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Expected time to recruitment in a Single Grade Man Power System with two Thresholds

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ABSTRACT

In this paper, a single graded marketing organization which is subject to loss of manpower due to its policy decisions is considered. A mathematical model is constructed using an univariate recruitment policy based on the shock model approach involving two thresholds (one is optional and the another one is mandatory) via order statistics for interdecision time. The performance measures of the time to recruitment are obtained. Also the new findings are verified with numerical illustration.

Keywords: Manpower planning; shock model; univariate recruitment policy; order statistics.

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1. INTRODUCTION

The concept of manpower planning is a very important role in Decision making problems. Many mathematical models have been discussed using different kinds of wastages and different type of distributions (see [1], [2]). Since then several authors [7]-[9] contributed to the development of the problem of time to recruitment in a single graded marketing organization involving only one threshold under different conditions. Since the number of exits in every policy decision-making epoch is unpredictable at the time, at which the cumulative loss of manhour crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon threshold crossing. Recently, Muthaiyan et. al. [6] obtained the mean and variance of time to recruitment when the interdecision time form an order statistics in a single grade manpower system with a mandatory exponential threshold for the loss of manpower. Quite recently Esther Clara [3] introduced and studied the concept of alertness in the recruitment policy which involves two thresholds (one is optional and the other is mandatory) in a single grade manpower system. Recall that, if the cumulative loss of man-hour crosses the optional threshold, the organization may or may not go for recruitment. However, the recruitment is necessary whenever the cumulative loss of man-hour crosses the mandatory threshold. In view of this policy, the organization can plan the decision upon the time for recruitment. This present paper extends the results in [3] for a single grade manpower system for different distributions of thresholds. This paper is organized as follows: In the second section, the model description is given. The mean and variance of the time to recruitment are obtained in the next section. Finally, the analytical results are numerically verified by assuming specific distributions, and the relevant conclusion is stated.

2. MODEL DESCRIPTION AND ANALYSIS

Consider an organization with single grade taking decisions at random epoch in the interval (0, ∞) and at every policy decision epoch a random number of persons quit the organization. There is an associated loss of man-hour if a person quits. It is assumed that the loss of man-hour is linear and cumulative. Let X_i (i = 1, 2, …) be the loss of man-hour due to the i^{th} decision epoch forming a sequence of independent and identically distributed exponential random variables with mean $\frac{1}{c}$ (c > 0) and the corresponding probability density functions are respectively denoted by g(·). If U_i (i = 1, 2, …) is a continuous random variable denoting inter – decision time between $(i-1)^{th}$ and i^{th} decisions with mean $\frac{1}{\lambda}$ $(\lambda > 0)$, then the corresponding cumulative distribution function (resp. the probability density function) is denoted by F(·) (resp. f(·)). Let $U_{(1)}$ (resp. $U_{(k)}$) be the smallest (resp. largest) order statistic with probability density function $f_{u(1)}(\cdot)$ (resp. $f_{u(k)}(\cdot)$). Let Y(resp. Z) be a positive continuous random variable denoting optional (resp. mandatory) thresholds for the loss of man-hour in single grade. If T is the time to recruitment in the organization with cumulative distribution function L(·), then the probability density function (resp. mean, variance) is denoted by l(·), (resp. E(T), V(T)). Let $F_k(\cdot)$ be the k-fold convolution of F(·). Also $l^*(\cdot)$, $f^*(\cdot)$, $f^*_{u(1)}(\cdot)$, $f^*_{u(k)}(\cdot)$ and $g^*(\cdot)$ denote the Laplace transform of l(·), f(·), $f_{u(1)}(\cdot)$, $f_{u(k)}(\cdot)$ and g(·), respectively. Let $V_k(t)$ be the probability that exactly k decision epochs exist in (0, t]. We recall from Renewal theory [5] that, $V_k(t)= F_k(t) - F_{k+1}(t)$ with $F_0(t)=1$. Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hour crosses the optional threshold Y. The univariate recruitment policy employed in this paper is as follows: If the total loss of man-hour exceeds the optional threshold level Y, the organization may or may not go for recruitment. But if the total loss of man-hour exceeds the mandatory threshol*d* Z,the recruitment is must.

Now, we recall some basic results which was used in this paper.

