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# An Analysis of Internal Gravity Wave Tunnelling in a Stratified Region Along with Rotation

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## ABSTRACT

The internal gravity wave tunnelling in presence of earth's rotation is studied for different density barriers. An exponential approximation used reveals the existence of evanescence in the barrier region which signifies the trapping of wave energy in the tunnelling region. The Transmission coefficients are computed for different density barriers and the comparative study shows that across the locally mixed region the transmission is enhanced. The asymptotic analysis of the transmission co-efficient using the rotational parameter reveals the convergence and the graphs shows that the transmission decreases continuously and leads to the non-rotating case. The results are compared with the non-rotational case and we observe that the evanescence caused by the rotation makes the waves travel more along the horizontal direction than in the vertical direction.

## Keywords

Alfvén-gravity wave; Density barrier; stratified fluid; transmission coefficient; Evanescence

## Academic Discipline And Sub-Disciplines

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## 1. Introduction

Gravity waves in the atmosphere provide significant and important dynamic coupling between horizontally and vertically separated atmospheric regions. Even under ideal (windless and inviscid) conditions, upward propagation through exponentially decreasing atmospheric density will result in exponential increase in wave velocity perturbations, conserving kinetic energy with altitude [1]. Understanding the local propagation characteristics of gravity waves is very important, because under certain conditions, gravity waves are able to transport energy and momentum vertically, and under other conditions they can be trapped (ducted), confining the major flow of wave energy and momentum to a limited range of altitude, and allowing long-range horizontal propagation [2]. Gravity waves propagate in the vertical direction when the vertical wavenumber is real and the magnitude of intrinsic frequency is less than the Brunt-väisälä frequency, N, and they become evanescent when the vertical wavenumber is imaginary and the intrinsic frequency exceeds the Brunt-väisälä frequency, N [3].

Earlier studies of gravity wave trapping was confined to three mechanisms namely variation in atmospheric structure [4], the variation in dissipation ([5], [6],[7]) and the variation in the background wind [8]. In this paper we provide an analytic prediction for the transmission co-efficient of internal gravity waves crossing a region in which is reduced (weakly stratified region). The results of this paper are of atmospheric, climatic and biophysical importance. An analytic theory for nonhydrostatic internal wave tunnelling through a weakly stratified fluid layer was derived by Sutherland and Yewchuk [9] and in this paper we study the effect of rotation. The transmission co-efficient of an internal gravity waves in the presence of rotation crossing over a barrier is computed and the effect of rotation on the transmission co-efficient is studied. We have also analysed at the large and small-time behavior associated with the rotational parameter which is inherent in the transmission of the internal gravity waves.

## 2. Mathematical Formulation

We study the wave tunnelling in a non-conducting fluid in presence of earth's rotation with vertical density stratification and the transient disturbance produced by temporary extraneous forces which is horizontally and temporally periodic in the form:

$$f(x,y,z,t) = \hat{f}(z)\exp[i(kx + ly - \omega t)] \quad (1)$$

It may be shown that with these assumptions along with the well defined horizontal wave number $k(>0)$ and phase velocity $\omega/k$, the vertical disturbance velocity, $\hat{w}$ satisfy the following linearized equation:

$$\frac{d^2\hat{w}}{dz^2} + \frac{\alpha^2\left(\frac{N^2}{\omega^2} - 1\right)}{\left(1 - \frac{4\Omega^2}{\omega^2}\right)}\hat{w} = 0. \quad (2)$$

Here, the stratification of the mean flow is described in terms of a single parameter which may vary with $z$, the Brunt-väisälä frequency $N$, defined by $N(z) = \left(-\frac{g}{\rho_b}\frac{d\rho_b}{dz}\right)^{\frac{1}{2}}$ and the rotation of earth is described by the parameter $\Omega$.

The solution of the equation (2.2) is given by

$$\hat{w} = Ae^{i\eta z} + Be^{-i\eta z} \quad , \quad (3)$$

where $\eta = -\alpha\left(\frac{N^2 - \omega^2}{\omega^2 - 4\Omega^2}\right)^{\frac{1}{2}}$, $\alpha = \left(l^2 + k^2\right)^{\frac{1}{2}}$, $A$ and $B$ are arbitrary constants. The vertical wave number for $|z| > L/2$, it is defined to be negative so that the incident wave and transmitted wave propagate upward. We seek to interpret these solutions as upward or downward propagating waves [10] in the following subsection which plays a significant role in understanding tunnelling, transmission and reflection of waves in the presence of weakly stratified regions.

## 3. Transmission across a barrier of density stratification

We study this for the following cases of density stratification:

(i) Uniform density fluid of finite depth $-L/2 < z < +L/2$ sandwiched on either side by a stratified fluid extending to infinity, called $N^2$-barrrier1.

(ii) Weakly stratified fluid of finite depth $-L/2 < z < +L/2$ sandwiched on either side by a stratified fluid extending to infinity with density discontinuity at the two interfaces, called $N^2$-barrrier2.

