

## Feynman rules for Four Bosons Electromagnetism

J. Chauca<sup>1</sup>, R. Doria<sup>2</sup>, I. Soares<sup>3</sup>

[1]CBPF, Rio de Janeiro, Brazil; Aprendanet, Petrópolis, Brazil [2]Quarks, Petrópolis, Brazil [3]CBPF, Rio de Janeiro, Brazil; Quarks, Petrópolis, Brazil

### ABSTRACT

A whole electromagnetism carrying four electric charge messengers is studied. Based on light invariance and conservation of electric charge, it provides a fields set  $\{A_\mu, U_\mu, V_\mu^\pm\}$ . Something beyond Maxwell appears. The usual photon is accomplished by others electric charge porters, which are a massive photon plus two charged photons. They carry electromagnetic processes with charge exchange  $\Delta Q = 0$  and  $|\Delta Q| = 1$ .

There is still room for an electromagnetism on electric charge transmission to be understood. Through such so-called four bosons electromagnetism a new way to conduct the electric charge is proposed. It says that the electromagnetic phenomena is something more than Maxwell's charge distribution. It establishes the presence of four fields association responsible for the electric charge transmission. It develops a quanta set which means electromagnetism based on eight messengers with spin-1 and spin-0 to be analysed.

Thus given such fields collection  $A_{\mu} \equiv \{A_\mu, U_\mu, V_\mu^\pm\}$  one studies the corresponding propagations and interactions. Derive the corresponding Feynman rules for this electric charge transmission. The model shows itself renormalizable and unitary. New features are obtained as selfinteracting photons. The photon is no more necessarily coupled to the electric charge. A diversity of coupling constants is obtained. The electromagnetism universality is on the *ubiquous* photon and not on the electric charge as coupling constant.

### 1 Introduction

The main theory explaining the electromagnetic phenomena is due to Maxwell who summarized two centuries of experimental findings into his equations. Maxwell's theory [1] works very well, as proven by experiments. But it has some limitations too. In fact, three basic considerations can be done. Firstly, the masslessness of the photon can not be proven, only a lower experimental limit can be given. Also, the superposition principle can be proven only to a certain precision. Thirdly, adopting Maxwell's theory in the quantum framework is not straightforward and is somewhat artificial.

There is still room for an electromagnetism beyond Maxwell. Currently there are 38 extended models [2]. Based on two basic electromagnetic postulates which are light invariance and electric charge conservation this work studies a model where while Maxwell focus on charges distribution its objective drives on electric charge transportation. For this, from an abelian whole model [3], a four bosons electromagnetism is developed [4]. It proposes an electric charge transmission where beyond the photon it adds three additional electromagnetic messengers.

Thus from Maxwell photon field one moves for a fields set  $\{A_\mu, U_\mu, V_\mu^\pm\}$  electromagnetism. It proposes the electromagnetic transmission through the usual photon, plus a massive photon and two charged photons. They build up a whole electromagnetic system carrying the electric charge for different physical processes involving  $\Delta Q = 0$  and  $|\Delta Q| = 1$ . From an enlarged abelian gauge symmetry  $U(1) \otimes SO(2)$  transforming under a common gauge parameter, an electric charge porter electromagnetism is derived. From [4], one gets

$$\begin{aligned} A_\mu' &= A_\mu + k_1 \partial_\mu \alpha, \\ U_\mu' &= U_\mu + k_2 \partial_\mu \alpha, \\ V_\mu^{+'} &= e^{iq\alpha} V_\mu^+ + k_+ \partial_\mu \alpha, \\ V_\mu^{-'} &= e^{-iq\alpha} V_\mu^- + k_- \partial_\mu \alpha. \end{aligned} \tag{1.1}$$

where  $\alpha(x)$  means the gauge parameter,  $q = eq_v$  the value of the coupled electric charge ( $e$  being the electric

<sup>1</sup> jlchm2@yahoo.com

<sup>2</sup> doria@aprendanet.com.br

<sup>3</sup> w.inacio@hotmail.com



charge and  $q_v$ , the charge intensity associated to fields  $V_\mu^\pm$ ,  $k_i$  are constants.

From eqs. (1.1), one gets

$$L = L_0 + L_I \quad (1.2)$$

where

$$L_0 = L_K + L_m + L_{GF} \quad (1.3)$$

and

$$L_I = L_I^3 + L_I^4. \quad (1.4)$$

Decomposing the kinetic term,  $L_K = L_K^A + L_K^S$ , the corresponding antisymmetric and symmetric parts are written as

$$L_K^A = a_1 F_{\mu\nu} F^{\mu\nu} + a_2 U_{\mu\nu} U^{\mu\nu} + 2a_3 V_{\mu\nu}^+ V^{\mu\nu-}, \quad (1.5)$$

$$\begin{aligned} L_K^S = & b_{(11)} S_{\mu\nu}^1 S^{\mu\nu 1} + b_{(22)} S_{\mu\nu}^2 S^{\mu\nu 2} \\ & + 2b_{(33)} S_{\mu\nu}^+ S^{\mu\nu-} + c_{(11)} S_\mu^{\mu 1} S_\nu^{v 1} + c_{(22)} S_\mu^{\mu 2} S_\nu^{v 2} \\ & + 2c_{(12)} S_\mu^{\mu 1} S_\nu^{v 2} + 2c_{(33)} S_\mu^{\mu+} S_\nu^{v-}, \end{aligned} \quad (1.6)$$

where the granular fields strength are defined as follows

$$\begin{aligned} F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu, & U_{\mu\nu} &\equiv \partial_\mu U_\nu - \partial_\nu U_\mu, & V_{\mu\nu}^\pm &\equiv \partial_\mu V_\nu^\pm - \partial_\nu V_\mu^\pm, \\ S_{\mu\nu}^1 &\equiv \partial_\mu A_\nu + \partial_\nu A_\mu, & S_{\mu\nu}^2 &\equiv \partial_\mu U_\nu + \partial_\nu U_\mu, & S_{\mu\nu}^\pm &\equiv \partial_\mu V_\nu^\pm + \partial_\nu V_\mu^\pm. \end{aligned} \quad (1.7)$$

Considering that physical fields are that ones which diagonalize the transverse sector, we should not consider terms like  $S_{\mu\nu}^1 S^{\mu\nu 2}$ . Consequently,  $A_\mu$ ,  $U_\mu$ ,  $V_\mu^\pm$  written at Eqs. (1.2)-(1.3) are the realistic fields involved with the enlarged abelian symmetry.

The mass term is written as

$$L_m = -\frac{1}{2} (m_2 U_\mu^2 U^\mu + 2m_3 V_\mu^+ V^{\mu-}) \quad (1.8)$$

In principle the gauge-fixing should given by

$$\begin{aligned} L_{GF} = & \frac{1}{4} \xi_{(11)} S_\mu^{\mu 1} S_\nu^{v 1} + \frac{1}{4} \xi_{(22)} S_\mu^{\mu 2} S_\nu^{v 2} + \frac{1}{2} \xi_{(12)} S_\mu^{\mu 1} S_\nu^{v 2} \\ & + \frac{1}{4} (\xi_{(33)} + \xi_{(44)}) S_\mu^{\mu+} S_\nu^{v-} + \frac{1}{4} (\xi_{(33)} - \xi_{(44)}) \text{Re}\{S_\mu^{\mu+} S_\nu^{v+}\} \\ & + \frac{\sqrt{2}}{2} S_\mu^{\mu 1} \text{Re}\{[\xi_{(13)} + i\xi_{(14)}] S_\nu^{v+}\} \\ & + \frac{\sqrt{2}}{2} S_\mu^{\mu 2} \text{Re}\{[\xi_{(23)} + i\xi_{(24)}] S_\nu^{v+}\} - \frac{1}{2} \xi_{(34)} \text{Im}\{S_\mu^{\mu+} S_\nu^{v+}\}. \end{aligned} \quad (1.9)$$

However, although eq. (1.9) fixes the gauge parameter it does not introduces the SO(2) invariance. For this, one has to consider the relationships  $\xi_{(33)} = \xi_{(44)}$ ,  $\xi_{(13)} = \xi_{(14)} = \xi_{(23)} = \xi_{(24)} = 0$ . It gives the final gauge fixing relationship



$$L_{GF} = \frac{1}{4} \xi_{(11)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 1} + \frac{1}{4} \xi_{(22)} S_{\mu}^{\mu 2} S_{\nu}^{\nu 2} + \frac{1}{2} \xi_{(12)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 2} + \frac{1}{2} \xi_{(33)} S_{\mu}^{\mu+} S_{\nu}^{\nu-} \quad (1.10)$$

Eq. (1.10) shows that from one gauge parameter one obtains a gauge fixing term composed by four different parameters. In a similar situation with spontaneous breaking symmetry models [5], the gauge fixing term provides the model with four free parameters. Another aspect from eq. (1.10), is that differently from the standard QED, the gauge fixing condition does not necessarily takes one degree of freedom. Consequently, by associating a common symmetry for four fields embedded in the Lorentz representation  $(\frac{1}{2}, \frac{1}{2})$ , one gets four particles with spin-1 plus four with spin-0.

