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Active Control of a Non-Linear Ship model with External and Parametric Excitation

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ABSTRACT

The response of a ship model with non-linearly coupled pitch and roll modes under modulated external and parametric solved and studied. The active control is applied to reduce the vibration of the system . The method of multiple scale perturbation technique is applied to obtain the periodic response equation near the primary resonance in the presence of internal resonance of the system. The objective of this research is focused on the stability of this periodic solution, dynamical properties and chaotic response. The stability of the obtained numerical solution is studied using both frequency response equation and phase-plane methods. The effects of some parameters on the vibrating system are investigated and reported in this paper.

Keywords: Pitch-roll ship ; Vibration; Nonlinear system; Periodic solution; Stability; Chaos; Phase-Plane.

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1- INTRODUCTION

Nayfeh and others [1- 4] studied the non-stationary responses of a non-linearly two-degree-of-freedom system under non-stationary excitation. The basic two-degree-of freedom ship model under a sinusoidal excitation with either a slowly varying amplitude or frequency . Nayfeh, Mook and Marshall's study [5-7]. They used first order multiple time scale method to analyze this system of ship model and a saturation phenomenon of the second mode has been reported. The authors [8, 9] have studied the same saturation phenomenon theoretically and experimentally. Many investigators have studied chaos in non-linear multi degree-of-freedom systems with internal resonance. For instance, quadratically coupled oscillators with 1:2 internal resonance [10,11], a harmonically excited mechanical system with one to one internal resonance [12], a parametrically and externally excited dynamical system with2:1 internal resonance [13,14] have been studied. Davis and Pan studied a system of a ship model using both first order multiple time scale and averaging method, and a more accurate analytical solution and bifurcation diagrams are obtained and reported [15-17]. They also studied the same system of ship model, when it has both a 2:1 internal resonance. Furthermore, the case of periodic response and chaotic response of the same system under modulated excitation has been studied. Also, the stability of the two modes of a ship motions near primary resonance in the presence of internal resonance has been studied and reported [10, 14, 17]. The stability of the numerical solution is investigated using both phase plane methods and frequency response equations. The stability of the proposed solution is determined applying Lyapunov's first method and the stability of the obtained numerical solution of the considered system is studied applying Runge-Kutta method [18,19]. Eissa and El-Bassiouny [20] construct a second-order uniform expansion of the non-linear rolling response of a ship in regular beam seas by method of multiple time scales. The analysis took into consideration linear, quadratic, cubic, quintus, and seven terms in the polynomial expansion of the relative roll angle. Eissa, El-Ganaini andHamed [21] Saturation phenomena may occur in non-linear vibrating systems. This phenomena is very useful in suppressing the undesired vibrations and saturation is investigated in a non-linear oscillating system subject to multi-parametric excitation. Kamel [22] studied the response of a two-degree-of-freedom system with quadratic coupling under a modulated amplitude sinusoidal excitation is studied and solved.. EL-Sayed, Kamel and Eissa [23] studied an application of passive vibration control to a non-linear spring pendulum system simulating a ship's roll motion. This leads to a four-degree-of-freedom (4-DOF) system subjected to multi external and parametric excitations. Sayed and Hamed [24] studied deals with the response of a two- degree-of-freedom(2DOF) system with quadratic coupling under parametric and harmonic excitations. The method of multiple scale perturbation technique is applied to solve the nonlinear differential equations and obtain approximate solutions up to and including the second order approximations.

Sayed and Kamel [25, 26] studied the effect of different controllers on the vibrating system and the saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. Kamel, El-Ganaini and Hamed[27] the coupling of two non-linear oscillators of the main system and absorber representing ultrasonic cutting process is investigated. This leads to a two-degree-of-freedom system subjected to multi-external excitation force. Dostal, Kreuzer and Navaratnam[28] studied Multi-degree-of-freedom ship motion and ship stability in random seas are of major interest for the development of new advanced intact stability criteria and improve the safety of new ship designs, but the results are relevant also for other engineering systems involving multiple scales.

We focus on roll-pitch and roll-heave motion in random seas. The random wave excitation is modeled by a non-white stationary process. This process is derived from a spectral description of the random seaway using traveling effective wave. The aim of this work is to control the main system behavior at simultaneous primary and internal resonance condition, where the system damage is probable. Multiple scale perturbation method is applied to obtain the solution up to the second order approximations and Some of resonance cases are investigated. The effects of natural and excitation frequencies on the response of the system are investigated and discussed.

2- MATHEMATICAL MODELING

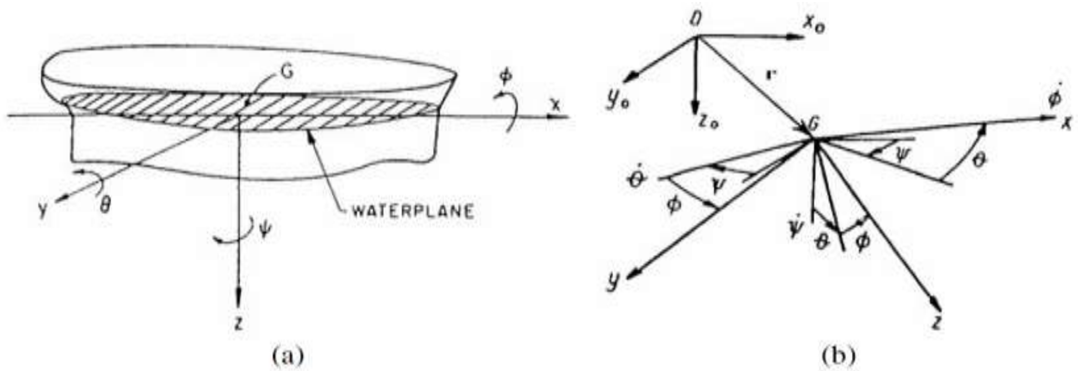

Fig.1a The body - fixed coordinates system $Ox_oy_oz_o$. b coordinates system Gxyz and $Ox_oy_oz_o$

In this paper, we consider a ship model with non linear coupled pitch and roll modes subjected to a sinusoidal harmonic excitation and parametric excitations [24]. The nonlinear system can be written as:

$$\ddot{X}+2\tilde{\mu}_1\dot{X}+\omega_1^2X+\tilde{\alpha}_1\dot{X}\dot{Y}=\tilde{G}_o\cos\Omega_1t+X\tilde{F}_1\sin\Omega_2t+T_1 \tag{1a}$$

$$\ddot{Y}+2\tilde{\mu}_2\dot{Y}+\omega_2^2Y+\tilde{\alpha}_2\dot{X}^2=\tilde{F}_o\cos\Omega_1t+Y\tilde{F}_2\sin\Omega_2t+T_2 \tag{1b}$$

where X and Y are the roll and pitch mode amplitudes, $\tilde{\mu}_1$ and $\tilde{\mu}_2$ the modal damping coefficients, ω_1 and ω_2 the natural angular frequencies of the roll and pitch modes, and Ω_1, Ω_2 the excitations or wave frequencies. $\tilde{G}_o$, $\tilde{F}_1$ and $\tilde{F}_o$, $\tilde{F}_2$ are the excitation force amplitudes of the roll and pitch modes, $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ non linear coefficients. $T_1=-\varepsilon G_1\dot{X}^3, and\ T_2=-\varepsilon G_2\dot{Y}^3$ are the absorbers of the system. All the coefficients in the above equations are complicated functions of the various ship moments of inertia, fluid parameters, boat speed, etc. The linear viscous damping forces, exciting forces and controller are assumed to be

$$\tilde{\mu}_n=\varepsilon\mu_n \qquad \tilde{F}_n=\varepsilon F_n \qquad \tilde{\alpha}_n=\varepsilon\alpha_n \qquad \tilde{G}_o=\varepsilon G_o \quad \tilde{F}_o=\varepsilon F_o \quad (n=1,2)$$

where ε is a small perturbation parameter and $0<\varepsilon<1$. G_1 and G_2 are gains of the absorbers.

