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Iterative Algorithm for the Effects of Atmospheric Refraction on the Equatorial Coordinates of a Star Valid for any Zenith Distance

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Abstract. In the present paper an efficient algorithm will be established for the effects of atmospheric refraction on the equatorial coordinates of a star valid for any zenith distance. The algorithm uses two new iterative schemes, the first is linear iterative scheme for the equatorial coordinates (α', δ') due to atmospheric refraction. The second is the quadratic Newton iterative scheme for obtaining the true zenith distance from Bennent formulae for the atmospheric refraction. Description of the algorithm was also given together with numerical applications.

Keywords: Atmospheric refraction; marine navigation; spherical astronomy; apparent place of stars.

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1. Introduction

Atmospheric refraction was mentioned as early as the first century A.D. by Cleomedes independently by Ptolemy (discussed in his *Optics*),ca.A.D.150.. and the refraction is the bending of light while passing from transparent homogenous medium to another transparent homogenous medium whose density is different from the first medium. The refraction follows some basic roles ,of these are :the incident ray ,,the refracted ray ,and the normal to the surface separating two media at the point O(say), all lie in same plane. The relation between the incident and the refracted angles ψ and ϕ respectively, is given by

$$\frac{\sin\psi}{\sin\varphi}=\mu$$

where μ is called the *refractive index* for the two media, μ is a constant quantity depending on the optical properties of the two media and can be determined by laboratory experiment. The value of μ changes for the same ray, that is the refractive index for the blue light is different refractive index for the red light. This phenomenon is known in optics as light *scattering* and it is not important in studying the effect of the refraction on astronomical observations. In the vacuum of space $\mu=1$. The value of μ_{air} depends on wavelength, temperature, and pressure as well. For example $\mu=1.000277$ is for green light $\approx 5500\mathrm{A}^{\mathrm{o}}$, for dry air for the conditions $\mathrm{T}=15^{\mathrm{o}}\,\mathrm{C}$ and pressure $\mathrm{P}=1.013\times10^{5}$ Pa. The value of μ for the atmosphere at the Earth's surface, at temperature 0^{o}C, and at atmospheric pressure 760 mm Hg is around 1.0002927 for the yellow light where the human eye has maximum sensitivity. Atmospheric refraction plays important roles in many applications of spherical astronomy, of these as for example, in top ocentric phenomena, such as the time of rising and setting of the Sun and Moon, and in theory in the prediction of local circumstances of eclipses. Also for observational reductions the effect of refraction on the equatorial coordinates of a star must be included.

Due to the importance of atmospheric refraction mentioned in few in the above, and due to the need of accurate equatorial coordinates of the stars, the present paper is devoted to establish an efficient algorithm for the effects of atmospheric refraction on the equatorial coordinates of a star valid for any zenith distance The algorithm uses two new iterative schemes, the first is linear iterative scheme for the equatorial coordinates (α',δ') due to atmospheric refraction. The second is the quadratic Newton iterative scheme for obtaining the true zenith distance from Bennent formulae for the atmospheric refraction. Description of the algorithm was also given together with numerical applications.

Basic formulations

Atmospheric refraction 2.1

The starlight moves in straight line until it meets the outer surface of the atmosphere, then it suffers through its passage in the Earth's atmosphere series of refractions called *astronomical refraction*. Since the air density of the upper layers of the Earth's atmosphere is very rarer, then its effect on the refraction is small compared with that of total refraction.

Consequently, the effective layers on the refraction are those at few tens of kilometers from the Earth's surface, and in these lyres, the ray is bending till it reaches the observer. So the star is observed in a direction not parallel to its true direction.

For illustrating the above points, consider Fig.1 where Z is the zenith, O is an observer on the Earth's surface and:

1- The straight pass of a starlight from the star S(say) till it enter the effective region of the Earth's atmosphere at the point A.

2-After the entrance at the point A, the ray continuously bending till it reaches the observer at the point O.

