ISSN 2347-1921

Reacting Laminar flow with Applied Magnetic Field in a Channel filled with Saturated Porous Media

Amos W.Ogunsola
Dept. of Pure and Applied Mathematics,Ladoke Akintola University of Technology, Ogbomoso, Nigeria,
amossolawale@yahoo.com
Benjamin A.Peter
Dept. of Pure and Applied Mathematics,Ladoke Akintola University of Technology, Ogbomoso, Nigeria,
pbenjamin2008@yahoo.ca

Abstract

In this work, we examined reacting laminar flow of a third grade fluid in a channel filled with saturated porous media under the effect of applied magnetic field and variable thermal conductivity. It is assumed that the fluid has temperature-dependent viscosity and reacts satisfying Arrhenius law. We employed Galerkin weighted residual method to solve the resulting non-linear equations. The results show the effects of variable viscosity parameter, Brinkman number, Reynolds number, Prandtl number, Darcy number, Hartmann number and $Frank - Kamenetskii$ parameter on the flow system.

Keywords: Non-Newtonian fluid; weighted residual method; Laminar flow; Magnetic field and Arrhenius reaction.

Council for Innovative Research
Peer Review Research Publishing System

Journal: Journal of Advances in Mathematics

Vol 8, No.1

editor@cirjam.com

www.cirworld.org, www.cirworld.com

INTRODUCTION

Flow of reactive fluids in porous media not only presents a theoretically challenging problem but also has a wide range of scientific, technological and engineering applications. This type of flow system can be found in, among others, packed bed chemical reactors, geothermal energy reservoirs, petroleum reservoirs, material processing industries, automobile exhaust systems, drying of food, waste disposal systems, insulation of buildings, groundwater movement, oil and gas production, to mention but just a few applications.

Heat transfer problem of Laminar flow and third grade fluids without heat source has been studied by several authors: Hayat et al [1] considered partial slip effect on the flow and heat transfer characteristics in a third grade fluid. Fosdick and Rajagopal [2] performed a complete thermodynamic analysis of constitutive equations for the third grade fluid involving heat transfer process. Massoudi and Christie [3] analyzed numerically the flow of a third grade fluid in a pipe without heat source where the shear viscosity was assumed to be temperature dependent. Olajuwon [4] examined the flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation.

Jayeoba and Okoya [5] employed analytical approximation to determine the velocity and temperature fields for steady flow of a third grade fluid in a pipe. Rilvin and Ericksen [6] analyzed stress deformation relation for isotropic materials. Motivated by the work of Szeri and Rajagopal [7] which examined the effects of variable viscosity parameter and viscous dissipation parameter on the flow of a Non-Newtonian fluid between heated parallel plates. Their results show that the temperature and velocity distribution remain sensibly invariant with respect to the variable viscosity parameter. Haroon et al [8] examined analysis of poiseuille flow of a reactive power-law fluid between parallel plates. The results show that the shear thinning/thickening behavior depends on the power-law index and the pressure gradient.

Motivated by the work of Lazarus [9] which studied the effects of variable viscosity on the velocity fluid and temperature fluid using semi-implicit finite difference scheme of unsteady Laminar flow in a channel filled with saturated porous media. The results show that the velocity fluid and temperature fluid increases as variable viscosity parameter increases. In this work, we considered a fully developed, steady and reacting flow of an incompressible fluid.

GOVERNING EQUATIONS AND METHOD OF SOLUTION

The basic governing equations are the conservation of mass, conservation of momentum and conservation of energy for an incompressible fluid.

Following [9] the momentum and energy equation are modified as follows:

$$div\, u = 0, \tag{1}$$

$$div\, T + \rho b = \rho \frac{dv}{dt} \tag{2}$$

Following Szeri and Rajagopal [7] an incompressible, homogeneous fluid of third grade is characterized by Cauchy stress τ of the following form:

$$\begin{aligned} \tau = &- pI + \mu(T)A_1 + \alpha_1(T)A_2 + \alpha_2(T)A_1^2 \\ &+ \beta_1(T)A_3 + \beta_2(T)[A_1A_2 + A_2A_1] + \beta_3(T)(trA_1^2)A_1. \end{aligned} \tag{3}$$

where $-pI$ denote the indeterminate part of the stress due to the constraint of incompressibility $\mu(T)$ is the coefficient of viscosity and $\alpha_1(T), \alpha_2(T)$ are material moduli, usually referred to as normal stress coefficients. The kinematic tensors A_1 , A_2 are defined by [7] through

$$A_1 = (grad\, v) + (grad\, v)^{\tau} \tag{4}$$

$$A_n = \frac{d}{dt}A_{n-1} + A_{n-1}(grad\, v) + (grad\, v)^T A_{n-1} \qquad n = 2,3. \tag{5}$$

