



Comparison of Quantities of Information in the Human Memory

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Abstract

A mathematical model of changing the amount of information in the abstract human memory is proposed in the presence of the subsequent "external discrete" training (filling the information). Under this model, the amount of information is a solution of impulsive differential equation with fixed moments of impulsive effects and variable structure. Sufficient conditions are proposed related to the moments and magnitudes of the impulsive effects (i.e., to the moments of discrete training and the volume of the received information), where the quantities of information in two different models of learning can be compared.

Indexing terms/Keywords: Information, Impulsive differential equation.

Mathematics Subject Classification: 34A37

Supporting Agencies:

1. Bulgarian National Scientific Research Fund, contract DM19/1 (2017).
2. Project BG05M2OP001-2.009-0015, University of Chemical Technology and Metallurgy-Sofia, Bulgaria.

Language: English

Date of Publication: 2018-27-10

ISSN: 2347-1921

Volume: 14 Issue: 02

Journal: *JOURNAL OF ADVANCES IN MATHEMATICS*

Website: <https://cirworld.com>



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How to Cite: Dishliev, A., Girginov, C., & Dishlieva, K. (2018). Comparison of Quantities of Information in the Human Memory. *JOURNAL OF ADVANCES IN MATHEMATICS*, 14(2), 8004-8012. Retrieved from <https://cirworld.com/index.php/jam/article/view/7840>



Introduction

In the articles [7], [8], and [10] the following concepts are introduced:

- Quantity of information $I = I(t)$, calculated at the moment $t > t_0$, where $t_0 \geq 0$ is an initial moment;
- Information storage coefficient $\alpha = \alpha(t)$, $t \geq 0$;
- Discreet, external information (in the form of short-term training)

$$\Delta I(t) = I(t+0) - I(t-0) = I(t+0) - I(t), \quad t = t_1, t_2, \dots,$$

where the moments t_1, t_2, \dots are fixed in advance and $t_0 < t_1 < t_2 < \dots$;

- Coefficient of proportional completing the information at each training $\beta = \beta(t, I(t)) > 0$, $t = t_1, t_2, \dots$.

In the above cited articles, the following hypotheses are used:

First hypothesis: The output numerical data and the obtained results are averaged, i.e. they refer to a "typical generalized representative" chosen by a group of learners placed under the same external conditions.

Second hypothesis: Information is quantified and its values at $t \geq t_0$ are expressed by the function $I = I(t) > 0$.

Third hypothesis: Every external filling of the information in the human memory (which takes place through an organized training) is realized in a relatively short time period. Therefore, we will assume that the changing of information by the external effects is done instantaneously in the form of impulses at the fixed moments t_1, t_2, \dots , $t_0 < t_1 < t_2 < \dots$. This means that the amount of information $I(t)$ in the case of short-terms discrete training is a peace-wise continuous function in $t \geq t_0$ with the points of discontinuity t_1, t_2, \dots .

Here, the new hypotheses are introduced:

Fourth hypothesis: The internal (continuous) change in the amount of information in each interval $t_{i-1} < t \leq t_i$, $i = 1, 2, \dots$, is proportional to the volume of information. The coefficients of proportionality in these intervals are expressed by the functions $\alpha_{i-1} = \alpha_{i-1}(t)$, $i = 1, 2, \dots$, defined in $t \geq 0$.

Fifth hypothesis: After each training, the increase of the amount of information satisfies the equalities

$$\Delta I(t_i) = I(t_i + 0) - I(t_i) = I_i, \quad i = 1, 2, \dots, \quad (1)$$

where the positive constants I_1, I_2, \dots are given in advance.

Sixth hypothesis: The amount of information in the human memory is limited above, i.e. there is a memory capacity. The increase in the amount of information as a result of short-term discrete training is limited from below, i.e. we have a minimal intake of information.