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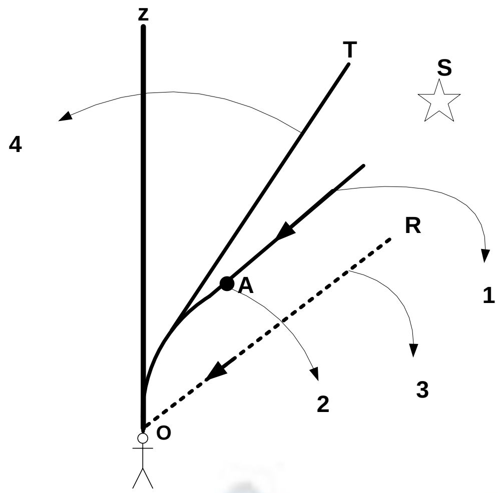

Fig.1.:The refraction of the starlight in the Earth's atmosphere

3- The parallel line to SA is the position of the star if there were no atmospherical refraction ,and this is the *true direction* of the star

4-The observer will see the star in the direction TO which is the direction of the tangent for the curve of the refracted ray at the point O and this is the *apparent* (*observed*) *direction* of the star.

From the above, it is clear that the refraction changes the true poison of the zenith of the star, that is ;*the zenith distance decreases due to the refraction*.

2.2 General theorem for the atmospheric refraction

Assumptions of the theorem

1-The Earth's is sregarded as sphere .

2-The atmosphere made up of a large number of thin spherical layers, concentric with the Earth's center.

3- Each layer has its own optical properties and ,in particular, its own refractive index

The formula

The numerical formula for the *mean refraction* for the standard conditions which are taken in practice to be: barometric pressure $=760$ mm Hg. and Temperature = 10^{o} C., is given by one of the following formulae

$$R = 58.294'' \tan\xi - 0.0668'' \tan^3\xi. \; , \tag{1}$$

$$R = 58.276'' \tan\xi - 0.0824'' \tan^3\xi \tag{2}$$

where ξ is the observed zenith distance. The first formula is given by Smart[1956] while the second one has been derived by Meeus[2000].

Clearly both formulae (1) and (2) are not valid when the observed zenith distance equal to 90^{o} . Also the formulae are insufficient when the zenith distance exceeds 75^{o} .

It appears that, at high altitudes, the refraction is proportional to the tangent of the zenithal distance.

2.3 Bennent formulae for the atmospheric refraction

In 1982 Bennent established a surprisingly simple formula for refraction, with good accuracy at all altitudes from 90^{o} to 0^{o}, this formula is :

$$R = \frac{1}{\tan\left(h_0 + \frac{7.31}{h_0 + 4.4}\right)} \; , \tag{3}$$

where $h_0 = 90 - \xi$ is the apparent (observed) altitude in degrees, and R in arc minute.

According to Bennett, this formula is accurate to 0.07 arc minutes for all values of h_0. The largest error, 0.07 arc minute, occurs at 12^o altitude.

Bennett also showed how his formula can be refined as follows calculate R from equation (3), then add the correction ΔR expressed in minutes of arc, where

$$\Delta R = -0.06 \sin(14.7R + 13), \tag{4}$$

the expression between parentheses is expressed in degrees. Calculated in this way, the maximum error is stated to only 0.015 arc minute, or $0.9''$, for the whole altitude range

2.4 The effects of the atmospheric pressure and temperature on the refraction

In the above refraction formulae we assume that the observations are made at the sea level, when the atmospheric pressure is 760 mmHg, and temperature $10^o C$. We can get the value of the refraction at different temperature and pressure as follows:

1-If the pressure at the Earth's surface in P mmHg and the temperature t in degree Celsius, then the values of R should be multiplied by the factor Δ_{PT}, where

$$\Delta_{PT} = \Delta_P \times \Delta_T \; ; \; \Delta_P = \frac{P}{760} \text{ and } \Delta_T = \frac{283}{273 + t}.$$