The interaction Lagrangian is decomposed in trilinear term and quadrilinear parts:  $L_I = L_3 + L_4$ . Physically, it is possible to separate them in antisymmetric and symmetric independent sectors:  $L_3 = L_3^A + L_3^S$  and  $L_4 = L_4^A + L_4^S$ . However, differently from Yang-Mills, each sector of this enlarged U(1) symmetry is separately gauge invariant. It gives

$$L_3^A = 4(b_1 F_{\mu\nu}^1 + b_2 U_{\mu\nu}^2) z_{\mu\nu}^{[12]\mu\nu} + 8b_3 \text{Re}\{ (z_{\mu\nu}^{[-1]\mu\nu} z_{\mu\nu}^{[-2]\mu\nu}) W_{\mu\nu}^+ \} \\ + 4 z_{\mu\nu}^{[+-]\mu\nu} (b_1 F_{\mu\nu}^1 + b_2 U_{\mu\nu}^2) + 4 z_{\mu\nu}^{(+)\mu\nu} (\beta_1 F_{\mu\nu}^1 + \beta_2 U_{\mu\nu}^2) \quad (1.11)$$

and

$$L_3^S = 2(\beta_1 S_{\mu\nu}^1 + \beta_2 S_{\mu\nu}^2) (z_{\mu\nu}^{(11)\mu\nu} + 2 z_{\mu\nu}^{(12)\mu\nu} + z_{\mu\nu}^{(22)\mu\nu}) \\ + 2(\rho_1 S_{\mu}^{\mu 1} + \rho_2 S_{\mu}^{\mu 2}) (z_{\mu\nu}^{(11)} + 2 z_{\mu\nu}^{(12)} + z_{\mu\nu}^{(22)}) \\ + 2[(\beta_1 + 4\rho_1) S_{\mu}^{\mu 1} + (\beta_2 + 4\rho_2) S_{\mu}^{\mu 2}] (\omega_{\mu\nu}^{(11)} + 2 \omega_{\mu\nu}^{(12)} + \omega_{\mu\nu}^{(22)}) \\ + 8\beta_3 \text{Re}\{ (z_{\mu\nu}^{(-1)\mu\nu} + z_{\mu\nu}^{(-2)\mu\nu}) S_{\mu\nu}^+ \} + 4 z_{\mu\nu}^{+-3\mu\nu} (\beta_1 S_{\mu\nu}^1 + \beta_2 S_{\mu\nu}^2) \\ + 8\rho_3 \text{Re}\{ (z_{\mu\nu}^{(-1)} + z_{\mu\nu}^{(-2)}) S_{\mu}^{\mu+} \} \\ + 8(\beta_3 + 4\rho_3) \text{Re}\{ (\omega_{\mu\nu}^{(-1)} + \omega_{\mu\nu}^{(-2)}) S_{\mu}^{\mu+} \} \\ + 4 z_{\mu\nu}^{+-3} (\rho_1 S_{\mu}^{\mu 1} + \rho_2 S_{\mu}^{\mu 2}) + 4(\beta_1 + 4\rho_1) \omega_{\mu\nu}^{+-3} S_{\mu}^{\mu 1} \\ + 4(\beta_2 + 4\rho_2) \omega_{\mu\nu}^{+-3} S_{\mu}^{\mu 2}. \quad (1.12)$$

Also,

$$L_4^A = 2 z_{\mu\nu}^{[12]} z_{\mu\nu}^{[12]\mu\nu} + 2 z_{\mu\nu}^{[12]} z_{\mu\nu}^{[21]\mu\nu} - 4 z_{\mu}^{\mu} z_{\nu}^{\nu} + 4 z_{\mu}^{\mu} z_{\nu}^{\nu} - 4 z_{\mu}^{\mu} z_{\nu}^{\nu} \\ + 4 z_{\mu\nu}^{[13+]} z_{\mu\nu}^{[13-]\mu\nu} + 4 z_{\mu\nu}^{[23+]} z_{\mu\nu}^{[23-]\mu\nu} + 8 z_{\mu\nu}^{[13+]} z_{\mu\nu}^{[23-]\mu\nu} \\ - 8 \text{Re}\{ z_{\mu}^{\mu} z_{\nu}^{\nu} \} + 8 z_{\mu\nu}^{[12]} \text{Re}\{ z_{\mu\nu}^{[+-]\mu\nu} \} + 8 \text{Im}\{ z_{\mu}^{\mu} z_{\nu}^{\nu} \} \\ + 2 z_{\mu\nu}^{[+-]} z_{\mu\nu}^{[+-]\mu\nu} - 2 z_{\mu\nu}^{[+-]} z_{\mu\nu}^{[+-]\mu\nu} - 4 z_{\mu}^{\mu} z_{\nu}^{\nu}, \quad (1.13)$$

and



$$\begin{aligned}
 L_4^S = & z_{\mu\nu}^{(11)} z^{\mu\nu(11)} + z_{\mu\nu}^{(22)} z^{\mu\nu(22)} + 2 z_{\mu\nu}^{(11)} \omega^{\mu\nu(11)} + 2 z_{\mu\nu}^{(22)} \omega^{\mu\nu(22)} \\
 & + 4 \omega_{\mu\nu}^{(11)} \omega^{\mu\nu(11)} + 4 \omega_{\mu\nu}^{(22)} \omega^{\mu\nu(22)} + 4 z_{\mu\nu}^{(11)} z^{\mu\nu(12)} + 4 z_{\mu\nu}^{(12)} z^{\mu\nu(22)} \\
 & + 8 z_{\mu\nu}^{(12)} \omega^{\mu\nu(11)} + 8 z_{\mu\nu}^{(22)} \omega^{\mu\nu(12)} + 16 \omega_{\mu\nu}^{(11)} \omega^{\mu\nu(12)} + 16 \omega_{\mu\nu}^{(12)} \omega^{\mu\nu(22)} \\
 & + 2 z_{\mu\nu}^{(11)} z^{\mu\nu(12)} + 2 z_{\mu\nu}^{(12)} z^{\mu\nu(22)} + 8 z_{\mu\nu}^{(12)} \omega^{\nu\mu(12)} + 16 \omega_{\mu\nu}^{(12)} \omega^{\nu\mu(12)} \\
 & + 4 z_{\mu}^{(11)} \omega_{\nu}^{\mu(22)} + 8 \omega_{\mu}^{(22)} \omega_{\nu}^{\mu(11)} + 2 z_{\mu\nu}^{(12)} z^{\nu\mu(12)} + 4 (z_{\mu\nu}^{(11)} + z_{\mu\nu}^{(22)}) z^{\mu\nu(11)} \\
 & + 4 z_{\mu}^{(13+)} z_{\nu}^{\mu(13-)} + 4 z_{\mu}^{(23+)} z_{\nu}^{\mu(23-)} + 16 z_{\mu}^{(13+)} \omega_{\nu}^{\mu(13-)} + 16 z_{\mu}^{(23+)} \omega_{\nu}^{\mu(23-)} \\
 & + 32 \omega_{\mu}^{(13+)} \omega_{\nu}^{\mu(13-)} + 32 \omega_{\mu}^{(23+)} \omega_{\nu}^{\mu(23-)} + 4 z_{\mu\nu}^{(13+)} z^{\mu\nu(13-)} + 4 z_{\mu\nu}^{(23+)} z^{\mu\nu(23-)} \\
 & + 8 z_{\mu\nu}^{(13+)} z^{\mu\nu(23-)} + 8 \{ z_{\mu}^{(11)} + z_{\mu}^{(22)} + 2 \omega_{\mu}^{(11)} + 2 \omega_{\mu}^{(22)} + 2 z_{\mu}^{(12)} + 4 \omega_{\mu}^{(12)} \} \omega_{\nu}^{\mu(11)} \\
 & + 8 z_{\mu\nu}^{(12)} \text{Re}\{ z^{\mu\nu(11)} \} + 8 \text{Re}\{ z_{\mu}^{(13+)} z_{\nu}^{\mu(23-)} \} + 32 \text{Re}\{ z_{\mu}^{(13+)} \omega_{\nu}^{\mu(13-)} \} \\
 & + 64 \text{Re}\{ \omega_{\mu}^{(13+)} \omega_{\nu}^{\mu(23-)} \} - 8 \text{Im}\{ z_{\mu\nu}^{(13+)} \omega^{\nu\mu(24-)} \} - 8 \text{Im}\{ z_{\mu\nu}^{(23+)} \omega^{\nu\mu(14-)} \} \\
 & - 32 \text{Im}\{ \omega_{\mu\nu}^{(13+)} \omega^{\nu\mu(24-)} \} + 32 \text{Im}\{ \omega_{\mu\nu}^{(14+)} \omega^{\nu\mu(23-)} \} + 4 \text{Im}\{ z_{\mu\nu}^{(13+)} z^{\nu\mu(24-)} \} \\
 & - 4 \text{Im}\{ z_{\mu\nu}^{(14+)} z^{\nu\mu(23-)} \} + 8 \text{Im}\{ z_{\mu\nu}^{(14+)} \omega^{\nu\mu(23-)} \} + 8 \text{Im}\{ z_{\mu\nu}^{(24+)} \omega^{\nu\mu(13-)} \} \\
 & + 2 \{ z_{\mu\nu}^{(+3)} z^{\mu\nu(+4)} + z_{\mu\nu}^{(+3)} z^{\mu\nu(-4)} \} - 8 \{ z_{\mu\nu}^{(+)} \omega^{\mu\nu(-)} - z_{\mu\nu}^{(+)} \omega^{\mu\nu(-)} \} \\
 & - 16 \{ \omega_{\mu\nu}^{(+)} \omega^{\mu\nu(-)} - \omega_{\mu\nu}^{(-)} \omega^{\mu\nu(+)} \} - 4 z_{\mu}^{(+)} z_{\nu}^{\mu(-)} + 8 z_{\mu}^{(+3)} \omega_{\nu}^{\mu(+4)} \\
 & + 16 \omega_{\mu}^{(+3)} \omega_{\nu}^{\mu(+4)}.
 \end{aligned} \tag{1.14}$$

