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Generalization of a fixed point theorem of Suzuki type in complete convex space

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Abstract. The aim of this paper is to generalize a fixed point theorem given by Popescu[20]. We also complement and extend some very recent results proved by Suzuki [T. Suzuki, A generalized Banach contraction principle that characterizes metric completeness, Proc. Amer. Math. Soc. 136 (2008) 1861 - 1869]. We furnish an example to validate our result.

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1. Introduction

Let (X, d) be a metric space, T a self-mapping on X and k a nonnegative real number such that the inequality $d(Tx,Ty) \le kd(x,y)$ holds for any $x, y \in X$. If $k < 1$ then T is said to be a contractive mapping and if k = 1, then T is said to be a nonexpansive mapping. The Banach theorem states that if X is complete, then every contractive mapping has a unique fixed point. There exists an ample vast literature about contractive and nonexpansive type mappings, where the contractive and nonexpansive conditions are replaced with more general conditions (see, for instance [1 - 13]).

In 1980 Gregus[15] proved the following result.

Theorem 1.1. Let C be a nonempty closed convex subset of a Banach space B and $T : C \to C$ a mapping that satisfies

$$\|Tx - Ty\| \le a\|x - y\| + b[\|x - Tx\| + \|y - Ty\|], \qquad (1)$$

for all $x, y \in C$, where $a > 0$, $b > 0$ and $a + 2b = 1$. Then T has a unique fixed point.

This result motivated many authors in further investigations. Abdeljawad and Karapinar[1], Ciric [4], Delbosco et al.[10], Diviccaro et al.[12], Fisher[13], Fisher and Sessa[14], Jungck[16], Li[17], Murthy et al.[18], Mukherjee and Verma[19], Raswan and Ahmed[22]. Ciric[5] has constructed an example to show that if the mapping T satisfies Bogin result 1.1[21] with b = 0 and if a and c are such that (1) holds, then T need not have a fixed point. The following remarkable generalization of the classical Banach contraction theorem, due to Suzuki[26], has lead to some important contributions in metric fixed point theory $(see,\ for\ instance, [11,20,23,25-28])$.

In 2009, Popescu[21] generalizes Theorem 1.1.

Theorem 1.2 Let X be a complete convex metric space and let $T : X \to X$ be a selfmapping. Assume that $1/2d(x,Tx) \le d(x,y)$

which implies $d(Tx,Ty) \le ad(x,y) + bd(x,Tx) + bd(y,Ty),$

where $a > 0$, $b > 0$ and and $a + 2b = 1$. Then T has a unique fixed point.

Recently, Tiwari et al. [29] proved a common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying an integral type contractive condition.

In this paper, we establish a common fixed point theorem of Suzuki type in complete convex metric space which generalizes Theorem 2.4 Popescu[21].

Now, we require following definition to prove our next result.

Definition 1.3 (Takahashi[28]) Let (X, d) be a metric space. A mapping $W : X \times X \times [0,1] \to X$ is said to be a convex structure on X if for each $(x, y, \lambda) \in X \times X \times [0,1]$ and $u \in X$ $d(u,W(x,y,\lambda)) \le \lambda d(u,x) + (1-\lambda)d(u,y)$. The metric space X together with W is called a convex metric space. It is obvious that in a convex metric space we have $d(x,W(x,y,\lambda)) = (1-\lambda)d(x,y)$ and $d(y,W(x,y,\lambda)) = \lambda d(x,y)$.

Now we prove a generalization of Popescu[21] theorem.