2.1-Perturbation analysis

The method of multiple time scale is applied to determine a first order uniform expansion for the solution of equations (1a) and (1b) as in the form:

$$X(t,\varepsilon)=x_o(T_o,T_1)+\varepsilon x_1(T_o,T_1)+\varepsilon^2x_2(T_o,T_1)+\varepsilon^3x_3(T_o,T_1)+O(\varepsilon^4) \tag{2a}$$

$$Y(t,\varepsilon)=y_o(T_o,T_1)+\varepsilon y_1(T_o,T_1)+\varepsilon^2y_2(T_o,T_1)+\varepsilon^3y_3(T_o,T_1)+O(\varepsilon^4) \tag{2b}$$

where ε is a small perturbation parameter, $T_o=t, T_1=\varepsilon t$ are fast and slow time scales respectively, and the time derivatives became

$$\frac{d}{dt}=D_o+\varepsilon D_1+\varepsilon^2D_2\ ,\frac{d^2}{dt^2}=D_o^2+2\varepsilon D_oD_1+\varepsilon^2(D_1^2+2D_oD_2) \tag{3}$$

Substituting equations (2a),(2b) and (3) in to equations (1a) and (1b) and equating the coefficients of the same power of ε in both sides, we obtain

$$(D_o^2+\omega_1^2)x_o=0 \tag{4a}$$

$$(D_o^2+\omega_2^2)y_o=0 \tag{4b}$$

$$(D_o^2+\omega_1^2)x_1=-2D_oD_1x_o-2\mu_1D_ox_o-\alpha_1(D_ox_o)(D_oy_o)+x_oF_1\sin\Omega_2t-G_1(D_ox_o)^3+G_0\cos\Omega_1t \quad (5a)$$

$$(D_o^2+\omega_2^2)y_1=-2D_oD_1y_o-2\mu_2D_oy_o-\alpha_2(D_ox_o)^2+F_o\cos\Omega_1t+y_oF_2\sin\Omega_2t-G_2(D_0y_0)^3 \quad (5b)$$

$$(D_o^2+\omega_1^2)x_2=-D_1^2x_o-2D_oD_1x_1-2\mu_1(D_ox_1+D_1x_o)-\alpha_1((D_ox_o)(D_oy_1)-3G_1(D_ox_o)^2(D_ox_1+D_1x_o)$$
$$+(D_ox_o)(D_1y_o)+(D_oy_o)(D_ox_1)+(D_oy_o)(D_1x_o))+x_1F_1\sin\Omega_2t \quad (6a)$$

$$(D_o^2+\omega_2^2)y_2=-D_1^2y_o-2D_oD_1y_1-2\mu_2(D_oy_1+D_1y_o)-2\alpha_2(D_ox_o)(D_ox_1+D_1x_o)+y_1F_2\sin\Omega_2t$$
$$-3G_2(D_oy_o)^2(D_oy_1+D_1y_o) \quad (6b)$$

$$(D_o^2+\omega_1^2)x_3=-D_1^2x_1-2D_oD_1x_2-2\mu_1(D_ox_2+D_1x_1)-\alpha_1((D_ox_1+D_1x_o)\times(D_oy_1+D_1y_o)$$
$$+(D_ox_o)(D_oy_2+D_1y_1)+(D_oy_o)(D_ox_2+D_1x_1))+x_2F_1\sin\Omega_2t-3G_1((D_ox_o)(D_ox_1+D_1x_o)^2$$
$$+(D_ox_o)^2(D_ox_2+D_1x_1)) \quad (7a)$$

$$(D_o^2+\omega_2^2)y_3=-D_1^2y_1-2D_oD_1y_2-2\mu_2(D_oy_2+D_1y_1)-\alpha_2((D_ox_1+D_1x_o)^2+2(D_ox_o)(D_ox_2+D_1x_1))$$
$$+y_2F_2\sin\Omega_2t-3G_2((D_oy_o)(D_oy_1+D_1y_o)^2+(D_oy_o)^2(D_oy_2+D_1y_1)) \quad (7b)$$

The general solution of equations (4a) and (4b) are given by

$$x_o(T_o,T_1)=A_o(T_1)e^{i\omega_1T_o}+\bar{A}_o(T_1)e^{-i\omega_1T_o} \quad (8a)$$

$$y_o(T_o,T_1)=B_o(T_1)e^{i\omega_2T_o}+\bar{B}_o(T_1)e^{-i\omega_2T_o} \quad (8b)$$

where A_o, B_o are complex function in T_1 and cc represents the complex conjugate of the preceding terms. Substituting equations (8a) and (8b) in to equations (5a) and (5b), and eliminating the secular terms, then the general solution obtained as

$$x_1(T_o,T_1)=C_1e^{i\omega_1T_o}+C_2e^{3i\omega_1T_o}+C_3e^{i\Omega_1T_o}+C_4e^{i(\omega_1+\omega_2)T_o}+C_5e^{i(\omega_1-\omega_2)T_o}+C_6e^{i(\Omega_2+\omega_1)T_o}$$
$$+C_7e^{i(\Omega_2-\omega_1)T_o}+cc \quad (9a)$$

$$y_1(T_o,T_1)=C_1^*e^{i\omega_2T_o}+C_2^*e^{3i\omega_2T_o}+C_3^*e^{2i\omega_1T_o}+C_4^*e^{i\Omega_1T_o}+C_5^*e^{i(\Omega_2+\omega_2)T_o}+C_6^*e^{i(\Omega_2-\omega_2)T_o}$$
$$+C_6^*e^{i(\Omega_2-\omega_2)T_o}+C_7^*+cc \quad (9b)$$

where C_i and C_i^* $(i=1,2,...,7)$ are complex function in T_1, cc are complex conjugate. Similarly, Substituting from equations (8a), (8b), (9a) and (9b) into equations (6a) and (6b) we get

$$x_2(T_o,T_1)=N_1e^{i\omega_1T_o}+N_2e^{3i\omega_1T_o}+N_3e^{5i\omega_1T_o}+N_4e^{i\Omega_1T_o}+N_5e^{i(\omega_1+\omega_2)T_o}+N_6\,e^{i(\omega_1-\omega_2)T_o}$$
$$+N_7e^{i(\omega_1+2\omega_2)T_o}+N_8e^{i(\omega_1-2\omega_2)T_o}+N_9e^{i(\omega_1+3\omega_2)T_o}+N_{10}e^{i(\omega_1-3\omega_2)T_o}+N_{11}e^{i(3\omega_1+\omega_2)T_o}$$
$$+N_{12}e^{i(3\omega_1-\omega_2)T_o}+N_{13}e^{i(\Omega_1+\Omega_2)T_o}+N_{14}e^{i(\Omega_1-\Omega_2)T_o}+N_{15}e^{i(\Omega_1+\omega_1)T_o}+N_{16}e^{i(\Omega_1-\omega_1)T_o}$$
$$+N_{17}e^{i(\Omega_1+2\omega_1)T_o}+N_{18}e^{i(\Omega_1-2\omega_1)T_o}+N_{19}e^{i(\Omega_1+\omega_2)T_o}+N_{20}e^{i(\Omega_1-\omega_2)T_o}+N_{21}e^{i(\Omega_2+\omega_1)T_o}$$
$$+N_{22}e^{i(\Omega_2-\omega_1)T_o}+N_{23}e^{i(\Omega_2+3\omega_1)T_o}+N_{24}\,e^{i(\Omega_2-3\omega_1)T_o}+N_{25}e^{i(2\Omega_2+\omega_1)T_o}+N_{26}e^{i(2\Omega_2-\omega_1)T_o}$$

$$+N_{27}e^{i(\Omega_2+(\omega_1+\omega_2))T_o}+N_{28}e^{i(\Omega_2-(\omega_1+\omega_2))T_o}+N_{29}e^{i(\Omega_2+(\omega_1-\omega_2))T_o}+N_{30}e^{i(\Omega_2-(\omega_1-\omega_2))T_o}+cc \quad (10a)$$

$$y_2(T_o,T_1)=M_1e^{i\omega_2T_o}+M_2e^{3i\omega_2T_o}+M_3e^{5i\omega_2T_o}+M_4e^{2i\omega_1T_o}+M_5e^{4i\omega_1T_o}+M_6e^{i(2\omega_1+\omega_2)T_o}$$
$$+M_7e^{i(2\omega_1-\omega_2)T_o}+M_8e^{2i(\omega_1+\omega_2)T_o}+M_9e^{2i(\omega_1-\omega_2)T_o}+M_{10}e^{i\Omega_1T_o}+M_{11}e^{i\Omega_2T_o}$$
$$+M_{12}e^{i(\Omega_1+\Omega_2)T_o}+M_{13}e^{i(\Omega_1-\Omega_2)T_o}+M_{14}e^{i(\Omega_1+\omega_1)T_o}+M_{15}e^{i(\Omega_1-\omega_1)T_o}+M_{16}e^{i(\Omega_1+2\omega_2)T_o}$$
$$+M_{17}e^{i(\Omega_1-2\omega_2)T_o}+M_{18}e^{i(\Omega_2+2\omega_1)T_o}+M_{19}e^{i(\Omega_2-2\omega_1)T_o}+M_{20}e^{i(\Omega_2+\omega_2)T_o}+M_{21}e^{i(\Omega_2-\omega_2)T_o}$$
$$+M_{22}e^{i(\Omega_2+3\omega_2)T_o}+M_{23}e^{i(\Omega_2-3\omega_2)T_o}+M_{24}e^{i(2\Omega_2+\omega_2)T_o}+M_{25}e^{i(2\Omega_2-\omega_2)T_o}+M_{26}+cc \quad (10b)$$