Preserving light invariance and electric charge conservation postulates, eq. (1.2) introduces through an U(1) × SO(2) symmetry, the minimal model for electric charge transmission. It introduces a richest electromagnetic world than Maxwell. Granular and collective fields, non-linearity, massive intermediary particles are among new aspects to be observed.

## 2 Perturbative Lagrangian

Our intention here is to study the quantum perspective to the involved fields and associated particles. So a next step is to organize the perturbative approach for the corresponding particles and their interactions. Taking the flavour notation  $A_{\mu l} \equiv \{A_{\mu}, U_{\mu}, V_{\mu}^{\pm}\}$ , eq. (1.2) is rewritten for the propagating sector as

$$\begin{aligned}
 L_0 = & a_{IJ} (\partial_{\mu} A_I^{\mu}) (\partial^{\mu} A_J^{\nu}) + b_{IJ} (\partial_{\mu} A_I^{\mu}) (\partial^{\nu} A_J^{\mu}) \\
 & + c_{IJ} (\partial_{\mu} A_I^{\mu}) (\partial_{\nu} A_J^{\nu}) + d_{IJ} A_{\mu}^I A^{\mu J}
 \end{aligned} \tag{2.1}$$

and for the interacting sector as

$$L_I = a_{IJK} (\partial_{\mu} A_I^{\mu}) A_J^{\mu} A_K^{\nu} + b_{IJK} (\partial_{\mu} A_I^{\mu}) A_{\nu J} A_K^{\nu} + a_{IJKL} A_{\mu}^I A_{\nu J} A_K^{\mu} A_L^{\nu} \tag{2.2}$$



Thus, from eq. (1.3), the kinetic term can be decomposed in three parts  $L_K = L_{K1} + L_{K2} + L_{K3}$ , where

$$L_{K1} = 2(a_1 + b_{(11)})\partial_\mu A_\nu \cdot \partial^\mu A^\nu + 2(b_{(22)} + a_2)\partial_\mu U_\nu \cdot \partial^\mu U^\nu + 4(a_3 + b_{(33)})\partial_\mu V_\nu^+ \cdot \partial^\mu V^{\nu-}, \quad (2.3)$$

$$L_{K2} = 2(b_{(11)} - a_1)\partial_\mu A_\nu \cdot \partial^\nu A^\mu + 2(b_{(22)} - a_2)\partial_\mu U_\nu \cdot \partial^\nu U^\mu + 4(b_{(33)} - a_3)\partial_\mu V_\nu^+ \cdot \partial^\nu V^{\mu-}, \quad (2.4)$$

and

$$L_{K3} = (4c_{(11)} + \xi_{(11)})\partial_\mu A^\mu \cdot \partial_\nu A^\nu + (4c_{(22)} + \xi_{(22)})\partial_\mu U^\mu \cdot \partial_\nu U^\nu + (8c_{(12)} + 2\xi_{(12)})\partial_\mu A^\mu \cdot \partial^\nu U^\nu + 2\xi_{(33)}\partial_\mu V^{\mu+} \cdot \partial_\nu V^{\mu-} + (\xi_{(33)} + i\xi_{(34)})\partial_\mu V^{\mu+} \cdot \partial_\nu V^{\mu+} + (\xi_{(33)} - i\xi_{(34)})\partial_\mu V^{\mu-} \cdot \partial_\nu V^{\mu-}. \quad (2.5)$$

The interaction term gives,  $L_I = L_I^3 + L_I^4$ , where

$$L_I^3 = L_{I(1)}^3 + L_{I(2)}^3 \quad (2.6)$$

with

$$\begin{aligned} L_{I(1)}^3 = & 4\beta_1\gamma_{(11)}\partial_\mu A_\nu \cdot A^\mu A^\nu + 4\beta_1\gamma_{(22)}\partial_\mu A_\nu \cdot U^\mu U^\nu + \\ & + 4(b_1\gamma_{[12]} + \beta_1\gamma_{(12)})\partial_\mu A_\nu \cdot A^\mu U^\nu + 4(\beta_1\gamma_{(12)} - b_1\gamma_{[12]})\partial_\mu A_\nu \cdot U^\mu A^\nu + \\ & + 4(\beta_2\gamma_{33} + i(\gamma_{[34]}b_1 - \gamma_{(34)}\beta_1))\partial_\mu A_\nu \cdot V^{\mu+}V^{\nu-} + \\ & + 4(\beta_2\gamma_{33} - i(\gamma_{[34]}b_1 - \gamma_{(34)}\beta_1))\partial_\mu A_\nu \cdot V^{\mu-}V^{\nu+} + \\ & + 4\beta_2\gamma_{(11)}\partial_\mu U_\nu \cdot A^\mu A^\nu + 4\beta_2\gamma_{(22)}\partial_\mu U_\nu \cdot U^\mu U^\nu + \\ & + 4(\beta_2\gamma_{(12)} + b_2\gamma_{[12]})\partial_\mu U_\nu \cdot A^\mu U^\nu + 4(\beta_2\gamma_{(12)} - b_2\gamma_{[12]})\partial_\mu U_\nu \cdot U^\mu A^\nu + \\ & + 4i(\gamma_{[34]}b_2 - \gamma_{(34)}\beta_2)\partial_\mu U_\nu \cdot V^{\mu+}V^{\nu-} + \\ & + 4i(\gamma_{[34]}b_2 - \gamma_{(34)}\beta_2)\partial_\mu U_\nu \cdot V^{\mu-}V^{\nu+} + \\ & + 4(\beta_3(\gamma_{(13)} - i\gamma_{(14)}) - b_3(\gamma_{[13]} - i\gamma_{[14]}))\partial_\mu V_{\nu+} \cdot V^{\mu-}A^\nu + \\ & + 4(\beta_3(\gamma_{(13)} - i\gamma_{(14)}) + b_3(\gamma_{[13]} - i\gamma_{[14]}))\partial_\mu V_{\nu+} \cdot A^\mu V^{\nu-} + \\ & + 4(\beta_3(\gamma_{(23)} - i\gamma_{(24)}) - b_3(\gamma_{[23]} - i\gamma_{[24]}))\partial_\mu V_{\nu+} \cdot V^{\mu-}U^\nu + \\ & + 4(\beta_3(\gamma_{(13)} - i\gamma_{(14)}) + b_3(\gamma_{[23]} - i\gamma_{[24]}))\partial_\mu V_{\nu+} \cdot U^\mu V^{\nu-} + \\ & + 4(\beta_3(\gamma_{(13)} + i\gamma_{(14)}) - b_3(\gamma_{[13]} + i\gamma_{[14]}))\partial_\mu V_{\nu+} \cdot V^{\mu-}A^\nu + \\ & + 4(\beta_3(\gamma_{(13)} + i\gamma_{(14)}) + b_3(\gamma_{[13]} + i\gamma_{[14]}))\partial_\mu V_{\nu+} \cdot A^\mu V^{\nu-} + \end{aligned}$$