(iii) Weakly stratified fluid of finite depth $-L/2 < z < +L/2$ sandwiched on either side by a stratified fluid extending to infinity with density being continuous at the interfaces, called locally mixed region.

Transmission across each of these barriers is described in the following subsections.

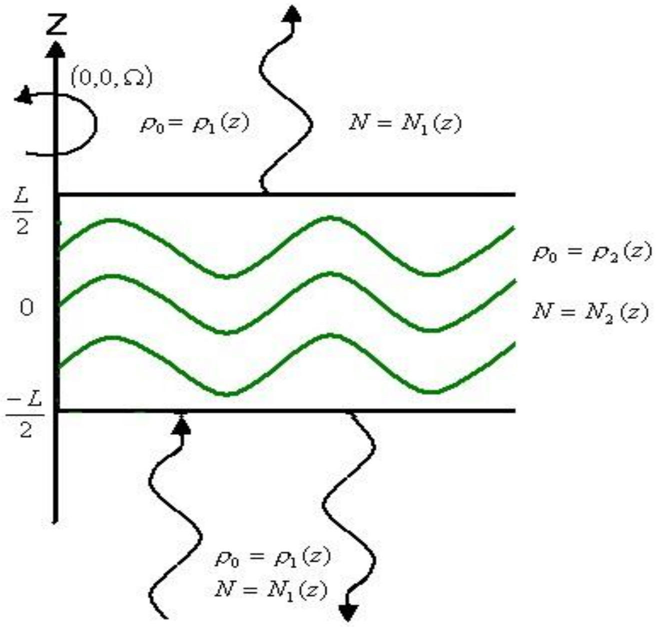


**FIGURE 1. Schematic representation of the system ( $N^2$-barrier1)**

### 3.1 *Transmission across the* $N^2$*-barrier1*

In this case we have a fluid of uniform density of finite depth L bounded on either side by a stratified non-conducting fluid extending to infinity. We assume

$$N^2 = \begin{cases} N_0^2, & |z| > \dfrac{L}{2} \\ 0, & |z| \le \dfrac{L}{2} \end{cases} \tag{4}$$

This is called '$N^2$-barrier1' of depth L as shown in figure 1. Now the solution of (2) can be written as

$$\hat{w} = \begin{cases} A_3 e^{i\eta z} & z > \dfrac{L}{2} \\ A_2 e^{\frac{z}{\delta}} + B_2 e^{\frac{-z}{\delta}} & \dfrac{-L}{2} \le z \le \dfrac{L}{2} \\ A_1 e^{i\eta z} + B_1 e^{-i\eta z} & z < \dfrac{L}{2} \end{cases}, \tag{5}$$

where $\eta = -\alpha\left(\dfrac{N_0^2 - \omega^2}{\omega^2 - 4\Omega^2}\right)^{\frac{1}{2}}$, $\alpha^2 = k^2 + l^2$ and $k = \dfrac{1}{\delta} = \alpha \Big/ \left(1 - \dfrac{4\Omega^2}{\omega^2}\right)^{\frac{1}{2}}$, is the well defined horizontal wave number, $\omega(\le N)$ is the wave frequency and we assume $\omega > 2\Omega$. We determine the transmission coefficient $|A_3/A_1|^2$ by using the boundary conditions at the interface $z = \pm L/2$ requiring that the vertical velocity and pressure

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are continuous [11] at each of the interfaces. For a fluid with zero mean flow this amounts to requiring pressure and velocity are continuous across each of the interfaces. Applying these boundary conditions to the solution (3) we get a system of four linear equations and solving for transmitted amplitude $A_3$, in terms of the incident amplitude $A_1$, gives a transmission coefficient $T_r = \left|A_3/A_1\right|^2$, which represents the fraction of energy transported across $N^2$-barrrier1 and is given by

$$T_r = \left[1 + \frac{\left(1+\eta^2\delta^2\right)^2}{4\eta^2\delta^2}\sinh^2\left(\frac{L}{\delta}\right)\right]^{-1} . \qquad (6)$$

Equation(2) obtained coincides with the result obtained by [9] for a non-conducting fluid. We note that $T_r$ given by (3.3) is a function of $\Omega$. The maximum value of $T_r$ at $\Omega = 0$ is given by

We note that $T_r$ decreases and becomes zero when $\Omega = \omega/2$ and from $\eta$ we find that when $\Omega = \omega/2$ the vertical wave number $\eta$ tends to $\infty$ the system becomes evanescent. Thus the effect of rotational parameter is to make the wave propagate horizontally rather than allow it to propagate upwards. We have plotted the graph of $T_r$ against $\Omega$ in figures 2 for various values of $N_0$ and $\omega$ and we find that as $\Omega$ increases $T_r$ decreases continuously and becomes 0 when $\Omega = \omega/2$.