The probability that the threshold level is not reached till t, that is,

$$P(T>t)=\sum_{k=0}^{\infty}V_k(t)P\left(\sum_{i=1}^{k}X_i\le Y\right)+p\sum_{k=0}^{\infty}V_k(t)P\left(Y<\sum_{i=1}^{k}X_i\right)\times P\left(\sum_{i=1}^{k}X_i\le Z\right).$$

Clearly we have $L(t)=1-P(T>t)$ and $l^*(s)=\frac{d}{dt}L(t)$. The random variables $U_1, U_2, U_3,\dots U_k$ be arranged in an increasing order so that we have a sequence of order statistics, namely, $U_{(1)}, U_{(2)}, U_{(3)},\dots U_{(k)}$. Note that the random variables $U_{(1)}, U_{(2)}, U_{(3)},\dots U_{(k)}$ are all not independent. The probability density function of $U_{(r)}$ is given by

$$f_{u(r)}(t)=r\,kC_r\left[F(t)\right]^{r-1}f(t)\left[1-F(t)\right]^{k-r},\quad r=1,2,3,\dots k\,.$$

3. MEAN AND VARIANCE OF TIME TO RECRUITMENT

Suppose the optional and mandatory thresholds follow exponential distribution (called model I). Then we have

$$P(T>t)=1-(1-\bar{g}(\theta_1))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1))^{k-1}+p\left(1-\bar{g}(\theta_1)\bar{g}(\theta_2)\right)\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1)\bar{g}(\theta_2))^{k-1}$$
$$-\,p(1-\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_2))^{k-1}$$

It is clear that

$$l^*(s) = \frac{(1-\bar{g}(\theta_1))f^*(s)}{1-\bar{g}(\theta_1)f^*(s)} + p\frac{(1-\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_2)f^*(s)} - p\frac{(1-\bar{g}(\theta_1)\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_1)\bar{g}(\theta_2)f^*(s)}$$

Now we shall discuss the following two cases:

Case (i): The probability density function $f(t)$ take it as a first order statistics. Then we have $f_{u(1)}(t) = kf(t)(1-F(t))^{k-1}$. Since $f(t) = \lambda e^{-\lambda t}$, $\bar{g}(\theta_1) = \frac{c}{c+\theta_1}$ and $\bar{g}(\theta_2) = \frac{c}{c+\theta_2}$, the probability density function of the first order statistics is given by $f^*_{u(1)}(s) = \frac{k\lambda}{s+k\lambda}$. Clearly we have

$$E(T) = -(\frac{d}{ds}l^*(s))_{s=0} = \frac{1}{k\lambda}\left\{\frac{(c+\theta_1)}{\theta_1} + \frac{p(c+\theta_2)}{\theta_2} - \frac{p(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}\right\} \quad \text{and}$$

$$E(T^2) = (\frac{d^2}{ds^2}l^*(s))_{s=0} = \frac{2}{(k\lambda)^2}\left\{\frac{(c+\theta_1)^2}{{\theta_1}^2} + \frac{p(c+\theta_2)^2}{{\theta_2}^2} - \frac{p(c+\theta_1)^2(c+\theta_2)^2}{(c\theta_1+c\theta_2+\theta_1\theta_2)^2}\right\}.$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case (ii): The probability density function $f(t)$ take it as a k^{th} order statistics. Then we have $f_{u(k)}(t) = kf(t)(1-F(t))^{k-1}$. Since, $f(t) = \lambda e^{-\lambda t}$ $\bar{g}(\theta_1) = \frac{c}{c+\theta_1}$ and $\bar{g}(\theta_2) = \frac{c}{c+\theta_2}$, the probability density function of the k^{th} order statistics is given by $f^*_{u(k)}(s) = \frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)\ldots\ldots\ldots(s+k\lambda)}$.

Clearly we have $E(T) = \frac{N}{\lambda}\left\{\frac{(c+\theta_1)}{\theta_1} + \frac{p(c+\theta_2)}{\theta_2} - \frac{p(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}\right\}$ and

$$E(T^2) = \frac{1}{\lambda^2}\left\{\frac{2(c+\theta_1)^2N^2 - \theta_1(c+\theta_1)[N^2-M]}{{\theta_1}^2} - \frac{p(c+\theta_2)\theta_2[N^2-M]+2p(c+\theta_2)^2N^2}{{\theta_2}^2} + \frac{p(c+\theta_1)(c+\theta_2)(c\theta_1+c\theta_2+\theta_1\theta_2)[N^2-M]-2p(c+\theta_1)^2(c+\theta_2)^2N^2}{(c\theta_1+c\theta_2+\theta_1\theta_2)^2}\right\},$$

where $N = \sum_{n=1}^{k}\frac{1}{n}$ and $M = \sum_{n=1}^{k}\frac{1}{n^2}$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Suppose the distribution of optional threshold is having Setting Clock Back to Zero (SCBZ) property and mandatory threshold follows an exponential distribution (called model II). Then we have

$$P(T>t) = 1 - p(1-\bar{g}(\theta_1+\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1+\theta_2))^{k-1} + p(1-\bar{g}(\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_3))^{k-1}$$
$$-q(1-\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_2))^{k-1} - p^2(1-\bar{g}(\theta_1+\theta_2)\bar{g}(\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1+\theta_2)\bar{g}(\theta_3))^{k-1}$$
$$+pq(1-\bar{g}(\theta_2)\bar{g}(\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_2)\bar{g}(\theta_3))^{k-1}.$$