where $N_m(m=1,2,...,30)$ and $M_i\ (i=1,2,...,26)$ are complex functions in T_1, cc are complex conjugates. Similarly, substituting from equations (8a), (8b), (9a), (9b), (10a) and (10b)in to equations (7a) and (7b) we get,

$$x_3(T_o,T_1)=w_1e^{i\omega_1T_o}+w_2e^{3i\omega_1T_o}+w_3e^{5i\omega_1T_o}+w_4e^{7i\omega_1T_o}+w_5e^{i\Omega_1T_o}+w_6e^{2i\Omega_1T_o}$$
$$+w_7e^{i(\omega_1+\omega_2)T_o}+w_8e^{i(\omega_1-\omega_2)T_o}+w_9e^{i(\omega_1+2\omega_2)T_o}+w_{10}e^{i(\omega_1-2\omega_2)T_o}$$
$$+w_{11}e^{i(\omega_1+3\omega_2)T_o}+w_{12}e^{i(\omega_1-3\omega_2)T_o}+w_{13}e^{i(\omega_1+4\omega_2)T_o}+w_{14}e^{i(\omega_1-4\omega_2)T_o}$$
$$+w_{15}e^{i(\omega_1+5\omega_2)T_o}+w_{16}e^{i(\omega_1-5\omega_2)T_o}+w_{17}e^{i(\omega_1+6\omega_2)T_o}+w_{18}e^{i(\omega_1-6\omega_2)T_o}$$
$$+w_{19}e^{i(3\omega_1+\omega_2)T_o}+w_{20}e^{i(3\omega_1+\omega_2)T_o}+w_{21}e^{i(3\omega_1+2\omega_2)T_o}+w_{22}e^{i(3\omega_1-2\omega_2)T_o}$$
$$+w_{23}e^{3i(\omega_1+\omega_2)T_o}+w_{24}e^{3i(\omega_1-\omega_2)T_o}+w_{25}e^{i(5\omega_1+\omega_2)T_o}+w_{26}e^{i(5\omega_1-\omega_2)T_o}$$
$$+w_{27}e^{i(\Omega_1+\Omega_2)T_o}+w_{28}e^{i(\Omega_1-\Omega_2)T_o}+w_{29}e^{i(\Omega_1+2\Omega_2)T_o}+w_{30}e^{i(\Omega_1-2\Omega_2)T_o}$$
$$+w_{31}e^{i(\Omega_1+\omega_1)T_o}+w_{32}e^{i(\Omega_1-\omega_1)T_o}+w_{33}e^{i(\Omega_1+2\omega_1)T_o}+w_{34}e^{i(\Omega_1-2\omega_1)T_o}$$
$$+w_{35}e^{i(\Omega_1+3\omega_1)T_o}+w_{36}e^{i(\Omega_1-3\omega_1)T_o}+w_{37}e^{i(\Omega_1+4\omega_1)T_o}+w_{38}e^{i(\Omega_1-4\omega_1)T_o}$$
$$+w_{39}e^{i(\Omega_1+\omega_2)T_o}+w_{40}e^{i(\Omega_1-\omega_2)T_o}+w_{41}e^{i(\Omega_1+2\omega_2)T_o}+w_{42}e^{i(\Omega_1-2\omega_2)T_o}$$
$$+w_{43}e^{i(\Omega_1+3\omega_2)T_o}+w_{44}e^{i(\Omega_1-3\omega_2)T_o}+w_{45}e^{i(2\Omega_1+\omega_1)T_o}+w_{46}e^{i(2\Omega_1+\omega_1)T_o}$$
$$+w_{47}e^{i(\Omega_2+\omega_1)T_o}+w_{48}e^{i(\Omega_2-\omega_1)T_o}+w_{49}e^{i(\Omega_2+3\omega_1)T_o}+w_{50}e^{i(\Omega_2-\omega_1)T_o}$$
$$+w_{51}e^{i(\Omega_2+5\omega_1)T_o}+w_{52}e^{i(\Omega_2-5\omega_1)T_o}+w_{53}e^{i(2\Omega_2+\omega_1)T_o}+w_{54}e^{i(2\Omega_2-\omega_1)T_o}$$
$$+w_{55}e^{i(2\Omega_2+3\omega_1)T_o}+w_{56}e^{i(2\Omega_2-3\omega_1)T_o}+w_{57}e^{i(3\Omega_2+\omega_1)T_o}+w_{58}e^{i(3\Omega_2-\omega_1)T_o}$$
$$+w_{59}e^{i(\Omega_1+(\omega_1+\omega_2))T_o}+w_{60}e^{i(\Omega_1-(\omega_1+\omega_2))T_o}+w_{61}e^{i(\Omega_1+(\omega_1-\omega_2))T_o}+w_{62}e^{i(\Omega_1-(\omega_1-\omega_2))T_o}$$
$$+w_{63}e^{i(\Omega_1+(\omega_1+2\omega_2))T_o}+w_{64}e^{i(\Omega_1-(\omega_1+2\omega_2))T_o}+w_{65}e^{i(\Omega_1+(\omega_1-2\omega_2))T_o}+w_{66}e^{i(\Omega_1-(\omega_1-2\omega_2))T_o}$$
$$+w_{67}e^{i(\Omega_1+(2\omega_1+\omega_2))T_o}+w_{68}e^{i(\Omega_1-(2\omega_1+\omega_2))T_o}+w_{69}e^{i(\Omega_1+(2\omega_1-\omega_2))T_o}+w_{70}e^{i(\Omega_1-(2\omega_1-\omega_2))T_o}$$

$$+w_{71}e^{i(\Omega_2+(\omega_1+\omega_2))T_o}+w_{72}e^{i(\Omega_2-(\omega_1+\omega_2))T_o}+w_{73}e^{i(\Omega_2+(\omega_1-\omega_2))T_o}+w_{74}e^{i(\Omega_2-(\omega_1-\omega_2))T_o}$$
$$+w_{75}e^{i(\Omega_2+(\omega_1+2\omega_2))T_o}+w_{76}e^{i(\Omega_2-(\omega_1+2\omega_2))T_o}+w_{77}e^{i(\Omega_2+(\omega_1-2\omega_2))T_o}+w_{78}e^{i(\Omega_2-(\omega_1-2\omega_2))T_o}$$
$$+w_{79}e^{i(\Omega_2+(\omega_1+3\omega_2))T_o}+w_{80}e^{i(\Omega_2-(\omega_1+3\omega_2))T_o}+w_{81}e^{i(\Omega_2+(\omega_1-3\omega_2))T_o}+w_{82}e^{i(\Omega_2-(\omega_1-3\omega_2))T_o}$$
$$+w_{83}e^{i(\Omega_2+(3\omega_1+\omega_2))T_o}+w_{84}e^{i(\Omega_2-(3\omega_1+\omega_2))T_o}+w_{85}e^{i(\Omega_2+(3\omega_1-\omega_2))T_o}+w_{86}e^{i(\Omega_2-(3\omega_1-\omega_2))T_o}$$
$$+w_{87}e^{i(2\Omega_2+(\omega_1+\omega_2))T_o}+w_{88}e^{i(2\Omega_2-(\omega_1+\omega_2))T_o}+w_{89}e^{i(2\Omega_2+(\omega_1-\omega_2))T_o}+w_{90}e^{i(2\Omega_2-(\omega_1-\omega_2))T_o}$$
$$+w_{91}e^{i((\Omega_1+\Omega_2)+\omega_1)T_o}+w_{92}e^{i((\Omega_1+\Omega_2)-\omega_1)T_o}+w_{93}e^{i((\Omega_1-\Omega_2)+\omega_1)T_o}+w_{94}e^{i((\Omega_1-\Omega_2)-\omega_1)T_o}$$
$$+w_{95}e^{i((\Omega_1+\Omega_2)+2\omega_1)T_o}+w_{96}e^{i((\Omega_1+\Omega_2)-2\omega_1)T_o}+w_{97}e^{i((\Omega_1-\Omega_2)+2\omega_1)T_o}+w_{98}e^{i((\Omega_1-\Omega_2)-2\omega_1)T_o}$$
$$+w_{99}e^{i((\Omega_1+\Omega_2)+\omega_2)T_o}+w_{100}e^{i((\Omega_1+\Omega_2)-\omega_2)T_o}+w_{101}e^{i((\Omega_1-\Omega_2)+\omega_2)T_o}+w_{102}e^{i((\Omega_1-\Omega_2)-\omega_2)T_o}+cc \qquad (11a)$$