$$\left(T_r\right)_{\max} = \left[1 + \frac{N_0^4}{4\omega^2\left(N_0^2 - \omega^2\right)} Sinh^2\left(L\alpha\right)\right]^{-1}, \qquad (7)$$

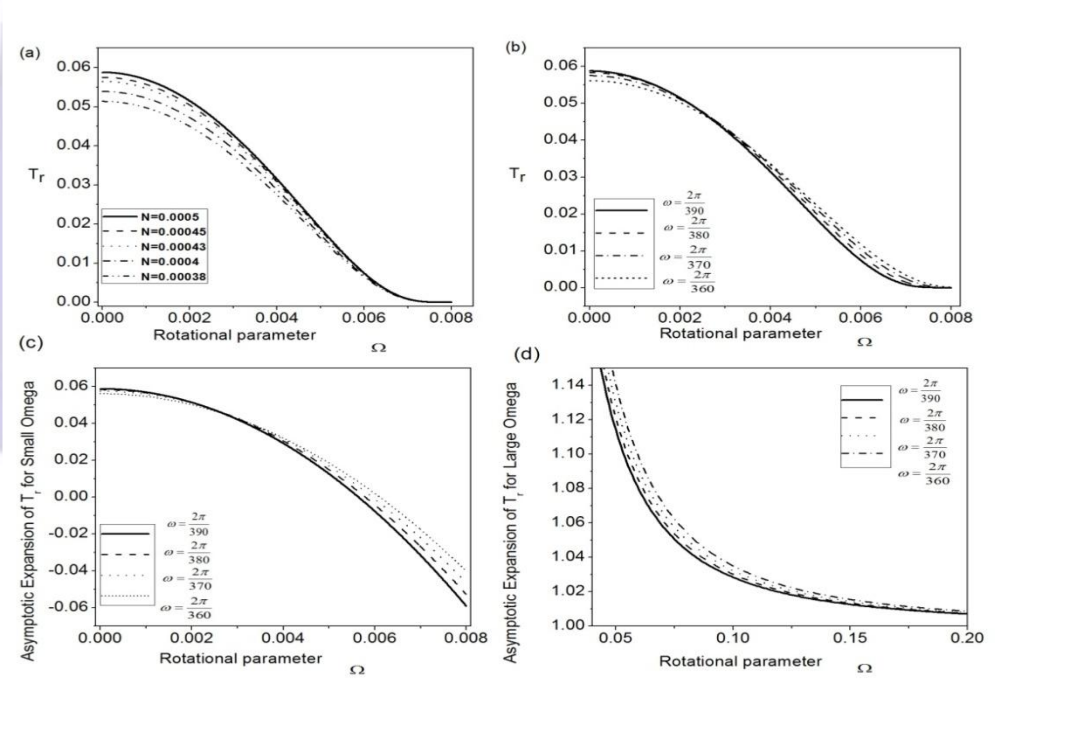


**FIGURE 2. (a) Variation of transmission coefficient $T_r$ with $\Omega$ for $2\Omega > N_0$ for $\omega = 2\pi/390$, $L = 5$, $k = 2\pi/15$ (b) Variation of Transmission coefficient $T_r$ with $\Omega$ for $2\Omega > N_0$ $N_0 = 0.0005$, $k = 2\pi/15$ $L = 5$ (c) Asymptotic expansion of transmission coefficient $T_r$ for small $\Omega$ (d) Asymptotic expansion of transmission coefficient $T_r$ for large $\Omega$**

The variation of the transmission coefficient $T_r$ in terms of $\Omega^2$ for $\Omega << 1$ using asymptotic expansion is in the form

$$(T_r)_{small\Omega} = (T_s)^{-1} - \frac{N_0^4 \alpha L \sinh(2\alpha L)}{2\omega^4 \left(N_0^2 - \omega^2\right)} \frac{1}{T_0^2} \Omega^2 + ----------- + \infty , \quad (8)$$

where,

$$T_s = \left(1 + \frac{N_0^4}{4\omega^2 \left(N_0^2 - \omega^2\right)} \sinh^2(\alpha L)\right), \alpha = \left(k^2 + l\right)^{\frac{1}{2}} .$$

$T_s$ corresponds to the transmission coefficient in the absence of magnetic field obtained by Sutherland and Yewchuk [9] with $l$ being zero. In the above expansion the coefficient of $\Omega^2$ is negative when $N_0 > \omega$. Thus when $N_0 > \omega$, $(T_r)_{small\Omega}$ decreases continuously. In the limit $\Omega \to 0$, the transmission coefficient $(T_r)_{small\Omega} \to T_s$. The variation of the transmission coefficient $T_r$ with $\Omega$ in terms of $\Omega^2$ for $\Omega >> 1$ using asymptotic expansion is in the form.

$$(T_r)_{large\Omega} = 1 + \frac{N_0^4 \alpha^2 L^2}{16\left(N_0^2 - \omega^2\right)} \frac{1}{\Omega^2} + -------------------- + \infty , \quad (9)$$

In the limit $\Omega \to \infty$, the transmission coefficient $(T_r)_{large\Omega} \to 1$.

