Cycles Cohomology and Geometrical Correspondences of Derived Categories to Field Equations

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Abstract

The integral geometry methods are the techniques could be the more naturally applied to study of the characterization of the moduli stacks and solution classes (represented cohomologically) obtained under the study of the kernels of the differential operators of the corresponding field theory equations to the space-time. Then through a functorial process a classification of differential operators is obtained through of the co-cycles spaces that are generalized Verma modules to the space-time, characterizing the solutions of the field equations. This extension can be given by a global Langlands correspondence between the Hecke sheaves category on an adequate moduli stack and the holomorphic bundles category with a special connection (Deligne connection). Using the classification theorem given by geometrical Langlands correspondences are given various examples on the information that the geometrical invariants and dualities give through moduli problems and Lie groups acting.

Indexing terms/Keywords: Cycles Cohomology, Field Equations, Field Ramifications, Generalized Verma Modules, Geometrical Langlands Correspondence, Hecke Category, Moduli Stack, Spec Functors.

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1. Introduction.

Equivalence spaces modulo a congruence relation are constructed on field solutions to establish an Universe theory that includes the quantum field theory (QFT), super-symmetries (SUSY), and the incorporating of the heterotic strings theory using sheaves of corresponding differential operators of the same field equations and coherent $D-$modules sheaves [1].

This identification will use to construct on the base of these equivalences the corresponding Zuckerman functor that is one of the universal functors of Harish-Chandra derived sheaves to the geometrical Langlands programme in mirror theory [2, 3]. The incorporating of the geometrical Langlands ramifications will establish extensions of a connection beyond of the holomorphicity to much of the complex vector bundles that can be constructed to a wide stack of physical phenomena, in the searching to obtain field solutions through of the curvature and torsion tensors of the space-time.

The obtained development includes complexes of infinite dimension $D-$modules, generalizing for this via, to the BRST-cohomology, the connection in this context. With it, the integrable system class can be extended in mathematical physics and with it, the possibility to obtain a general theory of integrals to study of the space-time (integral operator cohomology [4]) considering the kernels of the germs of the sheaves corresponding to the complex vector bundles, and therefore the measure of much of their field observables [5].

Having these Langlands correspondences we can to tend a bridge to complete a classification of the different operators to the field equations using on the base the Verma modules that are classifying spaces, the differential operators of $SO(1, n+1)$, where elements of the Lie algebra $\mathfrak{sl}(1, n+1)$, are differential operators of the modern mathematical physics [1]. The cosmological problem that exist is to reduce the number of the field equations that are resoluble under the same gauge field (Verma modules) and extend the gauge solutions to other fields using the symmetries of topological groups that define their interactions.

This extension can be given for a global Langlands correspondence between categories of Hecke sheaves on an adequate moduli stack and the category of holomorphic ${}^{L}G-$bundles with a special connection (Deligne connection). The corresponding $D-$modules can be viewed as sheaves of conformal blocks (or co-invariants) being images under a version of the generalized Penrose transform [1, 6], naturally arising in the frame of the conformal field theory.

Main Result.

We consider the results obtained in [2, 7, 8] on kernels of differential operators and complex cohomology class to the space-time, likewise as the theorem 4. 1, [8] to use the geometrical Langlands correspondence:

$$\mathcal{D}_{\mathrm{BRST}}(\mathrm{Oper}^{\leq n}_{{}^{L}G}(\mathcal{D}_y)) \cong \mathcal{D}^{\times}(\mathrm{Bun}_G(\Sigma)) \tag{1}$$

Then we have the following result.

Theorem (F. Bulnes) 2. 1 [8]. The derived category of quasi- $G-$equivariants $D_{G/H}$ $-$modules formed with the extended and generalized Verma modules given for ${}^{L}\Phi^{\mu}(\mathcal{M}) = \mathcal{M}\boxtimes\rho^{\mu}(\mathbb{V}),\ \forall\mathbb{V}\in(Loc_{L}),$ can be identified for a critically twisted sheaves category of D-modules on the moduli stack $\mathrm{Bun}_{G,y}, \forall y\in X$ (singularity) identified by the Hecke category $\mathcal{H}_{G,\mathcal{K},y}$, (geometrical Langlands correspondence), if this is an image of integral transforms acting on ramifications of the Hecke category $\mathcal{H}_G,\ \forall\lambda\in\mathfrak{h}*$ (for example $\mathcal{H}_{G,\lambda}$) on the flag manifold $G/\mathrm{B},$ with weight corresponding to twisted differential operators on $\mathrm{Bun}_{G,y}$.