Here $\frac{d}{dt}$ denotes material time derivative and v is the velocity vector. The above model contains, as a special subclass, the classical linearly viscous model (the case when all the coefficients expect μ are set equal to zero).

$$\rho \frac{\partial u}{\partial \bar{t}} = -\frac{\partial p}{\partial \bar{x}} + \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}(T)\frac{\partial u}{\partial \bar{y}}\right) + \left(6\beta_3\left(\frac{\partial u}{\partial \bar{y}}\right)^2 \frac{\partial^2 u}{\partial \bar{y}^2}\right) + \alpha_1 \frac{\partial^3 u}{\partial \bar{y}^2 \partial \bar{t}} - \frac{\overline{\mu_{ef}}(T)u}{\rho K} - \sigma B_0^2 u \tag{6}$$

$$\rho cp\frac{\partial T}{\partial t}=k\frac{\partial}{\partial \bar{y}}\left(\frac{\partial T}{\partial \bar{y}}\right)+\left(\frac{\partial u}{\partial \bar{y}}\right)^2\left(\bar{\mu}(T)+2\beta_3\left(\frac{\partial u}{\partial \bar{y}}\right)^2\right)+Q_c+\frac{\overline{\mu_{ef}}(T)u^2}{K}+\sigma B_0^2u^2 \tag{7}$$

$$Q_c=QC_0K_0(T),K_0(T)=J\left(\frac{hT}{vl}\right)^m\exp\left(-\frac{E}{RT}\right) \tag{8}$$

Together with the initial and boundary conditions

$$u(\bar{y},0)=0,T(\bar{y},0)=T_0 \tag{9}$$

$$u(0,\bar{t})=0,\ -k\frac{\partial T}{\partial \bar{y}}(0,\bar{t})=h_1\left[T_a-T(0,\bar{t})\right] \tag{10}$$

$$u(a,\bar{t})=0,\ -k\frac{\partial T}{\partial \bar{y}}(0,\bar{t})=h_2\left[T(a,\bar{t})-T_a\right] \tag{11}$$

where Q is the heat release per units mass, E is the activating energy, R is the universal gas constant, θ -is the dimensionless temperature, K is the permeability of the porous media, k is the thermal conductivity, ρ is the density, C_p is the specific heat at constant pressure, μ is the dynamic viscosity, $\mu\left(\frac{\partial u}{\partial r}\right)^2$ is the viscous heating effect, direction, μ_{ef} is the effective viscosity, m is the dimensionless numerical exponent, e^T is the thermal expansion, T_0 is the fluid initial temperature, T_a is the ambient temperature, T is the absolute temperature within the boundary layer, $T_1,\ T_2,\ldots..T_\infty$ - Temperature at the plate, h is the Boltzmann's constant, C_0 is the initial concentration of the reactant species, a is the channel width, l is the Plank's number, h_1 is the heat transfer coefficient at lower plate, h_2 is the heat transfer coefficient at the upper plate and α_1 & β_3 are the material coefficients.

From equation (25) we seek variable thermal conductivity $k(T)$ of the form

$$k(T)=k_0e^{-\gamma\theta} \tag{12}$$

we introduce the following dimensionless variables and parameters

$$\varepsilon=\frac{RT_0}{E},\theta=\frac{E(T-T_0)}{R\theta_0^{\ 2}},T=\frac{\theta RT_0^{\ 2}}{E}+T_0,T-T_0=\varepsilon T_0\theta,T=\varepsilon T_0\theta+T_0=T_0(\varepsilon\theta+1)$$

$$\frac{\bar{y}}{a}=y,\frac{\bar{u}}{u_0}=u,Da=\frac{K}{a^2},Bi_2=\frac{h_2a}{k},Ha^2=\frac{\sigma B_0^2a^2}{\mu_0},\theta_a=\frac{E(T-T_0)}{RT_0^2},\Pr=\frac{\mu_0c_p}{k},$$

