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On Rough Covexsity Sets

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Abstract

In this paper, we introduced new concepts for surely and possibly, start shaped (convex) set. Also for rough star shaped(rough convex) set. We established the necessary and sufficient conditions for a set to be star shaped (convex) set or rough star shaped(rough convex) set. Finally, we introduced new concepts for star shaped (convex) relation.

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1. Introduction

The study of convex sets is a branch of geometry, analysis, linear algebra that has numerous connections with other areas of mathematics. It is also relevant to several areas of science and technology. Any two distinct points x, y of real vector space E determine a unique line. It consists of all point of the form (1- λ) $x+\lambda y, \lambda$ ranging over all real numbers. These points for which $0\le\lambda$ and for which $0\le\lambda\le 1$ form respectively the ray from x through y and the segment γ_{xy} from x to y.

Definition 1.1[5] A set S is star shaped relative to a point $x\in S$ if for each $y\in S$ it is true that $\gamma_{xy}\subset S$.

Definition1.2[5] A set S is convex if for each pair of points x and y in S it is true that $\gamma_{xy}\subset S$.

Definition1.3[3] If (U, R) is an information system where U is a universe set and R is equivalence relation. Then we can define $\underline{R}-$ lower and $\overline{R}$ -upper approximation of A as follows.

$\underline{R}(A)=\{x\in U \mid R,[x]_R\subseteq A\}$. $\overline{R}(A)=\{x\in U \mid R,[x]_R\cap A\neq\phi\}$ where $[x]_R$ is the equivalence class containing an element $x\in X$.

Definition1.4[2] Let X be a universe set and R be a general relation over X. Then the lower and upper approximations of $A\subseteq X$ are, respectively, defined as:

$\underline{L}(A)=\cup\{G, G\subseteq A, G$ is open set$\}$, $\overline{L}(A)=\cap\{F, F\supseteq A, F$ is closed set$\}$,

Proposition1.1[3] If (U, R) is an information system and $A, B\subseteq U$ .Then, we have:

1) $\underline{R}(A)\subseteq A\subseteq\overline{R}(A).$

2) $\underline{R}(\phi)=\overline{R}(\phi)=\phi, \underline{R}(U)=\overline{R}(U)=U.$

3) $\overline{R}(A\cup B)=\overline{R}(A)\cup\overline{R}(B).$

4) $\underline{R}(A\cap B)=\underline{R}(A)\cap\underline{R}(B).$

5) If $A\subseteq B$, then $\underline{R}(A)\subseteq\underline{R}(B)$ and $\overline{R}(A)\subseteq\overline{R}(B).$

6) $\underline{R}(A\cup B)\supseteq\underline{R}(A)\cup\underline{R}(B).$

7) $\overline{R}(A\cap B)\subseteq\overline{R}(A)\cap\overline{R}(B).$

8) $\overline{R}(A^C)=(\underline{R}(A))^C.$

9) $\underline{R}(A^C)=(\overline{R}(A))^C.$

10) $\overline{R}(\overline{R}(A))=\underline{R}(\overline{R}(A))=\overline{R}(A).$

11) $\underline{R}(\underline{R}(A))=\overline{R}(\underline{R}(A))=\underline{R}(A),$

Where A^C is the complement of A.

Definition1.5[4] The set A is roughly -included in B if and only if $\underline{R}(A) \subseteq \underline{R}(B)$ and $\overline{R}(A) \subseteq \overline{R}(B)$.

Definition1.6[4] If (U, R) is an information system and $A, B \subseteq U$. Then, we have:

1) $x \underline{\in}_R A$ if and only if $x \in \underline{R}(A)$.

2) $x \overline{\in}_R A$ if and only if $x \in \overline{R}(A)$, where $\underline{\in}_R$ and $\overline{\in}_R$ are denoted to surely belong and possibly belong respectively.

Proposition 1.2 [4] If (U, R) is an information system and $A, B \subseteq U$. Then

1) $x \underline{\in}_R A$ implies $x \in A$ implies $x \overline{\in}_R A$.

2) $A \subseteq B$ implies $x \underline{\in}_R A$ implies $x \underline{\in}_R B$ and $x \overline{\in}_R A$ implies $x \overline{\in}_R B$.

3) $x \overline{\in}_R (A \cup B)$ if and only if $x \overline{\in}_R A$ or $x \overline{\in}_R B$.

4) $x \underline{\in}_R (A \cap B)$ if and only if $x \underline{\in}_R A$ and $x \underline{\in}_R B$.

5) $x \underline{\in}_R A$ or $x \underline{\in}_R B$ implies $x \underline{\in}_R (A \cup B)$.

6) $x \overline{\in}_R (A \cap B)$ if and only if $x \overline{\in}_R A$ and $x \overline{\in}_R B$.

7) $x \overline{\in}_R A$ if and only if $x \underline{\in}_R A^C$.