### *3.2 Transmission across the $N^2$ -barrier2*

In the second case we assume a weakly stratified fluid of finite depth L bounded on either side by a strongly stratified fluid extending to infinity on either side. In this case we have

$$N^2 = \begin{cases} N_0^2, & |z| > \dfrac{L}{2} \\ N_1^2, & |z| \le \dfrac{L}{2}. \end{cases} \quad (10)$$

This is called '$N^2$-barrier2' of depth L. With this the solution of (2) takes the form

$$\hat{w} = \begin{cases} A_3 e^{i\eta z} & z > \dfrac{L}{2} \\ A_2 e^{\xi z} + B_2 e^{-\xi z} & \dfrac{-L}{2} \le z \le \dfrac{L}{2} \\ A_1 e^{i\eta z} + B_1 e^{-i\eta z} & z < \dfrac{L}{2} \end{cases} , \quad (11)$$

where, $\eta = -\alpha \left(\frac{N_0^2 - \omega^2}{\omega^2 - 4\Omega^2}\right)^{\frac{1}{2}}$, $\xi = \alpha \left(\frac{\omega^2 - N_1^2}{\omega^2 - 4\Omega^2}\right)^{\frac{1}{2}}$ and $k = \frac{1}{\delta} = \alpha \Big/ \left(1 - \frac{4\Omega^2}{\omega^2}\right)^{\frac{1}{2}}$. Kinematic and dynamic boundary conditions are used at the interface $z = \pm L/2$, since the vertical velocity and pressure being continuous across the interface. The transmission coefficient is given by

$$T_{rb} = \left[1 + \frac{\left(\xi^2 + \eta^2\right)^2}{4\eta^2 \xi^2} \sinh^2(\xi L)\right]^{-1} . \quad (12)$$

$T_{rb}$ in (12) reaches the transmission obtained by Sutherland & Yewchuk [9] for $N^2$-barrier2 for a non-rotating case. In the limit $N_1 \to 0$ (12) reduces to the transmission coefficient (6) obtained for $N^2$-barrier1 in section 3.1.

The variation of $T_{rb}$ with $\Omega$ is discussed below based on the vertical wave numbers $\xi$ (in the region $|z| \le L/2$) and $\eta$ ( in the region $|z| > L/2$) are real or imaginary. Here we discuss the case in which $\eta$ and $\xi$ are real. From the expressions for $\eta$ and $\xi$ we have shown that for $\eta$ to be real $N_0 > \omega$ and $\omega > 2\Omega$ and $\xi$ to be real $\omega > N_1$ and $\omega > 2\Omega$ for we have assumed that $N_1 \le \omega \le N_0$. Since $\eta$ and $\xi$ are real we have vertically propagating waves in the region $|z| > L/2$ and evanescent waves in the region $|z| \le L/2$. This is similar to the waves that exist in these regions in $N^2$-barrier1 considered in section 3.1. We have plotted the graph of $T_{rb}$ against $\Omega$ in figures 3.