$$\begin{aligned}
 &+4(\beta_3(\gamma_{(23)}+i\gamma_{(24)})-b_3(\gamma_{[23]}+i\gamma_{[24]}))\partial_\mu V_{\nu+}\cdot V^\mu U^\nu + \\
 &+4(\beta_3(\gamma_{(13)}+i\gamma_{(14)})+b_3(\gamma_{[23]}+i\gamma_{[24]}))\partial_\mu V_{\nu+}\cdot U^\mu V^{\nu-}, \tag{2.7}
 \end{aligned}$$

$$\begin{aligned}
 L_{I(2)}^3 = &4(\gamma_{11}\rho_1 + \tau_{11}(\beta_1 + 4\rho_1))\partial_\mu A^\mu A_\nu A^\nu + \\
 &+8(\gamma_{12}\rho_1 + \tau_{12}(\beta_1 + 4\rho_1))\partial_\mu A^\mu A_\nu U^\nu + \\
 &+4(\gamma_{22}\rho_1 + \tau_{22}(\beta_1 + 4\rho_1))\partial_\mu A^\mu U_\nu U^\nu + \\
 &+8(\gamma_{33}\rho_1 + \tau_{33}(\beta_1 + 4\rho_1))\partial_\mu A^\mu V_\nu^+ V^{\nu-} + \\
 &+4(\gamma_{11}\rho_2 + \tau_{11}(\beta_2 + 4\rho_2))\partial_\mu U^\mu A_\nu A^\nu + \\
 &+8(\gamma_{12}\rho_2 + \tau_{12}(\beta_2 + 4\rho_2))\partial_\mu U^\mu A_\nu U^\nu + \\
 &+4(\gamma_{22}\rho_2 + \tau_{22}(\beta_2 + 4\rho_2))\partial_\mu U^\mu U_\nu U^\nu + \\
 &+8(\gamma_{33}\rho_2 + \tau_{33}(\beta_2 + 4\rho_2))\partial_\mu U^\mu V_\nu^+ V^{\nu-} \\
 &+8((\gamma_{[23]}-i\gamma_{[24]})\rho_3 + (\tau_{[23]}-i\tau_{[24]})(\beta_3 + 4\rho_3))\partial_\mu V^{+\mu} V_\nu^- A^\nu + \\
 &+8((\gamma_{[13]}-i\gamma_{[14]})\rho_3 + (\tau_{[13]}-i\tau_{[14]})(\beta_3 + 4\rho_3))\partial_\mu V^{+\mu} V_\nu^- U^\nu + \\
 &+8((\gamma_{[23]}+i\gamma_{[24]})\rho_3 + (\tau_{[23]}+i\tau_{[24]})(\beta_3 + 4\rho_3))\partial_\mu V^{-\mu} V_\nu^+ A^\nu + \\
 &+8((\gamma_{[13]}+i\gamma_{[14]})\rho_3 + (\tau_{[13]}+i\tau_{[14]})(\beta_3 + 4\rho_3))\partial_\mu V^{-\mu} V_\nu^+ U^\nu. \tag{2.8}
 \end{aligned}$$

Similarly,

$$L_I^4 = L_{I(1)}^4 + L_{I(2)}^4, \tag{2.9}$$

where

$$\begin{aligned}
 L_{I(1)}^4 = &(2\gamma_{(11)}\gamma_{(22)} + 8\gamma_{(12)}\tau_{(12)} + 16\tau_{(12)}^2 + 2\gamma_{(12)}^2 + 2\gamma_{[12]}\gamma_{[21]})A_\mu U_\nu A^\nu U^\mu + \\
 &+(4\gamma_{(11)}\gamma_{(33)} - 4\gamma_{[13]}^2 + 32\tau_{(13)}^2 + 48\tau_{(13)}\gamma_{(13)} + 4\gamma_{(13)}^2)A_\mu A_\nu V^{+\mu} V^{\nu-} + \\
 &+(4\gamma_{(22)}\gamma_{(33)} - 4\gamma_{[23]}^2 + 32\tau_{(23)}^2 + 16\tau_{(23)}\gamma_{(23)} + 4\gamma_{(23)}^2)U_\mu U_\nu V^{+\mu} V^{\nu-} \\
 &+(4i\tau_{(13)}\tau_{(24)} - 4i\tau_{(13)}\gamma_{(24)} - 16i\tau_{(13)}\tau_{(23)} + \\
 &+4i\tau_{(14)}\gamma_{(23)} - 4i\tau_{(23)}\gamma_{(14)} + 4i\tau_{(24)}\gamma_{(13)} - 2i\gamma_{(13)}\gamma_{(24)} + \\
 &+2i\gamma_{(14)}\gamma_{(23)} + 4\gamma_{(12)}\gamma_{(33)} + 4i\gamma_{[12]}\gamma_{[34]} + 4i\gamma_{[13]}\gamma_{[24]} - 4\gamma_{[13]}\gamma_{[23]} + \\
 &+32\tau_{(13)}\tau_{(23)} + 4\gamma_{(13)}\gamma_{(23)})A_\mu U_\nu V^{-\mu} V^{\nu+} + \\
 &+(16i\tau_{(13)}\tau_{(24)} - 4i\tau_{(13)}\gamma_{(24)} - 16i\tau_{(13)}\tau_{(23)} + \\
 &+4i\tau_{(14)}\gamma_{(23)} - 4i\tau_{(23)}\gamma_{(14)} + 4i\tau_{(24)}\gamma_{(13)} - 2i\gamma_{(13)}\gamma_{(24)} + \\
 &+2i\gamma_{(14)}\gamma_{(23)} + 4\gamma_{(12)}\gamma_{(33)} - 4i\gamma_{[12]}\gamma_{[34]} - 4i\gamma_{[13]}\gamma_{[24]} - 4\gamma_{[13]}\gamma_{[23]} +
 \end{aligned}$$



$$\begin{aligned}
 &+ 32\tau_{(13)}\tau_{(23)} + 4\gamma_{(13)}\gamma_{(23)})A_\mu U_\nu V^{+\mu}V^{-\nu} + \\
 &(-4\gamma_{[34]}^2 + 16\tau_{(33)}\tau_{(44)} + 8\gamma_{(33)}\tau_{(44)} + 4\gamma_{(34)}^2)V_\mu^+V_\nu^-V^{-\mu}V^{+\nu}, \tag{2.10}
 \end{aligned}$$

and

$$\begin{aligned}
 L_{I(2)}^4 = &(\gamma_{(11)}^2 + 2\tau_{(11)}\gamma_{(11)} + 4\tau_{(11)}^2)A_\mu A_\nu A^\mu A^\nu + \\
 &+(\gamma_{(22)}^2 + 2\tau_{(22)}\gamma_{(22)} + 4\tau_{(22)}^2)U_\mu U_\nu U^\mu U^\nu + \\
 &+(4\gamma_{(11)}\gamma_{(12)} + 8\tau_{(11)}\gamma_{(12)} + 16\tau_{(11)}\tau_{(22)})A_\mu A_\nu A^\mu U^\nu + \\
 &+(4\gamma_{(11)}\gamma_{(12)} + 8\tau_{(11)}\gamma_{(12)} + 16\tau_{(11)}\tau_{(22)})A_\mu A_\nu A^\mu U^\nu + \\
 &+(4\gamma_{(12)}\gamma_{(22)} + 8\tau_{(22)}\gamma_{(12)} + 16\tau_{(22)}\tau_{(12)})A_\mu U_\nu U^\mu U^\nu + \\
 &+(8\tau_{(11)}\tau_{(22)} + 4\gamma_{(11)}\tau_{(22)} + 2\gamma_{[12]}^2)A_\mu A^\mu U_\nu U^\nu + \\
 &+(4\gamma_{[13]}^2 + 4\gamma_{(13)}^2 + 48\tau_{(11)}\tau_{(33)} + 8\tau_{(33)}\gamma_{(11)})A_\mu A^\mu V_\nu^+V^{-\nu} + \\
 &+(4\gamma_{[23]}^2 + 4\gamma_{(23)}^2 + 16\tau_{(22)}\tau_{(33)} + 8\gamma_{(22)}\tau_{(33)})U_\mu U^\mu V_\nu^+V^{-\nu} + \\
 &+ 16\gamma_{(12)}\tau_{(33)}A_\mu U^\mu V_\nu^+V^{-\mu} + \\
 &+(16\tau_{(34)}^2 + 8\tau_{(34)}\gamma_{(34)} + 2\gamma_{(33)}\gamma_{(44)} - 2\gamma_{[34]}^2)V_\mu^+V^{+\mu}V_\nu^-V^{-\nu} + \\
 &+(16\tau_{(34)}^2 + 8\tau_{(34)}\gamma_{(34)} + 2\gamma_{(33)}\gamma_{(44)} - 2\gamma_{[34]}^2)V_\mu^+V^{-\mu}V_\nu^+V^{-\nu}.
 \end{aligned}$$