It is clear that

$$l^*(s) = p\frac{(1-\bar{g}(\theta_1+\theta_2))f^*(s)}{1-\bar{g}(\theta_1+\theta_2)f^*(s)} - p\frac{(1-\bar{g}(\theta_3))f^*(s)}{1-\bar{g}(\theta_3)f^*(s)} + q\frac{(1-\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_2)f^*(s)}$$
$$-pq\frac{(1-\bar{g}(\theta_2)\bar{g}(\theta_3))f^*(s)}{1-\bar{g}(\theta_2)\bar{g}(\theta_3)f^*(s)} + p^2\frac{(1-\bar{g}(\theta_1+\theta_2)\bar{g}(\theta_3))f^*(s)}{1-\bar{g}(\theta_1+\theta_2)\bar{g}(\theta_3)f^*(s)}$$

Case (i): The probability density function $f(t)$ take it as a first order statistics. Then we have $f_{u(1)}(t) = kf(t)(1-F(t))^{k-1}$. Since $f(t) = \lambda e^{-\lambda t}$, $\bar{g}(\theta_1+\theta_2) = \frac{c}{c+\theta_1+\theta_2}$, $\bar{g}(\theta_2) = \frac{c}{c+\theta_2}$ and $\bar{g}(\theta_3) = \frac{c}{c+\theta_3}$, the probability density function of the first order statistics is given by

$$f^*_{u(1)}(s) = \frac{k\lambda}{s+k\lambda}.$$

Clearly we have

$$E(T)=\frac{1}{k\lambda}\left\{\frac{p(c+\theta_1+\theta_2)}{\theta_1+\theta_2}-\frac{p(c+\theta_3)}{\theta_3}+\frac{q(c+\theta_2)}{\theta_2}-\frac{pq\,(c+\theta_2)(c+\theta_3)}{(c\theta_2+c\theta_3+\theta_2\theta_3)}+\frac{p^2(c+\theta_1+\theta_2)(c+\theta_3)}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_3+\theta_2\theta_3)}\right\} \text{ and}$$

$$E(T^2)=\frac{2}{(k\lambda)^2}\left\{\frac{p(c+\theta_1+\theta_2)^2}{(\theta_1+\theta_2)^2}-\frac{p(c+\theta_3)^2}{{\theta_3}^2}+\frac{q(c+\theta_2)^2}{{\theta_2}^2}-\frac{pq\,(c+\theta_2)^2(c+\theta_3)^2}{(c\theta_2+c\theta_3+\theta_2\theta_3)^2}+\frac{p^2(c+\theta_1+\theta_2)^2(c+\theta_3)^2}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_3+\theta_2\theta_3)^2}\right\}.$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case (ii): The probability density function f(t) take it as a kth order statistics. Then we have $f_{u(k)}(t)=kf(t)(F(t))^{k-1}$.

Since $f(t)=\lambda e^{-\lambda t}$, $\bar{g}(\theta_1+\theta_2)=\frac{c}{c+\theta_1+\theta_2}$, $\bar{g}(\theta_2)=\frac{c}{c+\theta_2}$ and $\bar{g}(\theta_3)=\frac{c}{c+\theta_3}$, the probability density function of the kth order statistics is given by $f^*_{u(k)}(s)=\frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)\ldots\ldots\ldots\ldots(s+k\lambda)}$. Clearly we have

$$E(T)=\frac{N}{\lambda}\left\{\frac{p(c+\theta_1+\theta_2)}{\theta_1+\theta_2}-\frac{p(c+\theta_3)}{\theta_3}+\frac{q(c+\theta_2)}{\theta_2}-\frac{pq\,(c+\theta_2)(c+\theta_3)}{(c\theta_2+c\theta_3+\theta_2\theta_3)}+\frac{p^2(c+\theta_1+\theta_2)(c+\theta_3)}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_3+\theta_2\theta_3)}\right\} \text{ and}$$

$$E(T^2)=\frac{N^2-M}{\lambda^2}\left\{\frac{-p(c+\theta_1+\theta_2)}{\theta_1+\theta_2}+\frac{p(c+\theta_3)}{\theta_3}-\frac{q(c+\theta_2)}{\theta_2}+\frac{pq\,(c+\theta_2)(c+\theta_3)}{(c\theta_2+c\theta_3+\theta_2\theta_3)}-\frac{p^2(c+\theta_1+\theta_2)(c+\theta_3)}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_3+\theta_2\theta_3)}\right\}+$$
$$\frac{2N^2}{\lambda^2}\left\{\frac{p(c+\theta_1+\theta_2)^2}{(\theta_1+\theta_2)^2}-\frac{p(c+\theta_3)^2}{{\theta_3}^2}+\frac{q(c+\theta_2)^2}{{\theta_2}^2}-\frac{pq\,(c+\theta_2)^2(c+\theta_3)^2}{(c\theta_2+c\theta_3+\theta_2\theta_3)^2}+\frac{p^2(c+\theta_1+\theta_2)^2(c+\theta_3)^2}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_3+\theta_2\theta_3)^2}\right\},$$

where $N=\sum_{n=1}^{k}\frac{1}{n}$ and $M=\sum_{n=1}^{k}\frac{1}{n^2}$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

