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FIRST – ORDER STRONG DIFFERENTIAL SUBORDINATION AND SUPERORDINATION PROPERTIES FOR ANALYTIC FUNCTIONS ASSOCIATED WITH LINEAR OPERATOR

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ABSTRACT

In the present paper we study of some properties first - order strong differential subordination and superordination for analytic functions associated with Ruscheweyh derivative operator which are obtained by considering suitable classes of admissible functions.

Indexing terms/Keywords

Analytic function; univalent function; strong differential subordination; strong differential superordination; Ruscheweyh derivative; admissible functions; Hadmard product.

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1- INTRODUCTION AND PRELIMINARIES

Let $\mathcal{H}(U)$ denote the class of analytic function in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{H}[a, n]$ be the subclass of $\mathcal{H}(U)$ consisting of the functions of the form $f(z) = a + a_n z^n + a_{n+1} z^{n+1}$ ..., where $a \in \mathbb{C}$ and $n \in N$. Also $\mathcal{H}_0 = \mathcal{H}[0,1]$ and $\mathcal{H}_1 = \mathcal{H}[1,1]$.

Let A denote the subclass of functions of $\mathcal{H}(U)$ consisting of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n , \qquad (z \in U) \tag{1.1}$$

which are normalized analytic univalent in U.

Let $f, g \in \mathcal{H}(U)$, we say that a function f is subordinate to g or g is said to be superordinate to f, if there exists a Schwarz function $w(z)$ which is analytic in U, with $w(0) = 0$, $|w(z)| < 1$, $for\ all\ (z \in U)$, such that $f(z) = g(w(z))$. In such case, we write $f \prec g$ or $f(z) \prec g(z)$.Furthermore, if the function g is univalent in U, then we have the following equivalent (see [3,7]) :

$f(z) \prec g(z), (z \in U) \Leftrightarrow f(0) = g(0)$ and $f(U) \subset g(U)$.

For the function f given by (1.1) and the function g given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n , (z \in U),$$

the Hadmard product (or convolation) of f and g is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z).$$

Now, for functions $f(z) \in A$ in the form (1.1) we define the Ruscheweyh derivative operator [14], $D^{\lambda} f(z) : A \to A$ as follows $D^0 f(z) = f(z), D' f(z) = z f'(z)$, $D^{\lambda} f = f(z) * \frac{1}{(1-z)^{\lambda+1}}$

$$= z + \sum_{n=2}^{\infty} B_n(\lambda) a_n z^n , \tag{1.2}$$

where $B_n(\lambda) = \frac{(\lambda+1)(\lambda+2)\dots(\lambda+n-1)}{(n-1)!}, \lambda > -1, z \in U.$

We note that

$$z\left(D^{\lambda} f(z)\right)' = (\lambda + 1) D^{\lambda+1} f(z) - \lambda D^{\lambda} f(z), \tag{1.3}$$

such type of study was carried out by several different authors, like Dinggong and Liu [4], and Lupus [5].

The notion of differential superordination was introduced in 2003 [8] by Miller and Mocanu as a dual concept differential subordination in 2000 [7]. The notion of strong differential superordination was introduced by Antonino and Romaguera in 2006 [2] as a dual concept differential subordination in 1994 [1] which were developed by (G.I. Oros, 2007 [9]), (G.I. Oros, Oros, 2009 [10]) and (G.I. Oros, and Oros, 2009[11]).

To prove our main results, we need the following definitions and Lemmas.

DIFFINTION 1.1 [9,12] Let $H(z, \xi)$ be analytic in $U \times \overline{U}$ and let $f(z)$ analytic and univalent in U. The function $H(z, \xi)$ is strongly subordinate to $f(z)$ written $H(z, \xi) \prec\prec f(z)$, or $f(z)$ is said to be strongly superordinate to $H(z, \xi)$, written $f(z) \prec\prec H(z, \xi)$ if for $\xi \in \overline{U}$, $H(z, \xi)$ as a function of z is subordinate to $f(z)$. We note that : $H(z, \xi) \prec\prec f(z) (z \in U, \xi \in \overline{U}) \Leftrightarrow H(0, \xi) = f(0)$ and $H(U \times \overline{U}) \subset f(U)$ and if $H(z, \xi)$ is univalent, then : $f(z) \prec\prec H(z, \xi), (z \in U, \xi \in \overline{U}) \Leftrightarrow f(0) = H(0, \xi)$ and $f(U) \subset H(U \times \overline{U})$.