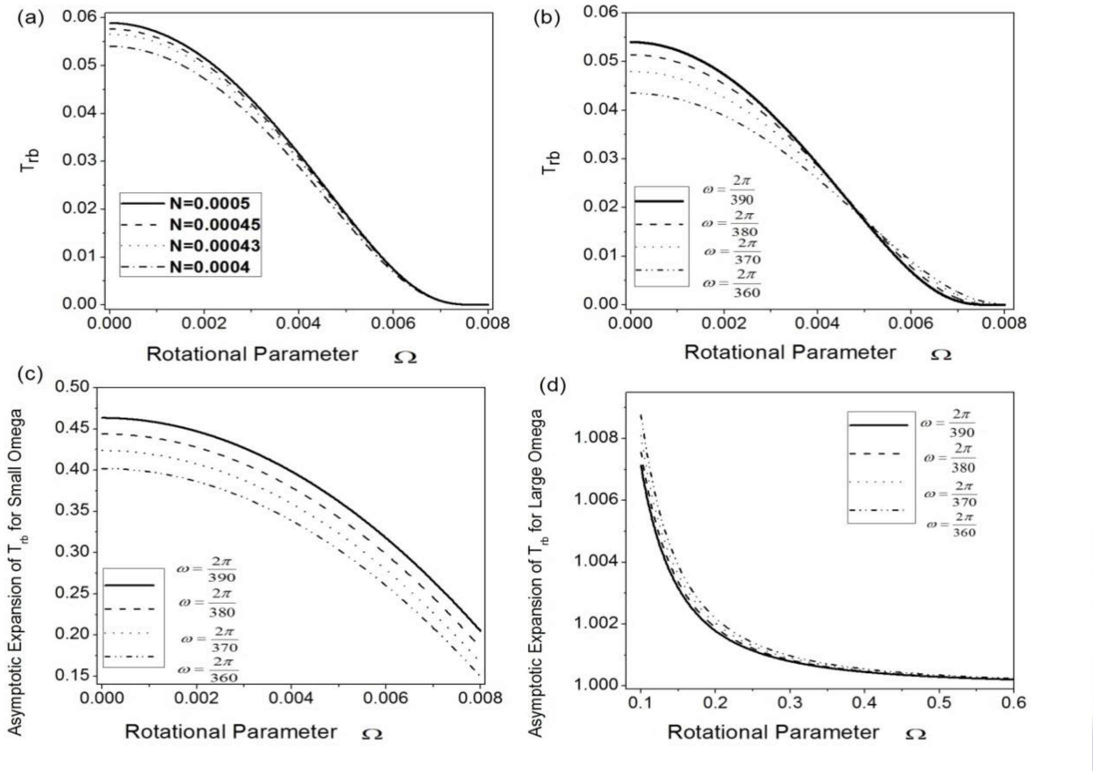


**FIGURE 3. (a) Variation of transmission coefficient $T_r$ with $\Omega$ for $\omega > 2\Omega$ and $\omega = 2\pi/390$, $k = 2\pi/15$, $L = 5$, (b) Variation of transmission coefficient $T_{rb}$ with $\Omega$ for $2\Omega > N_0$ $N_0 = 0.0004$, $k = 2\pi/15$, $L = 5$ (c) Asymptotic expansion of transmission coefficient $T_r$ for small $\Omega$ (d) Asymptotic expansion of transmission coefficient $T_r$ for large $\Omega$.**

In figure3 we find that the variation $T_{rb}$ with $\Omega$ is similar to that of $N^2$-barrier1 in section 3.1. As $\Omega$ increases from 0 , $T_{rb}$ decreases to zero. The variation of the transmission coefficient $T_{rb}$ in terms of $\Omega^2$ for $\Omega^2 << 1$ using asymptotic expansion is in the form

$$(T_{rb})_{small} = (T_{sb})^{-1} + \frac{\left(N_0^2 - N_1^2\right)^2 \beta\alpha L a_0 \left(1 - \frac{N_1^2}{\omega^2}\right)^{\frac{1}{2}} \sinh\left(\alpha L \left(1 - \frac{N_1^2}{\omega^2}\right)^{\frac{1}{2}}\right)}{4\omega^2\left(N_0^2 - \omega^2\right)\left(\omega^2 - N_1^2\right) T_{sb}^2} \Omega^2 + - - + \infty$$

( 13)

where

$$T_{sb} = \left(1 + \frac{\left(\gamma^2 + \beta^2\right)^2}{4\gamma^2\beta^2}\sinh^2\left(\beta L\right)\right)^{-1},\ a_0 = \frac{k^2}{\omega^2}.$$

The variation of $T_{rb}$ with small values of $\Omega$ depends on the coefficient of $\Omega^2$ in (13). If the coefficient is positive $T_{rb}$ increases with $\Omega$ and if it is negative it decreases with $\Omega$. From the coefficient of $\Omega^2$ we find that its sign depend on the sign of $\left(N_0^2 - \omega^2\right)$ and $\left(\omega^2 - N_1^2\right)$ which are always positive since we have considered $N_1 \le \omega \le N_0$. In the limit $\Omega \to 0$ its value coincides with that obtained in the case of Sutherland and Yewchuk [9].

In figure 3 we find that as $\Omega$ increases from 0, $T_{rb}$ increases and reaches maximum and becomes zero instantaneously. The variation of the transmission coefficient $T_{rb}$ in terms of $\Omega^2$ for $\Omega^2 >> 1$ using asymptotic expansion is in the form

$$\left(T_{rb}\right)_{large\Omega} = 1 + \frac{\left(N_0^2 - N_1^2\right)^2 \alpha^2 L^2}{16\left(N_0^2 - \omega^2\right)}\frac{1}{\Omega^2} + ------------------ + \infty \quad (14)$$

In the limit $\Omega \to \infty$, the transmission coefficient $\left(T_{rb}\right)_{large\Omega} \to 1$.

