An algorithm for solving fractional Zakharov-Kuznetsv equations

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Abstract:

By using the fractional power series method, we give an algorithm for solving fractional Zakharov-Kuznetsv equations . Compared to the other method, the fractional power series method is more derect , effective and the algorithm can be implemented as a computer program.

Keyword: fractional Zakharov-Kuznetsv equations; power series method; Caputo fractional derivative.

1. Introduction

Fractional differential equations are encountered in various disciplines, such as fluid mechanics, viscoelasticity, chemistry, biology, physics and engineering [1,2].

In general, there exists no method that yields an exact solution for fractional differential equations. Thus, great attention has been given to finding approximate analytical or numerical solutions of fractional differential equations.

Recently, some analytical techniques to handle nonlinear fractional differential equations have been proposed, for example, Adomain decomposition method (ADM)[3], the differential transform method (DTM)[12], homotopy perturbation method (HPM)[4], variational iteration method (VIM) [5] , homotopy analysis method (HAM)[6], and fractional power series method (FPSM)[9,10].

In this paper, by using fractional power series method, we give an algorithm for solving fractional Zakharov-Kuznetsv equations [11]. The fractional Zakharov-Kuznetsv equations considered are of the form:

$$D_t^{\alpha}u+(u^2)_x+\frac{1}{8}(u^2)_{xxx}+\frac{1}{8}(u^2)_{xyy}=0, \qquad (1)$$

where $u=u(x,y,t)$ and D^{α} is the Caputo derivative of order α , and $0<\alpha\le 1$. The equation (1) governs the behavior of weakly nonlinear ion acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field [12,13,14].

Compared to the other method, the fractional power series method is more derect , effective and the algorithm can be implemented as a computer program [11-14].

2. Basic definitions

In order to obtain our results it is necessary to give some definitions:

Definition 1.([10]) A power series representation of the form

$$\sum_{n=0}^{\infty}c_n(t-t_0)^{n\alpha}=c_0+c_1(t-t_0)^{\alpha}+c_2(t-t_0)^{2\alpha}+\cdots, \qquad (2)$$

where $0\le m-1<\alpha\le m, m\in N^{+}$ and $t\ge t_0$ is called a fractional power series (FPS) about t_0 , where t is a variable and c_n are the coeffients of the series.

Definition 2. ([2,9]) The fractional derivative of $f(x)$ in Caputo sense is defined as

$$D^{\alpha} f(x)=\frac{1}{\Gamma(m-\alpha)}\int_0^x (x-s)^{m-\alpha-1} f^{(m)}(s)ds, \tag{3}$$

for $m-1<\alpha\le m, m\in N^{+}, x>0.$

Also, the following theorem would be necessary in order to use fractional power series method.

Theorem 1. ([10]) Suppose that the FPS $\sum_{n=0}^{\infty} c_n t^{n\alpha}$ has radius of convergence $R>0$. If $f(t)$ is a function defined by $f(t)=\sum_{n=0}^{\infty} c_n t^{n\alpha}$ on $0\le t<R$, then for $m-1<\alpha\le m$ and $0\le t<R$, we have:

$$D^{\alpha} f(t)=\sum_{n=0}^{\infty} c_n \frac{\Gamma(n\alpha+1)}{\Gamma((n-1)\alpha+1)} t^{(n-1)\alpha}. \tag{4}$$

3. Algorithm for solving equation (1)

In order to use the fractional power series method, we suppose that the solution of (1) takes the following form:

$$u(x,y,t)=\sum_{k=0}^{\infty} u_k(x,y)t^{\alpha k}=u_0(x,y)+u_1(x,y)t^{\alpha}+u_2(x,y)t^{2\alpha}+\cdots. \tag{5}$$

By Theorem 1, on the one hand, we have

$$D_t^{\alpha} u(x,y,t)=\sum_{k=1}^{\infty} \frac{u_k(x,y)\Gamma(\alpha k+1)}{\Gamma(\alpha(k-1)+1)} t^{\alpha(k-1)}. \tag{6}$$