$$t=\frac{\bar{t}\mu_0}{\rho a^2},\ \operatorname{Re}=\frac{\mu_0u_0}{\rho a^2},\bar{\mu}=\frac{\mu}{\mu_*},Bi_1=\frac{h_1a}{k},S^2=\frac{1}{Da},x=\frac{\bar{x}}{a},p=\frac{\bar{p}\rho a^2}{\mu_0^2},G=-\frac{\partial\bar{p}}{\partial\bar{x}}$$

$$\gamma=\frac{\beta_3\mu_0}{\rho^2a^4},\alpha=\frac{bRT_0^2}{E},\delta=\frac{\alpha_1}{\rho a^2},\psi=\frac{l^mh^2T_0^{m-2}}{v^mh^mRK\mu_*V^2}\frac{EQC_0A}{(RT_0^2+T_0E)}e^{-E/RT_0} \tag{13}$$

$\Gamma=\frac{l^mv^m\mu_0^3}{T_0^mh^m}\frac{QC_0A}{QC_0Aa^4\rho}e^{E/RT_0}$, ψ being $Frank-Kamenetskii$ parameter for the system.

Substituting (13) into (6) and (7), considering a steady case we obtain

ISSN 2347-1921

$$G - Ha^2u - S^2u + \frac{1}{\mathrm{Re}}\frac{d^2u}{dy^2} - \alpha\theta' u' + 6\gamma u''(u')^2 = 0 \qquad (14)$$

$$\frac{1}{\mathrm{Pr}}\frac{d}{dy}\left(e^{-\gamma_1\theta}\theta'\right) + \Gamma\left[Ha^2u^2 + S^2u^2 + (u')^2\left(Br + 2\gamma u'^2\right)\right] + \psi(1+\varepsilon\theta)^m e^{\frac{\theta}{(1+\varepsilon\theta)}} = 0 \qquad (15)$$

Following [9] Equations (14) and (15) are to be solved subject to the boundary conditions:

$$u(0) = u(1) = 0, Bi_1\theta(0) - Bi_1\theta_a, Bi_2\theta_a - Bi_2\theta(1) \qquad (16)$$

We now proceed to solve equations (14) and (15) subject to (16) numerically using Galerkin-Weighted Residual Method as follows:

let $u = \sum_{i=0}^{2} A_i e^y, \theta = \sum_{i=0}^{2} B_i e^{\left(-i/4\right)y}$ (17)

A maple 14 pseudo code was used to perform the iterative computation and results are presented in Figures 1-6 as follows:

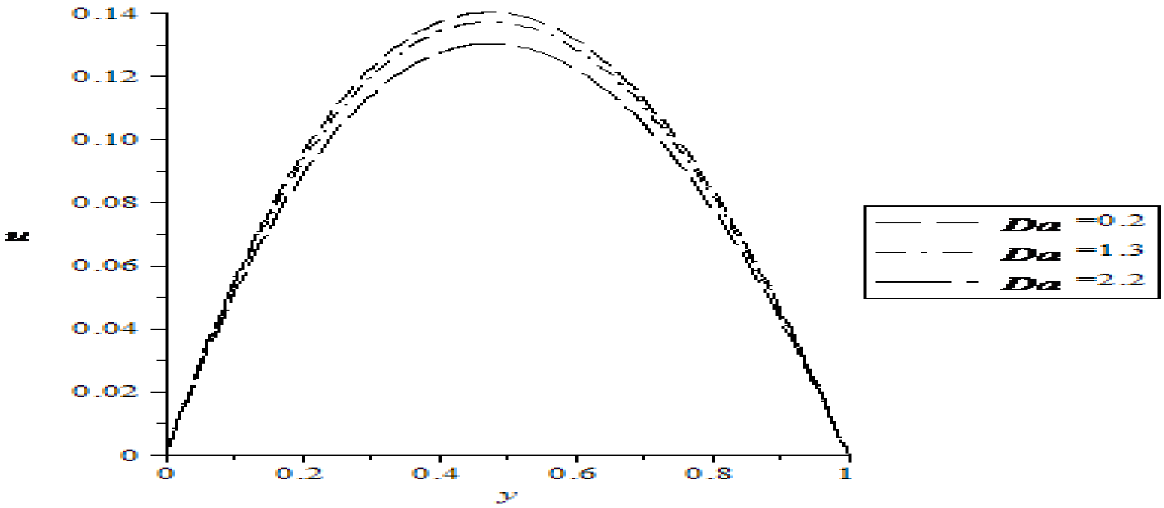

Fig .1: Graph of the velocity function u against the similarity variable y when

$Br = 0.5, G = 2.0, \mathrm{Re} = 0.75, \mathrm{Pr} = 1.0$.