### *3.3 Transmission across locally mixed region*

In this case, $\rho_b\left(z\right)$ is assumed to vary continuously, even though its slope is discontinuous at $z = \pm\frac{L}{2}$. More realistically, localized mixed regions within a stratified fluid are better represented by a discontinuous density profile in the form:

$$\rho_b = \begin{cases} \rho_0\left(1 - \dfrac{z}{H_1}\right) & |z| \le \dfrac{L}{2} \\ \rho_0\left(1 - \dfrac{z}{H_0}\right) & |z| > \dfrac{L}{2} \end{cases}. \quad (15)$$

This is called '$N^2$-barrier3' of depth L as shown in figure1. Where $H_0 \equiv \frac{g}{N_0^2}$ *and* $H_1 \equiv \frac{g}{N_1^2}$ measure the strength of stratification respectively outside and within a partially mixed region of depth L. Consistent with the Boussinesq approximation, we assume $H_0, H_1 \rangle\rangle L, k^{-1}$ . The corresponding squared buoyancy frequency is the same as that for the generalization of the $N^2$-barrier except for infinite spikes at $z = \pm\frac{L}{2}$ where the density changes discontinuously by $\Delta\rho_b = \rho_0\left[\frac{\left(N_0^2 - N_1^2\right)}{g}\right]\left(\frac{L}{2}\right)$. The prescribed $N^2$-barrier in this case is as follows:

$$N^2 = \begin{cases} N_0^2, & |z| > \dfrac{L}{2} \\ N_1^2, & |z| \le \dfrac{L}{2} \end{cases}, \quad (16)$$

where $N_1 \le \omega \le N_0$, requiring velocity and pressure to be continuous across the interface we compute the transmission coefficient in the case is given by:

$$T_{rmix} = \left[ 1 + \left( \frac{\left(\xi^2 + \eta^2\right)^2}{4\eta^2 \xi^2} \sinh^2\left(\xi L\right) \Gamma_{rmix}^2 \right)^2 \right]^{-1}, \qquad (17)$$

in which $\Gamma_{rmix} = \left[ 1 + \frac{L^2 \alpha^2 N_0^2 \left(1-\sigma^2\right)}{4\left(\omega^2 - 4\Omega^2\right)} - L\xi Coth\left(L\xi\right) \right].$

In the limit $\Omega \to 0$ $T_{rmix}$ in (17) reduces to non-conducting fluid results of Sutherland & Yewchuk [9]. From (17) we note that $T_{rb}$ varies with $\Omega$. The variation of $T_{rb}$ with $\Omega$ is discussed below based on the vertical wave numbers $\xi$ (in the region $|z| \le L/2$) and $\eta$ ( in the region $|z| > L/2$) are real or imaginary and we consider the following case for $\eta$ and $\xi$ real from the expressions for $\eta$ and $\xi$. We have shown that for $\eta$ to be real when $N_0 > \omega$ and $\omega > 2\Omega$ and $\xi$ to be real when $\omega > N_1$ and $\omega > 2\Omega$ for we have assumed that $N_1 \le \omega \le N_0$. Since $\eta$ and $\xi$ are real we have vertically propagating waves in the region $|z| > L/2$ and evanescent waves in the region $|z| \le L/2$. This is similar to the waves that exist in these regions in $N^2$-barrier1 considered in section 3.1. The transmission coefficient is given by (17). However we have plotted the graph of $T_{rmix}$ against $\Omega$ in figure 3 when $\omega > 2\Omega$.