### 3 Feynman rules

A non-linear electromagnetism is obtained. A next step is to express the corresponding Feynman rules. The effective action of the classical field is defined by the functional Legendre transformation,

$$\Gamma(A) = W[J] - \int d^4x J_I(x)A_I(x) \tag{3.1}$$

where

$$W = -i \ln Z, \tag{3.2}$$

$$Z[J] = \int \mathcal{D}A_\mu e^{i \int d^4x [L(A) + J_\mu A^\mu]} \tag{3.3}$$

Expanding in powers of  $A_\mu$ , one gets

$$\Gamma(A) = \frac{i}{n!} \int d^4x_1 \dots d^4x_n \Gamma_{I\dots N}^{(n)\mu\nu\dots\rho}(x_1, \dots, x_n) A_{\mu I}(x_1) A_{\nu J}(x_2) \dots A_{\rho N}(x_n) \tag{3.4}$$

where  $\Gamma_{I\dots N}^{(n)\mu\nu\dots\rho}(x_1, \dots, x_n)$  is the sum over all 1PI Feynman diagrams with  $n$ -external lines. In terms of a functional Taylor series (3.4) yields

$$\Gamma_{I\dots N}^{(n)\mu\nu\dots\rho}(x_1, \dots, x_n) = \frac{1}{i} \frac{\delta^n \Gamma(A)}{\delta A_I(x_1) \dots A_N(x_n)} \Big|_{A=0}. \tag{3.5}$$

An advantage of this formalism, written through the effective action  $\Gamma$ , relies on the fact that it allows an expansion

in terms of the number of loops. This series is written in terms of powers of the Planck's constant,

$$\Gamma = \sum_{L=0}^{\infty} \hbar^{L-1} \Gamma(L). \quad (3.6)$$

Physical interpretations from (3.4) are in general more explicit in momentum space. Thus in order to calculate the Feynman rules we will follow the Fourier transform conventions,

$$f(x) = \int \frac{d^4 k}{(2\pi)^4} f(k) e^{ikx},$$

$$f(k) = \int d^4 x f(x) e^{-ikx}$$

and

$$\begin{aligned} (2\pi)^4 \delta(k_1 + \dots + k_n) \Gamma^{(n)}(k_1; \dots; k_n) &\equiv \\ &\equiv \int d^4 x_1 \dots d^4 x_n \Gamma^{(n)}(x_1; \dots; x_n) e^{-ix_1 k_1} \dots e^{-ix_n k_n}. \end{aligned} \quad (3.7)$$

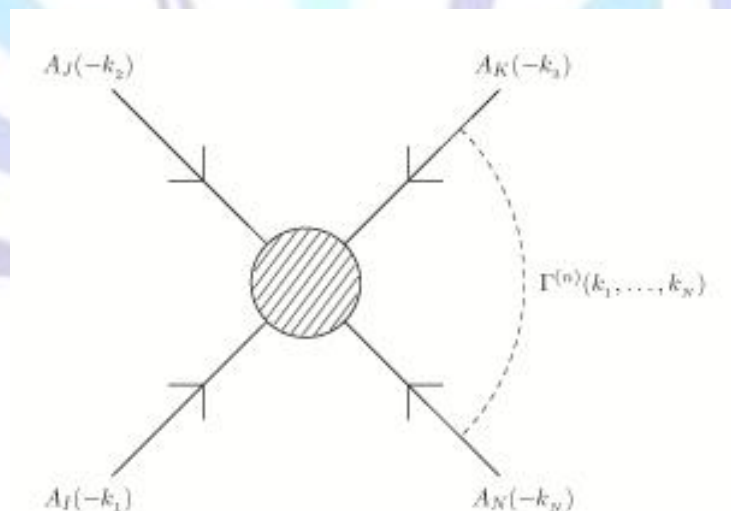
It yields,

$$\Gamma(A) = \sum_{n=1}^{\infty} \frac{i}{n!} \int \frac{d^4 k_1}{(2\pi)^4} \dots \frac{d^4 k_{n-1}}{(2\pi)^4} \Gamma^{(n)\mu\nu\dots\rho}_{U\dots N}(k_1, \dots, k_n) A_I(-k_1) \dots A_N(-k_n) \quad (3.8)$$

where  $k_1 + k_2 + \dots + k_n = 0$ .

The conventions for the momentum flows are indicated in Fig. 1. (3.4) and (3.8) show that propagators and vertices can be read off as coefficients of Fourier transform of the fields. This useful derivation of the Feynman rules is because at tree level,  $\Gamma(A)$  means the classical action plus the external source terms. So, by taking the action,

$S(A) = \int d^4 x L[A_I, \partial A_I]$ , and making its Fourier transform, we can read off  $\Gamma^{(n)\mu\nu\dots\rho}_{U\dots N}(k_1, \dots, k_n)$  by adjusting the factors  $(i)$  and  $(\frac{1}{n!})$ .



**Fig. 1. The convention to compute the effective action.**

The quadratic part of the Lagrangian containing four potential fields is

$$L_0 = -A_{\mu} [(a_{\mu} W - m_{\mu}^2) \eta^{\mu\nu} + (b_{\mu} + c_{\mu}) \partial^{\mu} \partial^{\nu}] A_{\nu}. \quad (3.9)$$

Thus, taking the Fourier transform  $\hat{A}$  and using the delta function one gets



$$L_0 = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \Phi^t(-k) \mathcal{O} \Phi(k)$$

where

$$\Phi^t \equiv (A_{\mu 1}, A_{\mu 2}, \dots, A_{\mu N}),$$

$$\mathcal{O} = -i\Gamma^{(2)\mu\nu},$$

$$\mathcal{O} \equiv (Ak^2 + M)\eta^{\mu\nu} + (B + C)k^\mu k^\nu. \quad (3.10)$$

Propagators are obtained from the block matrix inverse of the kinetic part. Observe that  $A, B, C, D$  are  $n \times n$  real and symmetric matrices. Each matrix can be individually diagonalized by an orthogonal transformation. However, the diagonalization is not in general simultaneous. Writing in terms of the transverse and longitudinal projection operators

$$\langle T A_{\mu l} A_{\nu j} \rangle_T = i \left[ \frac{1}{Ak^2 + M} \right]_{ll}^{-1} \left( \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \quad (3.11)$$

$$\begin{aligned} \langle T A_{\mu l} A_{\nu j} \rangle_L &= i \left[ \frac{1}{\square B k^2 + M} \right]_{ll}^{-1} \frac{k_\mu k_\nu}{k^2} \\ &= \frac{i \text{cof}(k^2 - \square B \quad M)_{ik}}{\det(k^2 - \square B \quad M)} (\square B)^{-1}_{kj} \frac{k^\mu k^\nu}{k^2} \end{aligned} \quad (3.12)$$

where  $\square B = A + B + C$ .

For the vertices, the correspondence between the interacting terms that appear in the Lagrangian and the Feynman vertices is not one-to-one. The introduction of more potential fields in the same group enlarges the possibilities for playing with the Lorentz indices and also appear different possibilities for distributing the flavour indices. Thus, it appears a kind of topology of gauge invariance where a determined graph incorporates different contributions from the Lagrangian terms.

