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Traveling Wave Solutions For The Couple Boiti-Leon-Pempinelli System By Using Extended Jacobian Elliptic Function Expansion Method

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ABSTRACT

In this work, an extended Jacobian elliptic function expansion method is proposed for constructing the exact solutions of nonlinear evolution equations. The validity and reliability of the method are tested by its applications to the Couple Boiti-Leon-Pempinelli System which plays an important role in mathematical physics.

Keywords

Extended Jacobian elliptic function expansion method; The Couple Boiti-Leon-Pempinelli System; Traveling wave solutions; hyperbolic solutions.

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1- Introduction

The nonlinear partial differential equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, Optics, Plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, tanh – sech method [2]-[4], extended tanh - method [5]-[7], sine - cosine method [8]-[10], modified simple equation method [11, 12], the exp $\left(exp(-\phi(\xi))\right)$ −expansion method [13]-[15], the extended exp $\left(exp(-\phi(\xi))\right)$ − expansion method [16], $\left(\frac{G'}{G}\right)$ −expansion method [17]-[19], Jacobi elliptic function method [20]- [23] and so on.

The objective of this article is to apply the extended Jacobian elliptic function expansion method for finding the exact traveling wave solution the Couple Boiti-Leon-Pempinelli System which play an important role in mathematical physics. The rest of this paper is organized as follows: In Section **2**, we give the description of the extended Jacobi elliptic function expansion method In Section **3**, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section **4**, conclusions are given.

2- Description of method

Consider the following nonlinear evolution equation

$$P(u,u_t,u_x,u_{tt},u_{xx},\ldots\ldots)=0, \quad (2.1)$$

Since, P is a polynomial in $u(x,t)$ and its partial derivatives. In the following, we give the main steps of this method

Step 1. We use the traveling wave solution in the form

$$u(x,t)=u(\xi), \qquad \xi=x-ct, \quad (2.2)$$

where c is a positive constant, to reduce Eq. (2.1) to the following ODE:

$$p(u,u',u'',u''',\ldots\ldots)=0, \quad (2.3)$$

where P is a polynomial in u (ξ) and its total derivatives, while $u'=\dfrac{du}{d\xi}$.

Step 2. Making good use of ten Jacobian elliptic functions we assume that (2.3) have the solutions in these forms:

$$u(\xi)=a_0+\sum_{j=1}^{N} f_i^{\,j-1}(\xi)\left[a_j f_i(\xi)+b_j g_i(\xi)\right],\ i=1,2,3,\ldots \quad (2.4)$$

With

$$\begin{aligned}
f_1(\xi)&=sn\xi,\ g_1(\xi)=cn\xi,\\
f_2(\xi)&=sn\xi,\ g_2(\xi)=dn\xi,\\
f_3(\xi)&=ns\xi,\ g_3(\xi)=cs\xi,\\
f_4(\xi)&=ns\xi,\ g_4(\xi)=ds\xi,\\
f_5(\xi)&=sc\xi,\ g_5(\xi)=nc\xi,\\
f_6(\xi)&=sd\xi,\ g_6(\xi)=nd\xi,
\end{aligned} \quad (2.5)$$

where $sn\xi, cn\xi, dn\xi,$ are the Jacobian elliptic sine function, The jacobian elliptic cosine function and the Jacobian elliptic function of the third kind and other Jacobian functions which is denoted by Glaisher's symbols and are generated by these three kinds of functions, namely

$$ns\xi=\frac{1}{sn\xi},nc\xi=\frac{1}{cn\xi},nd\xi=\frac{1}{dn\xi},sc\xi=\frac{cn\xi}{sn\xi},$$
$$cs\xi=\frac{sn\xi}{cn\xi},ds\xi=\frac{dn\xi}{sn\xi},sd\xi=\frac{sn\xi}{dn\xi}, \tag{2.6}$$

those have the relations

$$sn^2+cn^2=1,\ dn^2\xi+m^2sn^2\xi=1,\ ns^2\xi=1+cs^2\xi,$$
$$ns^2\xi=m^2+ds^2\xi,\ sc^2\xi+1=nc^2\xi,\ m^2sd^2+1=nd^2\xi, \tag{2.7}$$

with the modulus m (0 < m < 1). In addition we know that

$$\frac{d}{d\xi}sn\xi=cn\xi dn\xi,\frac{d}{d\xi}cn\xi=-sn\xi dn\xi,\frac{d}{d\xi}dn\xi=-m^2sn\xi cn\xi. \tag{2.8}$$

The derivatives of other Jacobian elliptic functions are obtained by using Eq. (2.8). To balance the highest order linear term with nonlinear term we define the degree of u as D[u] = n which gives rise to the degrees of other expressions as

$$D\left[\frac{d^qu}{d\xi^q}\right]=n+q,\ D\left[u^p\left(\frac{d^qu}{d\xi^q}\right)^s\right]=np+s\left(n+q\right). \tag{2.9}$$

According the rules, we can balance the highest order linear term and nonlinear term in Eq. (2.3) so that n in Eq. (2.4) can be determined.