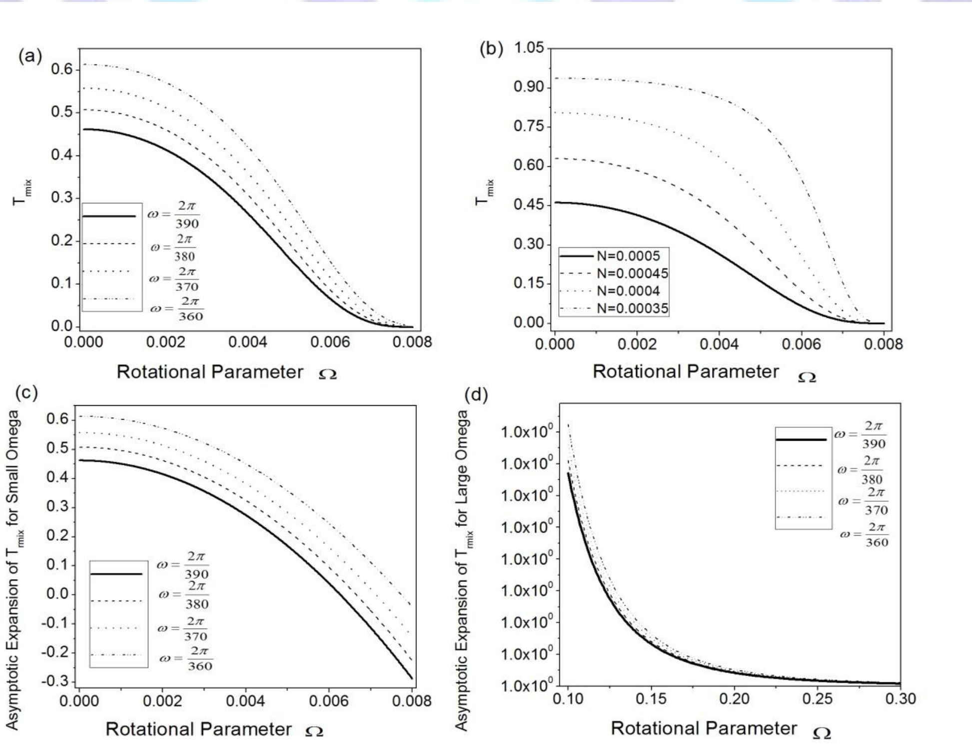


**FIGURE 4. (a) Variation of transmission coefficient $T_{rmix}$ with $\Omega$ for $\omega > 2\Omega$ $\omega = 2\pi/390$, $k = 2\pi/15$, $L = 5$ (b) Variation of transmission coefficient $T_{rmix}$ with $\Omega$ for $2\Omega > N_0$ $N_0 = 0.0004$, $k = 2\pi/15$, $L = 5$ .(c) Asymptotic expansion of transmission coefficient $T_{rmix}$ for small $\Omega$ (d) Asymptotic expansion of transmission coefficient $T_{rmix}$ for large $\Omega$.**

From the figure 5(a) we find that the variation $T_{rmix}$ with $\Omega$ of $N^2$-barrier1 in section 3.1. $T_{rmix}$ reaches maximum at $\Omega = 0$ then decreases zero at $\Omega = \omega/2$.

The variation of the transmission coefficient $T_{rmix}$ in terms of $\Omega^2$ for $\Omega^2 << 1$ using asymptotic expansion is in the form

$$\left(T_{rmix}\right)_{small} = \left(T_{smix}\right)^{-1} + \frac{\left(N_0^2 - N_1^2\right)^2 \left[2\tau\tau_1 \sinh^2\left(\beta L\right) - \frac{\beta L a_0}{2} \sinh\left(2\beta L\right)\tau^2 \varepsilon\right]}{4\left(N_0^2 - \omega^2\right)\left(\omega^2 - N_1^2\right) T_{smix}^2} \Omega^2 + - - + \infty \qquad (18)$$

where,

$$T_{smix} = \left(1 + \frac{\left(N_0^2 - N_1^2\right)^2}{4\omega^2 \left(N_0^2 - \omega^2\right)\left(\omega^2 - N_1^2\right)} \sinh^2\left(\beta L\right) \tau_s{}^2\right)$$

$$\tau = \left[1 + \frac{L^2 \alpha^2 N_0^2}{4\omega^2}\left(1 - \frac{N_1^2}{N_0^2}\right) - L\beta \coth\left(L\beta\right)\right]$$

$$\tau_1 = \left[\frac{L\alpha\, a_0}{2}\left(1 - \frac{N_1^2}{\omega^2}\right)^{\frac{1}{2}} \coth(\beta L) - \frac{L^2\alpha^2\left(N_0^2 - N_1^2\right)}{4\omega^2} a_0 - \beta^2 L^2 \frac{a_1}{2} \cos ec^2 h(\beta L)\right],$$

$$\beta = \alpha\left(1 - \frac{N_1^2}{\omega^2}\right)^{\frac{1}{2}},$$

In the limit $\Omega \to 0$ the above equation reduces to the equation obtained in the case of Sutherland (2004). The variation of the transmission coefficient $T_{rmix}$ in terms of $\Omega^2$ for $\Omega^2 >> 1$ is in the form

$$\left(T_{rmix}\right)_{small} = 1 + \frac{\left(N_0^2 - N_1^2\right)^2 \alpha^2 L^2 \Gamma_1^2}{16\left(N_0^2 - \omega^2\right)} \frac{1}{\Omega^2} + - - - - - - - - - - - - - - - - + \infty \qquad (19)$$

where, In the limit $\Omega \to \infty$ transmits maximum without any reflection.

To understand how the transmission coefficient changes from $N^2$-barrier1 to $N^2$-barrier2 we have plotted graphs of $T_r$ and $T_{rb}$ against $\Omega$ and also to understand how the transmission coefficient changes from $N^2$-barrier 2 to $N^2$-barrier 3 we have plotted graphs of $T_{rmix}$ and $T_{rb}$ against $\Omega$ in figure 5.