PRELIMINARY REMARKS

In the article [10], formulated above Fifth hypothesis is replaced by the supposition:

Hypothesis [10]: After each training, the quantity increase of information satisfies the equality

$$\Delta I(t_i) = \beta(t_i, I(t_i - 0)) \cdot I(t_i - 0) = \beta(t_i, I(t_i)) \cdot I(t_i), \quad i = 1, 2, \dots, \quad (2)$$



where $\beta = \beta(t, I(t)) > 0$, $t \geq 0$, is a coefficient (function determined in advance) of the proportional filling of the information.

Clear that, the Fifth hypothesis formulated here is a special case of Hypothesis [10]. For this purpose, it is enough to substitute

$$\beta(t_i, I(t_i)) = \frac{I_i}{I(t_i)}, \quad i = 1, 2, \dots,$$

in equality (2) in order to obtain equality (1) of the Fifth hypothesis.

In the papers [8] and [10], we assume that the coefficients of proportionality in each intervals $t_{i-1} < t \leq t_i$, $i = 1, 2, \dots$, is the same, i.e. the equalities $\alpha_0(t) = \alpha_1(t) = \dots$, $t \geq 0$ are valid.

In [10] the general problem that modeled the dynamics of the amount of information in the human memory at the presence of the relevant hypotheses is found. We have:

$$\frac{dI}{dt} = \alpha(t - t_i)I, \quad t_{i-1} < t \leq t_i;$$

$$\Delta I(t_i) = \beta(t_i, I(t_i))I(t_i), \quad i = 1, 2, \dots;$$

$$I(t_0) = I_0.$$

In this study, provided that the five hypotheses formulated in the preceding paragraph are valid, as a special case of the mathematical model in [10], we can write the following initial value problem of differential equations with impulses that model the dynamics of the amount of information in the human memory:

$$\frac{dI}{dt} = \alpha_{i-1}(t - t_i)I, \quad t_{i-1} < t \leq t_i; \quad (3)$$

$$\Delta I(t_i) = I_i, \quad i = 1, 2, \dots; \quad (4)$$

$$I(t_0) = I_0. \quad (5)$$

The problem above is a main object of this paper. We denote its solution by $I(t; t_0, t_1, \dots, I_0, I_1, \dots)$. The next inequalities are valid:

$$I_0 = I(t_0; t_0, t_1, \dots, I_0, I_1, \dots); \quad I(t; t_0, t_1, \dots, I_0, I_1, \dots) \geq 0, \quad t \geq t_0.$$

We will use the notation $I(t; t_0, I_0) = I(t; t_0, t_1, \dots, I_0, I_1, \dots)$ if the solution is not subjected to the impulsive effect up to the moment $t > t_0$. If the impulsive effects are in number i to the same moment $t > t_0$, we will use also the notation $I(t; t_0, t_1, \dots, t_i, I_0, I_1, \dots, I_i) = I(t; t_0, t_1, \dots, I_0, I_1, \dots)$ for the solution of problem (3), (4), (5). The general solution of the above initial value problem has the form (see [10])



$$I_i^{*+} = I(t_i^*; t_0^*, t_1^*, \dots, t_{i-1}^*, I_0^*, I_1^*, \dots, I_{i-1}^*) + I_i^* < I(t_i^{**}; t_0^{**}, t_1^{**}, \dots, t_{i-1}^{**}, I_0^{**}, I_1^{**}, \dots, I_{i-1}^{**}) + I_i^{**} = I_i^{**+} \quad (7)$$

there exists a constant T , $T > \max\{t_i^*, t_i^{**}\}$, such that for each $t > T$, it is satisfied

$$I(t; t_0^*, t_1^*, \dots, t_i^*, I_0^*, I_1^*, \dots, I_i^*) < I(t; t_0^{**}, t_1^{**}, \dots, t_i^{**}, I_0^{**}, I_1^{**}, \dots, I_i^{**}).$$