2-If the pressure at the Earth's surface P in mb(Millbrae) and the temperature t in degree Celsius, then the values of R should be multiplied by the factor Δ_{PT}, where

$$\Delta_{PT} = \Delta_P \times \Delta_T \; ; \; \Delta_P = \frac{P}{1010} \text{ and } \Delta_T = \frac{283}{273 + t}.$$

3. Computational developments

In this section the computational developments needed for the design of our basic algorithm of the present paper will be established through the following subsections

3.1 Transformations between the altitude and azimuth to the hour angle and declination

Transformations formulae

Assume that (a, A) are the altitude and azimuth of a star X and (H, δ) are its hour angle and declination. From the known spherical triangle PZX we get

$$\sin\delta = \sin a \sin\varphi + \cos a \cos\varphi \cos A \tag{5}$$

$$\sin a = \sin\delta \sin\varphi + \cos\delta \cos\varphi \cos H \tag{6}$$

$$\cos A = \frac{\sin\delta - \sin\varphi \sin a}{\cos\varphi \cos a} \tag{7}$$

$$\cos H = \frac{\sin a - \sin\varphi \sin\delta}{\cos\varphi \cos\delta} \tag{8}$$

where φ is the given observer's latitude, then

➤ Equations (5), (8) could be used to find (H, δ) from (a, A).

➤ Equations (6) ,(7) could be used to find (a,A).from (H,δ). .

Before applying these equations, we have to remember the following important rule between the measurement of the azimuth A and the hour angle H.

The measurement rule between A and H.

- *If the star's azimuth is west, the hour angle is between* $0°, 180°$ or $0^h, 12^h$ (*and vice versa*).
- *If the star's azimuth is east, the hour angle is between* $180°, 360°$ or $12^h, 24^h$ (*and vice versa*) .

Mathematically, these means that:

- *If* $\sin A \succ 0$ *then* $H = 360 - H$ *,where* H *on the right side is that computed from Equation* (8).*But ,if* $\sin A \prec 0$ *,then* H *remains as it is.*
- *If* $\sin H \succ 0$ *then* $A = 360 - A$, *where* A *on the right side is that computed from Equation* (7).*But ,if* $\sin H \prec 0$ *,then A remains as it is.*

For practical applications we can unified (may be to the first time) the two sets of Equations [(5),(8)], [(6),(7)] with the above measurement rule in the form: let

➤ (ξ, η) are the *required* coordinates,

➤ (g, h) are the *given* coordinates,

(ξ, η) could be obtained according to the above measurement rule from the following equations:

$$\xi = \sin^{-1}\{X\} \tag{9}$$

$$\eta = \begin{cases} \cos^{-1}\{Y\} & \text{if} \quad \sin g < 0 \\ 360° - \cos^{-1}\{Y\} & \text{if} \quad \sin g > 0 \end{cases} \tag{10}$$

where

$$S = \sin\varphi \quad , \quad C = \cos\varphi \tag{11}$$

$$\sin\xi = S\sin h + C\cos h \cos g = X \tag{12}$$

$$\cos\eta = \frac{\sin h - S\sin\xi}{C\cos\xi} = Y. \tag{13}$$

3.2 The effect of refraction on the right accession and declination of a star

Let (α, δ) be the right accession and declination of a star respectively, and (α', δ') its corresponding coordinates due the atmospheric refraction R. It could be shown that

[e.g.Smart 1956]

$$\Delta\alpha = \alpha - \alpha' = -R\sec\delta'\cos\varphi\ \sin H'\csc\xi \tag{14}$$

$$\Delta\delta = \delta - \delta' = -R(\sin\varphi - \cos\xi\sin\delta')\csc\xi\ \sec\delta' \tag{15}$$

where H' is the hour angle associated with α' is written in terms of the local sidereal time LST as:

$$H' = LST - \alpha'. \tag{16}$$

Since the unknown coordinates (α', δ') appear also in the right hand side of the above equations this leads us to purpose the following linear iterative scheme:

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$$y_1 = \alpha$$
$$y_{i+1} = \alpha + R\ \sec\delta'\cos\varphi\ \ \sin(LST\text{-}y)\csc\xi\ \ ; i = 1,2,3,4, \quad (17)$$

then $\alpha' = y_5$ Also , δ' could be determined from linear iterative scheme:

$$x_1 = \delta$$
$$x_{i+1} = \delta + R(\sin\varphi\text{-}\cos\xi\sin x_i)\csc\xi\ \ \sec x_i ; i = 1,2,3,4, \quad (18)$$

then $\delta' = x_5$. Note that the four iterations in Equations (17) and (18) are sufficient for computing (α', δ') because in general $\Delta\alpha$ and $\Delta\delta$ are small.

3.3 The value of the atmospheric refraction R if the true zenith distance z is known using Bennent's formulae

Substituting $\xi = z - R$ into Equation (3) it becomes

$$G(R) = R - \tan\left(z - R - \frac{7.31}{94.4 - z + R}\right) = 0, \quad (19)$$

where ξ and z are respectively the apparent and the true zenith distances.

Equation (19) is a transcendental equationin R, and usually solved by iterative methods, for example Newton iterative method.

For the application of Newton method we need beside $G(R)$ the following :

1-The first derivative of the function $G(R)$ with respect to R and is given as

$$G'(R) = -\left[1 - \frac{7.31}{(94.4 - z + R)^2}\right]\sec^2\left(z - R - \frac{7.31}{(94.4 - z + R)}\right).$$

2-The maximum number of iterations "itmax"(say).

3- The initial guess R_0 of the root of Equation (19). We take $R_0 = 0$, assuming the solution of Equation (19) is $\tilde{R}$,then we can check its accuracy by the condition that

$$|G(\tilde{R})| = \varepsilon, \quad (20)$$

where ε is an accuracy criterion around 10^{-15}.

4. Computational Algorithm

In this section, we shall develop an efficient iterative algorithm for the effects of atmospheric refraction on the equatorial coordinates of a star valid for any zenith distance .The design of the algorithm is given as follows

4. 1 Algorithm I

●Input

➤P; The value of atmospheric pressure at the date in question in In-Hg, (or any other unit)

➤T: The value of temperature at the date in question in C^o (or any other unit).

➤ α : The right ascension of the star in degrees.

➤ δ : The declination of the star in degrees.

➤ ϕ :The latitude in degrees.

➤LST: The local sidereal time at the date in question in degrees

● Output

α' : The right ascension of the star due to the refraction in radian

δ' : The declination of the star due to the refraction in radian

● **Computational steps**

1-Using $g = LST - \alpha$ and $h = \delta$ into the algorithm of Section 3.1 to find $z = (90^0 - a) \times 180 / \pi$.

2-Using algorithm of Section 3.3.to find R by Newton iterative method with itmax$= 6.$.

3- $\Delta_P = P \times 25.4 / 760$; $\Delta_T = 283/(273 + T)$

4- $R = R \times \Delta_P \times \Delta_T$

5- $\xi = z - R / 3600.$

6- Using algorithm of Section 3.2 to compute both (α', δ') in radian

7-End

4. 2 Numerical example

In order to apply algorithm I, we generate 30 real random values of :

P: Between 10 and 50 In-Hg.

T: Between 0 and 50 C^o.

α : Between 0 and 360 degrees

δ : Between 0 and 90 degrees

ϕ : Between 0 and 90 degrees

LST: Between 4 and 23.5 hours.

Table I of Appendix A list the input and output of algorithm I

In concluding the present paper ,we stress ,an efficient algorithm was established for the effects of atmospheric refraction on the equatorial coordinates of a star valid for any zenith distance The algorithm uses two new iterative schemes, the first is linear iterative scheme for the equatorial coordinates (α', δ') due to atmospheric refraction. The second is the quadratic Newton iterative scheme for obtaining the true zenith distance from Bennent formulae for the atmospheric refraction. Description of the algorithm was also given together with numerical applications .