ISSN 2347-1921

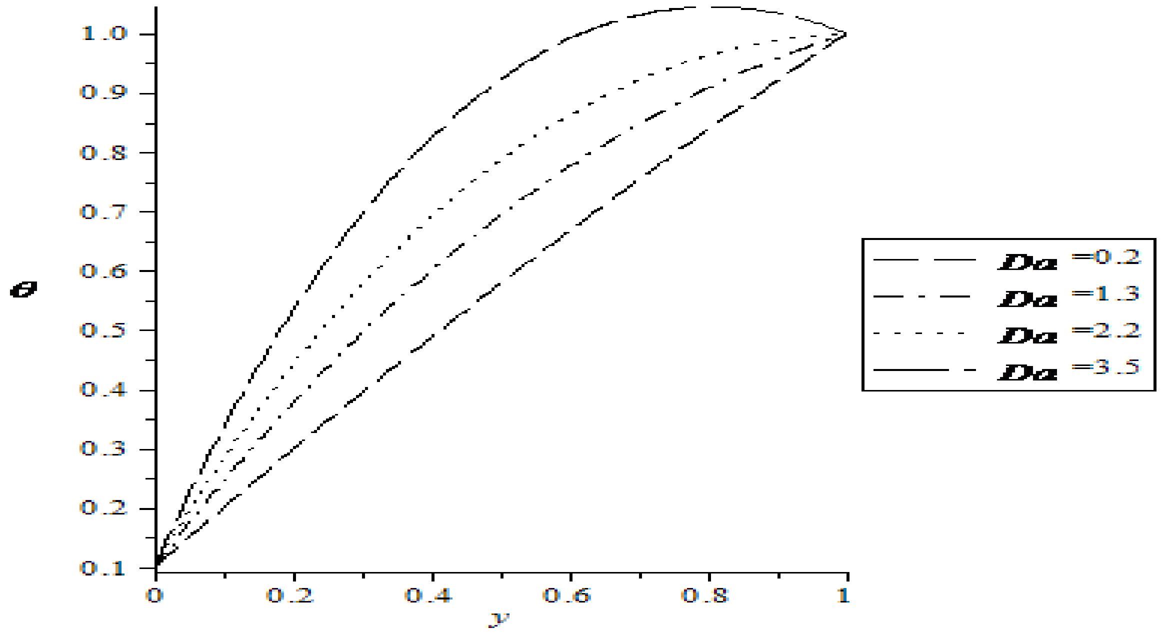

Fig.2: Graph of the temperature function θ against the similarity variable y of Da