2. Main results

Theorem 2.1. Let X be a complete convex metric space and let $T : X \to X$ be a selfmapping. Assume that $1/2d(x,Tx) \le d(x,y)$ which implies

$$\begin{aligned} &d(Tx,Ty) + p\,max\{d(x,y), d(x,Tx) + d(y,Ty)\} \\ &\le ad(x,y) + bd(x,Tx) + bd(y,Ty), \qquad (2) \end{aligned}$$

where $a > 0$, $b > 0$, $p \ge 0$ and $a + 2b - p = 1$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$, $u_n = T^n x_0$ and $d_n = d(u_n, u_{n+1})$. Since $1/2d(T^n x_0, T^{n+1}x_0) \le d(T^n x_0, T^{n+1} x_0)$, we have from the assumption that

$$d_{n+1} + p\,max\{d_n, d_n + d_{n+1}\} \le ad_n + bd_n + bd_{n+1}.$$

Thus, $d_{n+1} \leq \dfrac{a+b-p}{1-b+p} d_n = d_n$ for all $n \geq 0$. So, d_n is a decreasing sequence.

Let $t = W(Tx_0, T^2x_0, 1/2)$ and $z = W(T^2x_0, T^3x_0, 1/2)$. Suppose that $d(Tx_0, z) < 1/2d(Tx_0, T^2x_0)$.

Since $d(T^2x_0, z) = 1/2d(T^2x_0, T^3x_0) \leq 1/2d(Tx_0, T^2x_0),$ it follows that

$$d(Tx_0, T^2x_0) \leq d(Tx_0, z) + d(T^2x_0, z)$$

$$< d(Tx_0, T^2x_0),$$

which is a contradiction.

Thus we have $d(Tx_0, z) \geq 1/2d(Tx_0, T^2x_0)$. From the assumption we obtain

$$d(T^2x_0, Tz) + p\, max\{d(Tx_0, z), d(Tx_0, T^2x_0) + d(z, Tz)\}$$

$$\leq ad(Tx_0, z) + bd(Tx_0, T^2x_0) + bd(z, Tz).$$

Since

$$1/2d(T^2x_0, T^3x_0) = d(T^2x_0, z) \leq d(T^2x_0, z),$$

we also obtain,

$$d(T^3x_0, Tz) + p\, max\{d(T^2x_0, z), d(T^2x_0, T^3x_0) + d(z, Tz)\}$$

$$\leq ad(T^2x_0, z) + bd(T^2x_0, T^3x_0) + bd(z, Tz).$$

Therefore,

$$d(z, Tz) \leq 1/2d(Tz, T^2x_0) + 1/2d(Tz, T^3x_0)$$

$$\leq 1/2[ad(Tx_0, z) + bd(Tx_0, T^2x_0) + bd(z, Tz)$$

$$-p\, max\{d(Tx_0, z), d(Tx_0, T^2x_0) + d(z, Tz)\}]$$

$$+1/2[ad(T^2x_0, z) + bd(T^2x_0, T^3x_0)$$

$$+bd(z, T_z) - p\, max\{d(T^2x_0, z), d(T^2x_0, T^3x_0) + d(z, Tz)\}].$$

Thus we get

$$(2-2b)d(z, Tz) \leq ad(z, Tx_0) + ad(z, T^2x_0) + 2bd(x_0, Tx_0) - 2pd(z, Tz) - 2pd(x_0, Tx_0)$$

or

$$(2-2b+2p)d(z, Tz) \leq ad(z, Tx_0) + ad(z, T^2x_0) + 2bd(x_0, Tx_0) - 2pd(x_0, Tx_0)$$

and so,

$$d(z, Tz) \leq \frac{a}{1+a+p} d(z, Tx_0) + \frac{a}{2(1+a+p)} d(T^2x_0, T^3x_0) + \frac{2b-2p}{1+a+p} d(x_0, Tx_0)$$

$$\leq \frac{a}{1+a+p}d(z,Tx_0)+\frac{a}{2(1+a+p)}d_0+\frac{1-a-p}{1+a+p}d_0$$

$$=\frac{a}{1+a+p}d(z,Tx_0)+\frac{2-a-2p}{2(1+a+p)}d_0.$$

Next we consider two cases.