To demonstrate this result we used the inherent arguments of the demonstration realized to the theorem 4. 1 [8] to characterize the complexes to this case [2], with the modality of the functors defined for ${}^{L}\Phi^{\mu}$, and using the geometrical Langlands equivalence given for (1). Then applying the Penrose transform on ${}^{L}\mathrm{Bun}_{G}$, we have

$$\mathrm{End}_{\mathfrak{g}_{\kappa c}} \mathbb{V}_{\lambda} \cong \mathrm{FunOp}_{{}^{L}G}{}^{\lambda}, \tag{2}$$

where $\mathrm{FunOp}_{{}^{L}G}{}^{\lambda} \subset \mathrm{FunOp}_{G}(D^{\times})$.

Considering some conjectures in ramification theory [8], we arrive to that the unique objects that are coefficients of the cohomological space of zero dimension are certain Verma modules. Then considering the Springer fiber, we characterize the derived categories that will give the Hecke category of the Verma modules ${}^{L}\Phi^{\mu}(\mathbb{V}_{w(\lambda+\rho)\text{-}r}(\chi_{y}))$, $\forall w \in W$, which are the ramifications of the Hecke category $\mathcal{H}_{G}$, $\forall \lambda \in \mathfrak{h}*$. This demonstrates the theorem and proposes a classification of the differential operators to the field equations (see the table).

The following table is a corollary of the theorem.

Table 1. Classification.

Generalized Verma Module $\mathcal{M}\boxtimes\rho^{\mu}(\mathbb{V})$,	**Stack Moduli**	**Differential Operators**
$H_{\bullet}^{n,n}(M, \mathcal{L}_{\lambda})$	Homogeneous line Bundles (with tensorization with form space)	$\mathfrak{sl}(n, n+1)$
$H_{[\mathbb{P}^{1}/\infty]}^{1,0}(\mathbb{P}^{1}, \mathcal{O})$ $n-2<0$	$\mathcal{L}_{\lambda} \otimes \Omega^{n}$	$\mathfrak{sl}(2, \mathbb{C})$
$\mathbb{V}_{w(\lambda+\rho)-\rho}(\chi_{y})$	$\mathrm{Bun}_{G,y}$	$\tilde{\mathfrak{g}}$
$H_{[\infty]}^{1,0}(\mathbb{P}^{1}, \mathcal{O}(n))$ $n \leq 0$	Homogeneous line Bundles	$\mathfrak{sl}(2, \mathbb{C})$
$H_{[C_{W}/\partial C_{W}]}^{n,l(w)}(\mathbb{P}^{1}, \mathcal{L}_{\lambda})$ $n \geq 2$, $*$	Holomorphic Vector Bundles	$\tilde{\mathfrak{g}}/\mathfrak{b}$

$H_{\bullet}^{n,n}(M,\mathcal{O})$	$\mathrm{Bun}_{G,y,n}$	$\mathfrak{so}(n,n+1)/\mathfrak{b}$
$H_{\bullet}^{n,m}(M,\mathcal{O})$ $m=n+1,n$	Higgs Bundles	$\mathfrak{so}(n,n+1)$
$H_{\infty}^{1}(\mathcal{NS},\mathbb{M})$ $H^{0}(\mathbb{CP}^{1},\hat{\mathcal{O}}_{\mathbb{P}^{1}}^{\mathrm{ch}})$ $case: \quad n=2, K=\mathbb{R}$	Bosonic Bundles	$\mathfrak{sl}(n,n+1)/\mathfrak{b}$

2. **Some Field Equations and their Ramifications as corollaries of the Theorem 2. 1.**

We consider the following example.

Example 3. 1. (Higgs Bundles). We consider the Abelian Hodge theory approach [9], and let $\mathrm{Loc}_{^{L}G}(C)$, with C, the curve of their corresponding ramification with the same geometry to construct the pair or Langlands data (M,δ), where $M=\mathrm{Bun}_{\mathrm{Higgs}}$, and

$$\delta : M \to M \otimes \Omega^{1}_{\mathrm{Higgs}/B}, \tag{3}$$

The differential field equation is

$$\square \nabla \phi = 0, \tag{4}$$

This connection is a meromorphic relative flat connection acting along of fibers of the Hitchin mapping:

$$h : \mathrm{Bun}_{\mathrm{Higgs}} \to B, \tag{5}$$

Furthermore by construction the bundle with connection (M,δ), is a Hecke eigen D-module with eigenvalue (E,∇), (as the found in the theorem 4. 1 [10]) but with respect to an Abelianized version