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Appendix A

Table I:The effect of refraction on the The right ascension and declination of some stars

P(Hg)	T(C^o)	α^o	δ^o	ϕ^o	LST(h)	α'^o	δ'^o	$\Delta\alpha^o$	$\Delta\delta^o$
18.7096	25.2285	340.586	75.6465	17.5127	4.12676	340.773	75.6432	−0.186693	0.00326499
28.7132	32.3226	27.4713	48.2766	62.3308	22.6633	27.4618	48.2831	0.00943643	−0.00651428
44.3142	30.9181	302.53	70.9737	14.3449	21.4646	302.56	70.95	−0.029944	0.0237761
21.7634	35.6595	109.938	71.3004	47.2506	4.08604	109.908	71.2967	0.0302747	0.00361171
44.1812	2.24614	130.55	33.6414	48.8639	7.97074	130.547	33.6458	0.00246022	−0.00439723
11.886	11.7739	321.046	50.8589	42.725	11.1251	320.977	50.9877	0.0695907	−0.12871
23.7579	10.8711	50.7051	80.8285	44.3159	5.23356	50.7475	80.8181	−0.0424076	0.0104266
38.0564	9.10689	94.8356	6.53178	25.9434	13.069	94.7502	6.48805	0.0853696	0.0437235
41.9271	25.693	143.784	43.0908	73.9643	22.8106	143.78	43.1213	0.00414177	−0.0305627
29.1153	5.22227	136.747	31.9527	51.2204	17.0681	136.812	32.0358	−0.0650827	−0.0830957
23.3385	24.337	213.612	18.8149	83.397	12.178	213.609	18.8503	0.00243649	−0.0354255
14.7717	23.008	323.249	0.656034	24.55	20.4983	323.244	0.66337	0.00447447	−0.00733603
43.3447	10.2407	262.29	48.2991	69.5226	5.92168	262.288	48.3295	0.00206543	−0.0304162
21.4196	38.9299	104.367	66.1843	0.831179	16.4567	104.289	66.1463	0.078108	0.0379971
23.8182	2.24194	180.695	12.5746	16.5215	10.8361	180.69	12.5759	0.00502951	−0.0013018
20.3451	43.0378	320.513	71.9546	4.52672	4.35414	322.446	72.1169	−1.93315	−0.162308
24.0114	16.6098	170.017	52.284	28.7123	7.07492	169.983	52.2837	0.0337296	0.000306243
38.9267	3.86181	70.6306	63.3835	12.2858	5.22039	70.6381	63.3636	−0.00752138	0.019925
10.732	10.2829	7.46697	86.1749	10.6948	22.9966	7.11178	86.1175	0.355192	0.057392
36.478	45.2131	0.668495	15.2193	83.0765	11.1532	0.671503	15.3222	−0.00300703	−0.102819
36.2353	29.6737	218.959	43.5764	35.3098	19.6935	218.992	43.5851	−0.0329418	−0.00861727
19.1673	40.8596	228.947	71.8284	67.2566	18.7395	228.963	71.8294	−0.0161074	−0.00102272
13.5166	8.5168	342.787	33.9026	53.8076	21.6982	342.784	33.9087	0.00364788	−0.00612183
49.4985	35.9592	6.36487	68.0451	16.8348	6.41926	6.51659	68.0512	−0.151716	−0.00616285
28.3324	33.9806	87.7911	49.9654	51.6625	18.3461	87.7815	50.0405	0.00958538	−0.0750846
36.8776	1.04559	323.382	8.69733	31.1111	16.227	323.322	8.73159	0.0598685	−0.0342541
29.2125	18.0337	270.031	54.4767	65.8594	22.5252	270.043	54.4844	−0.01254	−0.00778059
32.5986	10.0834	71.5871	72.3131	67.8041	13.9086	71.6042	72.324	−0.0170522	−0.0109464
29.4109	40.7915	317.861	67.608	1.78742	20.9675	317.855	67.5721	0.00602456	0.0359038
16.665	22.7893	311.925	83.6651	9.85526	20.968	311.947	83.6102	−0.0228274	0.0548306