$$y_3(T_o,T_1)=Z_1e^{i\omega_2T_o}+Z_2e^{2i\omega_2T_o}+Z_3e^{3i\omega_2T_o}+Z_4e^{5i\omega_2T_o}+Z_5e^{6i\omega_2T_o}+Z_6e^{7i\omega_2T_o}$$
$$+Z_7e^{2i\omega_1T_o}+Z_8e^{4i\omega_1T_o}+Z_9e^{6i\omega_1T_o}+Z_{10}e^{i\Omega_1T_o}+Z_{11}e^{2i\Omega_1T_o}+Z_{12}e^{i\Omega_2T_o}$$
$$+Z_{13}e^{2i\Omega_2T_o}+Z_{14}e^{i(\omega_1+2\omega_2)T_o}+Z_{15}e^{i(\omega_1-2\omega_2)T_o}+Z_{16}e^{i(\omega_1+3\omega_2)T_o}$$
$$+Z_{17}e^{i(\omega_1-3\omega_2)T_o}+Z_{18}e^{i(\omega_1+4\omega_2)T_o}+Z_{19}e^{i(\omega_1-4\omega_2)T_o}+Z_{20}e^{i(2\omega_1+\omega_2)T_o}$$
$$+Z_{21}e^{i(2\omega_1-\omega_2)T_o}+Z_{22}e^{2i(\omega_1+\omega_2)T_o}+Z_{23}e^{2i(\omega_1-\omega_2)T_o}+Z_{24}e^{i(2\omega_1+3\omega_2)T_o}$$
$$+Z_{25}e^{i(2\omega_1-3\omega_2)T_o}+Z_{26}e^{i(2\omega_1+4\omega_2)T_o}+Z_{27}e^{i(2\omega_1-4\omega_2)T_o}+Z_{28}e^{i(4\omega_1+\omega_2)T_o}$$
$$+Z_{29}e^{i(4\omega_1-\omega_2)T_o}+Z_{30}e^{2i(2\omega_1+\omega_2)T_o}+Z_{31}e^{2i(2\omega_1-\omega_2)T_o}+Z_{32}e^{i(\Omega_1+\omega_1)T_o}$$
$$+Z_{33}e^{i(\Omega_1-\omega_1)T_o}+Z_{34}e^{i(\Omega_1+2\omega_1)T_o}+Z_{35}e^{i(\Omega_1-2\omega_1)T_o}+Z_{36}e^{i(\Omega_1+3\omega_1)T_o}$$
$$+Z_{37}e^{i(\Omega_1-3\omega_1)T_o}+Z_{38}e^{i(\Omega_1+2\omega_2)T_o}+Z_{39}e^{i(\Omega_1-2\omega_2)T_o}+Z_{40}e^{i(\Omega_1+3\omega_2)T_o}$$
$$+Z_{41}e^{i(\Omega_1-3\omega_2)T_o}+Z_{42}e^{i(\Omega_1+4\omega_2)T_o}+Z_{43}e^{i(\Omega_1-4\omega_2)T_o}+Z_{44}e^{i(2\Omega_1+\omega_2)T_o}$$
$$+Z_{45}e^{i(2\Omega_1-\omega_2)T_o}+Z_{46}e^{i(\Omega_2+2\omega_1)T_o}+Z_{47}e^{i(\Omega_2-2\omega_1)T_o}+Z_{48}e^{i(\Omega_2+4\omega_1)T_o}$$
$$+Z_{49}e^{i(\Omega_2-4\omega_1)T_o}+Z_{50}e^{i(\Omega_2+\omega_2)T_o}+Z_{51}e^{i(\Omega_2-\omega_2)T_o}+Z_{52}e^{i(\Omega_2+2\omega_2)T_o}$$
$$+Z_{53}e^{i(\Omega_2-2\omega_2)T_o}+Z_{54}e^{i(\Omega_2+3\omega_2)T_o}+Z_{55}e^{i(\Omega_2-3\omega_2)T_o}+Z_{56}e^{i(\Omega_2+5\omega_2)T_o}$$
$$+Z_{57}e^{i(\Omega_2-5\omega_2)T_o}+Z_{58}e^{2i(\Omega_2+\omega_1)T_o}+Z_{59}e^{2i(\Omega_2-\omega_1)T_o}+Z_{60}e^{i(2\Omega_2+\omega_2)T_o}$$
$$+Z_{61}e^{i(2\Omega_2-\omega_2)T_o}+Z_{62}e^{i(2\Omega_2+3\omega_2)T_o}+Z_{63}e^{i(2\Omega_2-3\omega_2)T_o}+Z_{64}e^{i(3\Omega_2+\omega_2)T_o}$$
$$+Z_{65}e^{i(3\Omega_2-\omega_2)T_o}+Z_{66}e^{i(\Omega_1+(\omega_1+\omega_2))T_o}+Z_{67}e^{i(\Omega_1-(\omega_1+\omega_2))T_o}+Z_{68}e^{i(\Omega_1+(\omega_1-\omega_2))T_o}$$
$$+Z_{69}e^{i(\Omega_1-(\omega_1-\omega_2))T_o}+Z_{70}e^{i(\Omega_1+(\omega_1+2\omega_2))T_o}+Z_{71}e^{i(\Omega_1-(\omega_1+2\omega_2))T_o}+Z_{72}e^{i(\Omega_1+(\omega_1-2\omega_2))T_o}$$

$$+Z_{73}e^{i(\Omega_1-(\omega_1-2\omega_2))T_o}+Z_{74}e^{i(\Omega_1+(2\omega_1+\omega_2))T_o}+Z_{75}e^{i(\Omega_1-(2\omega_1+\omega_2))T_o}+Z_{76}e^{i(\Omega_1+(2\omega_1-\omega_2))T_o}$$

$$+Z_{77}e^{i(\Omega_1-(2\omega_1-\omega_2))T_o}+Z_{78}e^{i(\Omega_2+(\omega_1+3\omega_2))T_o}+Z_{79}e^{i(\Omega_2-(\omega_1+3\omega_2))T_o}+Z_{80}e^{i(\Omega_2+(\omega_1-3\omega_2))T_o}$$

$$+Z_{82}e^{i(\Omega_2+(2\omega_1+\omega_2))T_o}+Z_{83}e^{i(\Omega_2-(2\omega_1+\omega_2))T_o}+Z_{84}e^{i(\Omega_2+(2\omega_1-\omega_2))T_o}+Z_{85}e^{i(\Omega_2-(2\omega_1-\omega_2))T_o}$$

$$+Z_{86}e^{i(\Omega_2+2(\omega_1+\omega_2))T_o}+Z_{87}e^{i(\Omega_2-2(\omega_1+\omega_2))T_o}+Z_{88}e^{i(\Omega_2+2(\omega_1-\omega_2))T_o}+Z_{89}e^{i(\Omega_2-2(\omega_1-\omega_2))T_o}$$

$$+Z_{90}e^{i(\Omega_1+\Omega_2)T_o}+Z_{91}e^{i(\Omega-\Omega_2)T_o}+Z_{92}e^{i(\Omega_1+2\Omega_2)T_o}+Z_{93}e^{i(\Omega_1-2\Omega_2)T_o}$$

$$+Z_{94}e^{i((\Omega_1+\Omega_2)+\omega_1)T_o}+Z_{95}e^{i((\Omega_1+\Omega_2)-\omega_1)T_o}+Z_{96}e^{i((\Omega_1-\Omega_2)+\omega_1)T_o}+Z_{97}e^{i((\Omega_1-\Omega_2)-\omega_1)T_o}$$

$$+Z_{98}e^{i((\Omega_1+\Omega_2)+2\omega_2)T_o}+Z_{99}e^{i((\Omega_1+\Omega_2)-2\omega_2)T_o}+Z_{100}e^{i((\Omega_1-\Omega_2)+2\omega_2)T_o}+Z_{101}e^{i((\Omega_1-\Omega_2)-2\omega_2)T_o}$$

$$+Z_{102}+cc \tag{11b}$$

where $w_m\,(m=1,2,...,103)$ and $Z_i\ (i=1,2,...,102)$ are complex functions in T_1, cc are complex conjugates.