$$^{L}\Phi'^{i}_{Abelian} : \mathrm{D}^{b}(\mathrm{Bun}_{\mathrm{Higgs}},\mathcal{O}) \to \mathrm{D}^{b}(\mathrm{Bun}_{\mathrm{Higgs}} \times C,\mathcal{O}), \tag{6}$$

of Hecke functors. These are defined again for $i=1,2,3,\ldots,n-1$, as integral transforms with respect to the trivial local system on the Abelianized Hecke correspondences $\mathcal{H}_{G,\lambda}$, with $\mathrm{Bun}_{\mathrm{Higgs}}$, and $\mathrm{Bun}_{\mathrm{Higgs}} \times C$, in the classic scheme of the double fibration of the integral geometry (Penrose transforms!) on micro-local objects of these bundles.

Example 3. 2. (Extension of a connection on singularities). We consider a complex Riemannian manifold $\mathbb{M}$, with a singularity such that exists around of this singularity a decomposing of factors lines bundle (or zeros of homogeneous polynomial) that can to give integrals in $H^{\bullet}(\mathbb{M}, \mathcal{O}(\lambda))$, to certain differential field equation

$$(\bar{\partial} + \nabla_s)\phi = 0, \tag{7}$$

where ∇_s, is the singular connection component.

Then by the theorem 2. 1, to each integral weight $\lambda \in \mathfrak{h}^*$, we have the invariant line bundle $\ell_\lambda = G \times \mathbb{C}_\lambda$, on G/B. Here we have one of the Verma modules ${}^L\Phi^{\mu}(\mathbb{V}_{w(\lambda+\rho)\text{-}\rho}(\chi_y))$, and their cycles in this case are complex hyper-lines [8]. Indeed, the line bundle $\mathcal{L}_\lambda$, on Bun_{G,I_y}, is defined in such way that their restriction to each fiber of the projection p, is isomorphic to ℓ_λ. Then one solution characterized by the functors established in the theorem 2. 1, of the section 2, is the integral transform:

$$\mathcal{P} : H^{\bullet}(\mathbb{M}, \Omega^n \otimes \widetilde{\mathcal{L}}_\lambda) \cong \ker(U, \bar{\partial} + Q), \tag{8}$$

where $\widetilde{\mathcal{L}}_\lambda = \mathcal{L}_\lambda \otimes p^*$. Here $\bar{\partial} + Q$, can be viewed as the connection of $\mathrm{Deligne} + \mathrm{other\ thing}$, [1] [jmss] belonging to one "twisted" sub-category of $D-$modules on the moduli stack with eigenvalues $E\big|_{X\setminus\{y_1,\ldots,y_n\}}$. Then their geometrical Langlands ramification in the points $\{y_1, \ldots, y_n\}$, is the enveloping of the Langlands correspondence of loop groups obtained of the context of the group ${}^L G$, of the moduli space $\mathscr{M}_{\mathrm{Flat}}({}^L G, C)$. The property of being "twisted" is demonstrated by the argument of the Penrose transform that involve to the twistor transform in (8). The twistor transform followed of Penrose $\mathcal{P}(\mathcal{T})$, evaluates the kernels of the cohomological groups that are isomorphisms in the sense of the equivalences of the Kashiwara theorem [12-14].

Example 3. 3. (Electromagnetic basting of Space-Time Carpet). We consider a non-compact Minkowskian space such that in their electromagnetic field appear gauge fields in the quotient group $SL(2, \mathbb{C})/U(1)$, and in the group $SU(n)$. Then when the gauge group is $SU(2)$, and G, is a non-compact maximal subgroup of