Case 1 Suppose that $1/2d(x_0,Tx_0)\leq (x_0,T^2x_0).$ By the assumption we get

$$d(T^3x_0,Tx_0)+p\,max\{d(x_0,T^2x_0),d(x_0,Tx_0)+d(T^2x_0,T^3x_0)\}$$

$$\leq ad(x_0,T^2x_0)+bd(x_0,Tx_0)+bd(T^2x_0,T^3x_0)$$

$$\leq a[d(x_0,Tx_0)+d(Tx_0,T^2x_0)]+2bd_0-2pd_0$$

$$\leq (2a+2b-2p)d_0=(1+a-p)d_0.$$

Since

$$d(z,Tx_0)\leq 1/2d(Tx_0,T^2x_0)+1/2d(Tx_0,T^3x_0),$$

we obtain

$$d(z,Tx_0)\leq 1/2(2+a-p)d_0.$$

Therefore

$$d(z,T_z)\leq \frac{a}{1+a+p}\frac{2+a-p}{2}d_0+\frac{2-a-2p}{2(1+a+p)}d_0$$

$$=\frac{a^2-2(a-p)+2-a}{2(1+a+p)}d_0.$$

Since $a^2-2(a-p)+2-a<2(1+a+p)$, taking $k_1=\frac{a^2-2(a-p)+2-a}{2(1+a+p)}$ we have $k_1<1$ and $d(z,T_z)\leq k_1d_0$.

Case 2 Suppose that $1/2d(x_0,Tx_0)>d(x_0,T^2x_0)$. We will show that there exists $k_2<1$ such that $d(t,Tt)\leq k_2d_0$. Assume that $d(x_0,t)<1/2d(x_0,Tx_0)$.

If $d(Tx_0,t)\leq 1/2d(x_0,Tx_0),$ then we have

$$d(x_0,Tx_0)\leq d(x_0,t)+d(Tx_0,t)<d(x_0,Tx_0).$$

which is a contradiction. So, we get $d(Tx_0,t)>1/2d(x_0,Tx_0)$. Since $d(Tx_0,t)=1/2d(Tx_0,T^2x_0)$, we obtain that $d(Tx_0,T^2x_0)>d(x_0,Tx_0)$, which is also a contradiction. Therefore, we must have $1/2d(x_0,Tx_0)\leq d(x_0,t)$. By the assumption we get

$$d(Tx_0,Tt)+p\,max\{d(x_0,t),[d(t,Tt)+d(x_0,Tx_0)]\}$$

$$\leq ad(x_0,t)+bd(t,Tt)+bd(x_0,Tx_0).$$

Since $1/2d(Tx_0,T^2x_0)=1/2d(x_0,t)\leq d(x_0,t)$, we also get

$$d(T^2x_0,Tt)+p\,max\{d(Tx_0,t),d(t,Tt)+d(Tx_0,T^2x_0)\}$$

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$$\leq ad(Tx_0,t)+bd(t,Tt)+bd(Tx_0,T^2x_0).$$

Hence we obtain

$$d(t,Tt)\leq 1/2d(Tx_0,Tt)+1/2d(T^2x_0,Tt)$$

$$\leq 1/2[ad(x_0,t)+bd(t,Tt)+bd(x_0,Tx_0)-p\,max\{d(x_0,t),d(t,Tt)+d(x_0,Tx_0)\}]$$

$$+1/2[ad(Tx_0,t)+bd(t,Tt)+bd(Tx_0,T^2x_0)$$

$$-p\,max\{d(Tx_0,t),d(t,Tt)+d(Tx_0,T^2x_0)\}].$$

Thus

$$(1-b+p)d(t,Tt)\leq a/2d(x_0,t)+a/2d(Tx_0,t)+bd(x_0,Tx_0)-pd(x_0,Tx_0)$$

$$=a/2d(x_0,t)+a/4d(Tx_0,T^2x_0)+bd(x_0,Tx_0)-pd(x_0,Tx_0)$$

$$\leq a/2d(x_0,t)+(a/4+b-p)d_0.$$

Since

$$d(x_0,t)\leq 1/2d(x_0,Tx_0)+1/2d(x_0,T^2x_0)<1/2d_0+1/4d_0$$

$$=3/4d_0,$$

we obtain $(1-b+p)d(t,Tt)\leq(3a/8+a/4+b-p)d_0$ which implies $d(t,Tt)\leq\frac{5/8a+b-p}{1-b+p}d_0$.

