DYNAMICAL CHAOS IN Φ^6-RAYLEIGH OSCILLATOR WITH THREE WELLS DRIVEN AN AMPLITUDE MODULATED FORCE.

Y. A.Amer[1],S.M.Ahmed[2],ManarM.Dahshan[2]and N.M.Ali[3]

[1] Mathematics Department, Faculty of science, Zagazig University, Egypt.

[2] Mathematics Department, Faculty of science, El-Arish University, Egypt.

[3]Mathematics Department, Faculty of science, Suez Canal University, Egypt.

ABSTRACT

Chaotic behavior of Φ^6-Rayleigh oscillator with three wells is investigated. The method of multiple scale method is used to solve the system up to 3rd order approximation. Effect of parameters is studied numerically; all resonance cases are studied numerically to obtain the worst case. Stability of the system is investigated using both phase plane and frequency response curves. Comparison between the approximate solution and numerical solution is obtained

KEYWORDS: Vibration control; nonlinear oscillation; perturbation technique; sub-harmonic resonance.

1. INTRODUCTION

The Φ^6 Rayleigh oscillator which we have, is one-degree- of freedom nonlinear oscillator has been investigated and studied for quite a long time. Vibration method such as multiple scales has been applied to obtain approximate solutions. The method of multiple scales is one of the important perturbation techniques widely used. The method yields transient solutions as well as steady state solutions in contrast to some other techniques which yield only the steady state solution. Siewe, Tchawoua and Rajasekar [1] studied the effect on chaotic behavior of Φ^6-Rayleigh oscillator with three wells, they used Melnikov theorem to detect the conditions for possible occurrence of chaos. Chin et al. [2] studied the dynamics of a buckled beam possessing a two-to-one internal resonance when the higher mode is subjected to principal parametric excitation.

EL-Bassiounyet. al. [3] studied the system of two degree of freedom with quadratic and cubic nonlinearities and subjected to parametric and external excitations, the method of multiple scale perturbation technique (MSPT) is used to analyze the response of this system. Xu and Chung [4] investigated the mechanism of the action of the time delay in a non-autonomous system.

Eissa and Amer [5] studied the vibration control of a second order system simulating the first mode of a cantilever beam vibration subject to primary and sub-harmonic resonance. Zhu et. al. [6] studied the non-linear dynamics of a two-degree-of freedom vibrating system having both non-linear damping and non-linear spring stiffness. Eissaet. al. [7] investigated saturation phenomena in non-linear oscillating systems subject to multi-parametric and/ or external excitations. Eissaet. al. [8] studied both passive and active control in some non-linear differential equations described the vibration of aircraft wing subjected to: multi-excitation force, multi-parametric excitations are considered. Chatterjee [25] studied a theoretical basis of time delayed acceleration feedback control of linear and nonlinear vibrations of mechanical oscillators. Kamelet. al. [10-11] studied the vibration in ultrasonic machining that described by two different systems of non-linear differential equations.

EL-Gohary and El0Ganaini [12] studied the vibration suppression of a damping structure subject to multi-parametric excitation forces. Leung et. al. [13] analyze the steady state bifurcation of a periodically excites system, in which three kinds of delayed feedback controls are considered to discuss the effects. Eissaet. al. [14] time delay is considered in active suppression of nonlinear vibrations applying saturation based controller. They studied its effect on the system behavior. Amer and Abdel slam [15,16] studied the stability and control of dynamical system subjected to multi external forces, and vibration reduction of nonlinear dynamical system at combined resonance subject to tuned excitation.Thomsen [17] considered a string under similar conditions and used a sliding-mass non-linear absorber. Rong et al. [18] studied the non-linear saturated phenomenon which found its applications in vibration control. So several designed non-linear vibration absorbers on the basis of saturation phenomena have shown their feasibility and efficiency in practice.

Warminski et al. [19] studied the application of the non-linear saturation control (NSC) algorithm for a self-excited strongly non-linear beam structure driven by an external force. Orhan and Peter [20] investigate the effect of excitation and damping parameters on the super harmonic and primary resonance responses of a slender cantilever beam undergoing flapping motion. Elena et al. [21] studied the formal analysis and description of the steady-state behavior of an electrostatic vibration energy harvester operating in constant-charge mode and using different types of electromechanical transducers. Beltra-Carbajal et al. [22] studied the application of a passive/active Duffing-type vibration absorber for Duffing mechanical systems directly affected by unknown harmonic excitation forces, possibly resonant.