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[1] $\mathrm{D}_{\mathrm{coh}}({}^L\mathrm{Bun}, \mathcal{D}) \cong \ker(U, \bar{\partial} + \nabla_s)$, then their images under the inverse Penrose transform are elements in sheaves of the category $\mathcal{D}_{\mathrm{BRST}}(\mathrm{Oper}^{\leq n}_{{}^L G})$, since by [**11**] $Q^2_{BRST} = 0$, which is equivalent to the application of Cousin cohomology and their involved twistor transform haves kernel isomorphic (this could be in $Z(\tilde{\mathfrak{g}}_{\kappa_c})$) to $\mathrm{FunOp}_L(\mathcal{D}^{\times})$. By the *Opers* theory $\mathrm{Op}_L(\mathcal{D}^{\times}) \cong \mathrm{proj}(\mathcal{D}^{\times}) \times \oplus \Omega_{\mathcal{K}}^{\otimes(d_j+1)}$, where $\Omega_{\mathcal{K}}^{\otimes n}$, is the space of $n-$differentials on $\mathcal{D}^{\times}$, and $\mathrm{proj}(\mathcal{D}^{\times})$, is the $\Omega_{\mathcal{K}}^{\otimes 2}-$ torsor of projective connections on $\mathcal{D}^{\times}$, which is conformed ∇_s. Also we can see the content of the space $\mathrm{Loc}_{{}^L G}(\mathcal{D}^{\times}) = \{\partial_t + A(t) \forall A(t) \in {}^L\mathfrak{g}((t))\}/{}^L G(t)$. To the case exposed here (that is to say ℓ_λ), $\mathrm{Loc}_{{}^L G}(\mathbb{C}^{\times})$. This is much seemed to the analytic continuation studied in complex variable. In this case is more complicated, since $\nabla + ramifications$, can be viewed as images under functors of the type $\underline{\Phi} + Geometrical \quad hypothesis$, using our integral transforms.

$SO(3,1),$ (considering in general the quotient G/H) the unique invariant solutions are Abelian Maxwell fields [15]. Then this produce to the field operator $Q,$ [11], the field equation:

$$Q_{\mathrm{BRST}}^2 = 0, \tag{9}$$

(zero branes stability), that is to say, the D - module of the Q - brane is the torus which is isomorphic to $U(1)$ [2]. The mosaic of the space-time to these electromagnetic fields is the of symmetrical electromagnetic waves. The corresponding moduli space is $\mathscr{M}_{g,0}(\mathbb{P}^3, d).$ Then a solution of the field equation (9) is:

$$\mathcal{P} : H^0(\mathbb{M}, O(-k)) \cong \ker(U, Q_{\mathrm{BRST}}), \tag{10}$$

In particular, $g = 0,$ and $d = 1,$ then (9), is the classical Penrose transform which gives the solution to the Laplace equation

$$\nabla^2 \phi = 0, \tag{11}$$

in a neighborhood $U \subset \mathbb{M} \cong S^4$. In this case the Verma modules are the of table 1, $H^n_{\bullet}(M, \mathcal{L}_{\lambda}),$ with $M = \mathbb{M},$ $\mathcal{L}_{\lambda} = O(-2).$ Here, $\mathbb{M},$ is a complex manifold which also can be modeled as the Grassmanian manifold $G_{2,4}(\mathbb{C}).$ Their corresponding moduli space is $\mathscr{M}_{0,0}(\mathbb{P}^3, 1).$ To the stacking of the space-time is necessary to have the strings $\mathcal{T},$ and $\mathcal{S},$ and the electromagnetic carpet of the space-time is given in the figure 1. These are solutions to (9), to two types of Q - branes.

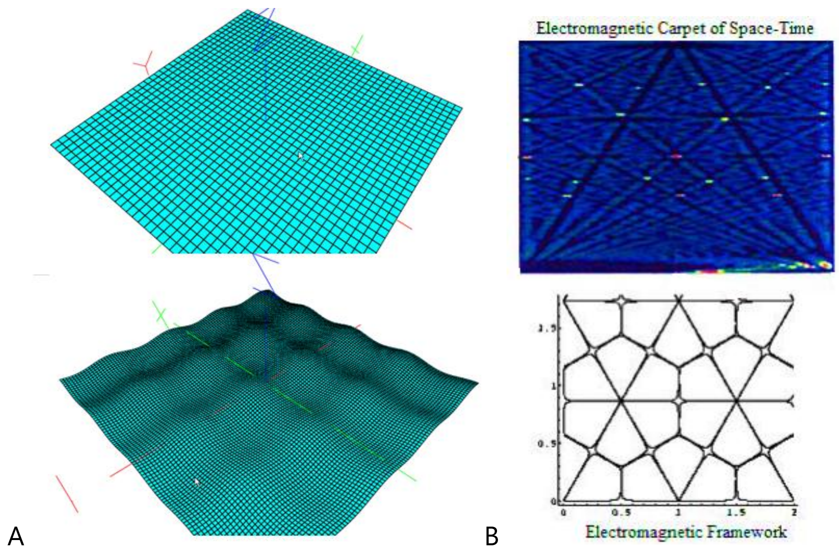


Fig. 1. Re-normalizing of the Group in semi-groups of electrodynamical quotients from $SL(2, \mathbb{C})/U(1),$ when $Q_{\mathrm{BRST}}^2 = 0,$ [16]. A). Stack to moduli space $\mathscr{M}_{0,0}(\mathbb{P}^3, 1),$ solutions. B). Stack to moduli space $\mathscr{M}_{g,0}(\mathbb{P}^3, d) = \mathbb{P}^{3|4},$ solutions.