From above-proposed solution, the reported resonance cases are :

(a) Trivial resonance: $\Omega_1\cong\Omega_2\cong\omega_1\cong\omega_2=0$

(b) Primary resonance: $\Omega_1\cong\omega_1,\ \Omega_2\cong\omega_2$

(c) Internal resonance: (1) $\omega_1\cong n\omega_2$ (2) $\omega_2\cong n\omega_1$, n=1, 2,3,4,5 (3) $n\omega_1\cong m\omega_2$, n=2, 3, m=3, 5

(d) Sub-harmonic resonance: (1) $\Omega_1\cong n\omega_1$ (2) $\Omega_1\cong n\omega_2$, n=2, 3, 4, 5 (3) $\Omega_2\cong m\omega_1$ (4) $\Omega_2\cong m\omega_2$, m=2, 4, 6

(e) Combined resonance: (1) $\Omega_1\cong\omega_1\pm\omega_2$ (2) $\Omega_1\cong 2(\omega_1\pm\omega_2)$ (3) $\Omega_1\cong(2\omega_1\pm 3\omega_2)$

(4) $\Omega_1\cong(\omega_1\pm 2\omega_2)$ (5) $\Omega_1\cong(2\omega_1\pm\omega_2)$ (6) $2\Omega_2\cong(2\omega_1\pm\omega_2)$ (7) $\Omega_2\cong 2(\omega_1\pm\omega_2)$

(8) $\Omega_2\cong(4\omega_1\pm\omega_2)$ (9) $\Omega_2\cong(2\omega_1\pm\omega_2)$

(f) Simultaneous resonance:

Any combination of the above resonance cases is considered as a simultaneous resonance one.

2.2-Stability Analysis

Stability analysis is limited to the first order approximation. Then all the solution coefficients are functions in T_1 only. Applying an absorber to the main system at internal resonance $\omega_2\cong 2\omega_1$. Introducing the detuning parameters σ_1 and σ_2 in the primary and internal resonance to convert the small-divisor terms into the secular terms according to

$$\Omega_1\cong\omega_1+\varepsilon\sigma_1 \text{ and } \omega_2\cong 2\omega_1+\varepsilon\sigma_2 \tag{12}$$

Eliminating the secular terms of both x_1 and y_1 of equations (5a) and (5b), leads to the solvability conditions, and noting that A_o and B_o are functions in T_1 only, we get

$$[-2i\omega_1(D_1A_o+\mu_1A_o)-3iG_1\omega_1^3A_o^2\bar{A}_o]e^{i\omega_1T_o}+\frac{G_o}{2}e^{i\Omega_1T_o}-\alpha_1\omega_1\omega_2\bar{A}_oB_oe^{i(\omega_1-\omega_2)T_o}=0 \tag{13a}$$

$$[-2i\,\omega_2(D_1B_o + \mu_2 B_o) - 3iG_2\omega_2^3 B_o^2 \bar{B}_o]e^{i\omega_2 T_o} + \alpha_2\omega_1^2 A_o^2 e^{2i\omega_1 T_o} = 0 \quad (13b)$$

Putting the polar form

$$A_o = \frac{1}{2}a_1(T_1)e^{i\gamma_1(T_1)}\ ,\ B_o = \frac{1}{2}a_2(T_1)e^{i\gamma_2(T_1)} \quad (14)$$

where a_1, a_2, γ_1 and γ_2 are real. Substituting Equations (14) into equations (13a), (13b), and separating real and imaginary parts we get the following

$$a_1' = -\mu_1 a_1 - \frac{3}{8}G_1\omega_1^2 a_1^3 + \frac{G_o}{2\omega_1}\sin\theta_1 - \frac{\alpha_1}{4}\omega_2 a_1 a_2 \sin\theta_2 \quad (15a)$$

$$a_1\gamma_1' = \frac{\alpha_1}{4}\omega_2 a_1 a_2 \cos\theta_2 - \frac{G_o}{2\omega_1}\cos\theta_1 \quad (15b)$$

$$a_2' = -\mu_2 a_2 - \frac{3}{8}G_2\omega_2^2 a_2^3 - \frac{\alpha_2\omega_1^2 a_1^2}{4\omega_2}\sin\theta_2 \quad (15c)$$

$$a_2\gamma_2' = -\frac{\alpha_2\omega_1^2 a_1^2}{4\omega_2}\cos\theta_2 \quad (15d)$$

where $\theta_1 = (\sigma_2 T_1 + \gamma_2 - 2\gamma_1)$ and $\theta_2 = \sigma_1 T_1 - \gamma_1$. For steady state solutions, $a_1' = a_2' = \theta_1' = \theta_2' = 0,$ and equations (15a), (15b), (15c) and (15d) becomes

$$0 = -\mu_1 a_1 - \frac{3}{8}G_1\omega_1^2 a_1^3 + \frac{G_o}{2\omega_1}\sin\theta_1 - \frac{\alpha_1}{4}\omega_2 a_1 a_2 \sin\theta_2 \quad (16a)$$

$$a_1\sigma_1 = \frac{\alpha_1}{4}\omega_2 a_1 a_2 \cos\theta_2 - \frac{G_o}{2\omega_1}\cos\theta_1 \quad (16b)$$

$$0 = -\mu_2 a_2 - \frac{3}{8}G_2\omega_2^2 a_2^3 - \frac{\alpha_2\omega_1^2 a_1^2}{4\omega_2}\sin\theta_2 \quad (16c)$$

$$(2\sigma_1 - \sigma_2)a_2 = -\frac{\alpha_2\omega_1^2 a_1^2}{4\omega_2}\cos\theta_2 \quad (16d)$$

Squaring equations (16a), (16b) and adding the result, we get the corresponding frequency response equations (FRE) are

$$a_1^2\sigma_1^2 + (\mu_1 a_1 + \frac{3}{8}\omega_1^2 G_1 a_1^3)^2 - \frac{G_o^2}{4\omega_1^2} - \frac{1}{16}\alpha_1^2\omega_2^2 a_1^2 a_2^2 + \frac{G_o\alpha_1\omega_2}{\omega_1}a_1 a_2 = 0 \quad (17)$$

Similarly, from equations (16c)and (16d), we get

$$a_2^2(2\sigma_1 - \sigma_2)^2 + (\mu_2 a_2 + \frac{3}{8}G_2\omega_2^2 a_2^3)^2 - (\frac{\alpha_2\omega_1^2 a_1^2}{4\omega_2})^2 = 0 \quad (18)$$

From equations (17) and (18) we have the following cases:

(i) Case 1 $a_1 \neq 0,\ a_2 = 0$

$$a_1^2\sigma_1^2 + (\mu_1 a_1 + \frac{3}{8}\omega_1^2 G_1 a_1^3)^2 - \frac{G_o^2}{4\omega_1^2} = 0 \quad (19)$$

(ii) Case 2 $a_1 = 0,\ a_2 \neq 0$

$$a_2^2(2\sigma_1 - \sigma_2)^2 + (\mu_2 a_2 + \frac{3}{8}G_2\omega_2^2 a_2^3)^2 = 0 \quad (20)$$

(iii) Case 3 $a_1 \neq 0,\ a_2 \neq 0$ represented by equations (17) and (18).The steady state solution of the obtained fixed points will be determined as follows:

Let A_o and B_o Expressed in Cartesian form as following:

$$A_0(T_1) = \frac{1}{2}(p_1 - iq_1)e^{i\nu_1 T_1}\ ,\ B_0(T_1) = \frac{1}{2}(p_2 - iq_2)e^{i\nu_2 T_1} \quad (21)$$

where p_n and q_n, (n=1,2) are real values and. Inserting equations (19) in to the linear form of equations (13a) ,(13b) and separating real and imaginary parts, the following system of equations is obtained as:

$$p_1' + \nu_1 p_1 - \eta_1 q_1 = 0 \quad (22a)$$

$$q_1' + \nu_1 q_1 + \eta_1 p_1 = 0 \quad (22b)$$

$$p_2' + \nu_2 p_2 - \eta_2 q_2 = 0 \quad (22c)$$

$$q_2' + \nu_2 q_2 + \eta_2 p_2 = 0 \quad (22d)$$

where $\eta_1 = \frac{\mu_1}{\omega_1},\ \eta_2 = \frac{\mu_2}{\omega_2}\ ,\ \nu_1 = \frac{\sigma_1}{\omega_1},\ \ \nu_2 = \frac{\sigma_2}{\omega_2}$

The stability of linear solution is investigated from the zero characteristics matrix

$$\begin{vmatrix} \lambda+\nu_1 & -\eta_1 & 0 & 0 \\ \eta_1 & \lambda+\nu_1 & 0 & 0 \\ 0 & 0 & \lambda+\nu_2 & -\eta_2 \\ 0 & 0 & \eta_2 & \lambda+\nu_2 \end{vmatrix} = 0$$

The eigen values are given by

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0$$

where, $r_1 = 2(\eta_1 + \eta_2),\ \ r_2 = \eta_1^2 + \eta_2^2 + 4\eta_1\eta_2 + \nu_1^2 + \nu_2^2$

$$r_3 = 2\eta_1\eta_2(\eta_1 + \eta_2) - 2\eta_2\nu_1^2 + 2\eta_1\nu_2^2, r_4 = (\eta_1^2 + \nu_1^2)(\nu_2^2 + \eta_2^2)$$

According to the Routh-Hurwitz criterion, the linear solution is stable if the following are satisfied

$$r_1 > 0, \quad r_1 r_2 - r_3 > 0, \quad r_3(r_1 r_2 - r_3) - r_1^2 r_4 > 0, \quad r_4 > 0$$

4- RESULTS AND DISCUSSION

A non –linear control low is proposed to suppress the vibration of the pitch-roll ship under modulated external and parametric excitation. The system is modeled by two second order non-linear ordinary differential equations and the

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control is based on cubic velocity feedback. The numerical solutions are given by Runge–kutta 4^{th} order method at non resonance case as shown in Fig. 2, which consider as basic case. From this figure it can be seen that the amplitudes of roll(x)and pitch (y)are about 0.02 and 0.012 respectively.