when $Br = 0.5, \mathrm{Re} = \mathrm{Pr} = 1.0.$

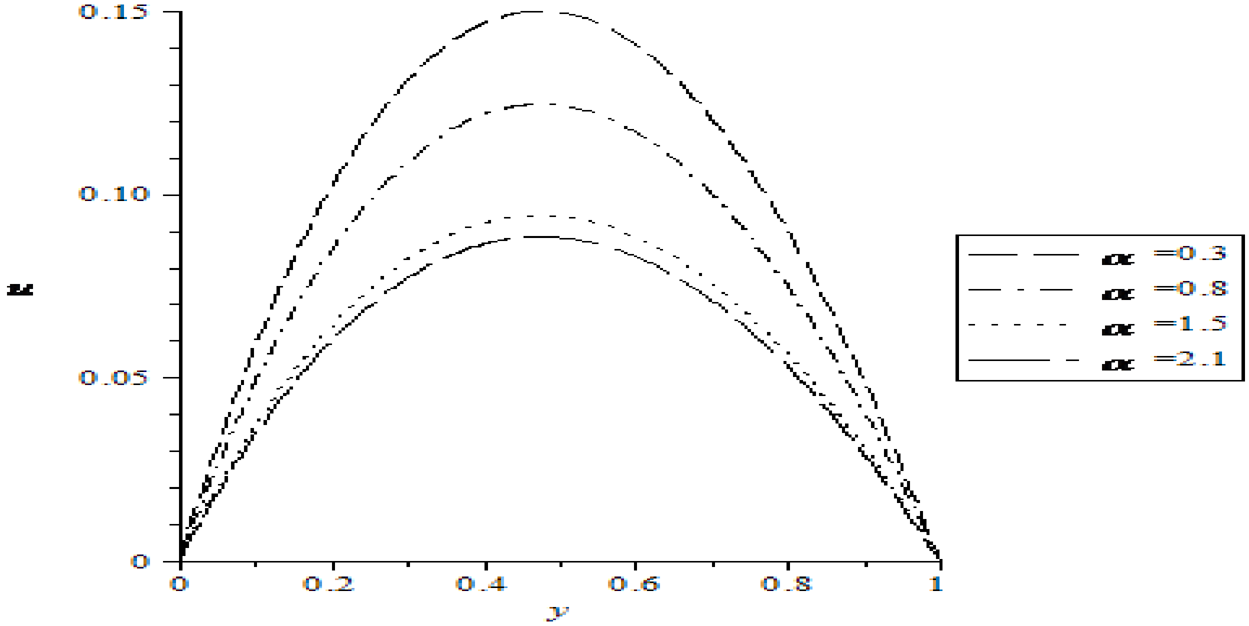

Fig .3: Graph of the velocity function u against the similarity variable y when

$\Gamma = 0.5, m \geq 0.1, S = 0.5, \gamma = 1.2, G = -3.0, \mathrm{Re} = 0.25$

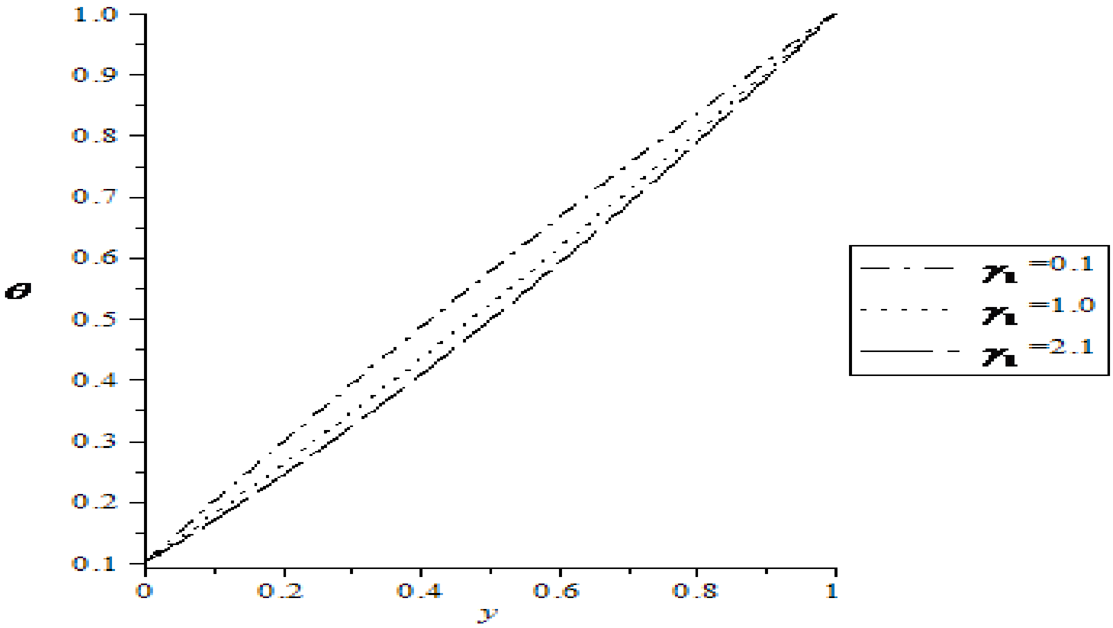

Fig.4: Graph of the temperature function θ against the similarity variable y of

$\Gamma = 0.5, \psi = 0.25, \Pr = 0.75, \text{Re} = 0.2, \gamma = 0.01, m = 3.0, \varepsilon \geq 0, Bi_1 = 0.5, Bi_2 = 1.0, \theta_a = 1.2, S = 0.01.$