[2] The co-weight lattice of $G,$ is defined as the lattice of homomorphisms from $U(1),$ to a maximal torus $\mathbb{T},$ of $G.$ The weight lattice of $G,$ is the lattice of homomorphisms from $\mathbb{T},$ to $U(1).$

Example 3. 4. (**Dualities).** Mirror equivalences to sources and holes of the quantum electromagnetic fields. The first image corresponds to the hole or singularity of the $U(1)-$ connections. The two images correspond to the source of the gauge $-U(1)$ field field. Both geometrical objects are equivalent inside of the dualities of the moduli spaces $\mathscr{M}_{\mathrm{FLAT}}({}^{L}G, C),$ and the Hitchin moduli space $\mathscr{M}_{H}({}^{L}G, C),$ (Figure 2). The geometrical correspondence is to this case given by the moduli identity:

$$\mathscr{M}_{H}(G, C) = \mathscr{M}_{\mathrm{FLAT}}(G, C)K, \tag{12}$$

Where $K,$ a vector homogeneous bundle of lines, whose polynomials have zeros equal to poles or singularities of field.

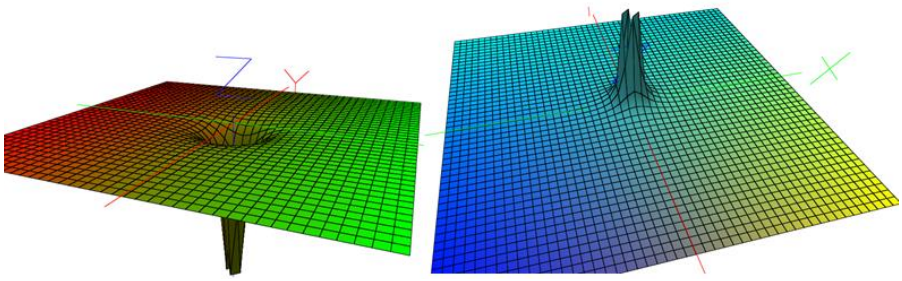


Fig. 2. Mirror equivalences between sources and holes in the space-time.

A re-interpretation of the singularities of the electromagnetic type in the space-time are given as sources of electromagnetic radiative fields[3] where through the twistor geometry these singularities can be zeros (or roots) of certain polynomials of the homogeneous lines bundles. Also these can be poles of certain surfaces, where can be projected fields whose origin are charges Q (see the figure 3). For example considering the sphere S^2, which we can identify through twistor theory as twistor space $\mathbb{P}\mathcal{T},$ with their two orbits $\mathbb{P}\mathcal{T}^{+},$ and $\mathbb{P}\mathcal{T}^{-},$ are projectivized the poles as $\mathbb{P}\mathcal{N},$ and $\mathbb{P}S,$ to each semi-sphere $S^{+},$ and $S^{-},$ identified these with the two orbits of $\mathbb{P}\mathcal{T}.$ Likewise the line $S^1,$ (Ecuador circle) divide to the holomorphic functions (H^0 $-$elements) in $S^2,$ in their parts of positive or negative frequency in $S^{+},$ and $S^{-},$ respectively. If we consider to the signals that come or go of the Riemann sphere $S^2,$ through the solutions of the complex equations

$$DAe^{i\omega t} = 0, \tag{13}$$

these can be interpreted as signals emitted or received by a encoder in $S^2,$ of an field energy signal coming from of the space-time.

Example 3. 5. (Gravitational Diffeomorphism and their decoupling from Electromagnetic field component). We consider the following application given for the Brans-Dicke argumentation[4], from a point of view of the variation of the gravitational constant, which varies from the place in time. Then assuming the

[3] These could be multi-radiative electromagnetic fields to hyper-transmission-reception of signals.

