and the stability of Jensen's functional equation, J. Inequal. Pure Appl. Math. 4 (2003), Article 4.

[4] Y.J. Cho, R. Saadati and J. Vahidi, Approximation of homomorphisms and derivations on non-Archimedean Lie *C**-algebras via fixed point method, DiscreatDyn. Nat. Soc. 2012 (2012), Article ID 373904, 9 pp.

[5] P. Găvruta, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, J. Math. Anal. Appl. 184 (1994), 431-436.

[6] M.E. Gordji and A. Fazeli, Stability and superstability of homomorphisms on C*-ternary algebras, An. Stiint. Univ. “Ovidius" Constanta Ser. Math. 20 (2012), 173-188.

[7] M.E. Gordji and G.H. Kim, Approximate *n*-Lie homomorphisms and Jordan *n*-Lie homomor- phisms on *n*-Lie algebras, Abs. Appl. Anal. 2012, Article ID 279632, 11pp.

[8] D.H. Hyers, On the stability of the linear functional equation, Proc. Natl. Acad. Sci. 27 (1941), 222-224.

[9] S.-M. Jung, Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis, Hadronic Press, Palm Harbor, 2001.

[10] S.-M. Jung, Hyers-Ulam stability of a system of first order linear differential equations with constant coefficients, J. Math.Anal. Appl. 320 (2006), 549-561.

[11] S.S. Kim, J.M. Rassias, Y.J. Cho and S.H. Kim, Stability of n-Lie homomorphisms and Jordann-Lie homomorphisms on n-Lie algebras, J. Math. Phys. 54 (2013), 053501.

[12] G.Y. Kim, S.S Kim and S.H. Kim, On the stability of *-derivations on Lie C*-algebras, J. Adv. Phy. 5 (2014), 801-806.

[13] Y. Li, Y. Shen, Hyers-Ulam stability of linear differential equations of second order, Appl. Math. Lett. 23 (2010), 306-309.

[14] N. Lungu, C. Crăciun, Ulam-Hyers-Rassias stability of a hyperbolic partial differential equation, ISRN Math. Anal. 2012 (2012), Article ID 609754, 10pp.

[15] A. Najati, C. Park and R.J. Lee, Homomorphisms and derivations in C*-ternary algebras, Abs. Appl. Anal.2009 (2009), Artical ID 612392.

[16] A. Najati and A. Ranjbari, Stability of homomorphisms for a 3D Cauchy-Jensen type functional equation on C*-ternary algebras, J. Math.Anal. Appl. 341 (2008), 62-79.

[17] C. Park, Homomorphisms between PoissinJC*-algebras, Bull. Brazil. Math. Soc. 36 (2005), 79-97.

[18] C. Park, Lie *-homomorphisms between Lie C*-algebras and Lie *-derivations on Lie C*-algebras, J. Math. Anal. Appl. 293 (2004), 419-434.

[19] C. Park, G.Z. Eskandani, H. Vaezi and D.Y. Shin, Hyers-Ulam stability of derivations on proper Jordan CQ*-algebras, J. Inequal. Appl. 2012, 2012:114.

[20] J.M. Rassias and H.M. Kim, Approximate homomorphisms and derivations between C*-ternary algebras, J. Math.Phys. 49.063507 (2008).

[21] Th.M. Rassias, On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. 72 (1978), 297-300.

[22] R. Saadati, Gh. Sadeghi and Th.M. Rassias, Approximate generalized additive mappings in proper multi-CQ*-algebras, Filomat. 28 (2014), 677-694.

[23] C. Trapani, Quasi-∗-algebras of operators and their applications, Rev. Math. Phys. 7 (1995), 1303-1332.

[24] S.M. Ulam, Problems in Modern Mathematics, Science Editions, John Wiley & Sons, Inc., New York, USA, 1964.