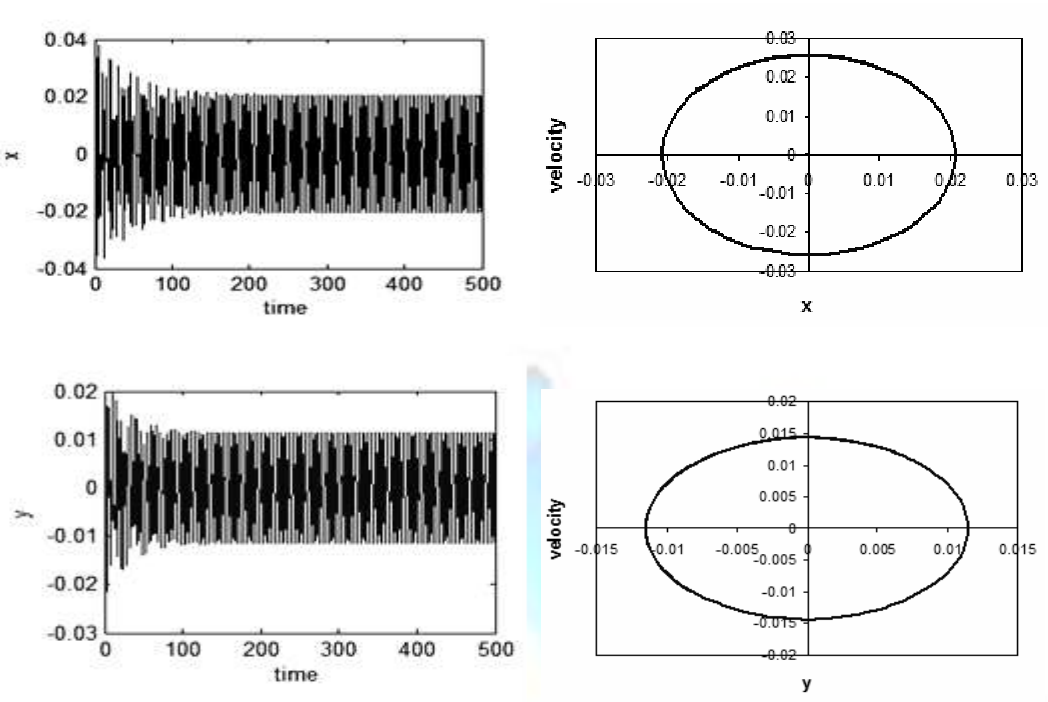


Fig.2. The Plot of basic case

4.1- Effect of parameters

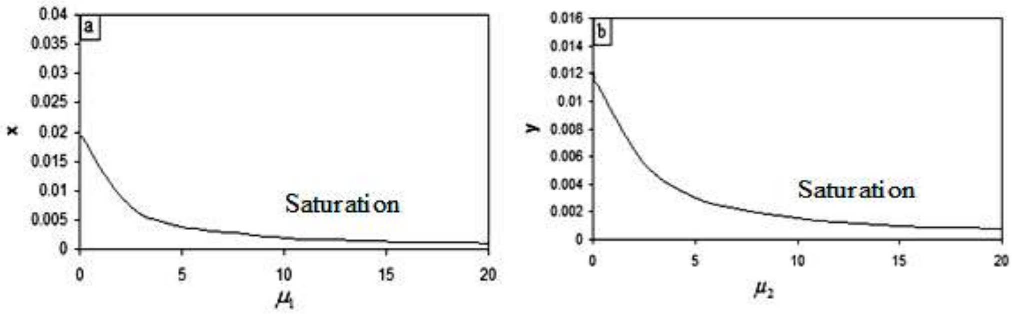
i) For the positive damping coefficients the amplitudes of roll and pitch are monotonic decreasing functions on μ_1 and μ_2 respectively as shown in Figs. 3a, 3b, and more increasing values of μ_1 and μ_2 leads to saturation phenomena.

ii) From Figs. 3c, 3d, we can see that the steady state amplitude of the roll and pitch are monotonic decreasing functions of the nonlinear parameters α_1 and α_2.

iii) The steady state amplitude of the roll and pitch are monotonic increasing functions of the excitation force amplitude G_o , F_o, $F_1$and F_2, which leads to the system is becomes un stable. as shown in Figs. 3e, 3f, 3g, 3h, and 3i respectively.

4.2- Effect of the control

From Figs. 3j and 3k, the steady state amplitude of the roll and pitch are monotonic decreasing functions of the gains of control G_1 and G_2 respectively. but more increasing of values G_1 and G_2 leads to saturation phenomena.



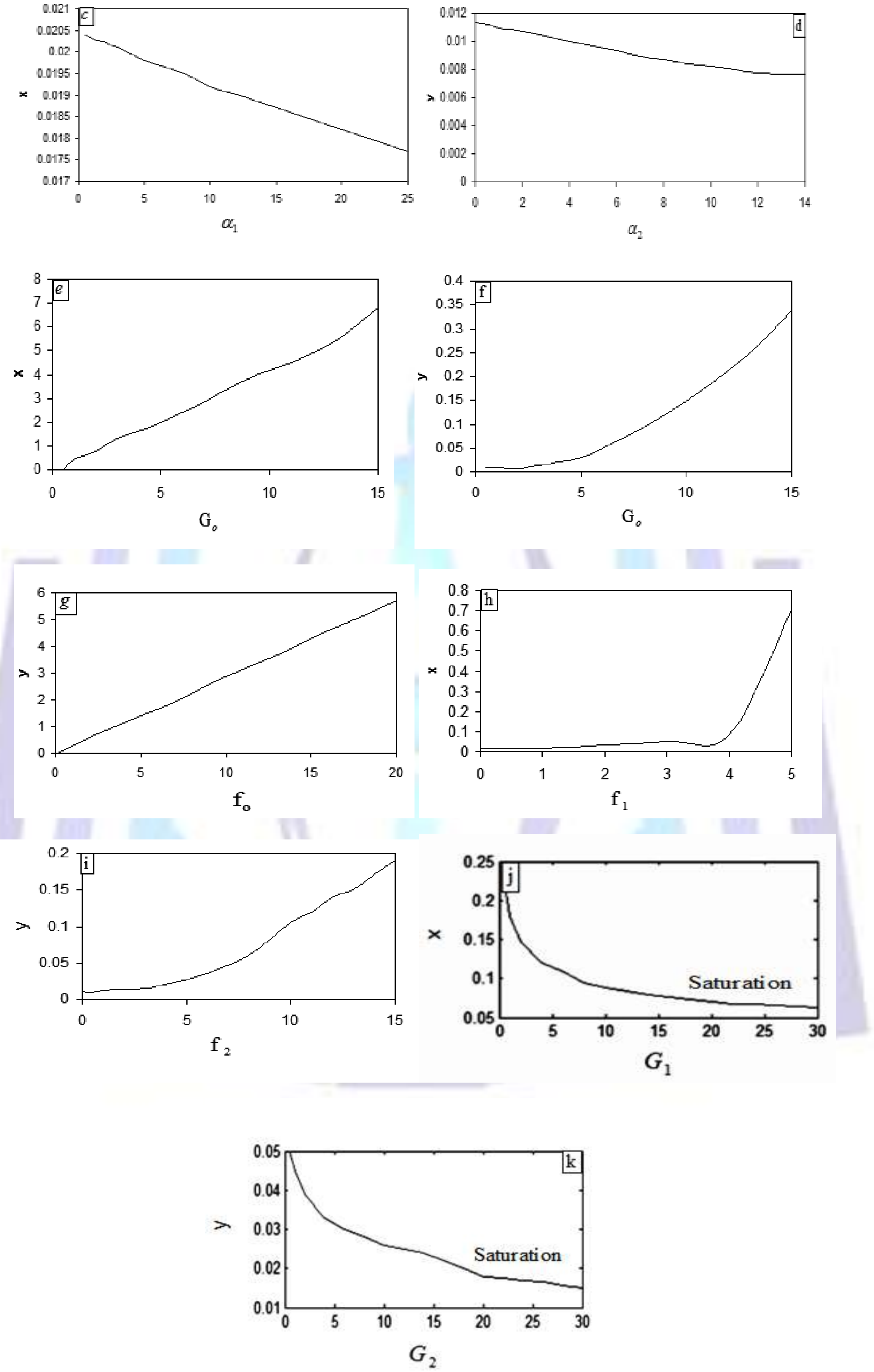


Fig.3. Effects of parameters

4.3- Resonance cases

All resonance cases of the system which are studied numerically obtain the worst case. From table 1, we find that the worst resonance case is simultaneous primary resonance $\Omega_1 \cong \omega_1, \omega_2 \cong 2\omega_1$, so we take this case to study the effect of control on the system Fig.4, shows the steady state amplitudes of the system at $\Omega_1 \cong \omega_1, \omega_2 \cong 2\omega_1$, without control ($G_1 = G_2 = 0$), from this figure we see that the amplitude of the roll is increased to about 0.67 (34 times) and the amplitude of the pitch is increased to about 0.064(6times) respectively, of the basic case which shown in Fig.2.