[4] As mentioned before, the Brans–Dicke theory of gravitation is a theoretical framework to explain gravitation from a point of view of electromagnetic wave to explain the variation of the gravitational constant that is assumed in this theory as function of a time, possibly is an inverse time. The gravitational interaction is mediated by a scalar field and also the corresponding tensor field of general relativity. Then the scalar field can vary from place to place and in time.

gravitational field with invariant due the gravitational diffeomorphism $T_{\phi}E \rightarrow T_{(\square + m^2)\phi}E$, where E, is the "energy space" which has topological space structure isomorphic to Einstein manifold, is had that:

$$\partial^{\lambda}G_{\mu\nu\lambda}(x) - \partial_{\nu}G^{\lambda}_{\mu\lambda}(x) + m^2(\phi_{\mu\nu}(x) + \frac{1}{2}g_{\mu\nu}\phi(x)) = T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T(x), \tag{14}$$

whose action is constant then the gravitational waves are evidenced from the remote source (figure 10). But if is affected by a dilaton action, the corresponding scalar field action due the electromagnetic action of the dilaton could "dilute the gravity" action trying vacuum in the space-time. Likewise, if we consider one of the Brans-Dicke equations, for example,

$$\square\phi = \frac{8\pi}{3 + 2\omega}T, \tag{15}$$

The equation says that the trace of the stress-energy acts as the source for the scalar field ϕ. But electromagnetic fields contribute only a traceless term to the stress-energy tensor, which implies that in a region of space-time containing only electromagnetic field the right side of (15) vanishes and the curved space-time obeys the wave equation.

But, this electromagnetic wave is propagated infinitely (see figure 4). In such case, we can say that the field is a long-range field.

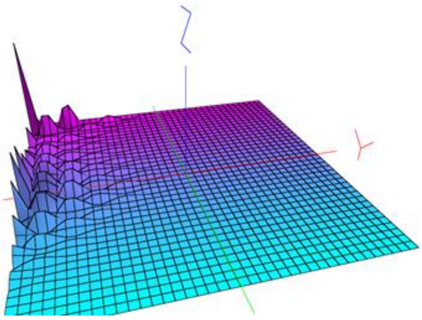


Fig. 4. Variation of the gravitational constant.

Therefore, the moduli space of the electromagnetic dilaton actions on space-time

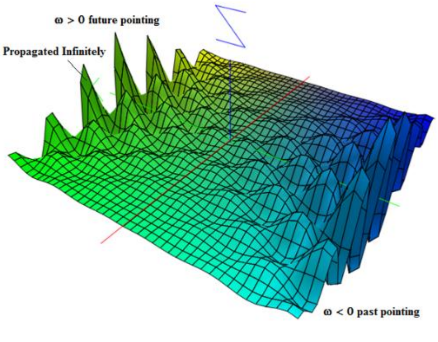


Fig. 5. 2-Dimensional of electromagnetic wave solutions on space-time without matter-energy (long-range field) [16].

In the figure 5, we have electromagnetic waves in conformal actions of the group $SU(2,2)$, on a 2-dimensional flat model of the space-time. The ultra-hyperbolic wave equation is satisfied. In both sides of an axis appear the auto-dual Maxwell fields of positive frequency and negative frequency on $\mathbb{M}$, respectively that go being added in each time to each orbit. This corresponds to partial waves expansions in 2-dimensions.

Example 3. 6. (Gravitational waves as oscillations in the space-time curvature/spin). We consider now $G = GL(1,\mathbb{C}) \cong^{L} G.$ We consider the bundle stack given for[5]

$$\mathbb{M} = \mathrm{Pic}(C), \tag{16}$$

which is known as Picard variety of C. Then the Hecke functor is the mapping

$$\Phi^1 : D_{\mathrm{coh}}(\mathrm{Pic}(C), \mathcal{D}) \to D_{\mathrm{coh}}(C \times \mathrm{Pic}(C), \mathcal{D}), \tag{17}$$

which is pull-back of the Abel-Jacobi mapping:

$$\mathrm{aj} : \mathrm{C} \times \mathrm{Pic}^{d}(C) \to \mathrm{Pic}^{d+1}(C), \tag{18}$$

with correspondence rule

$$(L, x) \mapsto L(x), \tag{19}$$

In this case, the geometrical Langlands correspondence comes give as:

$$\mathfrak{c}(\mathbb{L}) = \left\{ \begin{matrix} \text{The unique translation invariant rank on local systemon variety} \\ \mathrm{Pic}(C), \text{ whose restriction on each component } \mathrm{Pic}^{d}(C), \text{ has the} \\ \text{same monodromy as } \mathbb{L}. \end{matrix} \right\}, \tag{20}$$

where $\mathbb{L}$, is the space of the Langlands data (bundle and connection) (L, ∇), that is a rank one local system on C. Due to that $\pi_1(\mathrm{Pic}^{d}(C))$, is the abelianization of $\pi_1(C)$, and the monodromy of the space $\mathbb{L}$, is Abelian, we can view this space $\mathbb{L}$, as a local system on each component $\mathrm{Pic}^{d}(C)$, of $\mathrm{Pic}(C)$.

