ISSN 2347-1921
Volume 13 Number 3
Journal of Advance in Mathematics

On time fractional Cahn-Allen equation

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ABSTRACT

In [1], Ozkan Güner et al. obtained some exact solutions of the time fractional Cahn-Allen equation.

By using the method proposed in [10], we have tested these solutions and have found that they are not the solutions of this equation.

Keywords: time fractional Cahn-Allen equation; exact solution,Exp-function method; First integral method

SUBJECT CLASSIFICATION 35J05, 35J10, 33E12, 33R11, 35A22

INTRODUCTION

In [1], Ozkan Güner et al. studied the following time fractional Cahn-Allen equation：

$$D_t^{\alpha}u - u_{xx} - u + u^3 = 0, \qquad (1)$$

where $0 < \alpha \leq 1$, and

$$D_t^{\alpha}u(x,t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha}[u(x,\tau)-u(x,0)]d\tau. \qquad (2)$$

Where D_t^{α} denotes Jumarie's modified Riemann–Liouville fractional derivative [2]. Eq. (1) arises in many scientific applications such as quantum mechanics and plasma physics [3-8]. They obtained some analytical exact solutions by using Exp-function method, the (G'/G)-expansion method and First integral method[3]. However, we have observed that these solutions are not true. Here we list two exact solution obtained in [1] as follows:

$$u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh(\frac{\sqrt{2}}{2}x + \frac{3}{2\Gamma(1+\alpha)}t^{\alpha}), \qquad (3)$$

$$u(x,t) = \frac{1}{2} - \frac{1}{2}\tanh(\frac{\sqrt{2}}{2}x - \frac{3}{2\Gamma(1+\alpha)}t^{\alpha}). \qquad (4)$$

In section 2, by the method proposed in [10], we will prove that the functions (3) and (4) are not the solutions of the Eq. (1).

ANALYSIS AND RESULTS

By Eq. (2), we can rewrite the Eq. (1) as:

$$\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha}(u(x,\tau)-u(x,0))d\tau = u_{xx}+u-u^3. \qquad (5)$$

For simplicity, we choose $\alpha = 0.5$ in Eq.(1) for checking the obtained solutions (3) and (4).

If the function (4) is a solution of the fractional differential Eq. (1), then the function

$$u(x,t)=\frac{1}{2}-\frac{1}{2}\tanh(\frac{\sqrt{2}}{2}x-\frac{3}{2\Gamma(1.5)}t^{0.5}) \qquad (6)$$

satisfies the following equation:

$$\frac{1}{\Gamma(0.5)}\frac{d}{dt}\int_0^t (t-\tau)^{-0.5}(u(x,\tau)-u(x,0))d\tau = u_{xx}+u-u^3. \qquad (7)$$

Integrating both sides of the eq. (7) with respect to t from 0 to 1, we have

$$\int_0^1 (1-\tau)^{-0.5}(u(x,\tau)-u(x,0))\,d\tau = \Gamma(0.5)\int_0^1 (u_{xx}+u-u^3)\,dt. \qquad (8)$$

Take $x=0$ in Eq.(8), we have:

$$\int_0^1 \frac{1}{2}(1-\tau)^{-0.5}\tanh(\frac{3\tau^{0.5}}{2\Gamma(1.5)})\,d\tau$$

$$=\Gamma(0.5)\int_0^1 (u_{xx}+u-u^3)\big|_{x=0}\,dt. \qquad (9)$$

By Maple software, we obtain that left side of Eq.(9) approximately equals 0.832725 and right approximately equals 0.087684.

Thus, the function (4) is not a solution of the Eq.(1). Similarly we can prove that the function (3) does not satisfy Eq. (1).

DISCUSSION AND CONCLUSIONS

Different from integer-order differential equation, for a given fractional differential equation, it is very difficulty to test whether or not a function satisfies it. In this paper, by using the method proposed in [11], we have tested the functions (3) and (4) and have found that they are not the solutions of the Eq. (1).

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