But $5/8a+b-p<1-b+p$, so taking $k_2=\frac{5/8a+b-p}{1-b+p}$ we have $k_2<1$ and $d(t,Tt)\leq k_2d_0$.

Hence in all cases there exists $k<1$ and $y\in X$ such that $d(y,Ty)<kd(x_0,Tx_0)$. Therefore $inf\,\{d(x,Tx):x\in X\}=0$.

Next, we will prove that the infimum is attained. Take the following system of sets: $K_n:=x\in X:d(x,Tx)\leq r/(2n)$, TK_n and $\overline{TK_n}$, where $\overline{TK_n}$ is the closure of $TK_n, r=(1-a+p)/(1+b-p)$, $n\geq 1$. For any $x,y\in K_n$ such that $1/2d(x,Tx)\leq d(x,y)$ we have

$$d(x,y)\leq d(x,Tx)+d(Tx,Ty)+d(y,Ty)-p\,max\{d(x,Tx),d(Tx,Ty),d(y,Ty)\}$$

$$\leq\frac{r}{2n}+ad(x,y)+bd(x,Tx)+bd(y,Ty)+\frac{r}{2n}-p\,max\{d(x,Tx)+d(y,T_y)\}$$

$$\leq ad(x,y)+\frac{(1+b)r}{n}-2pd(x,y)$$

$$\leq ad(x,y)+\frac{(1+b)r}{n}-p\frac{r}{n}.$$

Thus

$$d(x,y)\leq\frac{1+b-p}{1-a}.\frac{r}{n}$$

$$=\frac{1}{n}.$$

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If $1/2d(x,Tx) > d(x,y)$ we have $d(x,y) < \frac{1}{2}.\frac{r}{2n} < \frac{1}{n}$.

Hence $d(x,y) \le 1/n$ for all $x, y \in K_n$, that is $diam(K_n) \le 1/n$. Since $d(Tx,T^2x) \le (x,Tx)$ we have $T(K_n) \subseteq K_n$ and then, $diam(T(K_n)) = diam(T(K_n)) \le 1/n$.

Supposing $y \in K_n$ then for any $\varepsilon > 0$ there exists $y' \in K_n$ such that $d(y,Ty') < \varepsilon.$

If $d(y,y') < 1/2d(y',Ty')$ and $d(y,Ty') < 1/2d(Ty',T^2y')$, then

$$d(y',Ty') < d(y,y') + d(y,Ty')$$

$$< 1/2[d(y',Ty') + d(Ty',T^2y')] \le d(y',Ty').$$

which is a contradiction. Hence, either $d(y,y') \le 1/2d(y',Ty')$ or $d(y,Ty') \le 1/2d(Ty',T^2y')$.

In the first case, we have by the assumption

$$d(Ty,Ty') + p\, max\{d(y,y'), d(y,Ty) + d(y',Ty'\}$$

$$\le ad(y,y') + bd(y,Ty) + bd(y',Ty').$$

Thus

$$d(y,Ty) \le d(y,Ty') + d(Ty,Ty')$$

$$\le \varepsilon + ad(y,y') + bd(y,Ty) + \frac{br}{2n} - pd(y,Ty) - p\frac{r}{2n}$$

$$\le \varepsilon + a[d(y,Ty') + d(Ty',y')] + (b-p)d(y,Ty) + \frac{br}{2n} - \frac{pr}{2n}.$$

Hence

$$d(y,Ty) \le \frac{(a+1)\varepsilon}{1-b+p} + \frac{b-p}{1-b+p}\frac{r}{2n} < \frac{a+3}{1-b+p}\varepsilon + \frac{r}{2n}.$$