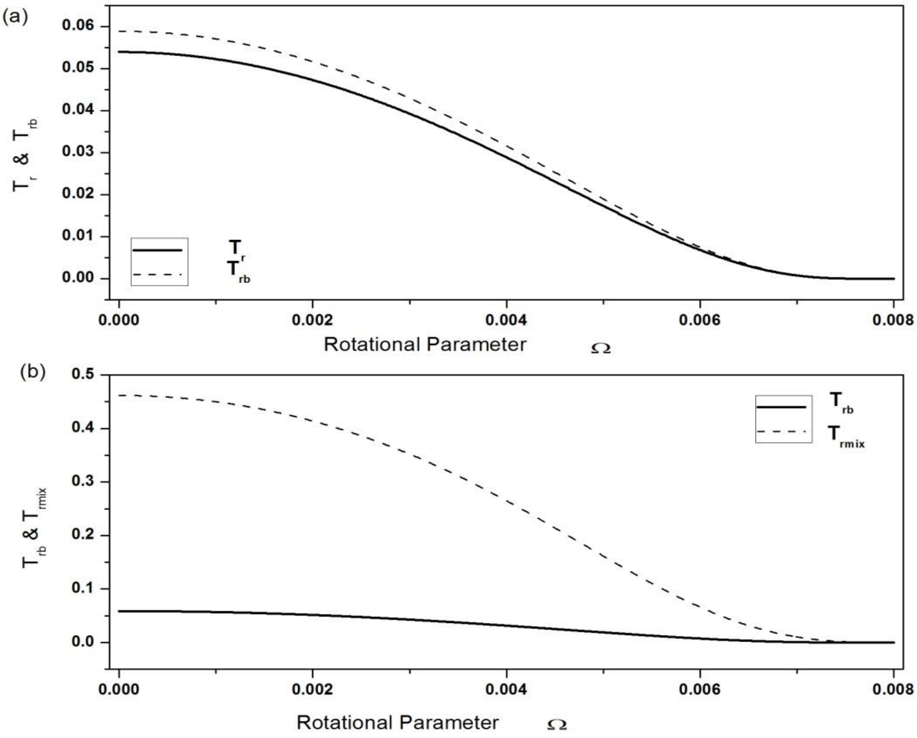


**FIGURE 5. (a) Comparison of transmission coefficients $T_r$ and $T_{rb}$ for $N^2$-barrier1 and $N^2$-barrier2 for $N_1 = 0.00002$, $k = 2\pi/15$, $\omega = 2\pi/390$ $L = 5$. (b) Comparison of transmission coefficients $T_{rmix}$ and $T_{rb}$ for $N^2$-barrier2 and $N^2$-barrier3 for $N_1 = 0.00002$, $k = 2\pi/15$, $\omega = 2\pi/390$, $L = 5$, $N_0 = 0.0004$.**

From the figure 5(a) we find that the transmission coefficient $T_{rb}$ is higher for $N^2$-barrier2. This is because in region $|z| \le L/2$ the waves propagate in $N^2$-barrier2 and evanescent in $N^2$-barrier1.

From 5(b) we find that the transmission coefficient in $N^2$-barrier 3 is higher when $\Omega$ is small and lower when $\Omega$ is large. This is because as $\Omega$ increases gravity waves propagate more and more horizontally (i.e. along the direction of the fluid line) rather than vertically.

## 4. Results and Conclusions

As we have investigated the internal gravity wave tunnelling through an incompressible, inviscid, Boussinesq, non-conducting incompressible fluid for a rotating system. We have derived the transmission co-efficients (which represents the fraction of incident energy transported) analytically for three different $N^2$-barriers in sections 3.1, 3.2 and 3.3.

In section 3.1 we have obtained the transmission coefficient for $N^2$-barrier1. We have shown that the transmission coefficient varies with $\Omega$ for various values of $N_0^2$, $N_1^2$ and $\omega^2$. We have found that the maximum value of $T_r$ is a function of Brunt-väisälä frequency $N_0$, the wave frequency $\omega$ but independent of the rotational parameter $\Omega$ and when $\Omega = 0$, $T_r$ has obtained the maximum which is evident from figures 2. As $\Omega$ increases further to $\omega/2$ the transmission coefficient decreases to 0. We also note from expression for $\eta$ when $\Omega = \omega/2$ the vertical wave number $\eta$ tends to $\infty$ so that the waves become more and more horizontal. Thus the effect of rotational parameter is to make the wave to propagate along the fluid lines rather than allow it to propagate upwards.

In section 3.2 we have obtained the transmission coefficient for the effect of rotation on the transmission coefficient for four different cases have been identified based on the vertical wave numbers $\xi$ (in the region $|z| \le L/2$)

and $\eta$ ( in the region $|z| > L/2$ ) defined below (11) are real or imaginary. In case1, when $\eta$ and $\xi$ real we find that the variation $T_{rb}$ with $\Omega$ is similar to that in case1 of $N^2$-barrier1 in section 3.1. From the figure 5 we find that the transmission coefficient $T_{rb}$ is higher for $N^2$-barrier2. This is because in region $|z| \le L/2$ the waves propagate in $N^2$-barrier2 and evanescent in $N^2$-barrier1. In case 3, when $\eta$ and $\xi$ imaginary we find that the variation $T_{rb}$ attains maximum and becomes zero at $\Omega = \omega/2$.

In section 3.3 we find that, the effect of rotational parameter is to make the wave to propagate along the fluid lines rather than allow it to propagate upwards which is depicted in graphs. From the figure 5 we find that the transmission coefficient in $N^2$-barrier 3 is higher when $\Omega$ is small and lower when $\Omega$ is large. This is because as $\Omega$ increases gravity waves wave to propagate along the fluid lines rather than allow it to propagate upwards. The above results conclude that rotation accounts for the evancescence the barrier region.

## Acknowledgements

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