Thus three-gauge-boson proper vertices are systematized in three structures. They correspond graphs with three different fields, two fields being equal and the case where all fields interacting in a vertex are the same. Taking the Fourier transform of the action corresponding to

$$L_3 = a_{\mu K} \eta_{\nu\rho} (\partial_\mu A_I^\rho) A_J^\mu A_K^\nu + b_{\mu K} \eta_{\nu\rho} (\partial_\mu A_I^\mu) A_J^\rho A_K^\nu \quad (3.13)$$

one gets the following Feynman rule for the first case,

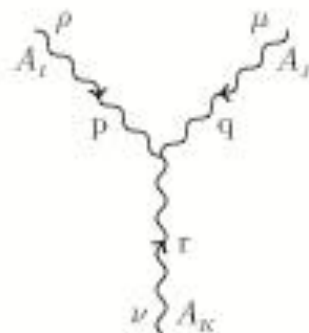


Fig. 2

$$\equiv a_{IK} \eta_{\nu\rho} p_\mu + b_{IK} \eta_{\nu\rho} p_\mu$$

$$\text{with } p+q+r=0.$$

$$(3.14)$$

For the case with two-equal fields, the vertex receive contributions from ten different terms. It gives,

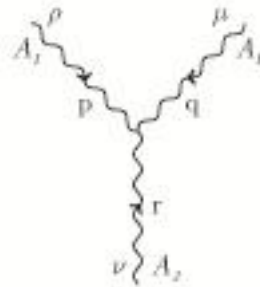


Fig. 3

$$\begin{aligned} &\equiv [a_{III} + a_{III} + a_{III}] \eta_{\nu\rho} p_\mu + [b_{III} + 2b_{III} + b_{III}] \eta_{\mu\nu} p_\rho \\ &+ a_{II} \eta_{\mu\nu} q_\rho + a_{III} \eta_{\nu\rho} r_\mu + b_{III} \eta_{\rho\nu} q_\mu \\ &+ a_{III} \eta_{\mu\rho} p_\nu + b_{III} \eta_{\mu\rho} r_\nu \\ &\text{with } p+q+r=0. \end{aligned}$$

$$(3.15)$$

Finally, when the three involved fields are equal, one gets the expression

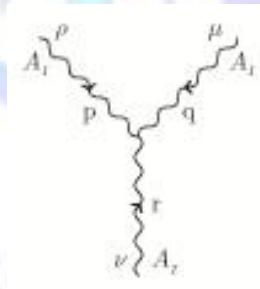


Fig. 4

$$\begin{aligned} &\equiv -[a_{III} + 2b_{III}] \cdot [\eta_{\mu\nu} p_\rho + \eta_{\nu\rho} q_\mu + \eta_{\mu\rho} r_\nu] \\ &\text{with } p+q+r=0. \end{aligned}$$

$$(3.16)$$

Similarly, for the four-boson vertex

$$L_4 = a_{IKL} \eta_{\mu\rho} \eta_{\nu\lambda} A_{\mu\lambda} A_{\nu\rho} A_{\rho K} A_{\lambda L}$$

$$(3.17)$$

which yields the following cases

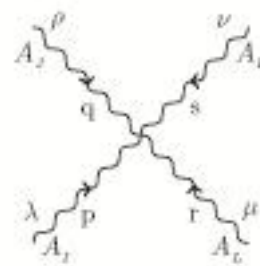


Fig. 5

$$\equiv -ia_{IKL}\eta_{\nu\rho}\eta_{\mu\lambda}$$

with  $p+q+r=0$ . (3.18)

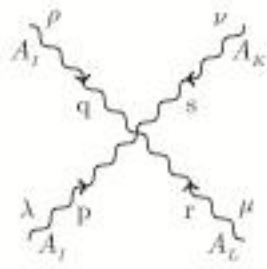


Fig. 6

$$\equiv -i[a_{IKL} + a_{IKL}]\eta_{\nu\rho}\eta_{\mu\lambda} - ia_{IKL}\eta_{\nu\lambda}\eta_{\mu\rho}$$

with  $p+q+r=0$ . (3.19)

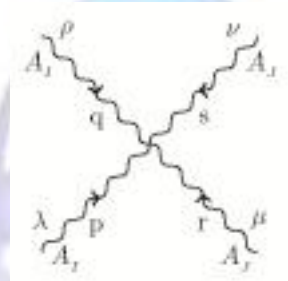


Fig. 7

$$\equiv -4i[a_{III} + a_{III}]\eta_{\nu\rho}\eta_{\mu\lambda} - 2ia_{III}\eta_{\nu\lambda}\eta_{\mu\rho}$$

with  $p+q+r=0$ . (3.20)

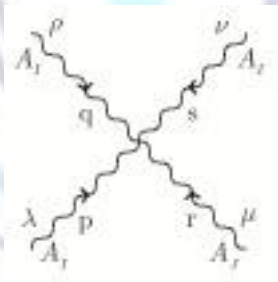


Fig. 8

$$\equiv -2ia_{III}[\eta_{\nu\rho}\eta_{\mu\lambda} + \eta_{\nu\mu}\eta_{\rho\lambda} + \eta_{\nu\lambda}\eta_{\mu\rho}]$$

with  $p+q+r=0$ . (3.21)

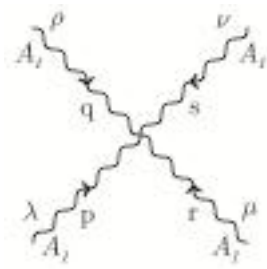


Fig. 9

$$\equiv -8ia_{III} [\eta_{\nu\rho}\eta_{\mu\lambda} + \eta_{\mu\nu}\eta_{\rho\lambda} + \eta_{\mu\rho}\eta_{\nu\lambda}]$$

$$\text{with } p + q + r = 0. \quad (3.22)$$

Four features can be taken from these Feynman rules. First, the above Feynman rules structures a new approach to a non-linear electromagnetism. The first relevant try was due to Born and Infeld in 1934 [6]. The difference here is that the non-linearity is on potential fields. Three and four vector bosons vertices are developed independently.

We should also observe that this non-linearity is not ruled by electric charge. Eqs. (3.13-3.22) are showing different coupling constants which can take any value without breaking gauge invariance. As consequence this welcomed fact release the model on facing the crucial result from electrodynamics which is the accuracy on the electron anomalous magnetic moment measurement. The possibility for a model adding new vertices to agree with the experimental value to more than 10 significant figures [7] is to be free for adjusting the coupling constants being included.

As a third aspect, the above graphs are showing on deviation from linearity in the quantum regime at tree level. A result for justifying processes as light-light scattering. Usually it requires the uncertainty principle support which allows the momentary creation of electron-positron couple on subsequent annihilation with creation of two photons. Eq. (3.20) is enough for considering two plane waves defined by the wave vectors  $k_1$  and  $k_2$  scatter, transforming into two different waves with vectors  $k_3$  and  $k_4$ .

Finally, we should observe on the presence of self-interaction photons. Eqs. (3.16) and (3.22) are showing three and four vertices where photons are coupled to themselves. The photon can not be taken as a single particle. A non point-like photon structure appears to be understood.

#### 4 Spin-1 and spin-0 propagators

The representation  $(\frac{1}{2}, \frac{1}{2})$  provides a fields collection  $\{A_{\mu l}\}$  which is consistent with the introduction of four electromagnetic messengers. So after Feynman rules be derived one has to derive the corresponding propagators, poles, residues and the quanta spectroscopy. Lorentz group introduces not only the space-time correlation but also the spin as quantum number. Given the fields set  $\{A_{\mu l}\} \equiv \{A_{\mu}, U_{\mu}, V_{\mu}^{\pm}\}$  it yields two families with spin 1 and spin 0 whose quantum numbers must be understood. From eq. (3.9) one gets,

$$L_0 = \frac{1}{2} A_{\mu l} (\mathbf{O}^{\mu\nu})_{ll} A_{\nu l}$$

where the propagator expression is  $i(\mathbf{O}^{\mu\nu})_{ll}^{-1}$ .