In this paper we studied vibration control of a nonlinear system under tuned excitation force. The method of multiple scale method is applied to obtain the approximate solution of the system. Vibration method is used to reduce the amplitude of vibration at the worst resonance case. The effect of different parameters are investigated, the comparison between the approximate solution and numerical solution is obtained.

2. MATHEMATICAL ANALYSIS

The nonlinear differential equation of Φ^6 Rayleigh oscillator one-degree- of freedom is given by:

$$\ddot{X} - \varepsilon c_1 \dot{X} + \varepsilon c_2 \dot{X}^3 + \omega_\circ^2 X + \varepsilon \lambda X^3 + \varepsilon \beta X^5 = \varepsilon\left(d_1 + 2d_2 \sin\Omega_2 t\right)\cos\Omega_1 t. \quad (1)$$

where c_1, c_2 represent damping coefficient terms, $\omega_\circ$ is the natural frequency, λ, β quantifying the nonlinearity stiffness, d_1 is unmodulated carrier amplitude, $2d_2$ is the degree of modulation, Ω_1, Ω_2 are the two frequencies of the force, this type of signal $\left(d_1 + 2d_2 \sin\Omega_2 t\right)\cos\Omega_1 t$. can be used in communications where the information signal is chosen to be an incoming amplitude modulated signal.

Equations (1) can be solved analytically using multiple time scale perturbation technique as :

$$\mathrm{X}(t;\varepsilon) = x_\circ(T_\circ, T_1) + \varepsilon x_1(T_\circ, T_1) + \varepsilon^2 x_2(T_\circ, T_1) \quad (2)$$

And the time derivatives become:

$$\frac{d}{dt} = D_\circ + \varepsilon D_1 + \varepsilon^2 D_2 + O(\varepsilon^3) \quad (3)$$

$$\frac{d^2}{dt^2} = D_\circ^2 + 2\varepsilon D_\circ D_1 + \varepsilon^2(\mathrm{D}_1^2 + 2\,\mathrm{D}_\circ) + O(\varepsilon^3) \quad (4)$$

From equations (2)-(4), into equation (1),and compare the coefficients of the same power of ε in the both sides, we obtain the following set of ordinary differential equations:

$$O(\varepsilon^0): (D_\circ^2 + \omega_\circ^2)x_\circ = 0 \quad (5)$$

$$\begin{aligned} O(\varepsilon): (D_\circ^2 + \omega_\circ^2)x_1 = & -2D_\circ D_1 x_\circ + c_1(D_\circ x_\circ) - \mathrm{c}_2(D_\circ x_\circ)^3 - \lambda x_\circ^3 - \beta x_\circ^5 \\ & + (d_1 + 2d_2 \sin\Omega_2 t)\cos\Omega_1 t \end{aligned} \quad (6)$$

$$\begin{aligned} O(\varepsilon^2): (D_\circ^2 + \omega_\circ^2)x_2 = & -2D_\circ D_1 x_1 - D_1^2 x_\circ + c_1 D_1 x_\circ - 3c_2(D_\circ x_\circ)^2\left[D_1 x_\circ + D_\circ x_1\right] \\ & + 3\lambda x_\circ^2 x_1 + 5\beta x_\circ^4 x_1 \end{aligned} \quad (7)$$

The general solution of Equ. (5) Is given by:

$$x_\circ = A(T_1)e^{i\omega_\circ T_\circ} + \overline{A}(T_1)e^{-i\omega_\circ T_\circ} \quad (8)$$

where A is unknown function in T_1. Substituting Equ. (8) intoEqu. (6) and eliminating the secular terms then the solution of (6) is given by:

$$x_1 = E_1 e^{3i\omega_\circ T_\circ} + E_2 e^{5i\omega_\circ T_\circ} + E_3 e^{i\Omega_1 T_\circ} + E_4 e^{i(\Omega_1+\Omega_2)T_\circ} + E_5 e^{i(\Omega_2-\Omega_1)T_\circ} + cc \quad (9)$$