Proof. Using equality (6) and inequality (7) for $t > \max\{t_i^*, t_i^{**}\}$, we consistently obtain

$$\begin{aligned} & I(t; t_0^*, t_1^*, \dots, t_i^*, I_0^*, I_1^*, \dots, I_i^*) - I(t; t_0^{**}, t_1^{**}, \dots, t_i^{**}, I_0^{**}, I_1^{**}, \dots, I_i^{**}) \\ &= I_i^{*+} \exp\left(\int_{t_i^*}^t \alpha_i(\tau - t_i^*) d\tau\right) - I_i^{**+} \exp\left(\int_{t_i^{**}}^t \alpha_i(\tau - t_i^{**}) d\tau\right) \\ &= I_i^{*+} \exp\left(\int_0^{t-t_i^*} \alpha_i(\tau) d\tau\right) - I_i^{**+} \exp\left(\int_0^{t-t_i^{**}} \alpha_i(\tau) d\tau\right) \\ &= I_i^{*+} \exp\left(\int_0^{t-t_i^{**}} \alpha_i(\tau) d\tau\right) \exp\left(\int_{t-t_i^{**}}^{t-t_i^*} \alpha_i(\tau) d\tau\right) - I_i^{**+} \exp\left(\int_0^{t-t_i^{**}} \alpha_i(\tau) d\tau\right) \\ &= \exp\left(\int_0^{t-t_i^{**}} \alpha_i(\tau) d\tau\right) \left(I_i^{*+} \exp\left(\int_{t-t_i^{**}}^{t-t_i^*} \alpha_i(\tau) d\tau\right) - I_i^{**+} \right). \end{aligned} \quad (8)$$

By the Conditions H1 and H2, we deduce that

$$\lim_{t \rightarrow \infty} \int_{t-t_i^{**}}^{t-t_i^*} \alpha_i(\tau) d\tau = (t_i^{**} - t_i^*) \lim_{t \rightarrow \infty} \alpha_i(\theta) = (t_i^{**} - t_i^*) \lim_{\theta \rightarrow \infty} \alpha_i(\theta) = 0$$

$$\Rightarrow \lim_{t \rightarrow \infty} \exp\left(\int_{t-t_i^{**}}^{t-t_i^*} \alpha_i(\tau) d\tau\right) = 1, \quad (9)$$

where the constant θ is between $t - t_i^*$ and $t - t_i^{**}$. Then by (8), having in mind the equality (9), condition H2 and inequality (7), we find successively

$$\begin{aligned} & \lim_{t \rightarrow \infty} \left(I(t; t_0^*, t_1^*, \dots, t_i^*, I_0^*, I_1^*, \dots, I_i^*) - I(t; t_0^{**}, t_1^{**}, \dots, t_i^{**}, I_0^{**}, I_1^{**}, \dots, I_i^{**}) \right) \\ &= \lim_{t \rightarrow \infty} \exp\left(\int_0^{t-t_i^{**}} \alpha_i(\tau) d\tau\right) \left(I_i^{*+} \lim_{t \rightarrow \infty} \exp\left(\int_{t-t_i^{**}}^{t-t_i^*} \alpha_i(\tau) d\tau\right) - I_i^{**+} \right) \\ &= \lim_{t \rightarrow \infty} \exp\left(\int_0^{t-t_i^{**}} \alpha_i(\tau) d\tau\right) (I_i^{*+} - I_i^{**+}) < 0. \end{aligned}$$

Since the solutions $I(t; t_0^*, t_1^*, \dots, t_i^*, I_0^*, I_1^*, \dots, I_i^*)$ and $I(t; t_0^{**}, t_1^{**}, \dots, t_i^{**}, I_0^{**}, I_1^{**}, \dots, I_i^{**})$ are continuous for $t > \max\{t_i^*, t_i^{**}\}$, then by the sign of the upper limit it follows that there is a constant $T > \max\{t_i^*, t_i^{**}\}$, such that for each $t \geq T$, the next inequality is valid

$$I(t; t_0^*, t_1^*, \dots, t_i^*, I_0^*, I_1^*, \dots, I_i^*) < I(t; t_0^{**}, t_1^{**}, \dots, t_i^{**}, I_0^{**}, I_1^{**}, \dots, I_i^{**}).$$

The theorem is proved.