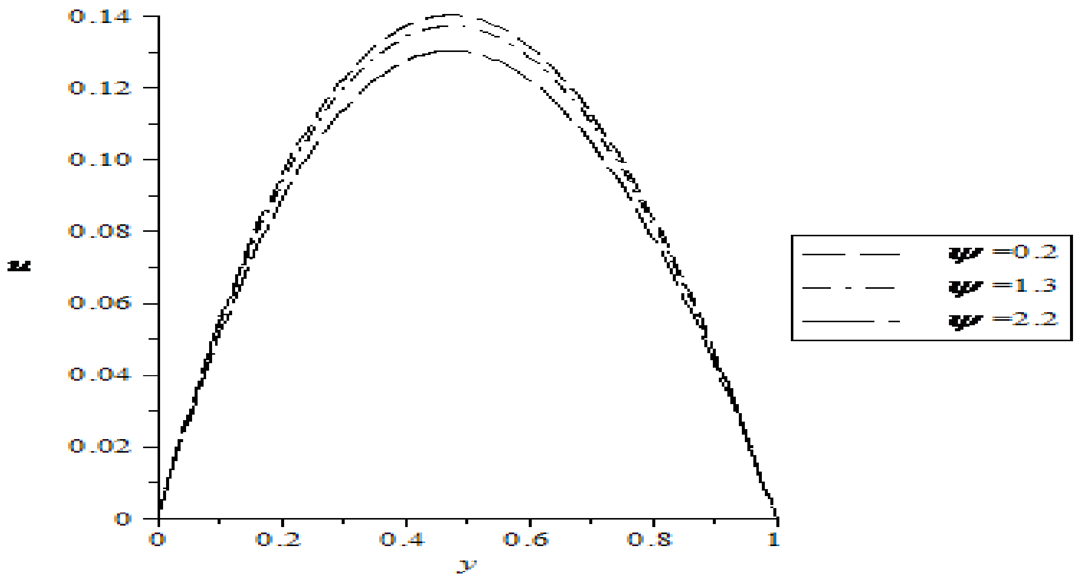

Figure 5: Graph of the velocity function u against the similarity variable y when

$\Gamma = 0.5, m \geq 0.1, S = 0.5, \gamma = 1.2, G = -3.0, \text{Re} = 0.25$

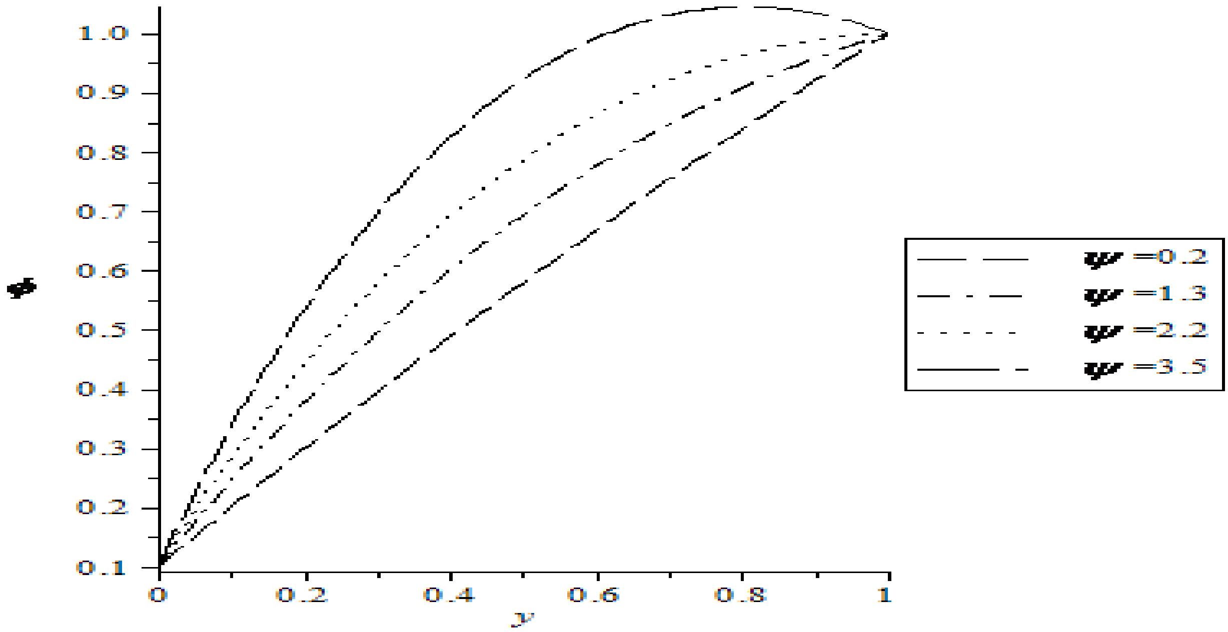

Figure 6: Graph of the temperature function θ against the similarity variable y of