Substituting eqs. (1.4-9) in eq. (3.10), we obtains

$$\mathbf{O}^{\mu\nu} = A\theta^{\mu\nu} + B\omega^{\mu\nu} \quad (4.1)$$

where  $A, B$  are  $4 \times 4$  matrices derived as

$$A = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & 0 & q \\ 0 & 0 & q & 0 \end{bmatrix},$$

with

$$p_1 \equiv -2(a_1 + b_{(11)})W, \quad p_2 \equiv -2(a_2 + b_{(22)})W - m_2^2, \quad q \equiv -2(a_3 + b_{(33)})W - m_3^2, \quad (4.2)$$

and

$$B = \begin{bmatrix} r_1 & s & 0 & 0 \\ s & r_2 & 0 & 0 \\ 0 & 0 & 0 & t \\ 0 & 0 & t & 0 \end{bmatrix},$$



with

$$\begin{aligned} r_1 &= -2\alpha_{11}W, & r_2 &= -2\alpha_{22}W - 2m_2^2, \\ s &= -2\alpha_{12}W, & r &= -2\alpha_{33}W - 2m_3^2, \end{aligned} \quad (4.3)$$

where

$$\begin{aligned} \alpha_{11} &= -4(b_{(11)} + c_{(11)}) + \frac{1}{4}\xi_{(11)}, \\ \alpha_{22} &= -4(b_{(11)} + c_{(11)}) + \frac{1}{4}\xi_{(22)}, \\ \alpha_{12} &= -2(c_{(12)} + \frac{1}{2}\xi_{(12)}), \\ \alpha_{33} &= -2(2b_{(33)} + c_{(33)} + \frac{1}{2}\xi_{(33)}). \end{aligned} \quad (4.4)$$

For the transversal sector, the corresponding propagators are

$$\langle A_\mu A_\nu \rangle_T = \frac{i}{k^2} R_{\mu\nu,A}^{(1)} \quad (4.5)$$

$$\langle U_\mu U_\nu \rangle_T = \frac{i}{k^2 - m_U^2} R_{\mu\nu,U}^{(1)} \quad (4.6)$$

$$\langle V_\mu^+ V_\nu^- \rangle_T = \langle V_\mu^- V_\nu^+ \rangle_T = \frac{i}{k^2 - m_V^2} R_{\mu\nu,V}^{(1)}. \quad (4.7)$$

where the corresponding residues expressions are

$$R_{\mu\nu,A}^{(1)} = \frac{1}{a_1 + b_{11}} \theta_{\mu\nu} \quad (4.8)$$

$$R_{\mu\nu,U}^{(1)} = \frac{1}{a_2 + b_{11}} \theta_{\mu\nu} \quad (4.9)$$

$$R_{\mu\nu,V}^{(1)} = \frac{1}{a_3 + b_{33}} \theta_{\mu\nu}. \quad (4.10)$$

Reading off propagators, one obtains information on mass and spin contents. Eqs. (4.5-4.7) are saying that the model carry four poles with spin-1. They are a massless photon, a massive photon with a mass  $m_U = \frac{m_2^2}{a_2 + b_{(11)}}$  and two charged photons with the same mass  $m_{V^\pm} = \frac{m_3^2}{a_3 + b_{(33)}}$ . Notice that due to the addition of a SO(2) symmetry to the primitive U(1) symmetry splits the coefficients that determines the masses. Coefficients that are related to neutral fields do not contribute on the charged physical masses and vice-versa.

A second step is to understand on the corresponding spin content. The spin study is on the residue of the propagators. Taking as reference system  $k^\mu = (m, 0, 0, 0)$ , one gets the general expression

$$R_{\mu\nu,I}^{(1)} \Big|_{k^\mu=(m,0,0,0)} = \frac{1}{r_I} \text{diag}(0, -1, -1, -1) \quad (4.11)$$



where eq. (4.11) is saying that for avoiding ghosts the condition is  $r_1 < 0$ . This means that given the relationship  $a_1 + b_{11} < 0$ , one gets three physically health degrees of freedom for the field  $A_\mu$ . Similarly for the other fields under  $a_2 + b_{22} < 0$  and  $a_3 + b_{33} < 0$ , respectively.

For the longitudinal sector, the corresponding propagators are

$$\langle A_\mu A_\nu \rangle_L = \frac{r_1}{\Delta}, \quad (4.12)$$

$$\langle U_\mu U_\nu \rangle_L = \frac{r_2}{\Delta}, \quad (4.13)$$

$$\langle A_\mu U_\nu \rangle_L = \langle U_\mu A_\nu \rangle_L = \frac{s}{\Delta} \quad (4.14)$$

$$\langle V_\mu^+ V_\nu^- \rangle_L = \frac{1}{r}, \quad (4.15)$$

with a common pole given by

$$\Delta = r_1 r_2 - s^2 = 4\alpha_{11} W \left[ \left( \alpha_{22} - \frac{\alpha_{12}^2}{\alpha_{11}} \right) W + m_2^2 \right], \quad (4.16)$$

and the longitudinal charged boson pole is  $r$  defined at eq. (4.3).

As result, there are four spin-0 quanta. They correspond to a massless longitudinal photon  $m_A^0 = 0$ , a scalar massive photon with mass  $m_U^0 = \frac{m_2^2}{\alpha_{22}}$ , and two charged massive scalars, with the common mass  $m_{V^\pm}^0 = \frac{m_3^2}{\alpha_{33}}$ .

Considering  $\xi_{12} = -2c_{12}$ , one gets

$$\langle A_\mu A_\nu \rangle_L = \frac{i}{k^2} R_{\mu\nu,A}^{(L)}, \quad (4.17)$$

$$\langle U_\mu U_\nu \rangle_L = \frac{i}{k^2 - m_U^2} R_{\mu\nu,U}^{(L)}, \quad (4.18)$$

$$\langle V_\mu^+ V_\nu^- \rangle_L = \langle V_\mu^- V_\nu^+ \rangle_L = \frac{i}{k^2 - m_V^2} R_{\mu\nu,V^\pm}^{(L)}, \quad (4.19)$$

where the residues are given by the following expressions

$$R_{\mu\nu,A}^{(0)} = \frac{1}{\alpha_{11}} \omega_{\mu\nu}, \quad (4.20)$$

$$\langle R_{\mu\nu,U}^{(0)} = \frac{1}{\alpha_{22}} \omega_{\mu\nu}, \quad (4.21)$$

$$R_{\mu\nu,V^\pm}^{(0)} = \frac{1}{\alpha_{33}} \omega_{\mu\nu}. \quad (4.22)$$

Taking again as reference system  $k^\mu = (m, 0, 0, 0)$  one gets the general expression



$$R_{\mu\nu,I}^{(0)} \Big|_{k^\mu=(m,0,0,0)} = \frac{1}{u_I} \text{diag}(1,0,0,0) \quad (4.23)$$

which requires  $\alpha_{11}$ ,  $\alpha_{22}$ ,  $\alpha_{33}$  be positive defined in order to avoid ghosts.

As an important result, considering that the parameters that define the transversal and longitudinal parameters are independent, one can get the solutions where the model does not depend on ghosts. This means that it is unitary.

Concluding on these two quantum numbers masses and spin, being studied through the poles and residues of propagators, we should also notice that they are explicitly showing on the antireductionistic character of the model. Their expressions are depending on the system as a whole. As result, one can control on tachyons and ghosts through free coefficients and also given their different poles and residues, the spin-1 and spin-0 quanta can be identified as different particles.

## 5 Discrete symmetries

A next aspect is to study on discrete symmetries at eq. (1.2). For this, as example, we are going to select the following possibilities:

- (i)  $J^{PC} = 1^{--}$ , as the photon,  $J/\psi$ ,  $\rho(770)$ . Call such particles as  $A_\mu$ .
- (ii)  $J^{PC} = 1^{+-}$ , as the  $b_1(1235)$ . Call such particles as  $B_\mu$ .
- (iii)  $J^{PC} = 1^{++}$ , as the  $\chi_1(3510)$ . Call such particles as  $C_\mu$ .

Then any composition between such possibilities or between themselves as

$$\begin{aligned} a_{CCC}(\partial_\mu C_\nu)C^\mu C^\nu & \quad a_{AACB}A_\mu A_\nu C^\mu B^\nu \\ a_{BBC}(\partial_\mu B_\nu)B^\mu C^\nu & \quad d_{AC}A_\mu C^\mu \end{aligned} \quad (5.1)$$

preserves CPT. This verification confirms the basic theorem where a local and Poincaré invariant Lagrangian as eq. (1.2) should preserve CPT invariance [8].