DEFINITION 1.2 [9,13] Let $\phi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$ and let $h(z)$ be univalent in U. If $p(z)$ is analytic in U for all $\xi \in \overline{U}$ and satisfies the following (first – order) strong differential subordination.

$$\phi(p(z), zp'(z); z; \xi) \prec\prec h(z), \qquad (z \in U, \xi \in \overline{U}) \tag{1.4}$$

then $p(z)$ is called a solution of the strong differential subordination. The univalent function $q(z)$ is called a dominate of the solutions of the strong differential subordination or more simply a dominant if $p(z) \prec q(z)$,

$(z \in U)$ for all $p(z)$ satisfying (1.4). A dominant $\check{q}(z)$ that satisfy $\check{q}(z) \prec q(z)$ for all dominants $q(z)$ of (1.4) is said to be the best dominant.

DIFFINITION 1.3 [9,12] Let $\psi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$ and let $h(z)$ be analytic in U. If $p(z)$ and $\psi\ (p(z), zp'(z); z\ ;\ \xi)$ are univalent in U for all $\xi \in \overline{U}$ and satisfy the (first – order) strong differential superordination

$$h(z) \prec\prec \psi\ (p(z), zp'(z); z\ ;\ \xi) \qquad (1.5)$$

then $p(z)$ is called a solution of the strong differential superordination. An analytic function $q(z)$ called a subordinate of the solutions of the strong differential superordination, or more simply a subordinate if $q(z) \prec p(z)$ for all $p(z)$ satisfying (1.5). A univalent subordinate $\check{q}(z)$ that satisfies $q(z) \prec \check{q}(z)$ for all subordinants $q(z)$ of (1.5) is said to be the best subordinant. Note that the best subordinant is unique up to a rotation of U. For Ω a set in $\mathbb{C}$ with ψ and $p(z)$ as given in Definition 1.3, suppose (1.5) is replaced by :

$$\Omega \subset \{\psi\ (p(z), zp'(z); z\ ;\ \xi), z \in U\ , \xi \in \overline{U}\} \qquad (1.6)$$

DIFFINITION 1.4 [7] We denoted by $\mathcal{Q}$ the set of functions f that are analytic and injective in $\overline{U} \backslash E(f)$, where

$$E(f) = \left\{\xi \in \partial U\ ;\ \lim_{z \to \xi} f(z) = \infty\right\}$$

and are such that $f'(\xi) \neq 0$ for $\xi \in \partial U \backslash E(f)$. The subclass of $\mathcal{Q}$ for which $f(0) = a$ is defined by $\mathcal{Q}(a), \mathcal{Q}(0) \equiv \mathcal{Q}_0, \mathcal{Q}(1) \equiv \mathcal{Q}_1$.

DIFFINITION 1.5 [13] Let Ω be a set in $\mathbb{C}$, $q \in \mathcal{Q}$ and $n \in N$. The class of admissible functions $\Psi_n[\Omega\ , \mathrm{q}]$ consists of those functions $\psi : C^2 \times U \times \overline{U} \to \mathbb{C}$ that satisfy the following admissibility condition :

$\psi(r, s; z) \notin \Omega$,

whenever $r = q(\zeta)$, $s = m\ \zeta\ q'(\zeta)$, where $z \in U, \zeta \in \partial U, \xi \in \overline{U}$ and $m \geq n \geq 1$. We write $\Psi_1[\Omega, q]$ as $\Psi[\Omega, q]$.

