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ANOTHER PROOF OF BEAL'S CONJECTURE

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ABSTRACT. Beal's Conjecture : The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z with μ, ξ and ν odd primes at least 3. A proof of this longstanding conjecture is given.

Beal's Conjecture: The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z with ξ, μ and ν odd primes at least 3. A history of this problem can be found in [1].

Suppose $z^\xi = x^\mu + y^\nu$ is true for any relatively prime positive integers x, y, z and odd primes ξ, μ and ν with ξ, μ, ν at least 3. When x, y and z are relatively prime, $(z^\xi), (x^\xi)$ and (y^ξ) are also relatively prime. Then $(z^\xi)^\xi = (x^\xi)^\mu + (y^\xi)^\nu$. That is, suppose $(z^\xi)^\xi = (x^\mu)^\xi + (y^\nu)^\xi$.

The Proof.

It is clear that if $(z^\xi)^\xi = (x^\mu)^\xi + (y^\nu)^\xi$, then either x^μ or y^ν or z^ξ is divisible by 2. Suppose z^ξ is divisible by 2. Then x^μ and y^ν are odd. Since $(z^\xi)^\xi = (x^\mu)^\xi + (y^\nu)^\xi$, $(z^\xi)^\xi$ is $2^{m\xi}$ times an odd integer, where m is an integer, and $(x^\mu)^\xi + (y^\nu)^\xi = (x^\mu + y^\nu)(\sum_{k=0}^{\xi-1}(x^\mu)^k(y^\nu)^{\xi-1-k})$, by prime factorization, $x^\mu + y^\nu$ is even. Hence,

$$x^\mu + y^\nu = 2^{m\xi}. \qquad (1)$$

Also,

$$x^\mu + y^\nu - z^\xi \equiv 0 \ (mod\ 2). \qquad (2)$$

So,

$$(x^\mu + y^\nu - z^\xi)^\xi \equiv 0 \ (mod\ 2^\xi);$$

and

$$(x^\mu + y^\nu)^\xi - (z^\xi)^\xi \equiv 0 \ (mod\ 2^\xi), \qquad (3)$$

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since, by expanding $(x^\mu + y^\nu - z^\xi)^\xi$ using binomial expansion,

$$(x^\mu + y^\nu - z^\xi)^\xi - ((x^\mu + y^\nu)^\xi - (z^\xi)^\xi) = \sum_{k=1}^{\xi-1} C(\xi,k)(x^\mu + y^\nu)^{\xi-k}(-z^\xi)^k.$$

Hence, in view of equation (2) and (3),

$$(z^\xi)^\xi - (x^\mu)^\xi - (y^\nu)^\xi = (x^\mu + y^\nu)^\xi - (x^\mu)^\xi - (y^\nu)^\xi$$

$$= \sum_{k=1}^{\xi-1} C(\xi,k)(x^\mu)^{\xi-k}(y^\nu)^k \equiv 0\ (mod\ 2^\xi). \qquad (4)$$

So, $y^\nu \equiv 0\ (mod\ 2)$ and $x^\mu \equiv 0\ (mod\ 2)$. That is, if z^ξ is even, z, x and y are even.

Now assume that x^μ is even and we have $(x^\mu)^\xi = (z^\xi)^\xi - (y^\nu)^\xi$. Since x^μ is even, z^ξ and y^ν are odd; $z^\xi - y^\nu = 2^{n\xi}$ for some integer n and hence

$$z^\xi - y^\nu - x^\mu \equiv 0\ (mod\ 2). \qquad (5)$$

So,

$$(z^\xi - y^\nu - x^\xi)^\xi \equiv 0\ (mod\ 2^\xi). \qquad (6)$$

Also

$$(z^\xi - y^\nu - x^\mu)^\xi - ((z^\xi - y^\nu)^\xi - (x^\mu)^\xi) = \sum_{k=1}^{\xi-1} C(\xi,k)(z^\xi - y^\nu)^{\xi-k}(-x^\mu)^k \equiv 0\ (mod\ 2^\xi). \qquad (7)$$

So,

$$(z^\xi - y^\nu)^\xi - (x^\mu)^\xi \equiv 0\ (mod\ 2^\xi). \qquad (8)$$

Hence,

$$(x^\mu)^\xi - (z^\xi)^\xi + (y^\nu)^\xi = (z^\xi - y^\nu)^\xi - (z^\xi)^\xi + (y^\nu)^\xi$$

$$= \sum_{k=1}^{\xi-1} C(\xi,k)(z^\xi)^{\xi-k}(-y^\nu))^k \equiv 0\ (mod\ 2^\xi)$$

So, $z^\xi \equiv 0\ (mod\ 2)$; and $y^\nu \equiv 0\ (mod\ 2)$ and hence z and y are even.

The case when y^ν is even is similar to the case when x^μ is even. So, if either x or y or z is even then, all are even which leads to a contradiction of the equation. Hence Beal's Conjecture is proved.

REFERENCES

[1] https://www.bealconjecture.com/

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