$\Gamma = 0.5, \psi = 0.25, \Pr = 0.75, \mathrm{Re} = 0.2, \gamma = 0.01, m = 3.0, \varepsilon \geq 0, Bi_1 = 0.5, Bi_2 = 1.0, \theta_a = 1.2, S = 0.01.$

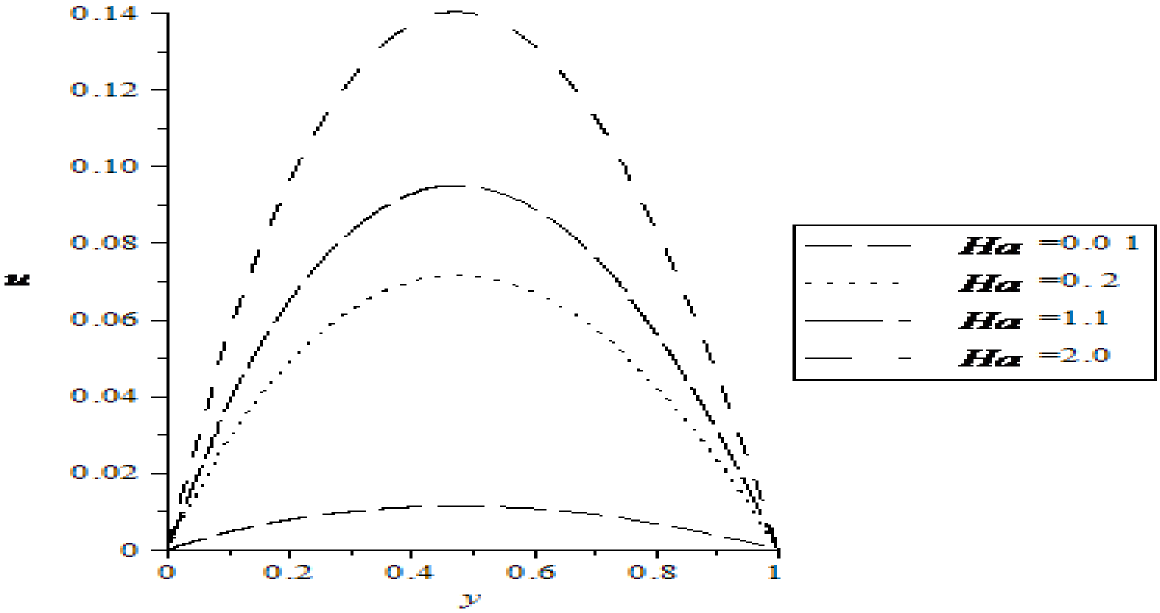

Figure 7: Graph of the velocity function u against the similarity variable y when