Similarly From equ. (8)& (9) in to (7), we get:

$$\begin{aligned} x_2 = & E_6 e^{9i\omega_\circ T_\circ} + E_7 e^{7i\omega_\circ T_\circ} + E_8 e^{5i\omega_\circ T_\circ} + E_9 e^{3i\omega_\circ T_\circ} + E_{10} e^{i\Omega_1 T_\circ} + E_{11} e^{i(\Omega_1+\Omega_2)T_\circ} + E_{12} e^{i(\Omega_1-\Omega_2)T_\circ} \\ & + E_{13} e^{i(\Omega_1+2\omega_\circ)T_\circ} + E_{14} e^{i(\Omega_1-2\omega_\circ)T_\circ} + E_{15} e^{i(\Omega_1+4\omega_\circ)T_\circ} + E_{16} e^{i(\Omega_1-4\omega_\circ)T_\circ} + E_{17} e^{i(\Omega_1+\Omega_2+2\omega_\circ)T_\circ} \\ & + E_{18} e^{i(\Omega_1+\Omega_2-2\omega_\circ)T_\circ} + E_{19} e^{i(\Omega_1+\Omega_2+4\omega_\circ)T_\circ} + E_{20} e^{i(\Omega_1+\Omega_2-4\omega_\circ)T_\circ} + E_{21} e^{i(\Omega_1-\Omega_2+2\omega_\circ)T_\circ} \\ & + E_{22} e^{i(\Omega_1-\Omega_2-2\omega_\circ)T_\circ} + E_{23} e^{i(\Omega_1-\Omega_2+4\omega_\circ)T_\circ} + cc \end{aligned} \quad (10)$$

where $E_j, (j = 1, \ldots, 23)$ are complex functions of T_1.

From the above derived solutions, the reported resonance cases are $\omega_\circ = \Omega_1$, : $\omega_\circ = \Omega_1 + \Omega_2$, $\omega_\circ = \Omega_2 - \Omega_1$ and $\omega_\circ = \Omega_1 - \Omega_2$

3.STABILTY ANALYSIS

We study the different resonance numerically to see the worst resonance, one of the worst cases has been chosen to study the system stability, the selected resonance case in this case we introduce $\omega_\circ = \Omega$, the detuning parameter σ according to

$$\omega_\circ = \Omega_1 + \varepsilon\sigma \quad (11)$$

Substituting Equ.(11) into Equ. (9), and eliminating the secular and small divisor terms of x_1,we get the following:

$$-2i\omega_\circ \dot{A} + ic_1\omega_\circ A - 3ic_2\omega_\circ^3 A^2\bar{A} - 3\lambda A^2\bar{A} - 10\beta A^3\bar{A}^2 + \frac{d_1}{2}e^{-i\sigma T_1} = 0 \quad (12)$$

To analyze the solution of Equ. (12), it is convenient to express A in a polar form:

$$A(T_1) = \frac{1}{2}a(T_1)e^{i\gamma(T_1)} \quad (13)$$

From Equ. (13),into Equ. (12) we get :

$$-i\omega_\circ \dot{a} + \omega_\circ \dot{\gamma} a + \frac{1}{2}ic_1\omega_\circ a - \frac{3}{8}ic_2\omega_\circ^3 a^3 - \frac{3}{8}\lambda a^3 - \frac{5}{16}\beta a^5 + \frac{d_1}{2}e^{-i(\sigma+\gamma)T_1} = 0 \quad (14)$$

$$-i\omega_\circ \dot{a} + \omega_\circ(\dot{\varphi} - \sigma)a + \frac{1}{2}ic_1\omega_\circ a - \frac{3}{8}ic_2\omega_\circ^3 a^3 - \frac{3}{8}\lambda a^3 - \frac{5}{16}\beta a^5 + \frac{d_1}{2}\left(\cos\varphi - i\sin\varphi\right) = 0 \quad (15)$$

where $\varphi = \sigma T_1 + \gamma$,and then $\dot{\gamma} = \dot{\varphi} - \sigma$, Comparing the real and imagine parts in the two sides, we have:

$$\dot{a} = \frac{1}{2}c_1 a - \frac{3}{8}c_2\omega_\circ^2 a^3 - \frac{d_1}{2\omega_\circ}\sin\varphi \quad (16)$$

$$a(\dot{\varphi} - \sigma) = \frac{3}{8\omega_\circ}\lambda a^3 + \frac{5}{16\omega_\circ}\beta a^5 - \frac{d_1}{2\omega_\circ}\cos\varphi \quad (17)$$