In the second case, from the assumption we obtain

$$d(Ty,T^2y') + p\, max\{d(y,Ty'), d(y,Ty), bd(Ty',T^2y')\}$$

$$\le ad(y,Ty') + bd(y,Ty) + bd(Ty',T^2y').$$

But $d(Ty',T^2y') \le 2d(y,Ty') = 2\varepsilon$. And then

$$d(y,Ty) \le d(y,T^2y') + d(T^2y',Ty)$$

$$\le d(y,Ty') + d(Ty',T^2y') + d(T^2y',Ty)$$

$$\le \varepsilon + 2\varepsilon + ad(y,Ty') + (b-p)d(y,Ty) + (b-p)\frac{r}{2n}$$

$$\le \varepsilon + 2\varepsilon + a\varepsilon + (b-p)d(y,Ty) + (b-p)\frac{r}{2n}$$

$$\le (a+3)\varepsilon + (b-p)d(y,Ty) + \frac{br}{2n} - \frac{pr}{2n}.$$

Hence

$$d(y,Ty) \le \frac{a+3}{1-b+p}\varepsilon + \frac{b-p}{1-b+p}\frac{r}{2n} < \frac{a+3}{1-b+p}\varepsilon + \frac{r}{2n} + \frac{r}{2n}.$$

Therefore, in all cases we proved that

$$d(y,Ty) \le \frac{a+3}{1-b+p}\varepsilon + \frac{r}{2n}.$$

Since $\varepsilon > 0$ is arbitrary, we get $d(y,Ty) \le r/(2n)$, i.e. $y \in K_n$. Hence we have $T(K_n) \subseteq K_n$. Therefore, $\{\overline{T(K_n)}\}$ is a decreasing sequence of closed nonempty sets with diam $(\overline{T(K_n)}) \to 0$ as $n \to \infty$. Thus they have a nonempty intersection. Since $\overline{T(K_n)} \subseteq K_n$, we obtain that there exists $u \in K_n$ for all n. This implies that u is a fixed point of T. If v is another fixed point of T, since $1/2d(u,Tu) = 0 \le d(u,v)$ we have

$$d(u,v) = d(Tu,Tv) \le ad(u,v) + (b-p)d(u,u) + (b-p)d(v,v)$$
$$= ad(u,v).$$

which is a contradiction.

Remark 2.2. If we put $p = 0$ in above, we get Theorem 2.4 of Popescu[21]. Now we finish the following to validate our result.

Example 2.3. Let $X = [-1,1]$ with the usual metric and let $T : X \to X$ be given as

$$Tx = \begin{cases} -x, & if \quad x \in [0\,,3/4) \bigcup (3/4,1] = U\,; \\ \frac{x}{2}, & if \quad x \in [-1\,,0) = V\,; \\ -1/6, & if \quad x = \frac{3}{4}. \end{cases}$$

We will prove that:

1. T has a unique fixed point.

2. T satisfies condition (4) with $a = 3/4, b = 1/4, p = 1/8$

i.e., $d(x,Tx)/2 \le d(x,y) \Rightarrow d(Tx,Ty) \le m(x,y)$, where $m(x,y) = \frac{3}{4}d(x,y) + \frac{1}{4}[d(x,Tx) + d(y,Ty)] - p\,max\{d(x,y), d(x,Tx) + d(y,Ty)\}$.

3. T does not satisfy Suzuki condition from Theorem 1.3 of Popescu[21].

4. T does not satisfy Gregus condition (2) with $a = \frac{3}{4}, b = \frac{1}{4}, p = \frac{1}{8}$.

Proof. 1 is obvious. Next, we consider the following to prove 2.

• For $x, y \in U$ then

$$m(x,y) = \frac{3}{4}|y-x| + \frac{1}{4}|2y+2x| - \frac{1}{8}|2y+2x| = \frac{3}{4}|y-x| + \frac{1}{4}|2y+2x| - (1/8)|2y+2x| = d(Tx,Ty)$$

$m(x,y) = (21/24)|y-x|$ and $d(Tx,Ty) = |y-x| \le m(x,y)$ and (2) holds.

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• If $x, y \in V$, then $m(x,y) = \frac{3}{4}|y-x| + \frac{1}{8}\frac{x+y}{2}$ and $d(Tx,Ty) = \frac{1}{2}|y-x| \le m(x,y)$ so (2) holds.