The Optional threshold follows an exponential distribution and the distribution of mandatory thresholds has SCBZ property (called Model III). Then we have

$$P(T>t)=1-(1-\bar{g}(\theta_1))\sum_{k=0}^{\infty}F_k(t)(\bar{g}(\theta_1))^{k-1}+p^2\{(1-\bar{g}(\theta_2+\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_2+\theta_3))^{k-1}$$
$$-(1-\bar{g}(\theta_1)\bar{g}(\theta_2+\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1)\bar{g}(\theta_2+\theta_3))^{k-1}\}$$
$$-pq\{(1-\bar{g}(\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_3))^{k-1}+pq(1-\bar{g}(\theta_1)\bar{g}(\theta_3))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1)\bar{g}(\theta_3))^{k-1}\}.$$

It is clear that

$$l^*(s)=\frac{(1-\bar{g}(\theta_1))f^*(s)}{1-\bar{g}(\theta_1)f^*(s)}+p^2\frac{(1-\bar{g}(\theta_2+\theta_3))f^*(s)}{1-\bar{g}(\theta_2+\theta_3)f^*(s)}-p^2\frac{(1-\bar{g}(\theta_1)\bar{g}(\theta_2+\theta_3))f^*(s)}{1-\bar{g}(\theta_1)\bar{g}(\theta_2+\theta_3)f^*(s)}$$
$$-pq\frac{(1-\bar{g}(\theta_3))f^*(s)}{1-\bar{g}(\theta_3)f^*(s)}-pq\frac{(1-\bar{g}(\theta_1)\bar{g}(\theta_3))f^*(s)}{1-\bar{g}(\theta_1)\bar{g}(\theta_3)f^*(s)}.$$

Case (i): The probability density function f (t) take it as a first order statistics. Then we have $f_{u(1)}(t)=kf(t)(1-F(t))^{k-1}$. Since $f(t)=\lambda e^{-\lambda t}$, $\bar{g}(\theta_1)=\frac{c}{c+\theta_1}$, $\bar{g}(\theta_2+\theta_3)=\frac{c}{c+\theta_2+\theta_3}$ and $\bar{g}(\theta_3)=\frac{c}{c+\theta_3}$, the probability density function of the first order statistics is given by $f^*_{u(1)}(s)=\frac{k\lambda}{s+k\lambda}$.

Clearly we have $E(T)=\frac{1}{k\lambda}\left\{\frac{(c+\theta_1)}{\theta_1}+\frac{p^2(c+\theta_2+\theta_3)}{\theta_2+\theta_3}-\frac{p^2(c+\theta_1)(c+\theta_2+\theta_3)}{(c+\theta_1)(\theta_2+\theta_3)+c\theta_1}+\frac{pq\,(c+\theta_3)}{\theta_3}-\frac{pq\,(c+\theta_1)(c+\theta_3)}{(c+\theta_1)\theta_3+c\theta_1}\right\}$ and

$$E(T^2)=\frac{2}{(k\lambda)^2}\left\{\frac{(c+\theta_1)^2}{{\theta_1}^2}+\frac{p^2(c+\theta_2+\theta_3)^2}{(\theta_2+\theta_3)^2}-\frac{p^2(c+\theta_1)^2(c+\theta_2+\theta_3)^2}{((c+\theta_1)(\theta_2+\theta_3)+c\theta_1)^2}+\frac{pq\,(c+\theta_3)^2}{{\theta_3}^2}-\frac{pq\,(c+\theta_1)^2(c+\theta_3)^2}{((c+\theta_1)\theta_3+c\theta_1)^2}\right\}.$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case (ii): The probability density function f(t) take it as a k th order statistics. Then we have $f_{u(k)}(t)=kf(t)(F(t))^{k-1}$. Since $f(t)=\lambda e^{-\lambda t}$, $\bar{g}(\theta_1)=\frac{c}{c+\theta_1}$, $\bar{g}(\theta_2+\theta_3)=\frac{c}{c+\theta_2+\theta_3}$ and $\bar{g}(\theta_3)=\frac{c}{c+\theta_3}$, the probability density function of the k th order statistics is given by