The following statement is an important consequence of the above theorem.



Теорема 2. Assume that:

1. The conditions H1 and H2 are satisfied.
2. The impulsive effects of Problem (3), (4) and (5) are finite.

Then:

- for each finite number impulsive effects i ;
- for each finite sequence of impulsive moments $0 < t_0 < t_1 < \dots < t_{i-1}$;
- for every two impulsive moments t_1^* and t_i^{**} , $t_{i-1} < t_1^* < t_i^{**}$;
- for every finite sequence of impulsive effects $I_0 > 0, I_1 > 0, \dots, I_i > 0$,

there exists a constant T , $T > t_i^{**}$, such that for each $t \geq T$, it is fulfilled

$$I(t; t_0, t_1, \dots, t_{i-1}, t_i^*, I_0, I_1, \dots, I_i) > I(t; t_0, t_1, \dots, t_{i-1}, t_i^{**}, I_0, I_1, \dots, I_i).$$

Proof. As $t_i^* < t_i^{**}$ and $\alpha_i(t) < 0$ at $t \geq 0$, it follows that

$$\begin{aligned} & I(t_i^*; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}) \\ &= I_{i-1}^+ \exp\left(\int_{t_{i-1}}^{t_i^*} \alpha_{i-1}(\tau - t_{i-1}) d\tau\right) \\ &> I_{i-1}^+ \exp\left(\int_{t_{i-1}}^{t_i^{**}} \alpha_{i-1}(\tau - t_{i-1}) d\tau\right) \\ &= I(t_i^{**}; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}) \\ &\Leftrightarrow I_i^{*+} = I(t_i^*; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}) + I_i \\ &> I(t_i^{**}; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}) + I_i = I_i^{**+}. \end{aligned}$$

From the last inequality and Theorem 1, we reach the statement of Theorem 2.

We introduce the condition:

H3. It is satisfied $\int_0^\infty \alpha_{i-1}(\tau) d\tau = A_{i-1} = \text{const} < 0$, $i = 1, 2, \dots$.

Remark 1. The existence of limits $\lim_{t \rightarrow \infty} \alpha_{i-1}(t) = 0$, $i = 1, 2, \dots$, follow by the equalities

$$\int_0^\infty \alpha_{i-1}(\tau) d\tau = A_{i-1},$$

i.e. condition H2 follows by condition H3. This means that the statements of the above two theorems will not change if in their formulation, condition H2 is replaced by condition H3.

If the number of the short-term trainings is i , then the magnitude $I_\infty = I_i^+ \exp(A_i)$ is called an amount of residual information in memory. If we experimentally determine the amount of residual information I_∞ , then a constant A_i is defined immediately. For more details, see the article [7].

Given the Sixth hypothesis, we introduce the condition:



H4. There exist positive constants I_{\min} and I_{\max} such that the following inequalities are valid $I_1 \geq I_{\min}, I_2 \geq I_{\min}, \dots$ and $I_1^+ \leq I_{\max}, I_2^+ \leq I_{\max}, \dots$.

H5. The inequalities $A_i \geq A_{i-1} + \ln \frac{I_{\max}}{I_{\min}}, i = 1, 2, \dots$ are fulfilled.

Theorem 3. Let:

1. The Conditions H1, H3, H4 and H5 are satisfied.
2. The impulsive effects of initial value problem (3), (4), (5) are finite.
Consider two solutions to the problem (3), (4), (5):

- $I(t; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1})$ with $i-1$ number of impulsive effects;
- $I(t; t_0, t_1, \dots, t_i, I_0, I_1, \dots, I_i)$ with i number of impulsive effects.