In addition we see that when $m\to 1,\ sn\xi, cn\xi\, and\ dn\xi$ degenerate as $\tanh\xi, \sec h\xi, \sec h\xi$, respectively, while when therefore Eq. (2.5) degenerate as the following forms

$$u(\xi)=a_0+\sum_{j=1}^{N}\tanh^{j-1}(\xi)\left[a_j\tanh(\xi)+b_j\,\mathrm{sech}(\xi)\right],$$
$$u(\xi)=a_0+\sum_{j=1}^{N}\coth^{j-1}(\xi)\left[a_j\coth(\xi)+b_j\,\mathrm{csch}(\xi)\right],$$
$$u(\xi)=a_0+\sum_{j=1}^{N}\tanh^{j-1}(\xi)\left[a_j\tan(\xi)+b_j\sec(\xi)\right], \tag{2.10}$$
$$u(\xi)=a_0+\sum_{j=1}^{N}\coth^{j-1}(\xi)\left[a_j\cot(\xi)+b_j\csc(\xi)\right],$$

Therefore the extended Jacobian elliptic function expansion method is more general than sine-cosine method, the tan-function method and Jacobian elliptic function expansion method.

3- The Couple Boiti-Leon-Pempinelli System

Consider the Couple Boiti-Leon-Pempinelli System [24] is in the form

$$\begin{cases} u_{ty}=(u^2-u_x)_{xy}+2v_{xxx}, \\ v_t=v_{xx}+2uv_x. \end{cases} \tag{3.1}$$

Using the transformation $u(x,y,t)=u(\xi)$since $\xi=x+y-ct,$ and integrate the first equation of the system (3.1) three times with zero constant of integration and substitute it into the second equation of system (3.1) we obtained

$$u''-3u^3-3cu^2-c^2u=0. \tag{3.2}$$

Balancing $u''$and u^3 $we\ get\ N=1.$ So that, we assume the solution of Eq. (3.2) be in the form

$$u(\xi)=a_0+a_1sn+b_1cn. \tag{3.3}$$

Where a_0, a_1, b_1 are arbitrary constants to be determined later. Substituting Eq. (3.3) and it's derivatives into Eq. (3.2) and equating all coefficients of $sn^3, sn^2\ cn, sn^2, sn\ cn, sn, cn, sn^0$ to zero, we obtain

$$\begin{cases} 2a_1m^2 - 2a_1^3 + 6a_1b_1^2 = 0, \\ 2m^2b_1 - 6a_1^2b_1 + 2b_1^3 = 0, \\ 6a_0a_1^2 + 6a_0a_1^2 - 3c(a_1^2 - b_1^2) = 0, \\ -12a_0a_1b_1 - 6ca_1b_1 = 0, \\ -a_1 - a_1m^2 - 6a_0^2a_1 - 6a_1b_1^2 - 6ca_0a_1 - c^2a_1 = 0, \\ -6a_0^2b_1 - 2b_1^3 6ca_0b_1 - c^2b_1 = 0, \\ -2a_0^3 - 6a_0b_1^2 - 3a_0^2 - c^2a_0 = 0. \end{cases} \tag{3.4}$$

Solving above system by using Maple 16, we obtained

$$c = -\sqrt{2+2m^2}\,, a_0 = \frac{1}{2}\sqrt{2+2m^2}\,, a_1 = \pm m, b_1 = 0.$$

So that, we get the exact traveling wave solution of Eq. (3.3)

$$u(\xi) = \frac{1}{2}\sqrt{2+2m^2} \ \ \pm m\ sn. \tag{3.5}$$

When m = 1, we get hyperbolic solution of Eq. (3.3)

4- Conclusion

We establish exact solutions for the Couple Boiti-Leon-Pempinelli System is the most fascinatin problems of modern mathematical physics. The extended Jacobian elliptic function expansion method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for the Couple Boiti-Leon-Pempinelli System which has been constructed using the extended Jacobian elliptic function expansion method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the system are new and different from those obtained in [24]. It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations.

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