$f^*_{u(k)}(s)=\frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)\ldots\ldots\ldots\ldots(s+k\lambda)}$. Clearly we have

$$E(T)=\frac{N}{\lambda}\left\{\frac{(c+\theta_1)}{\theta_1}+\frac{p^2(c+\theta_2+\theta_3)}{(\theta_2+\theta_3)}-\frac{p^2(c+\theta_1)(c+\theta_2+\theta_3)}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_2+\theta_1\theta_3)}+\frac{pq(c+\theta_3)}{\theta_3}-\frac{pq(c+\theta_1)(c+\theta_3)}{(c\theta_1+c\theta_3+\theta_1\theta_3)}\right\}$$ and

$$E(T^2)=\frac{1}{\lambda^2}\left\{\begin{array}{c}\frac{-\theta_1(c+\theta_1)(N^2-M)+2(c+\theta_1)^2N^2}{{\theta_1}^2}-\frac{p^2[(\theta_2+\theta_3)(c+\theta_2+\theta_3)(N^2-M)-2(c+\theta_2+\theta_3)^2N^2]}{(\theta_2+\theta_3)^2}\\ +\frac{p^2[(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_2+\theta_1\theta_3)(c+\theta_1)(c+\theta_2+\theta_3)(N^2-M)-2(c+\theta_1)^2(c+\theta_2+\theta_3)^2N^2]}{(c\theta_1+c\theta_2+c\theta_3+\theta_1\theta_2+\theta_1\theta_3)^2}\\ +\frac{pq[(\theta_3(c+\theta_3)(N^2-M)-2(c+\theta_3)^2N^2]}{{\theta_3}^2}+\frac{pq[(c\theta_1+c\theta_3+\theta_1\theta_3)(c+\theta_1)(c+\theta_3)(N^2-M)-2(c+\theta_1)^2(c+\theta_3)^2N^2]}{(c\theta_1+c\theta_2+\theta_1\theta_3)^2}\end{array}\right\},$$

where $N=\sum_{n=1}^{k}\frac{1}{n}$ *and* $M=\sum_{n=1}^{k}\frac{1}{n^2}$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

The distribution of optional threshold having exponential distribution and mandatory thresholds follows extended exponential distribution (called Model IV). Then we have

$$P(T>t)=1-(1-\bar{g}(\theta_1))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1))^{k-1}+p\{(1-\bar{g}(\theta_1)\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1)\bar{g}(\theta_2))^{k-1}$$
$$-(1-\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_2))^{k-1}+(1-\bar{g}(2\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(2\theta_2))^{k-1}$$
$$-(1-\bar{g}(\theta_1)\bar{g}(2\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1)\bar{g}(2\theta_2))^{k-1}\}.$$

It is clear that

$$l^*(s)=\frac{(1-\bar{g}(\theta_1))f^*(s)}{1-\bar{g}(\theta_1)f^*(s)}-p\left\{\frac{(1-\bar{g}(\theta_1)\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_1)\bar{g}(\theta_2)f^*(s)}-\frac{(1-\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_2)f^*(s)}+\frac{(1-\bar{g}(2\theta_2))f^*(s)}{1-\bar{g}(2\theta_2)f^*(s)}-\frac{(1-\bar{g}(\theta_1)\bar{g}(2\theta_2))f^*(s)}{1-\bar{g}(\theta_1)\bar{g}(2\theta_2)f^*(s)}\right\}.$$

Case (i): The probability density function f (t) take it as a first order statistics. Then we have $f_{u(1)}(t)=kf(t)(1-F(t))^{k-1}$. Since $f(t)=\lambda e,^{-\lambda t}$ $\bar{g}(\theta_1)=\frac{c}{c+\theta_1}$, $\bar{g}(\theta_2)=\frac{c}{c+\theta_2}$ and $\bar{g}(2\theta_2)=\frac{c}{c+2\theta_2}$ the probability density function of the first order statistics is given by $f^*_{u(1)}(s)=\frac{k\lambda}{s+k\lambda}$

Clearly we have $E(T)=\frac{1}{k\lambda}\left\{\frac{(c+\theta_1)}{\theta_1}-p\left(\frac{(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}-\frac{(c+\theta_2)}{\theta_2}+\frac{(c+2\theta_2)}{2\theta_2}-\frac{(c+\theta_1)(c+2\theta_2)}{(c\theta_1+2c\theta_2+2\theta_1\theta_2)}\right)\right\}$ and

$$E(T^2)=\frac{2}{(k\lambda)^2}\left\{\frac{(c+\theta_1)^2}{2{\theta_1}^2}-p\left(\frac{(c+\theta_1)^2(c+\theta_2)^2}{(c\theta_1+c\theta_2+\theta_1\theta_2)^2}-\frac{(c+\theta_2)^2}{{\theta_2}^2}+\frac{(c+2\theta_2)^2}{4{\theta_2}^2}-\frac{(c+\theta_1)^2(c+2\theta_2)^2}{(c\theta_1+2c\theta_2+2\theta_1\theta_2)^2}\right)\right\}$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case (ii): The probability density function f(t) take it as a k th order statistics. Then we have $f_{u(k)}(t)=kf(t)(F(t))^{k-1}$. Since $f(t)=\lambda e^{-\lambda t}$, $\bar{g}(\theta_1)=\frac{c}{c+\theta_1}$, $\bar{g}(\theta_2)=\frac{c}{c+\theta_2}$ and $\bar{g}(2\theta_2)=\frac{c}{c+2\theta_2}$, the probability density function of the k th order statistics is given by