Definition1.6[1] If $(X, \overset{M}{S}O(X))$ is a $S-$approximation space associated with general relation over a universe set X and $A \subseteq X$ .Then the simply lower and simply upper approximations of A are defined by

$\underline{B}_S(A) = \cup\left\{G : G \in \overset{M}{S}O(X), U \subseteq A\right\}$, $\overline{B}_s(A) = \cap\left\{F ; F \in \overset{M}{S}O(X), F \supseteq A\right\}$. Where $\overset{M}{S}O(X)$ is the class of simply open sets of the universe set X.

Definition1.7[1] If $(X, \overset{M}{S}O(X))$ is a $S-$approximation space associated with general relation over a universe set X and $A \subseteq X$ .For any $a \in X$, we define two membership relations, which are called lower belong and upper belong as follows:

1) a is lower belong of A (briefly $a \underline{\in} A$) iff $a \in \underline{B}_S(A)$.

2) a is upper belong of A (briefly $a \overline{\in} A$) iff $a \in \overline{B}_S(A)$.

Proposition 1.2[1] If $(X, \overset{M}{S}O(X))$ is a $S-$approximation space associated with general relation over a universe set X and $A \subseteq X$ .Then, we have:

1) $\underline{B}_s(A) \subseteq A \subseteq \overline{B}_s(A)$

2) $\overline{B}_s(A \cup B) = \overline{B}_s(A) \cup \overline{B}_s(B)$.

3) $\underline{B}_s(A \cap B) = \underline{B}_s(A) \cap \underline{B}_s(B)$.

4) If $A\subseteq B$, then $\overline{B}_s(A)\subseteq\overline{B}_s(B)$ and $\underline{B}_s(A)\subseteq\underline{B}_s(B)$.

5) $\underline{B}_s(A\cup B)\supseteq\underline{B}_s(A)\cup\underline{B}_s(B)$.

6) $\underline{B}_s(A)=[\overline{B}_s(A)^C]^C$ and $\overline{B}_s(A)=[\underline{B}_s(A^C)]^C$.

7) $\underline{B}_s(\underline{B}_s(A))=\underline{B}_s(A)$, and $\overline{B}_s(\overline{B}_s(A))=\overline{B}_s(A)$.

8) $\overline{B}_s(A\cap B)\subseteq\overline{B}_s(A)\cap\overline{B}_s(B)$.

9) $\underline{B}_s(\phi)=\overline{B}_s(\phi)=\phi$ and $\underline{B}_s(X)=\overline{B}_s(X)=X$.

Remark 1.1 The simply boundary region of the set A (briefly $\overset{M}{S}b(A)$) is $\overset{M}{S}b(A)=\overline{B}_s(A)-\underline{B}_S(A)$.

2. Main work

Definition 2.1 A set A is called surely (possibly) subset of B denoted by $A\subseteq_s B$ ($A\subseteq_p B$) iff for all $x\underline{\in}A$ ($x\overline{\in}A$),then $x\underline{\in}B$ ($x\overline{\in}B$) respectively

Lemma 2.1 If A and B are two subsets of X . Then, A is surely (possibly) subset of B iff $\underline{R}(A)\subseteq\underline{R}(B)$ ($\overline{R}(A)\subseteq\overline{R}(B)$), respectively.

1) If A is surely subset of B, then $A\subseteq_s B\leftrightarrow x\underline{\in}A\rightarrow x\underline{\in}B\leftrightarrow x\in\underline{R}(A)\rightarrow x\in\underline{R}(B)\leftrightarrow\underline{R}(A)\subseteq\underline{R}(B)$.

2) If A is possibly subset of B, then

$$A\subseteq_p B\leftrightarrow x\overline{\in}A\rightarrow x\overline{\in}B\leftrightarrow x\in\overline{R}(A)\rightarrow x\in\overline{R}(B)\leftrightarrow\overline{R}(A)\subseteq\overline{R}(B)$$

Remark 2.1 It is easy to see that $\phi\subseteq_s A$ and $\phi\subseteq_p A$, for any set A of X.

1) If $A\subseteq_s B$ and $C\subseteq_s D$. Then, $A\cap C\subseteq_s B\cap D$.

2) If $A\subseteq_p B$ and $C\subseteq_p D$. Then, $A\cup C\subseteq_p B\cup D$.

Proof:

1) Since $A\subseteq_s B$, and $C\subseteq_s D$. Then $\underline{R}(A)\subseteq\underline{R}(B)$ and $\underline{R}(C)\subseteq\underline{R}(D)$. Which leads to

$\underline{R}(A)\cap\underline{R}(C)\subseteq\underline{R}(B)\cap\underline{R}(D)\Rightarrow\underline{R}(A\cap C)\subseteq\underline{R}(B\cap D)$ .Therefore, $A\cap C\subseteq_s B\cap D$.

2) Since $A\subseteq_p B$, and $C\subseteq_p D$. Then $\overline{R}(A)\subseteq\overline{R}(B)$ and $\overline{R}(C)\subseteq\overline{R}(D)$. Which leads to

$\overline{R}(A)\cup\overline{R}(C)\subseteq\overline{R}(B)\cup\overline{R}(D)\Rightarrow\overline{R}(A\cup C)\subseteq\overline{R}(B\cup D)$ .Therefore $A\cup C\subseteq_p B\cup D$.