DEFINITION 1.6 [9,12] Let Ω be a set in $\mathbb{C}$ *and* $q \in \mathcal{H}\ [a, n]$ with $q'(z) \neq 0$. The class od admissible function $\Psi'_n[\Omega\ , \mathrm{q}]$ consists of those functions $\psi : C^4 \times U \times \overline{U} \to \mathbb{C}$ that satisfy the

$\psi(r, s; \zeta, \xi) \in \Omega$,

whenever $r = q(z)$, $s = \frac{z\ q'(z)}{m}$, where $z \in U, \zeta \in \partial U, \xi \in \overline{U}$ and $m \geq n \geq 1$. When $n = 1$ we write $\Psi'_1[\Omega, q]$ as $\Psi'[\Omega, q]$.

LIMMA 1.1 [13] Let $q \in \mathcal{Q}$, with $q(0) = a$, and let $p(z) = a + a_n z^n + \cdots$ be analytic in U with $p(z) \neq a$ and $n \geq 1$. If p is not subordinate to q, then there exist points $z_0 = r_0 e^{i\theta_0} \in U$ and $\zeta_0 \in \partial U \backslash E(q)$, and $m \geq n \geq 1$ for which $p(U_{r_0}) \subset q(U)$, such that :

i) $p(z_0) = q(\zeta_0)$;

ii) $z_0 p'(z_0) = m\zeta_0 q'(\zeta_0)$.

THEOREM 1.1 [12] Let $\phi \in \phi_n[\Omega\ , \mathrm{q}]$ with $q(0) = a$. If $p \in \mathcal{H}[a, n]$ satisfies $\phi(p(z), zp'(z); z; \xi) \in \Omega$, then

$p(z) \prec q(z)$.

THEOREM 1.2 [12] Let $\psi \in \phi'_n[\Omega\ , \mathrm{q}]$ with $q(0) = a$. If $p \in \mathcal{Q}(a)$ satisfies $\psi(p(z), zp'(z); z; \xi)$ is univalent in U for $\zeta \in \overline{U}$,

$\Omega \subset \{\psi(p(z), zp'(z); z; z \in U, \zeta \in \overline{U})\}$

Implies

$q(z) \prec p(z)$

2- SUBORDINATION RESULTS

<u>**THEOREM 2.1**</u> Let $\phi \in \phi_n[\Omega\ , \mathrm{q}]$. If $f \in A$ satisfies

$$\left\{\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};z;\zeta\,|\,z\in U\,;\,\zeta\in\overline{U}\right)\right\}\subset\Omega \tag{1.7}$$

then

$$\frac{D^{\lambda}f(z)}{z}\prec q(z)$$

PROOF. Let

$$p(z)=\frac{D^{\lambda}f(z)}{z},z\in U. \tag{1.8}$$

From (1.8), we have

$$D^{\lambda}f(z)=zp(z), \tag{1.9}$$

and differentiating (1.9), we obtain

$$\left(D^{\lambda}f(z)\right)'=p(z)+zp'(z). \tag{1.10}$$

Using the property (1.3) of the Ruscheweyh Derivative operator

$$D^{\lambda+1}f(z)=\frac{z\left(D^{\lambda}f(z)\right)'+\lambda D^{\lambda}f(z)}{\lambda+1},\lambda>-1,z\in U. \tag{1.11}$$

Using (1.9) and (1.10) in (1.11), we obtain

$$\frac{D^{\lambda+1}f(z)}{z}=p(z)+\frac{zp'(z)}{\lambda+1}. \tag{1.12}$$

Then (1.7) becomes

$$\left\{\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};z,\zeta\right)\right\}=\left\{\phi\left(p(z),p(z)+\frac{zp'(z)}{\lambda+1};z;\zeta\right)\right\}\subset\Omega. \tag{1.13}$$

Assume $p \nprec q$. By Lemma 1.1 there exist points $z_0=r_0e^{i\theta_0}\in U$ and $\zeta_0\in\partial U\backslash E(q)$, and $m\geq n\geq 1$, that satisfy

$p(z_0)=q(\zeta_0)$, $\quad z_0p(z_0)=m\zeta_0q'(\zeta_0)$.