$f^*_{u(k)}(s)=\frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)\ldots\ldots\ldots\ldots(s+k\lambda)}$. Clearly we have

$$E(T)=\frac{N}{\lambda}\left\{\frac{(c+\theta_1)}{\theta_1}-p\left(\frac{(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}-\frac{(c+\theta_2)}{\theta_2}+\frac{(c+2\theta_2)}{2\theta_2}-\frac{(c+\theta_1)(c+2\theta_2)}{(c\theta_1+2c\theta_2+2\theta_1\theta_2)}\right)\right\}$$

and

$$E(T^2)=\frac{N^2-M}{\lambda^2}\left\{\frac{-(c+\theta_1)}{\theta_1}+p\left(\frac{(c+\theta_1)(c+\theta_2)}{c\theta_1+c\theta_2+\theta_1\theta_2}-\frac{(c+\theta_2)}{\theta_2}+\frac{(c+2\theta_2)}{2\theta_2}-\frac{(c+\theta_1)(c+2\theta_2)}{(c\theta_1+2c\theta_2+2\theta_1\theta_2)}\right)\right\}+$$
$$\frac{2N^2}{\lambda^2}\left\{\frac{(c+\theta_1)^2}{{\theta_1}^2}-p\left(\frac{(c+\theta_1)^2(c+\theta_2)^2}{(c\theta_1+c\theta_2+\theta_1\theta_2)^2}-\frac{(c+\theta_2)^2}{{\theta_2}^2}+\frac{(c+2\theta_2)^2}{4{\theta_2}^2}-\frac{(c+\theta_1)^2(c+2\theta_2)^2}{(c\theta_1+2c\theta_2+2\theta_1\theta_2)^2}\right)\right\},$$

where $N=\sum_{n=1}^{k}\frac{1}{n}$ and $M=\sum_{n=1}^{k}\frac{1}{n^2}$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

The distribution of optional threshold having extended exponential distribution and mandatory thresholds follows exponential distribution (called Model V). Then we have

$$P(T>t)=1+(1-\bar{g}(2\theta_1))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(2\theta_1))^{k-1}-2(1-\bar{g}(\theta_1))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1))^{k-1}$$
$$+p\left\{\begin{array}{l}2(1-\bar{g}(\theta_1)\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_1)\bar{g}(\theta_2))^{k-1}-(1-\bar{g}(2\theta_1)\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(2\theta_1)\bar{g}(\theta_2))^{k-1}\\-(1-\bar{g}(\theta_2))\sum_{k=1}^{\infty}F_k(t)(\bar{g}(\theta_2))^{k-1}\end{array}\right\}$$

It is clear that

$$l^*(s)=\frac{2(1-\bar{g}(\theta_1))f^*(s)}{1-\bar{g}(\theta_1)f^*(s)}-\frac{(1-\bar{g}(2\theta_1))f^*(s)}{1-\bar{g}(2\theta_1)f^*(s)}-p\left\{\frac{2(1-\bar{g}(\theta_1)\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_1)\bar{g}(\theta_2)f^*(s)}-\frac{(1-\bar{g}(2\theta_1)\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(2\theta_1)\bar{g}(\theta_2)f^*(s)}-\frac{(1-\bar{g}(\theta_2))f^*(s)}{1-\bar{g}(\theta_2)f^*(s)}\right\}.$$

Case (i): The probability density function **f** (t) take it as a first order statistics. Then we have $f_{u(1)}(t)=kf(t)(1-F(t))^{k-1}$. Since $f(t)=\lambda e^{-\lambda t}$, $\bar{g}(\theta_1)=\frac{c}{c+\theta_1}$, $\bar{g}(2\theta_1)=\frac{c}{c+2\theta_1}$ and $\bar{g}(\theta_2)=\frac{c}{c+\theta_2}$, the probability density function of the first order statistics is given by

$f^*_{u(1)}(s)=\frac{k\lambda}{s+k\lambda}$. Clearly we have

$$E(T)=\frac{1}{k\lambda}\left\{\frac{2(c+\theta_1)}{\theta_1}-\frac{(c+2\theta_1)}{2\theta_1}+p\left(\frac{(c+\theta_2)}{\theta_2}-\frac{2(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}+\frac{(c+2\theta_1)(c+\theta_2)}{(2c\theta_1+c\theta_2+2\theta_1\theta_2)}\right)\right\}$$

and

$$E(T^2)=\frac{2}{(k\lambda)^2}\left\{\frac{2(c+\theta_1)^2}{{\theta_1}^2}-\frac{(c+2\theta_1)^2}{4{\theta_1}^2}-p\left(\frac{2(c+\theta_1)^2(c+\theta_2)^2}{(c\theta_1+c\theta_2+\theta_1\theta_2)^2}-\frac{(c+2\theta_1)^2(c+\theta_2)^2}{(2c\theta_1+c\theta_2+2\theta_1\theta_2)^2}-\frac{(c+\theta_2)^2}{{\theta_2}^2}\right)\right\}.$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case (ii): The probability density function **f**(t) take it as a k[th] order statistics. Then we have $f_{u(1)}(t)=kf(t)(1-F(t))^{k-1}$. Since $f(t)=\lambda e,^{-\lambda t}$ $\bar{g}(\theta_1)=\frac{c}{c+\theta_1}$, $\bar{g}(2\theta_1)=\frac{c}{c+2\theta_1}$ and $\bar{g}(\theta_2)=\frac{c}{c+\theta_2}$, the probability density function of the first order statistics is given by $f^*_{u(k)}(s)=\frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)\ldots\ldots\ldots\ldots(s+k\lambda)}$.