Fig.5 illustrate the response of the system with control, which shows that the steady state of the roll is decreased to about 0.14 and steady state of the pitch is decreased to about 0.005, which means that the control are effective and E_a is 4.7 and 12.8 respectively.

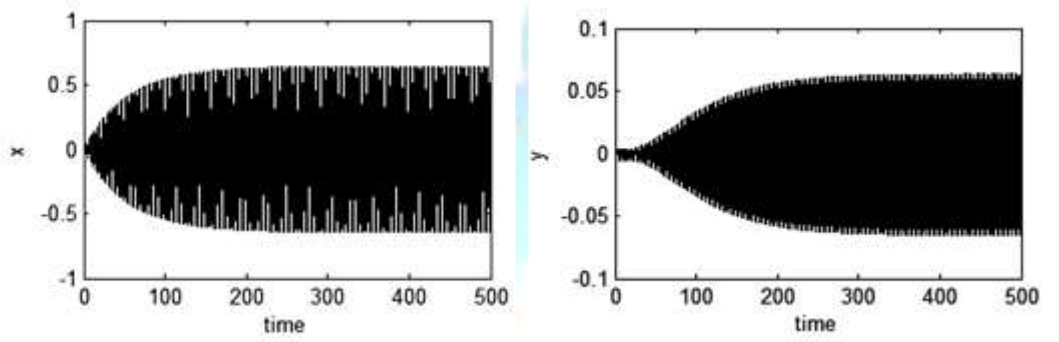


Fig. 4. Shows that the steady state amplitude without control at simultaneous primary resonance $\Omega_1 \cong \omega_1, \omega_2 \cong 2\omega_1$

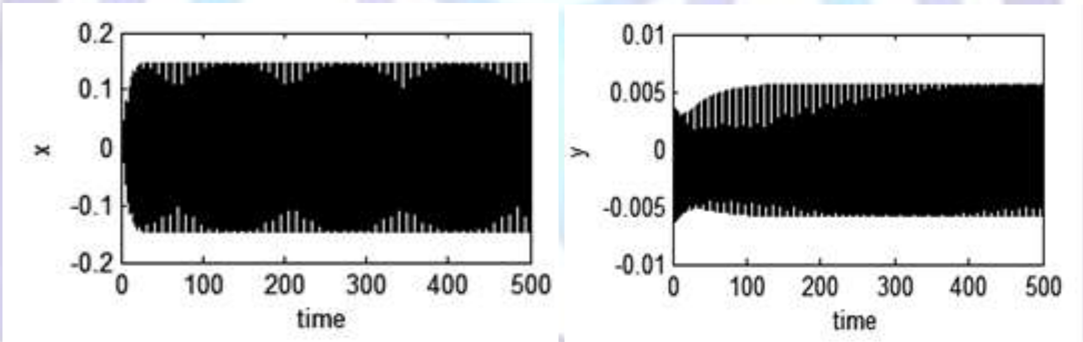


Fig. 5. The response for the system with absorber at the simultaneous primary resonance $\Omega_1 \cong \omega_1, \omega_2 \cong 2\omega_1$

To summaries the worst resonance cases with and without absorber

Table 1. Summary of worst resonance case

Cases	Without control		With control		E_a x	E_a y	Remarks
	x	y	x	y			
$\Omega_1 \cong \omega_1$	0.63	0.037	0.14	0.0368	4.5	1	Limit cycle
$\Omega_1 \cong \omega_2$	0.046	0.3	0.0458	0.10	1	3	Limit cycle
$\omega_1 \cong (\Omega_2 + \omega_2)/4$	0.99	0.020	0.22	0.012	4.5	1.7	Limit cycle
$\Omega_1 \cong \omega_1, \Omega_2 \cong \omega_1$	0.63	0.0358	0.14	0.0352	4.5	1	Limit cycle
$\Omega_1 \cong \omega_1, \omega_2 \cong \omega_1$	0.62	0.33	0.147	0.116	4.2	2.8	Limit cycle
$\Omega_1 \cong \omega_1, \omega_2 \cong 2\omega_1$	0.67	0.064	0.14	0.005	4.7	12.8	Multi Limit cycle
$\Omega_1 \cong \omega_1, \omega_1 \cong 3\omega_2/4$	0.65	0.018	0.14	0.012	4.6	1.5	Limit cycle
$\Omega_1 \cong \omega_1, \omega_2 \cong 3\Omega_2/2$	0.63	0.014	0.147	0.002	4.3	7	Limit cycle
$\Omega_2 \cong \omega_1, \omega_2 \cong 2\omega_1$	0.048	0.037	0.014	0.0112	3.4	3.3	Limit cycle
$\Omega_1 \cong \omega_1, \Omega_2 \cong \omega_1, \omega_2 \cong \omega_1$	0.63	0.33	0.136	0.12	4.6	2.75	Limit cycle

4.4- Frequency response curves

The frequency response equations 17-20 are nonlinear equations of the amplitudes of the roll (a_1) and the pitch (a_2) against the detuning parameters σ_1, σ_2 , which solved numerically as shown in Figs. (6-8) respectively. From these figures we find the amplitude of the roll is monotonic decreasing functions of the damping coefficient μ_1 and natural frequency ω_1 and the nonlinear parameter α_1 and the gain G_1 as shown in Figs. 6a, b, c and Figs. 8a, b, c. But the steady state amplitude of the roll(a_1) is monotonic increasing function in the excitation amplitude G_o as shown in Figs. 6d, 8d. Similarly, the steady state of the pitch (a_2) is monotonic decreasing function of the damping coefficient μ_2 and natural frequency ω_2 and the gain G_2 and the nonlinear parameter α_2 as shown in Figs. 7a, b, c and Figs. 8e, f, g, h which are a good agreement of the effect of parameter shown in Fig.2.

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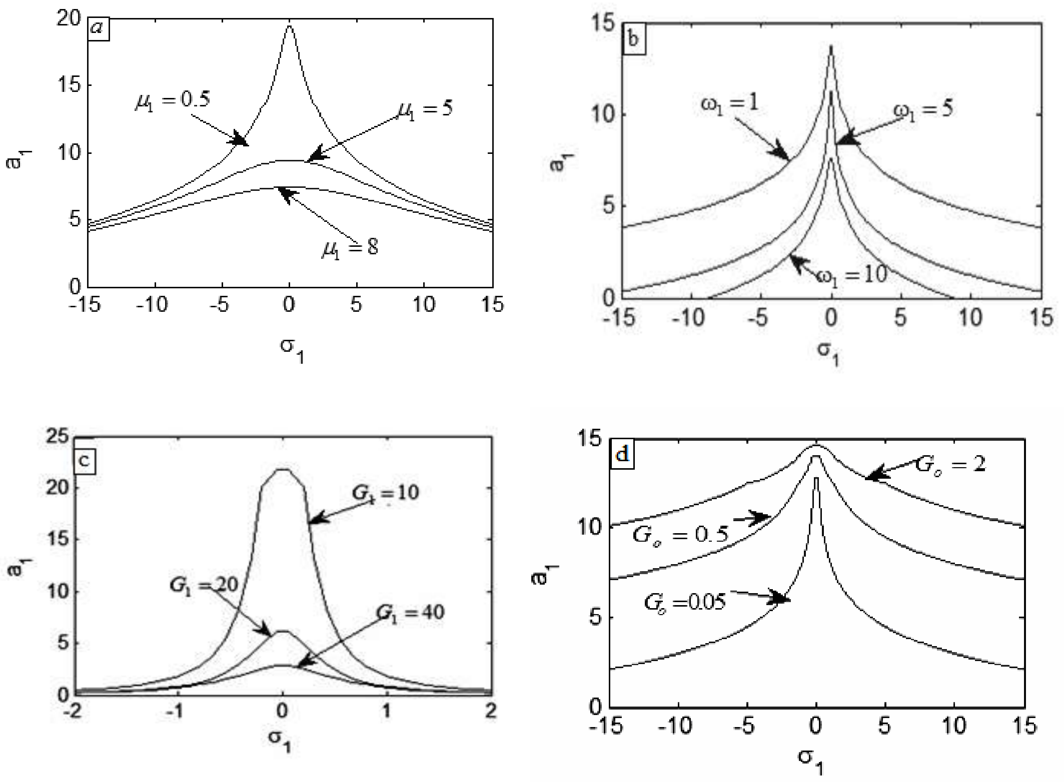