At steady state $\dot{a} = \dot{\varphi} = 0$,in to Eqs. (16) and (17),we find:

$$c_1 a - \frac{3}{4}c_2\omega_\circ^2 a^3 = \frac{d_1}{\omega_\circ}\sin\varphi \quad (18)$$

$$2a\sigma + \frac{3}{4\omega_\circ}\lambda a^3 + \frac{5}{8\omega_\circ}\beta a^5 = \frac{d_1}{\omega_\circ}\cos\varphi \quad (19)$$

Squaring (18), (19) and adding the results,we obtain:

$$4a^2\sigma^2 + a^4\left(\frac{3}{\omega_\circ}\lambda + \frac{5}{2\omega_\circ}\beta a^2\right)\sigma + \left(\frac{3}{4\omega_\circ}\lambda a^3 + \frac{5}{8\omega_\circ}\beta a^5\right)^2 + \left(c_1 a - \frac{3}{4}c_2\omega_\circ^2 a^3\right)^2 - \frac{d_1^2}{\omega_\circ^2} = 0 \quad (20)$$

3.1 Linear Solution:

Now, we study the stability of the linear solution we obtained fixed let us consider A in the form:

$$A(T_1) = \frac{1}{2}(p - iq)e^{i\delta T_1} \quad (21)$$

where p and q are real values, SubstitutingEqs.(21) in to linear parts of Eq. (12) ,we obtain:

$$-i\,\omega_\circ\left(\dot{p} + ip\delta - i\dot{q} + q\delta\right) + \frac{1}{2}ic_1\omega_\circ\left(p - iq\right) + \frac{d_1}{2} = 0 \quad (22)$$

Comparing the real and imagine parts in the two sides, we get:

$$\dot{p} = -q\sigma + \frac{1}{2}c_1 p \quad (23)$$

$$\dot{q} = p\sigma + \frac{1}{2}c_1 q + \frac{d_1}{2\omega_\circ} \quad (24)$$

Putting Eqs. (23)and (24) in to a matrix form:

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{1}{2}c_1 & -\sigma \\ \sigma & \frac{1}{2}c_1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{d_1}{2\omega_\circ} \end{pmatrix}$$

The stability of the linear solution is obtained from the zero characteristic equation:

$$\begin{vmatrix} \frac{1}{2}c_1 - \lambda & -\sigma \\ \sigma & \frac{1}{2}c_1 - \lambda \end{vmatrix} = 0,$$

Hence $\lambda_{1,2} = \frac{c_1}{2} \pm i\sigma$

Then, the system is said to be stable if and only if the real part of its Eigen value less than zero $c_1 < 0$.

3.2 NONLINEAR SOLUTION

To determine the stability of fixed points, one lets

$$a = a_\circ + a_1, \quad \varphi = \varphi_\circ + \varphi_1 \quad (25)$$

Substituting Equ. (25) into the nonlinear terms of Equ. (16) and (17):

$$(a_\circ + a_1)[\dot{\varphi}_\circ + \dot{\varphi}_1 - \sigma] = \frac{3}{8\omega_\circ}\lambda\left(a_\circ^3 + 3a_\circ^2 a_1 + 3a_\circ a_1^2 + a_1^3\right)$$
$$+\frac{5}{16}\left(a_\circ^5 + 5a_\circ^4 a_1 + 10a_\circ^3 a_1^2 + 10a_\circ^2 a_1^3 + 5a_\circ a_1^4 + a_1^5\right)$$
$$-\frac{d_1}{2\omega_\circ}\left(\cos\varphi_\circ \cos\varphi_1 - \sin\varphi_\circ \sin\varphi_1\right), \quad (26)$$

Since $a_\circ, \varphi_\circ$ are solutions ofEqu. (16) and (17), and a_1, φ_1 are perturbations which are assumed to be small comparing to $a_\circ, \varphi_\circ$, then:

$$\dot{a}_1 = \left(\frac{1}{2}c_1 - \frac{9}{8}\omega_\circ^2 c_2 a_\circ^2\right)a_1 + \left(\frac{d_1}{2\omega_\circ}\cos\varphi_\circ\right)\varphi_1 \quad (27)$$

$$\dot{\varphi}_1 = \left(\frac{1}{a_\circ}\sigma + \frac{9}{8\omega_\circ}\lambda a_\circ + \frac{25}{16\omega_\circ}\beta a_\circ^3\right)a_1 + \left(\frac{d_1}{2\omega_\circ a_\circ}\sin\varphi_\circ\right)\varphi_1 \quad (28)$$