Clearly we have

$$E(T)=\frac{N}{\lambda}\left\{\frac{2(c+\theta_1)}{\theta_1}-\frac{(c+2\theta_1)}{2\theta_1}+p\left(\frac{(c+\theta_2)}{\theta_2}-\frac{2(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}+\frac{(c+2\theta_1)(c+\theta_2)}{(2c\theta_1+c\theta_2+2\theta_1\theta_2)}\right)\right\}\text{ and}$$
$$E(T^2)=\frac{N^2-M}{\lambda^2}\left\{\frac{-2(c+\theta_1)}{\theta_1}+\frac{(c+2\theta_1)}{2\theta_1}-p\left(\frac{-2(c+\theta_1)(c+\theta_2)}{(c\theta_1+c\theta_2+\theta_1\theta_2)}+\frac{(c+2\theta_1)(c+\theta_2)}{(2c\theta_1+c\theta_2+2\theta_1\theta_2)}+\frac{(c+\theta_2)}{\theta_2}\right)\right\}+$$
$$\frac{N^2}{\lambda^2}\left\{\frac{4(c+\theta_1)^2}{{\theta_1}^2}-\frac{(c+2\theta_1)^2}{4{\theta_1}^2}-p\left(\frac{4(c+\theta_1)^2(c+\theta_2)^2}{(c\theta_1+c\theta_2+\theta_1\theta_2)^2}-\frac{2(c+2\theta_1)^2(c+\theta_2)^2}{(2c\theta_1+c\theta_2+2\theta_1\theta_2)^2}-\frac{2(c+\theta_2)^2}{{\theta_2}^2}\right)\right\},$$

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where $N = \sum_{n=1}^{k} \frac{1}{n}$ and $M = \sum_{n=1}^{k} \frac{1}{n^2}$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

The following tables give the mean and variance of the time to recruitment for all the models discussed in this paper by keeping $\theta_1 = 0.1$, $\theta_2 = 0.2$, $\theta_3 = 0.3$, $p = 0.2$ and $q = 0.4$ are fixed and varying k, c, λ one at a time.

Table 1: Effect of k, c, λ on the performance measures E(T) and V(T).

			Case(i)		Case(ii)	
k	c	λ	E(T)	V(T)	E(T)	V(T)
1	0.5	1	6.20589	35.94540	6.20589	35.94540
2	0.5	1	3.10294	8.98638	9.30883	74.67138
3	0.5	1	2.06863	3.99392	11.37746	108.40466
4	0.5	1	1.55147	2.24659	12.92893	137.91219
5	0.5	1	1.24117	1.43782	14.17011	164.13336
2	0.5	1	3.10294	8.98638	9.30883	74.67138
2	1	1	5.6875	30.05078	17.0625	259.08203
2	1.5	1	8.27128	63.46171	24.81384	554.61292
2	2	1	10.85484	109.21979	32.56452	961.26848
2	2.5	1	13.43831	167.32506	40.31494	1479.04819
2	0.5	1	3.10294	8.98638	9.30883	74.67138
2	0.5	2	1.55147	2.24659	4.65441	18.66788
2	0.5	3	1.03431	0.99849	3.10294	8.29683
2	0.5	4	0.77573	0.56165	2.32720	4.66699
2	0.5	5	0.62058	0.35946	1.86176	2.98686

Table 2: Effect of k, c, λ on the performance measures E(T) and V(T)

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			Case(i)		Case(ii)	
k	c	λ	E(T)	V(T)	E(T)	V(T)
1	0.5	1	1.32113	7.74805	1.32113	7.74805
2	0.5	1	0.66056	1.93702	1.98169	16.11201
3	0.5	1	0.44037	0.86089	2.42207	23.39981
4	0.5	1	0.33028	0.48435	2.75235	29.777543
5	0.5	1	0.26422	0.30991	3.01658	35.44111
2	0.5	1	0.66056	1.93702	1.98169	16.11201
2	1	1	1.13756	5.71552	3.41268	49.16466
2	1.5	1	1.61434	11.48725	4.84302	100.15666
2	2	1	2.09107	19.25226	6.27322	169.08816
2	2.5	1	2.56778	29.01058	7.70334	255.95975
2	0.5	1	0.66056	1.93702	1.98169	16.11201
2	0.5	2	0.33028	0.48435	0.99084	4.02800
2	0.5	3	0.22018	0.21522	0.66056	1.79022
2	0.5	4	0.16514	0.12105	0.49542	1.00699
2	0.5	5	0.13211	0.07747	0.39633	0.64448

Table 3: Effect of k, c, λ on the performance measures E(T) and V(T).