## 6 Photons possibilities

A possibility emerged from this work is the photon not be necessarily associated to the electric charge. Experimental collisions between photons are showing events that suggest new sorts of couplings for the photon. An increasing amount of data on the reaction  $\gamma\gamma \rightarrow$  mesons at low and intermediate energies has been available since 1987 [9]. Three-gamma modes were already detected. The known  $J/\psi(3097)$  decays obtaining three-photon final states are: the radiative decays towards a  $\pi^0$ ,  $\eta$ ,  $\eta'$  or  $\eta_C$ , followed by the decay of the meson into two photons, and the predicted direct electromagnetic decay  $J/\psi \rightarrow \gamma\gamma$ . The non-resonant QED process  $e^+e^- \rightarrow \gamma\gamma$  also contributes. In the five-photon final state, only the radiative decays towards a pair of  $\pi^0$  or  $\eta$  mesons were studied up to now. Thus, these facts are perhaps indicating that we are in front of a new physics for the photon whose systematization would be behind QED. For instance, reactions like

$$\begin{aligned} \gamma\gamma & \rightarrow \rho\rho \\ \gamma\gamma & \rightarrow K^{*0}\bar{K}^{*0} \\ \gamma\gamma & \rightarrow K^{*+}K^{*-} \\ \gamma\gamma & \rightarrow \pi^+\pi^-\pi^+\pi^- \end{aligned}$$

are indications from Tasso and Argus collaborations of a new structure for the photon to be described. Most of these reactions are expected to be understood through effective Lagrangians, QCD and  $q\bar{q}q\bar{q}$  models. Our search will be to justify them with such an extension of the U(1)-model. In this regard, a first hypothesis is based on the existence of vertices involving photons.

The presence of selfinteracting photons could perhaps be interpreted as a contribution from the non-point-like constitution of the photon. Graphs involving only photons as external lines would indicate the presence of partons inside the



photon. They would correspond to the photon structure function. A second class of graphs would be the ones involving the coupling of photons with vector mesons as  $\rho$ ,  $\omega$ ,  $\phi$ ,  $J/\psi$ ,  $\gamma$ , or with the weak interaction vector mesons  $W^\pm$  and  $Z^0$ . Thus, vector-photon reactions do not necessarily need to be reassessed by the effective Lagrangian approach [10]. A possible meaning for eq. (1.2) is to treat the  $\rho$ -meson, for instance, as a gauge boson and not as an approximate gauge boson associated with isospin [11]. Therefore, a challenge will be to compute such reactions not through perturbative QCD, but taking eq. (1.2) as guideline.

## 7 Conclusion

This work considers the electric charge participation on the physical processes as a whole. So preserving the electromagnetic postulates of light invariance and electric charge conservation, this work enlarges Maxwell to a four bosons electromagnetism. Instead of just considering on charge distribution, it considers the electromagnetic phenomena for charges transmission. It adds to Maxwell, physical processes exchanging  $|\Delta Q| = 1$  where the photon is no more the only one propagator of the electromagnetic phenomena.

Thus given its gauge invariance, renormalizability, unitarity a first question should ask on the origin of these four messengers. Literature is rich on vector bosons opportunities. We should start on investigating about situations where they already exist. For a neutral massive photon we select three cases. They are virtual photon,  $Z^0$  intermedating leptonic currents preserving parity, vectorial mesons. Also that physics proposes a  $Z^{0'}$  beyond Standard Model [12] and derived from superstrings theories [13].

Virtual photons make the function corresponding to a neutral massive photon. Reactions as  $e^+e^- \rightarrow \mu^+\mu^-$ ,  $\mu^-p \rightarrow \mu X$  ( $X$  means any hadronic function),  $\pi^+p \rightarrow e^+e^-$  are showing situations identified in the literature with virtual photon which could be intermediate by a possible neutral massive photon. A second case is reactions like  $p\bar{p} \rightarrow e^+e^-X$  where  $Z \rightarrow e^+e^-$  and  $X$  is a hadronic state. The general form for their coupling is  $L_{int} = [g_V \bar{l}\gamma^\mu l + g_A \bar{l}\gamma_\mu \gamma_5 l]Z_0$ . Then, the suggestion here is the appearance of a leptonic current purely electromagnetic ( $g_A = 0$ ). Finally, one can observe cases involving vectorial mesons as  $\Phi(ss)$ ,  $J/\psi(cc)$ ,  $Y(bb)$ . These particles can suffer decays to  $e^+e^-$  via massive neutral bosons. Similarly from  $\rho(u,d)$ ,  $\omega^\pm(u,d)$  decays to  $e^-(e^+)\nu_e$  via intermediate charged bosons.

Thus it is possible to build up a physics associating eight messengers responsible for the electric charge transmission  $\Delta Q = \pm 1$ . Based on section 3 spectroscopy, the model requires massless photon and scalar photon and other quanta with different masses. Consequently a variety of four bosons choices is possible. Given that the physical source is just the electric charge we can consider composite particles as intermediate bosons, without being classified as effective theory. This means that vectorial and scalar bosons can work together with photon to fill the four bosons picture. For instance, for spin-1 family ( $\gamma, J/\psi, \omega^\pm$ ) and for spin-0 family (*scalar photon*,  $\pi^0, \pi^\pm$ ). For instance, proton and neutron reactions including these 8 intermediating particles.

After such realm for introducing an electromagnetism with a new performance based on four bosons, we should comment on three features. They are non-linearity, self-interacting photons and electromagnetism beyond the electric charge as coupling constant. They redefine the meaning of electromagnetic interaction. They bring a new scale on energy and distance. There is a more rich electromagnetism to be considered [14].

Different subjects as condensed matter, plasma, non-linear optics, astrophysics are requiring a description based on a non-linear electromagnetism. While Maxwell laws are valid for the earth's electric field of 600 N/C and magnetic field of  $10^{-8}$  T the question is whether in a magnetar with  $E = 10^{24}$  N/C and  $B = 10^{12}$  T, non-linear effects should not be included. As we know, when gravity increases as in the case of Mercury perihelion or black holes, one has to move from Newton to general relativity approach. So the question is whether the electromagnetic model appropriate for earth should be the same for a magnetar or inside the atomic nucleus.

Eq. (1.2) develops a non-linear electromagnetism. Differently from most non-linear systems, as Born-Infeld and Euler-Heisenberg, a new approach to non-linearity is obtained. Instead of electric and magnetic fields the non-linearity is on potential fields. Although being an abelian model, classical non-linearities are present in the Lagrangian, section 1 and quantum non-linearities are studied through Feynman rules at section 3.

A second aspect is on photon selfinteraction. Photons interactions are being a challenge for modern electromagnetism. In section 3 one derives three and four photon vertices. A first phenomenology is on light-light scattering. Instead of studying this phenomena through loop contribution, one gets at tree level. It says that photons can interact with photons without the presence of electrons and with a coupling constant not depending on electric charge.

Feynman rules obtained in section 3 show the appearance of vertices with self-coupled photons. Although the



Landau-Yang and Furry theorems are constraints for vertices involving real photons, physically, such photon self-interaction and with other massive vector particles, may be interpreted as an indication of a non-point like structure for photons. The  $J/\psi$  decays are an important source of spectroscopic information to exploit about the relevance of such vertices. The  $J/\psi$  decay into photon-vector-vector is an interesting field of study to test a possible non-linear QED.

Thus, selfinteracting photons also rises the question on the photon nature. Are they point-like particles or composite? Photons combine the point-like nature of the leptons and the composite nature of the hadrons. The existence of a point-like component in the photon allows the study of high- $p_T$  reactions even at moderate photon energies and to compute those reactions in the frame of perturbative QCD. The contributions of the non-point-like component can be split into two parts. The first one is due to the presence of partons inside the photon, described by the photon structure functions that can be extracted from  $\gamma\gamma$  collisions in  $e^+e^-$  machines. This contribution is computable in perturbative QCD. The second one is related to the coupling of the photon to vector mesons. However, this contribution is not computable in the framework of perturbative QCD. Considering also that QCD does not have a good agreement with charm hadroproduction, it is propitious an investigation with a Lagrangian that works with non-linear photons.

A third aspect being considered are the relationship between photons and the electric charge. The question is how to associate a photon beyond electric charge. Since its isolation through a capacitor in 1745 by the german scientist Ewald von Kleist, the electric charge become the electromagnetic origin. Consequently the photon couplings should be associated to electric charge. This universality rules QED and the Standard Model [5]. Eq. (1.2) provokes this situation. Through a non-linear abelian model it shows in section 3 different types of vertices which coupling constants encompasses the electric charge. It says that electric charge as pure coupling constant appears only when fields are coupled to the global Noether conserved current.

Thus, eq. (1.2) dissociates the strict relationship between the photon and the electric charge as coupling constant. Given the photon singularity it is expected from it a new electromagnetic scale. This means an *ubiquous lux* as the universal reference and no more the electric charge coupling constant. For this, the four bosons electromagnetism preserves the electric charge conservation and develops the coupling constants diversity. It achieves a kind of light universality by performing interactions under different coupling constants. Perhaps, this is a good requirement for dark matter, a photon-matter interaction different from the fine structure constant. This photonic electromagnetism appears through eq. (1.2).

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