Fig.6 Frequency response curves of the first case ($a_1 \neq 0,\ a_2 = 0$)

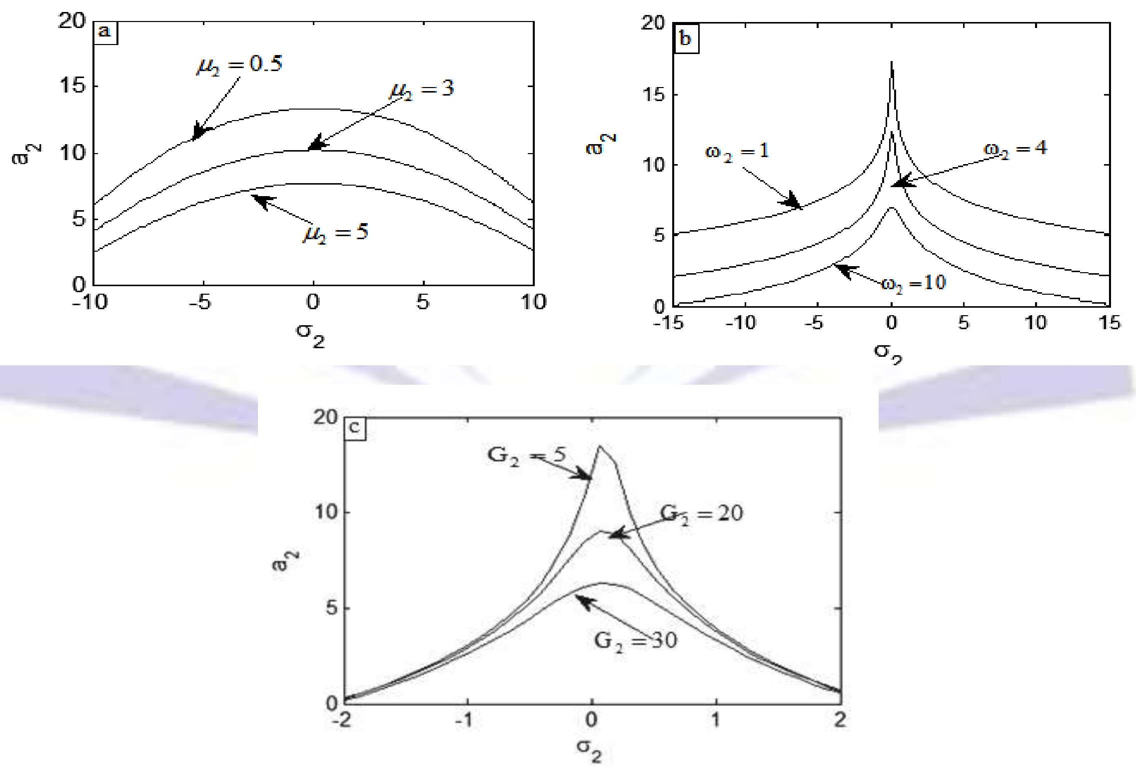

Fig. 7 Frequency response curves of the first case ($a_1 = 0,\ a_2 \neq 0$)

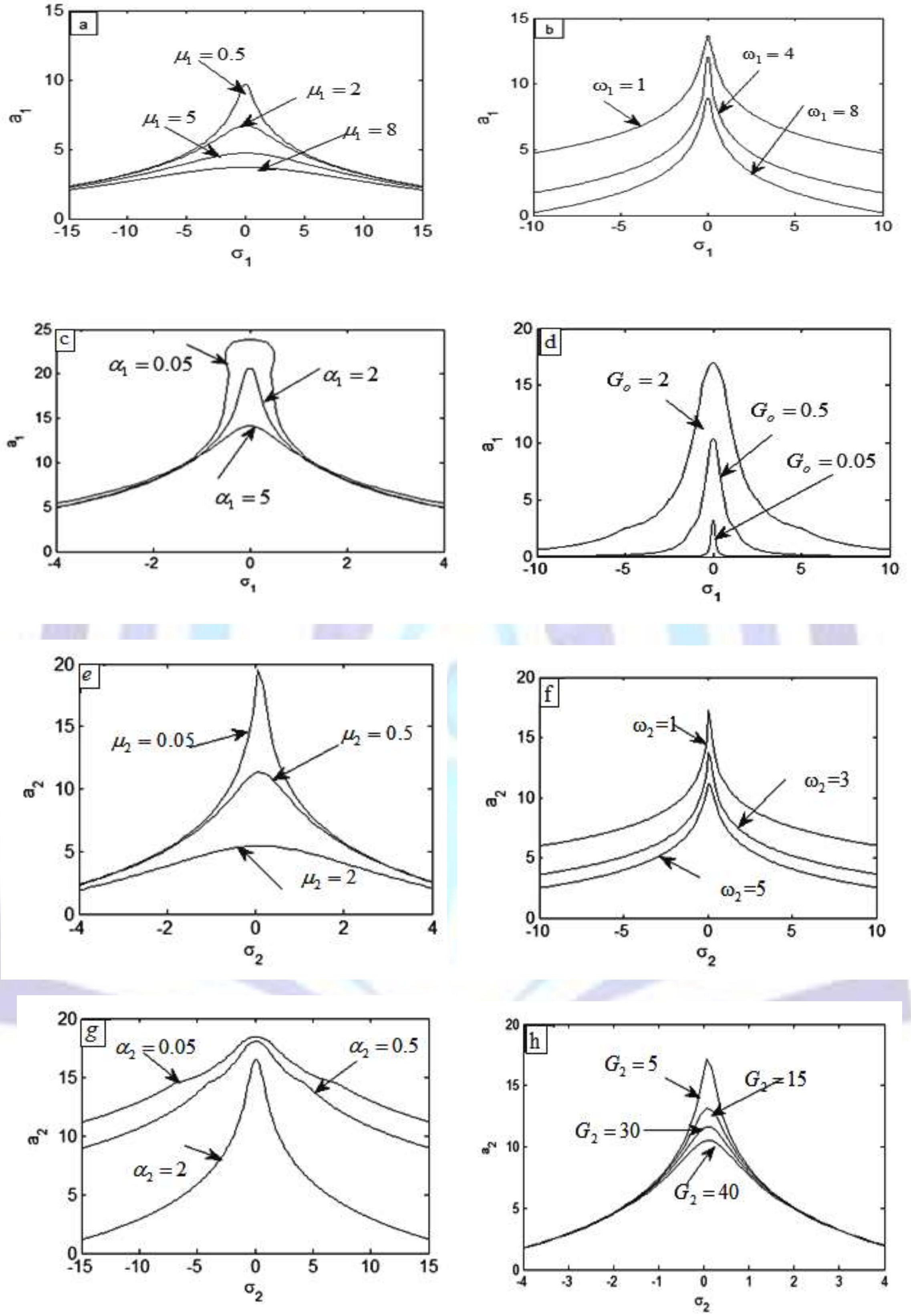

Fig.8. Frequency response curves of the first case $(a_1 \neq 0,\ a_2 \neq 0)$

5- CONCLUSIONS

The vibration of second order coupled system simulating the vibration of the roll-ship motion subjected to excitation forces is studied at the simultaneous primary resonance case. A control low based on cubic velocity feedback is proposed. The method of multiple scales is used to obtain the approximate solution of the system. The stability and effect of parameters are studied numerically. From the above study the following may be concluded

1- For the positive damping coefficients the amplitudes of roll and pitch are monotonic decreasing functions on μ_1 and μ_2 .

2- The steady state amplitude of the roll and pitch are monotonic decreasing functions of the nonlinear parameters α_1 and α_2.

3- The steady state amplitude of the roll and pitch are monotonic increasing functions of the excitation force amplitude G_o , F_o, F_1 and F_2, which leads to the system is unstable.

4- The worst resonance case is simultaneous primary resonance $\Omega_1 \cong \omega_1, \omega_2 \cong 2\omega_1$, which the amplitudes of the roll and the pitch are increased to about 0.67 (34 times) and 0.064 (6times) respectively, of the basic case .

5-The steady state amplitude of the roll(a_1) is monotonic decreasing functions of the gain G_1, the steady state of the pitch (a_2) is monotonic decreasing function of the gain G_2

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