Using these condition in Definition 1.5, we obtain

$$\phi\left(p(z_0),p(z_0)+\frac{1}{\lambda+1}z_0p'(z_0);\ z_0\right)=\phi\left(q(\zeta_0),q(\zeta_0)+\frac{m\zeta_0q'(\zeta)}{\lambda+1};\ z_0\right)\notin\Omega.$$

Since this contradicts (1.13), we must have $p\prec q$ by Theorem 1.1 or equivalent

$$\frac{D^{\lambda}f(z)}{z}\prec q(z).$$

COROLLARY 2.1 The conclusion of Theorem 2.1 can be written in the generalized form :

$$\left\{\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};w(z)\right)\right\}\subset\Omega,$$

then

$$\frac{D^{\lambda}f(z)}{z}\prec q(z),$$

where $w(z)$ is any mapping U onto U.

If $\Omega\neq\mathbb{C}$ is a simply connected domain, then $\Omega=h(U)$ for some conformed mapping h of U onto Ω. In this case the class $\Phi_n[h(U),q]$ is written as $\Phi_n[h,q]$.

The following result is an immediate consequence of Theorem 2.1.

THEOREM 2.2 Let $\phi\in\Phi_n[\Omega,\mathrm{q}]$. If $f\in A$, $\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};z,\zeta\right)$ is analytic in U, and

$$\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};z,\zeta\right) \prec\prec h(z) \qquad (1.14)$$

then

$$\frac{D^{\lambda}f(z)}{z} \prec q(z).$$

This result can be extended to those cases in which behavior of q on the boundary of U is unknown.

COROLLARY 2.2 Let $\Omega \subset \mathbb{C}$ and q be univalent in U with $q(0) = 1$. Let $\phi \in \Phi_n[\Omega, q_\rho]$ for some $\rho \in (0,1)$, where $q_\rho(z) = q(\rho z)$. If $f \in A$ satisfy :

$$\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};z,\zeta\right) \in \Omega$$

then

$$\frac{D^{\lambda}f(z)}{z} \prec q(z)$$

PROOF. From Theorem 2.1 yield $\frac{D^{\lambda}f(z)}{z} \prec q_\rho(z)$. The result is now deduced from $q_\rho(z) \prec q(z)$.

THEOREM 2.3. Let h and q be univalent in U with $q(0) = 0$ and set $q_\rho(z) = q(\rho z)$ and $h_\rho(z) = h(\rho z)$.

Let $\phi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$ satisfy one of the following conditions :

1) $\phi \in \Phi_n[\Omega, q_\rho]$ for some $\rho \in (0,1)$, or
2) There exist $\rho_0 \in (0,1)$ such that $\phi \in \Phi_n[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.

If $f \in A$ satisfies (1.14), then

$$\frac{D^{\lambda}f(z)}{z} \prec q(z)$$

PROOF.

Case (i) . By applying Theorem 2.1 we obtain $p \prec q_\rho$. Since $q_\rho \prec q$, we have $p(z) \prec q(z)$. i.e,

$$\frac{D^{\lambda}f(z)}{z} \prec q(z).$$

Case (ii) If we let $p_\rho(z) = p(\rho z)$, then

$$\phi\left(\frac{D^{\lambda}f(z)}{z},\frac{D^{\lambda+1}f(z)}{z};z\right) = \phi\left(p_\rho(z), p_\rho(z) + \frac{1}{\lambda+1}zp_\rho'(z);\ \rho z\right) = \phi\left(p(\rho z), p(\rho z) + \frac{1}{\lambda+1}zp'(\rho z);\ \rho z\right) \in h_\rho(U).$$

By using Corollary 2.1 with $w(z) = \rho z$, we obtain $p_\rho(z) \prec q_\rho(z)$ for $\rho \in (\rho_0, 1)$. By letting $\rho \to 1$ we obtain $p(z) \prec q(z)$. i.e,

$$\frac{D^{\lambda}f(z)}{z} \prec q(z).$$

The next theorem yields the best dominate of the differential subordination (1.7).

THEOREM 2.4 Let $h(z)$ be univalent in U and $\phi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$. Suppose that the differential equation

$$\phi\left(q(z), q(z) + \frac{1}{\lambda+1}zq'(z);\ z,\zeta\right) = h(z) \qquad (1.15)$$

has a solution q, whith $q(0) = 0$, and one of the following conditions is satisfied :

i) $q \in Q_0$ and $\phi \in \Phi_n[h, q]$,

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ii) q is univalent in U and $\phi \in \Phi_n[h, q_\rho]$ for some $\rho \in (0,1)$, or

iii) q is univalent in U and there exits $\rho_0 \in (0,1)$ such that $\phi \in \Phi_n[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.