The stability of a given fixed point to a disturbance proportional to exp(λ t) is determined by the roots of:

$$\begin{vmatrix} \frac{1}{2}c_1 - \frac{9}{8}c_2\omega_\circ^2 a_\circ^2 - \lambda_1 & \frac{d_1}{2\omega_\circ}\cos\varphi_\circ \\ \frac{1}{a_\circ}\sigma + \frac{9}{8}\lambda a_\circ + \frac{25}{16\omega_\circ}\beta a_\circ^3 & \frac{d_1}{2\omega_\circ a_\circ}\sin\varphi_\circ - \lambda_1 \end{vmatrix} = 0 \quad (29)$$

Consequently, a non-trivial solution is stable if and only if the real parts of both eigen values of the coefficient matrix (29) are less than zero.

4. NUMERICAL RESULTS

The nonlinear dynamical system is solved numerically using Maple; at non resonance case (basic case) as shown in Fig.1, we can see that the steady state amplitude is about 0.1 and the system is stable with dynamic chaos. All resonance cases are studied numerically as shown in Fig. 2, we can see that the worst case is the primary resonance case $(\omega_\circ = \Omega_1)$, the steady state amplitude is increasing to 2 times of the basic case, while the case $(\omega_\circ = \Omega_1 + \Omega_2)$ the steady state is decreasingabout3times of the basic case. Frequency response equation (20) is nonlinear algebraic equation of the amplitude a against the detuning parameter σ this equation is solved numerically as shown in Fig. 3a. The effect of different parameters are studied in Fig. 3b- 3g, from these figure we see that the amplitude of the main system is monotonic decreasing on nonlinear stiffness λ, natural frequency $\omega_\circ$, and damping coefficientsc$_1$, c$_2$as shown in Fig.3b, 3c,3e and 3f respectively , while the amplitude of the main system is monotonic increasing on the parameters β ,d$_1$as shown in fig.3d, 3g. . In Fig.4 we show the effects of all parameters at the worst case. For the damping coefficients c_1, c_2, Fig.4a, 4b show that steady state amplitude of the main system is monotonic decreasing function. For the nonlinear stiffness parameters λ, β, Fig.4c, 4d show that steady state amplitude of the main system is monotonic decreasing functions. In Fig.6 we show the effects of all parameters at the worst case,for the damping coefficients c_1, c_2, Fig.4a, 4b shows that steady state amplitude of the main system is monotonic decreasing function.For the non-linear parameters λ, β, Fig. 4c, 4d shows that steady state amplitude of the main system is monotonic decreasing function. Also there is a good agreement between approximate solution and numerical solution as shown in Fig. 5.

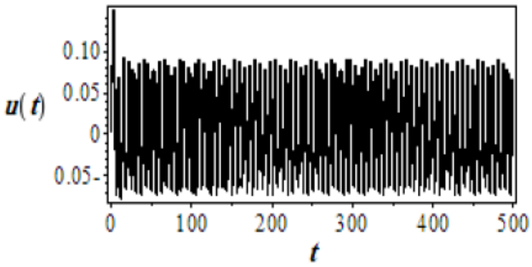
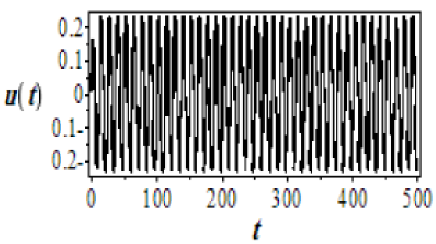
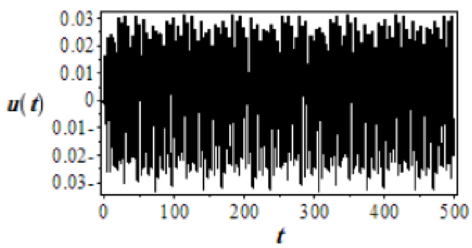


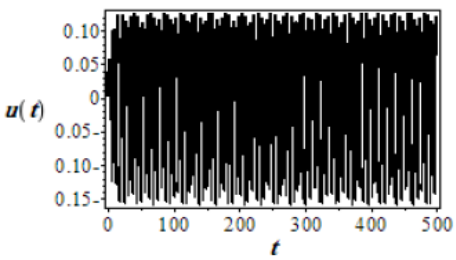
Fig.1. The plot of basic case.