$\Gamma = 0.5, m \geq 0.1, S = 0.5, \gamma = 1.2, G = -3.0, \mathrm{Re} = 0.25$

ISSN 2347-1921

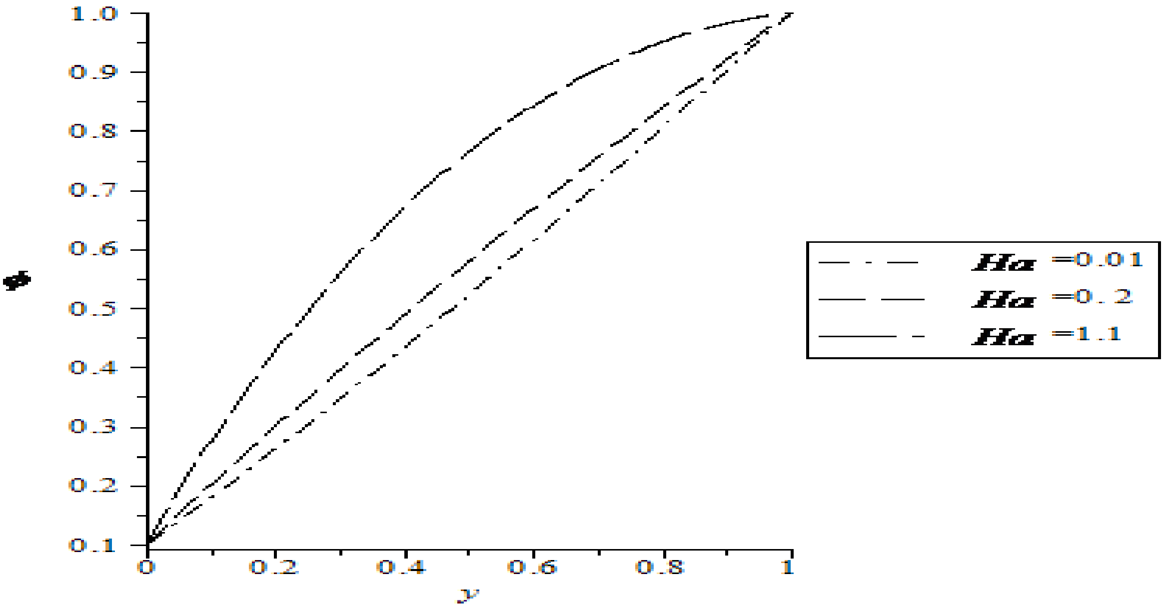

Figure 8: Graph of the temperature function θ against the similarity variable y of

$$\Gamma = 0.5, \psi = 0.25, \Pr = 0.75, \mathrm{Re} = 0.2, \gamma = 0.01, m = 3.0, \varepsilon \geq 0, Bi_1 = 0.5, Bi_2 = 1.0, \theta_a = 1.2, S = 0.01.$$

Discussion of Results

The study of heat transfer and reactive non-Newtonian fluids is extremely important due to its wide variety of practical applications in processes such as filtration of polymer solutions and soil remediation through the removal of liquid pollutants to mention but just a few. It is observed from Figures 1,3,5 and 7 that the velocity profile decreases with increase in each of α non-Newtonian parameter, Hartmann number, Darcy number and ψ $Frank - Kamenetskii$ parameter. It is also noticed from Figures 2,4,6 and 8 that the temperature profile increases as γ_1 variable thermal conductivity parameter, S Darcy number, Hartmann number and ψ $Frank - Kamenetskii$ parameter increases.

Conclusion

A comprehensive set of graphical results for velocity profile and temperature profile are discussed. It is observed that the temperature fluid increases as variable thermal conductivity parameter, Darcy number and $Frank - Kamenetskii$ parameter increases. We observed that there is a transient decrease in the fluid velocity with an increase in the fluid viscosity (which decreases the viscosity). A transient increase in the fluid temperature is observed with increase in $Frank - Kamenetskii$ parameter, α, Λ non-Newtonian parameters and Darcy number which decreases the porosity in the flow.

For engineering purpose, the flow model of our problem represents the oils well and as the α_1 viscosity parameter is increasing there is quick recovery of oil from the oils well. Also, the results of this work are of great interest in production processing, automobile engine, for the safety of life and proper handling of the materials during processing.

REFERENCES

[1] T. Hayat, M.A. Faroog, T.Javed and M. Sajid, Partial slip effects on the flow and heat transfer characteristics in third grade, Non-linear Analysis, Real World Applications. 10(2009),725-755.

[2] R.L. Fosdick and K.R. Rajagopal, Thermodynamics and stability of fluids of third grade , Proc. R. Soc. Lond. A 369(1980), 351-377.

[3] M. Massoudi and I.Christe, Heat transfer and flow of third grade fluid in a pipe, Math. Modeling Sci. Comput. 2(1993),1273-1275.

[4] B.I. Olajuwon, Flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation, Int. J. Non-Linear Science, 7 (1) (2009),50-56.

[5] O.J. Jayeoba and S.S. Okoya, Approximate analytical solutions for pipe flow of a third grade fluid with models of viscosities and heat generation/absorption, J. of Nigerian Mathematical Society. 31(2012), 207-228.

[6] Rivlin, R.S. and Erickson, J.I., Stress deformation relation for isotropic materials. J rat.Mech. Analysis,.4(1995),323-425.

[7] A.Z. Szeri and K.R. Rajagopal, Flow of a Non-Newtonian fluid between heated parallel plates. Int. J. Non-Linear Mechanics.20(2)(1985), 91-101.

ISSN 2347-1921

[8] T. Haroon, A.R. Ansari, A.M. Sidiqui and S.U. Jan, Analysis of poiseuille flow of a reactive power-law fluid between parallel plates, J. of Applied Mathematical Sciences.5 (55)(2011),2721-2746.

[9] Lazarus Rundora, Laminar flow in a channel filled with saturated porous media. A Ph.D. Thesis submitted to the Dept. of Mechanical Engineering, Cape Peninsula University of Technology, 2013.