• If $x \in U$, $y \in V$, then $m(x,y) = \frac{3}{4}|(y-x)| + \frac{1}{8}|2x + \frac{y}{2}| = \frac{13}{16}y - \frac{1}{2}x$ and $d(Tx,Ty) = x + \frac{y}{2}$

Since $x \ge 0$, $y < 0$ we have $\frac{13}{16}y - \frac{1}{2}x \ge x + \frac{y}{2}$

and $\frac{13}{16}y - \frac{1}{2}x \ge -x - \frac{y}{2}$, so (2) holds.

• If $x \in V$, $y \in U$, then $m(x,y) \ge d(Tx,Ty)$ like in (iii).

• For $x \in U$, $y = \frac{3}{4}$, then $m(x,y) = \frac{3}{4}|x - \frac{3}{4}| + \frac{2x}{8} + \frac{11}{96}$, $d(Tx,Ty) = |x - \frac{1}{6}|$. Since $\frac{1}{2}d(x,Tx) \le d(x,y)$ we have $x \le |x - \frac{3}{4}|$, so $x \le \frac{3}{8}$. Therefore $|x - \frac{1}{6}| \le \frac{1}{6}$ and $|x - \frac{3}{4}| \ge \frac{3}{8}$ Hence $m(x,y) \ge \frac{9}{32} + \frac{6}{64} - \frac{11}{96} = \frac{50}{192} > d(Tx,Ty)$ then $d(Tx,Ty) \le m(x,y)$ and (2) holds.

• For $x \in V$, $y = \frac{3}{4}$, then $m(x,y) = \frac{3}{4}(\frac{3}{4} - x) + \frac{1}{8}|\frac{x}{2}| + \frac{1}{8}(\frac{3}{4}) + \frac{1}{6} \ge \frac{9}{16} > \frac{1}{6}$ and $d(Tx,Ty) = |\frac{x}{2} + \frac{1}{6}| \ge \frac{1}{6} (\frac{x}{2} \in [-\frac{1}{2}, 0))$ (2) holds.

• If $x = \frac{3}{4}$, $y \in U$, then $m(x,y) = \frac{3}{4}|\frac{3}{4} - y| + \frac{1}{8}(\frac{3}{4} + \frac{1}{6}) + \frac{2y}{8}$ and $d(Tx,Ty) = |y - \frac{1}{6}|$. By $\frac{1}{2}d(x,Tx) \le d(x,y)$ we have $\frac{1}{2}(\frac{3}{4} + \frac{1}{6}) = \frac{11}{24} \le |y - \frac{3}{4}|$ So, $y \le \frac{17}{24}$. Therefore $m(x,y) \ge \frac{3}{4}.\frac{11}{24} > \frac{1}{6}$ and $d(Tx,Ty) \le \frac{1}{6}$. Hence (2) holds.

• If $x = \frac{3}{4}$, $y \in V$, then

$$m(x,y) \ge \frac{3}{4}d(Tx,Ty) = \frac{3}{4}(\frac{3}{4} - y) \ge \frac{9}{16} > \frac{1}{6} \text{ and } d(Tx,Ty) = |-\frac{1}{6} - \frac{y}{2}| \le \frac{1}{6} (\frac{y}{2} \in [-\frac{1}{2}, 0))$$ Hence (2) holds.

• If $x = y$ then (2) is obvious.

• If $x = 0$, $y = 1$, then $\theta(r)d(x,Tx) = 0 < 1 = d(x,y)$ and $d(Tx,Ty) = 1$, so condition from Theorem 1.3 of Popescu[21]. does not hold.

• If $x = \frac{3}{4}$, $y = 1$ we have $d(Tx,Ty) = \frac{2}{3}$ and $m(x,y) = \frac{3}{16} + \frac{1}{16} + \frac{1}{8}(\frac{3}{4} + \frac{1}{8}) = \frac{35}{96} < \frac{64}{96} = \frac{2}{3}$ so $d(Tx,Ty) > m(x,y)$. Therefore Gregus condition (1) does not hold.

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