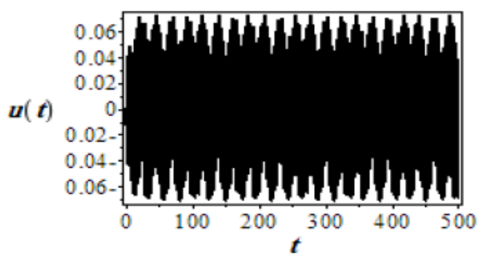
a) Primary resonance case $\omega_\circ = \Omega_1$.



b) Combined resonance case $\omega_\circ = \Omega_1 + \Omega_2$.



c) Combined resonance case $\omega_\circ = \Omega_1 - \Omega_2$.



d) Combined resonance case $\omega_\circ = \Omega_2 - \Omega_1$.

Fig. 3. Resonanse cases

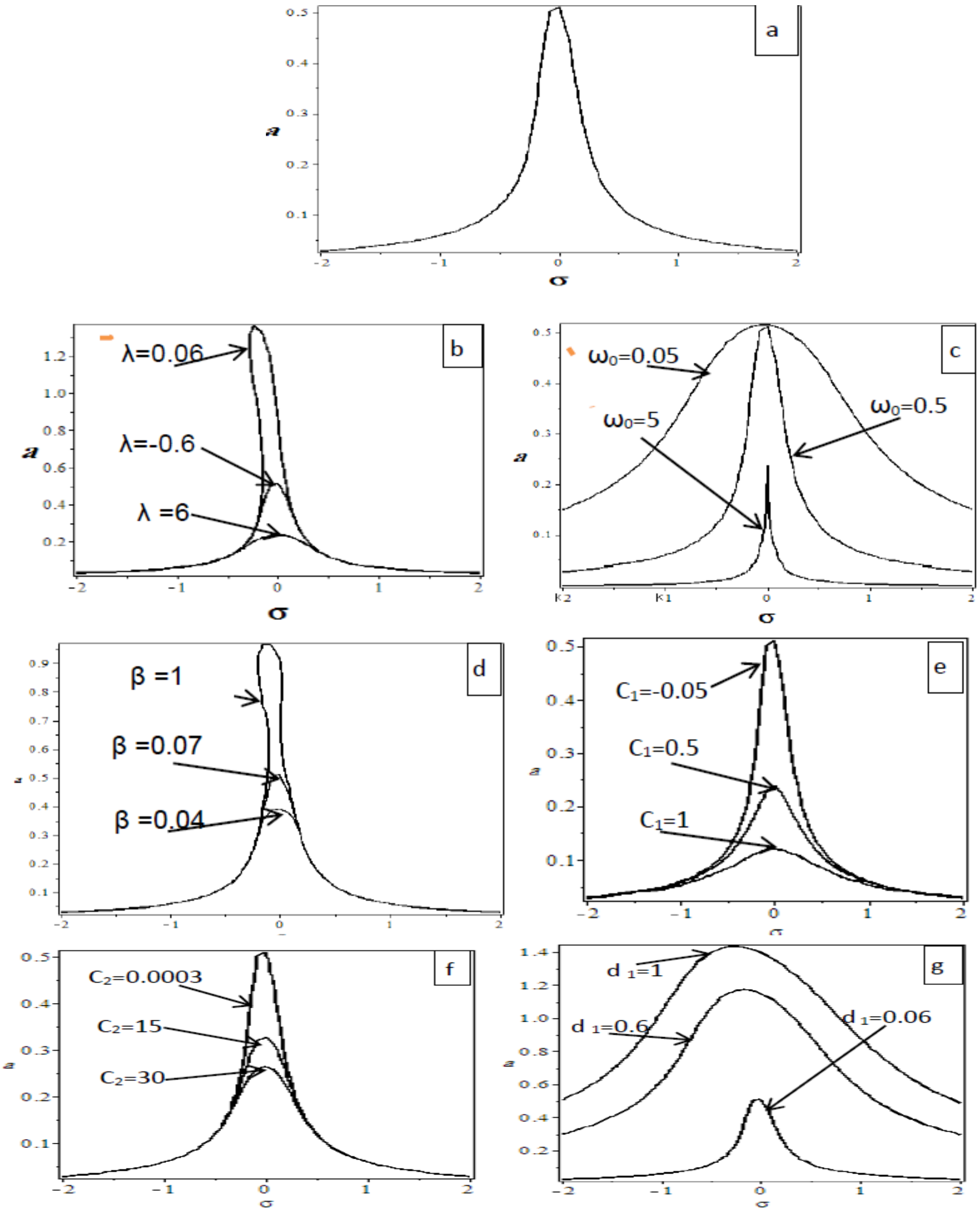
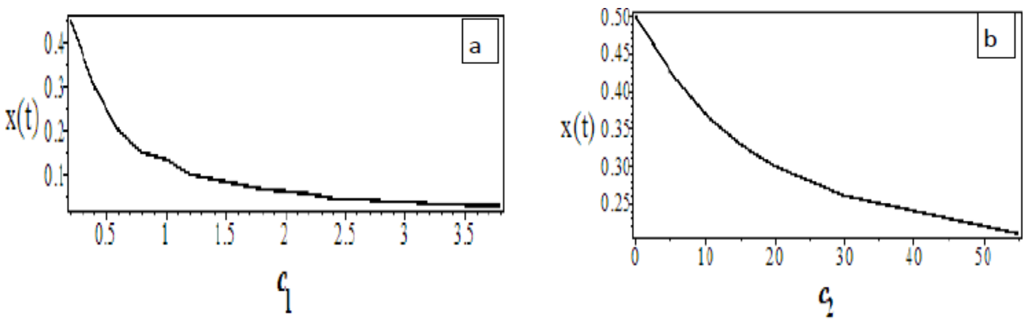