If $f \in A$ satisfies (1.14) and $\phi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right)$ is analytic in U, then

$$\frac{D^\lambda f(z)}{z} \prec q(z),$$

and q is best dominant.

PROOF. By applying Theorem 2.2 and Theorem 2.3 we deduce that q is dominant of (1.14). Since q satisfies (1.15), it is also a solution of (1.14) and therefore q will be the dominant of all dominants of (1.14). Hence q will be best dominate of (1.14).

3- SUPERORDINATION AND SANDWICH – TYPE RESULTS

THEOREM 3.1 Let $\psi \in \Psi_n'[\Omega, q]$. If $f \in A$, $\frac{D^\lambda f(z)}{z} \in Q_0$ and $\psi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right)$ is univalent in U, then

$$\Omega \subset \psi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z\right), \qquad (3.1)$$

implies

$$q(z) \prec \frac{D^\lambda f(z)}{z}, \quad z \in U.$$

PROOF. The same technic to proof Theorem 2.1.

Next, we consider the special situation when h is analytic on U and $h(U) = \Omega \neq \mathbb{C}$. In this case the class $\Phi_n[h(U), q]$ is $\Phi_n[h, q]$ and the following result is an immediate consequence of Theorem 3.1.

THEOREM 3.2 Let $q \in \mathcal{H}[0,1], h(z)$ be analytic in U and $\psi \in \Psi_n'[h, q]$. If $f(z) \in A$, $\frac{D^\lambda f(z)}{z} \in Q_0$ and $\psi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right)$ is univalent in U,

Then

$$h(z) \prec\prec \psi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right) \qquad (3.2)$$

implies

$$q(z) \prec \frac{D^\lambda f(z)}{z}.$$

Theorem 3.1 and Theorem 3.2 can only be used to obtain subordinations of differential superordination of the form (3.1) or (3.2). The following therefore proves the existence of the best subordinants of (3.2) for certain ϕ.

THEOREM 3.3 Let h be analytic in U and $\psi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$. Suppose that the differential equation

$$\phi\left(q(z), q(z) + \frac{1}{\lambda+1} z q'(z); z, \zeta\right) = h(z) \qquad (3.3)$$

has a solution $q \in Q_0$. If $\psi = \Psi_n'[h, q], f \in A$, $\frac{D^\lambda f(z)}{z} \in Q_0$ and $\psi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right)$ is univalent in U, then

$$h(z) \prec\prec \psi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right) \qquad (3.4)$$

implies

$$q(z) \prec \frac{D^\lambda f(z)}{z}$$

and q is the best subordinant.

PROOF. By applying Theorem 3.2, we deduce that q is a dominate of (3.2). Since q satisfies (3.3), it is also a solution of (3.2) and therefore q will be dominated by all dominates of (3.2). Hence q is the best dominates of (3.2).

Combining Theorem 2.2 and Theorem3.2, we obtain the following sandwich type Theorem.

THEOREM 3.4 Let $h_1(z)$ and $q_1(z)$ be analytic functions in U, $h_2 f(z)$ be a univalent function in U, $q_2(z) \in \mathcal{Q}_0$ with $q_1(0) = q_2(0) = 0$ and $\phi \in \Phi_n[h_2, q_2] \cap \Psi'_n[h_1, q_1]$. If $f \in A, \frac{D^\lambda f(z)}{z} \in \mathcal{H}[0,1] \cap \mathcal{Q}_0$ and $\phi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right)$ is univalent in U, then

$$h_1(z) \ll \phi\left(\frac{D^\lambda f(z)}{z}, \frac{D^{\lambda+1} f(z)}{z}; z, \zeta\right) \leq h_2(z)$$

implies

$$q_1(z) \prec \frac{D^\lambda f(z)}{z} \prec q_2(z)$$

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