			Case(i)		Case(ii)	
k	c	λ	E(T)	V(T)	E(T)	V(T)
1	0.5	1	6.05780	35.82893	6.05780	35.82894
2	0.5	1	3.0289	8.95723	9.0867	74.55729
3	0.5	1	2.01926	3.98101	11.10596	108.30943
4	0.5	1	1.51445	2.23930	12.62041	137.83899
5	0.5	1	1.21156	1.43315	13.83197	164.08143
2	0.5	1	3.0289	8.95723	9.0867	74.55729
2	1	1	5.54954	30.06246	16.64863	259.46280
2	1.5	1	8.06978	63.57581	24.97294	518.48715
2	2	1	10.58989	109.49797	31.76968	964.30142
2	2.5	1	13.10997	167.82841	39.32991	1484.23472
2	0.5	1	3.0289	8.95723	9.0867	74.55729
2	0.5	2	1.51445	2.23930	4.54335	18.63932
2	0.5	3	1.00963	0.99524	3.0289	8.28413
2	0.5	4	0.75722	0.55982	2.27167	4.65984

2	0.5	5	0.60578	0.35828	1.81734	2.98228

Table 4: Effect of k, c, λ on the performance measures E(T) and V(T).

			Case(i)		Case(ii)	
k	c	λ	E(T)	V(T)	E(T)	V(T)
1	0.5	1	6.12829	0.26445	6.12829	36.76529
2	0.5	1	3.06414	0.06613	9.19243	76.59370
3	0.5	1	2.04276	0.02939	11.23517	111.31528
4	0.5	1	1.53207	0.01653	12.76725	141.69787
5	0.5	1	1.22565	0.01059	13.99290	168.69859
2	0.5	1	3.06414	0.06613	9.19243	76.59370
2	1	1	5.62268	0.12220	16.86805	266.64555
2	1.5	1	8.18108	0.17026	24.54325	562.88761
2	2	1	10.73945	0.21007	32.21835	974.72276
2	2.5	1	13.29780	0.24186	39.89341	1498.99103
2	0.5	1	3.06414	0.06613	9.19243	76.59370
2	0.5	2	1.53207	0.01653	4.59621	19.14846
2	0.5	3	1.02138	0.00734	3.06414	8.51042
2	0.5	4	0.76603	0.00413	2.29810	4.78713
2	0.5	5	0.61282	0.00265	1.83848	3.06376

Table 5: Effect of k, c, λ on the performance measures E(T) and V(T).

			Case(i)		Case(ii)	
k	c	λ	E(T)	V(T)	E(T)	V(T)
1	0.5	1	8.62009	46.87834	8.62009	46.87835
2	0.5	1	4.31004	11.71962	12.93013	95.10634
3	0.5	1	2.87336	5.20872	15.80346	136.82351
4	0.5	1	2.15502	2.92989	17.95849	173.21836
5	0.5	1	1.72401	1.87515	19.68251	205.51789
2	0.5	1	4.31004	11.71962	12.93013	95.10634
2	1	1	8.10227	38.62116	24.30681	248.28597
2	1.5	1	11.89411	80.97081	35.68234	700.69842
2	2	1	15.68586	138.76843	47.05759	1212.04331
2	2.5	1	19.47758	212.01382	58.43275	1862.41818
2	0.5	1	4.31004	11.71962	12.93013	95.10634
2	0.5	2	2.15502	2.92989	6.46506	23.77664
2	0.5	3	1.43668	1.30218	4.31004	10.56739
2	0.5	4	1.07751	0.73247	3.23253	5.94415
2	0.5	5	0.86200	0.46879	2.58602	3.80428

FINDINGS

Influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment for all the models discussed in this paper are reported below for case(i) and case (ii) separately.

Case(i):

1. It is observed that if k, the number of decision epochs in (0, t] increases, the mean and variance of time to recruitment for all the models decreases.

2. If c increases, the average loss of man-hour decreases which, in turn, post pones the time to recruitment on the average and hence mean and variance of the time to recruitment increases.

3. As λ increases, the average interdecision time decreases which in turn shows that frequent decisions are made on the average and hence mean and variance of the time to recruitment decreases.

Case(ii):

1. It is observed that if k, the number of decision epochs in (0, t] increases, the mean and variance of time to recruitment for all the models increases.

2. Increase in parameter c, increases the mean and variance of time to recruitment.

3. As λ increases, the average interdecision time decreases which in turn show that frequent decisions are made on the average and hence mean and variance of the time to recruitment decreases.

Conclusion: From the above examples we conclude that the performance measures are fully depends on the values of k, c, λ. Moreover, the results show that the k^{th} order statistics is preferable.

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