Then:

- for each i ;
- for every finite sequence of impulsive moments $0 < t_0 < t_1 < \dots < t_i$;
- for every finite sequence of impulsive effects $I_0 > 0, I_1 > 0, \dots, I_i > 0$,

there exists a constant $T, T > t_i$, such that for every $t \geq T$, it is fulfilled

$$I(t; t_0, t_1, \dots, t_i, I_0, I_1, \dots, I_i) > I(t; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}).$$

Proof. For $t > t_i$, we have

$$\begin{aligned} & I(t; t_0, t_1, \dots, t_i, I_0, I_1, \dots, I_i) - I(t; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}) \\ &= I_i^+ \exp\left(\int_0^{t-t_i} \alpha_i(\tau) d\tau\right) - I_{i-1}^+ \exp\left(\int_0^{t-t_{i-1}} \alpha_{i-1}(\tau) d\tau\right). \end{aligned}$$

Therefore,

$$\begin{aligned} & \lim_{t \rightarrow \infty} \left(I(t; t_0, t_1, \dots, t_i, I_0, I_1, \dots, I_i) - I(t; t_0, t_1, \dots, t_{i-1}, I_0, I_1, \dots, I_{i-1}) \right) \\ &= I_i^+ \exp\left(\lim_{t \rightarrow \infty} \int_0^{t-t_i} \alpha_i(\tau) d\tau\right) - I_{i-1}^+ \exp\left(\lim_{t \rightarrow \infty} \int_0^{t-t_{i-1}} \alpha_{i-1}(\tau) d\tau\right) \\ &= I_i^+ \exp(A_i) - I_{i-1}^+ \exp(A_{i-1}) \\ &> I_{\min} \exp(A_i) - I_{\max} \exp(A_{i-1}) \\ &= I_{\min} \exp(A_{i-1}) \left(\exp(A_i - A_{i-1}) - \frac{I_{\max}}{I_{\min}} \right) > 0. \end{aligned}$$

The theorem is proved.

CONCLUSIONS

We will give an interpretation of the mathematical results obtained In the next few notes.



Conclusion 1. Let us consider two variants of discreetly filling in the information.

Both variants do not differ in:

- The number filling in the external information (training);
- The storage coefficients of the information (the latter means that the variants refer to the same abstract individual).

Both variants are permissible to differ in:

- The initial value conditions – the initial moment and the initial information quantity in the human memory;
- The moments in which discrete fill-in of the information takes place;
- The size of changing the information in discrete trainings;
- The amounts of information after the last filling in the information.

Comparing both options after a sufficiently long period of time, we obtain that the option with a bigger amount of information since the last filling the amount of information has a bigger volume. (see Theorem 1). In other words, the amount of information in memory, calculated immediately after the last training is a key factor for the future development of information.

Conclusion 2. Look again two variants of discreetly filling the information.

Both variants do not differ in:

- The initial value conditions;
- The number filling of external information;
- The storage coefficients of information;
- The size of changing the information in discrete trainings;
- From the first to the penultimate moment, in which the discrete fillings of information are carried out.

Both variants are permissible to differ in:

- The last moment of training.

Then (after a sufficiently long period of time) the amount of information is larger in the case of the later filling in the quantity of the information (see Theorem 2).

Conclusion 3. Look again two variants of discreetly filling the information.

Both variants do not differ in:

- The initial value conditions;
- The storage coefficients of information;
- The size of changing the information in discrete trainings;
- From the first to the penultimate moment, in which the discrete fillings of information are carried out.

Both variants are permissible to differ in:

- The number of trainings.

Then (after a sufficiently long period of time) the amount of information is greater in the case of another (additional) filling in the quantity of information (see Theorem 3).

Acknowledgements:

1. The authors are grateful for the funding of this research to the Bulgarian National Scientific Research Fund, under contract DM19/1 (2017).
2. The authors are grateful for funding of this work under project BG05M2OP001-2.009-0015.



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