Fig.3. Frequency response curves.



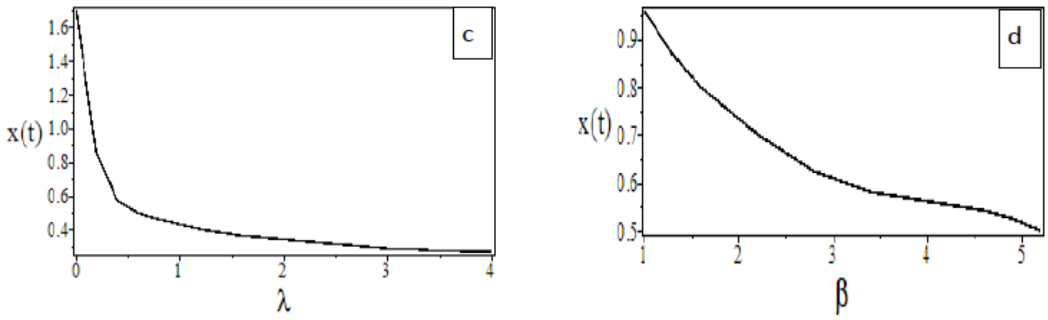


Fig.4. Effects of different parameters

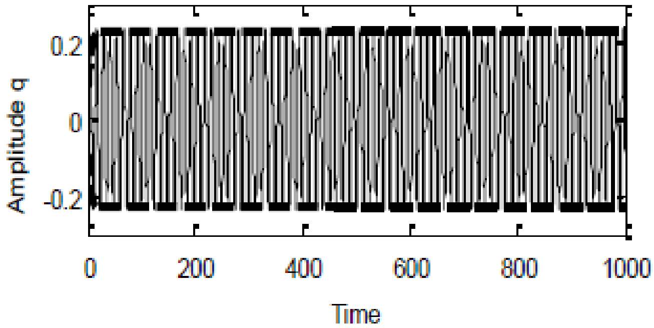


Fig. 5. Comparison between approximate solution (-----) and numerical solution (_____)

5. CONCLUSION

The vibration of non-linear dynamical system is studied, the worst resonance case is $(\omega_\circ = \Omega_1)$ .Hence the stability of the system is studied using the frequency response functions from the above study, and the following results are concluded:

1. The worst resonance case is $(\omega_\circ = \Omega_1)$ the steady state is increasing 2 times of the basic case.
2. The amplitude of the main system is monotonic decreasing function on natural frequency $\omega_\circ$ and damping coefficients c_1, c_2 .
3. The effect of parameters in the frequency response curves are a good agreement with the numerical solution as shown in Fig. 3 and Fog.4.
4. The approximate solution is a good agreement with the numerical solution as shown in Fig.5.

REFERENCES

[1] M. SieweSiewe, C.Tchawoua and S. Rajasekar," Homoclinicbifurcation and chaos in Φ^6-Rayleigh oscillator with three wells driven an amplitude modulated force",Tiruchirapalli 620 024, Tamilnadu, India.