On the other hand,

$$(u^2)_x=2u_0u_{0x}+2(u_{0x}u_1+u_0u_{1x})t^{\alpha}+2(u_{0x}u_2+u_0u_{2x}+u_1u_{1x})t^{2\alpha}+\cdots. \tag{7}$$

And

$$(u^2)_{xxx}=4u_{0x}u_{0xx}+2(u_0u_{0xxx}+u_{0x}u_{0xx})$$

$$+(2u_{0xxx}u_1+6u_{0xx}u_{1x}+6u_{0x}u_{1xx}+2u_0u_{1xxx})t^{\alpha}$$

$$+(2u_{0xxx}u_2+6u_{0xx}u_{2x}+6u_{0x}u_{2xx}+2u_0u_{2xxx}+6u_{1x}u_{1xx}+2u_1u_{1xxx})t^{2\alpha}+\cdots, \tag{8}$$

$$(u^2)_{xyy} = 4u_{0y}u_{0xy} + 2(u_0u_{0xyy} + u_{0x}u_{0yy})$$

$$+(2u_{0xyy}u_1 + 4u_{0xy}u_{1y} + 4u_{0y}u_{1xy} + 2u_0u_{1xyy} + 2u_{0x}u_{1yy} + 2u_{0yy}u_{1x})t^{\alpha}$$

$$+(2u_2u_{0xyy} + 4u_{0xy}u_{2y} + 2u_{0x}u_{2yy} + 2u_{0yy}u_{2xy} + 2u_{0y}u_{2xyy} + 2u_{0y}u_{2xy}$$

$$+2u_0u_{2xyy} + 2u_{1x}u_{1yy} + 4u_{1y}u_{1xy} + 2u_1u_{1xyy})t^{2\alpha} + \cdots. \quad (9)$$

Substituting (6), (7), (8) and (9) into (1), and comparing the coefficients of t^{α} in the both side, we get

$$u_1\Gamma(1+\alpha) + u_0^2 + \frac{1}{8}(4u_{0x}u_{0xx} + 2u_0u_{0xxx} + 2u_{0x}u_{0xx})$$

$$+\frac{1}{8}(2u_{0x}u_{0yy} + 2u_{0y}u_{0xy} + 2u_{0y}u_{0xy} + 2u_0u_{0xyy}) = 0, \quad (10)$$

and

$$u_2\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)} + 2(u_{0x}u_1 + u_{1x}u_0) + \frac{1}{8}(6u_{1x}u_{0xx} + 6u_{0x}u_{1xx} + 2u_0u_{1xxx} + 2u_1u_{0xxx})$$

$$+\frac{1}{8}(2u_1u_{0xyy} + 4u_{1y}u_{0xy} + 4u_{0y}u_{1xy} + 2u_{0x}u_{1yy} + 2u_{1x}u_{0yy} + 2u_0u_{1xyy}) = 0. \quad (11)$$

And so on .

Using initial value $u_0(x, y) = u(x, y, 0)$, from the above formula, we can determine the

$$u_k(x, y)\ (k = 1, 2, \cdots).$$

Therefore we obtain the approximate solution of equation (1):

$$u(x, y, t) = u_0(x, y) + u_1(x, y)t^{\alpha} + u_2(x, y)t^{2\alpha} + \cdots.$$

For example , if $u_0(x, y) = e^{x+y}$, then from (10) and (11), we get

$$u_1(x, y) = -\frac{3}{\Gamma(1+\alpha)}e^{2x+2y},$$

$$u_2(x, y) = -\frac{8\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}e^{2x+2y}$$

Hence we obtain the 2-order approximate solution of equation (1):

$$u(x,t) = e^{x+y} - \frac{3}{\Gamma(1+\alpha)} e^{2x+2y} t^{\alpha} - \frac{8\Gamma(1+\alpha)}{\Gamma(1+2\alpha)} e^{2x+2y} t^{2\alpha}.$$

Conclusion

By using the fractional power series method, we give an algorithm for solving fractional Zakharov-Kuznetsv equations . Compared to the ADM, DTM, HPM, VIM and HAM, the fractional power series method is more derect , effective and the algorithm can be implemented as a computer program.

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