Them likewise, considering the pull-back of the local system $\mathbb{L}$, to the various factors of the $d-\mathrm{th}$ Cartesian power $C^{\times d}$, of C, and tensor of these pull-backs to get rank one local system $\mathbb{L}^{\boxtimes d}$, where $\boxtimes$, is a micro-local tensor product.From a point of view of the field equations, each component of the correspondences space $\mathfrak{c}(\mathbb{L})$, on $\mathrm{Pic}^{d}(C)$, a trace of particles in the symplectic geometry that can be characterized in a Hamiltonian manifold, with the due quantization of the coherent sheaves of the differential operators of the field equations.

Likewise, using a Hitchin's abelianization we can induce the geometrical Langlands correspondence $\mathfrak{c}$, as was planted to the case of the group $G = GL(n, \mathbb{C})$, considering the correspondence $\mathfrak{c}$, as:

$$\mathfrak{c} = \mathrm{quant}_{\mathrm{Bun}} \circ \Phi \circ \mathrm{quant}_{C}^{-1}, \tag{21}$$

[5] Here $\mathrm{Pic}(C)$, is the moduli stack Bun.

Where Φ, is the Fourier-Mukai transform defined to this case as:

$$\Phi : D_{\mathrm{coh}}(\mathrm{quant}_{\mathrm{Bun}}, \mathcal{D}) \rightarrow D_{\mathrm{coh}}(C \times \mathrm{quant}_{\mathrm{Bun}}, \mathcal{D}), \tag{22}$$

Here, the quantization procedures $\mathrm{quant}_{\mathrm{Bun}}$, and $\mathrm{quant}_{\mathrm{C}}$, are appropriately understood non-Abelian Hodge correspondences. An adequate Hitchin mapping can give solution to the equations through Hamiltonian states, in the non-Abelian context of the Hodge theory [17] in hypercohomology:

$$d(da) = 0, \tag{23}$$

$\forall a \in \mathbb{C}[Op_{{}^L G}(D)]$, having as integral the integral transforms composition:

$$\mathfrak{c} \circ \Phi^{\mu} =^{L} \Phi^{\mu}, \tag{24}$$

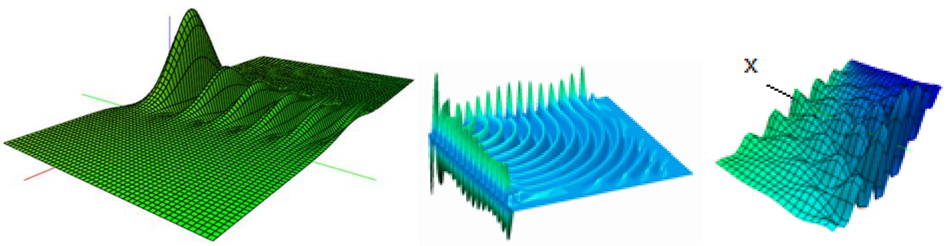
where the states of the quantum field are the cotangent vector (Higgs fields) $\mathbf{h}$, such that

$$\mathrm{Isom}\, d\mathbf{h} = 0, \tag{25}$$

Then by superposing of these states considering the field corresponding ramifications, we have:

$$\mathcal{H} = \mathbf{H^0}(\omega_{\mathrm{C}}^{\otimes 1}) \oplus \mathbf{H^0}(\omega_{\mathrm{C}}^{\otimes 2}) \oplus \ldots \oplus \mathbf{H^0}(\omega_{\mathrm{C}}^{\otimes n}), \tag{26}$$

Which has their re-interpretation as the curvature energy expressed through the H-states which can be written using the superposing principle to each connection $\omega_{\mathrm{C}}^{\otimes j}$, (with C, a curve) that describes the corresponding dilaton (photon). Likewise, in a Hamiltonian densities space [10] we have the figure 6A, considering a Hitchin base.



C)

A) B)

Fig. 6. A). Direct sum of H-states to establish the curvature measure by field ramification. B). The waves that are *spinor waves* which can be consigned in oscillations in the *space-time* in the presence of *curvature* to the change of particles spin. C). Gravitational waves produced by quantum gravity due the H-states on cylindrical surface. Their propagation is realized on axis X. These gravitational waves are originated for the oscillations in the space-time-curvature/spin (that is to say using causal fermions systems).