[2] C. Chin, A. H. Nayfeh, and W. Lacarbonara, Two-to-one internal resonances in parametrically excited buckled beam, AIAA paper no 97-1081, 1997.

[3] A. F. EL-Bassiouny, M. M. Kamel and A. Abdel-Khalik, "Two-to-one internal resonances in nonlinear two degree of freedom system with parametric and external excitations", Mathematics and Computers in Simulation, Vol.63, pp. 45-56, 2003.

[4] J. Xu and K. W. Chung, "Effect of time delayed position feedback on a van der pol oscillator", Physica D, Vol. 180, pp. 17-39, 2003.

[5] M. Eissa and Y. A. Amer, "Vibration control of a cantilever beam subject to both external and parametric excitation", Applied Mathematics and Computation, Vol. 152, pp. 611-619, 2004.

[6] S. J. Zhu, Y. F.Zheng and Y. M. Fu, "Analysis of non-linear damping dynamics of a two-degree-of-freedom vibration system with non-linear and non-linear spring", Journal of Sound and Vibration, Vol. 271, pp. 15-24,2004.

[7] M. Eissa, W. A. A.EL-Ganaini and Y. S. Hamed, " Saturation, stability and resonance of non-linear systems", Physica A, Vol. 356, pp. 341-358, 2005.

[8] S. EL-Serafi, M. Eissa, H. EL-Sherbiny and T. H. EL-Ghareeb, "Comparison between passive and active control of non-linear dynamical system", Japan Journal of Ind. And Appl. Math., Vol. 23, Issue2, pp. 139-161, 2006.

[9] S. Chatterjee, " Vibration control by recursive time-delayed acceleration feedback", Journal of Sound and Vibration, Vol. 317, pp. 67-90, 2008.

[10] M. M. Kamel, W. A.A. EL-Ganaini and Y. S. Hamed, "Vibration suppression in muti-tool ultrasonic machining to multi-external and parametric excitation", ActaMehanicaSinica, Vol. 25, pp. 403-415,2009.

[11] W. A. A. EL-Ganaini, M. M. Kamel and Y. S. Hamed, "Vibration reduction in ultrasonic machine to external and tuned excitation forces", Applied Mathematical Modeling, Vol. 33, Issue 6, pp. 2853-2863, 2009.

[12] H. A.EL-Gohary, W. A. A. EL-Ganaini, "Vibration suppression of a dynamical system to multi-parametric excitations via time-delay absorber", Applied Mathematical Modeling Vol. 36, pp. 35-45, 2012.

[13] A. Y. T. Leung, Z. J. Guo, A. Myers, " Steady state bifurcation of a periodically excited system under delayed feedback controls", Communications in Nonlinear Science and Numerical Simulation Vol. 17, pp. 5256-5272, 2012.

[14] N. A. Saeed, W. A. A. El-Ganini, M. Eissa, "Nonlinear time delay saturation-based controller for suppression of nonlinear beam vibrations", Applied Mathematical Modeling, Vol. 37, Issue 20, pp. 8846-8864, 2013.

[15] Y. A. Amer and M. N. Abdelslam, "Stability and control of dynamical system subjected to multi external forces", International Journal of Mathematics and Computer (IJMCAR), Vol. 3, Issue 4, 41-52, 2013.

[16] Y. A. Amer and M. N. Abdelslam, "Vibration reduction of nonlinear dynamical system at combined resonance subject to tuned excitation", Journal of Advances in Mathematics, Vol. 9, 1774-1786, 2014.

[17] J. J. Thomsen, Vibration suppression by using self-arranging mass: effects of adding restoring force, Journal of Sound and Vibration 197 (1996) 403-425

[18] H. Rong, X. Wang, W. Xu, and T. Fang, Saturation and resonance of non-linear system under bounded noise excitation absorber, Journal of Sound Vibration 291 (2006) 48-59.

[19] J. Warminski, M. P. Cartmell, A. Mitura, and M. Bochenski, Active vibration control of a non-linear beam with self-and external excitations, Journal of Shock and Vibration 20 (2013) 1033-1047.

[20] O. Orhan and J. A. Peter, Nonlinear response of flapping beam to resonant excitations under nonlinear damping, ActaMech (2015) 1-27.

[21] B. Elena, G. Dimitri, B. Philippe and F. Orla, Steady-State Oscillations in Resonant Electrostatic Vibration Energy Harvesters, IEEE Transactions on Circuits and Systems I, 60 (2015) 875-884.