In the case of spinor representation the corresponding H-states can be given as spinor waves (see figure 6B) which can be consigned in oscillations in the space-time-curvature/spin, to a microscopic deformation measured in $\mathcal{H}$.

Example 3. 7. (Maxwell twistor Framework). We consider the twistor Maxwell theory given by

$$0 \to \mathbb{C} \to \Omega^{\bullet}, \tag{27}$$

The first and last sheaves that are $\mathcal{O}$, and $\mathcal{O}(-4)$, and the cohomology space of type H^1, of these, gives respectively potential modulo gauge for left-handed and right-handed Maxwell fields $H^1(|\mathbb{P}^+|, \Omega^3)$, and $H^1(|\mathbb{P}^-|, \mathcal{O})$. But the constant sheaf $\mathbb{C}$, has relevance in the electromagnetic charge that live in (27), (as has been to some elaborate representation space of the conformal group, as for example $SU(2,2)$). In this sense we can consider the Penrose transform framework to obtain two pieces to $H^1(U'', \Omega^1)$, [18] indeed,

$$H^1(|\mathbb{P}^+|, \Omega^3) = \{\text{gauge - restricted gauge for right - handed potentials on U}\}, \tag{28}$$

and respectively

$$H^1(|\mathbb{P}^-|, \mathcal{O}). = \{\text{left - handed potentials on U}\}, \tag{29}$$

These two pieces are conformal blocks of electromagnetic representations of the space-time [18].

Also the Penrose transform of $H^1(U'', \Omega^2)$, is as (28)is a Verma module of type $H_{\bullet}^{n,n}(M, \mathcal{L}_{\lambda})$, as given in the table 1.

In these identifications, "gauge-restricted" refers to the imposition of the conformally invariant conditions as for example to field equations [10, 18]:

$$\square^2 f = 0, \tag{30}$$

And

$$\square \nabla^{\alpha} \Phi_{\alpha} = 0, \tag{31}$$

(see the figure 5). Then the Penrose transform of the complex (27) contains all the spaces of fields that one is the Maxwell theory table [18], where some are gauge to other fields.

Example 3. 8. (BRST-Cohomology). [**17**] We consider the field equations in BRST-cohomology:

$$\begin{cases} b_0 = \phi a, & \forall a \in \mathcal{O}(-D) \\ b_{1t} dz^t = \partial a, \end{cases} \tag{32}$$

have solutions such that $b_0 \bmod \operatorname{Im}\phi \in H^0(D, \mathcal{O}(D^{\vee}|_D \otimes \mathcal{O}_D)) = \operatorname{Ext}^1(\mathcal{O}_D, \mathcal{O})$, with D, a divisor on the complex line $\mathbb{C}$, that is to say:

$$0 \to \mathcal{O}(-D) \xrightarrow{\phi} \mathcal{O} \to \mathcal{O}_D \to 0, \tag{33}$$

Reciprocally $\mathrm{Ext}^1(\mathcal{O}_D,\mathcal{O}) = H^0(D,\mathcal{O}(D^\vee|_D \otimes \mathcal{O}_D)),$ with the field equations

$$\begin{cases}\bar{\partial}(b_{1r}dz^r)^r = 0,\\ \bar{\partial}b_0 = -\phi(b_1 dz),\end{cases} \tag{34}$$

which have solutions as the extended field $\Im_{BRST} = \bar{\partial} + \sum \phi^{\alpha\beta} = \bar{\partial} + \tilde{\varphi} = \mathrm{Oper}(\Im_{BRST}).$ Here precisely $\Im_{BRST}$, is the solution to the field equation with differential operators in $\mathcal{O}(D^\vee|_D \otimes \mathcal{O}_D)).$

3. Conclusions.

The theorem 2. 1, proposes a classification of the differential operators as points of a complex sheaf of quasi-coherent D-modules. The obtained functors ${}^L\Phi^\mu$, on generalized Verma modules of certain character that can to shape the bridge of the geometrical objects in physical stacks through the corresponding G-invariant vector bundles and the algebraic objects of the corresponding operators algebra, where these operators are the connections of the Langlands data or geometrical ramifications of the differential operators of the equations in field theory. The integral transforms methods permits explore in qualitative way, using cohomology to the algebraic structure of the operators that act in the geometrical stacks given through holomorphic vector bundles of the corresponding sheaves of the differential operators of the field equations. The bosons, fermions, tachyons, etcetera, are ramifications of connections of the field equations with an geometrical interpretation in extended holomorphic bundles and their corresponding context in derived geometry as deformed derived categories. These relations comes given for the geometrical Langlands correspondences.

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