Remark 2.2 It is easy to see, in the above lemma, that (1) is not true, for the union but (2) is not true for the intersection by using (6-7) in proposition (1.1), in general.

Definition 2.2 A subset $A\subseteq X$ is called surely (possibly) star shaped set w.r.t. x iff $x\underline{\in}A$ ($x\overline{\in}A$) and $\gamma_{xy}\subseteq_s A$

($\gamma_{xy} \subseteq_p A$) for all $y \in A$ ($y \in \bar{A}$).

Definition2.3 A subset $A \subseteq X$ is called surely (possibly) equal to a subset $B \subseteq X$ if $A \subseteq_s B$ and $B \subseteq_s A \leftrightarrow A =_s B$ (If $A \subseteq_p B$ and $B \subseteq_p A \leftrightarrow A =_p B$), where $=_s$ and $=_p$ are denote to surely and possibly equal.

Proposition 2.1 A subset $A \subseteq X$ is a star shaped set w.r.t. x iff $x \in A$ and $\bigcup_{y \in A} \gamma_{xy} = A.$

Proof: Let A be a star shaped set w.r.to x and $\bigcup_{y \in A} \gamma_{xy} \neq A.$ Then, there exists at least one point $y_\circ \in A$ and the segment $\gamma_{xy_\circ}$ contains at least one point z s.t. $z_\circ \notin A$. This means that $\gamma_{xy} \not\subseteq A$. Therefore, A is not a star shaped set w.r.to x . Which is a contradiction. Thus, if A is star shaped set w.r.to x, then $\bigcup_{y \in A} \gamma_{xy} = A.$ conversely, let $A = \bigcup_{y \in A} \gamma_{xy}, x \in A$ and A is not star shaped set w.r.to $x.$ Then, there exists at least one segment $\gamma_{xy_\circ}$ from x to $y_\circ \in A$ such that $\gamma_{xy_\circ} \not\subseteq A$. Therefore, there exists at least one point $z_\circ \in \gamma_{xy_\circ}$ and $z_\circ \notin A$. Thus, $A \neq \bigcup_{y \in A} \gamma_{xy}$. which is also a contradiction. Hence, if $A = \bigcup_{y \in A} \gamma_{xy}$, then A is star shaped set w.r.to $x.$

Remark 2.2 By the some manner, in the above proposition, we can prove the following proposition.

Proposition 2.2 A subset $A \subseteq X$ is surely (possibly)star shaped set w.r.to x iff $x \in \underline{A}$ ($x \in \bar{A}$) and $\bigcup_{y \in \underline{A}} \gamma_{xy} =_s A$ ($\bigcup_{y \in \bar{A}} \gamma_{xy} =_p A$), respectively.

Remark 2.3 If A and B are two star shaped set w.r.to the some point. Then $A \cap B$ and $A \cup B$ are star shaped sets w.r.to the some point. In this case, it is easy to see that $A \subseteq B$ or $B \subseteq A.$

Definition 2.4 The set of all points of A which the set A is star shaped w.r.to them is called the kernel of A and denoted by $\ker A. i.e. \ker A = \{x \in A \; : \; \gamma_{xy} \subseteq A,$ for all $y \in A\} = \{x \in A \; : \; \bigcup_{y \in A} \gamma_{xy} = A\}.$

Definition 2.5 A subset $A \subseteq X$ is surely (possibly) convex set iff for all x, $y \in \underline{A}$ ($x, y \in \bar{A}$) we have $\gamma_{xy} \subseteq_s A$ ($\gamma_{xy} \subseteq_p A$), respectively.

Proposition2.3 A set A is convex iff $\bigcup_{x, y \in A} \gamma_{xy} = A.$

Proof: Let A be a convex set and $\bigcup_{x, y \in A} \gamma_{xy} \neq A$. Then, there exists at least a point $z_\circ \in \gamma_{x_\circ y_\circ}$, $x_\circ, y_\circ \in A$ and $z_\circ \notin A.$ Therefore, $\gamma_{x_\circ y_\circ} \not\subseteq A.$ This means that A is not a convex set. Which is a contradiction. Hence, if A is a convex set, then $\bigcup_{x, y \in A} \gamma_{xy} = A.$ conversely, let $\bigcup_{x, y \in A} \gamma_{xy} = A.$, and A is not a convex set. Then, there exist, at least two points $x_\circ, y_\circ \in A$ such that $\gamma_{x_\circ y_\circ} \not\subseteq A.$ Thus $\bigcup_{x, y \in A} \gamma_{xy} \neq A.$ which is also a contradiction. Hence, if $\bigcup_{x, y \in A} \gamma_{xy} = A,$ then A is a convex set.

Remark 2.3 By the same manner, in the above Proposition, we can prove the following proposition.

Proposition 2.4 A subset $A \subseteq X$ is surely (possibly) convex set iff $\bigcup_{x, y \in \underline{A}} \gamma_{xy} =_s A$, ($\bigcup_{x, y \in \bar{A}} \gamma_{xy} =_p A$).

Proposition 2.5 If A, B are two surely (possibly) convex sets. Then $A \cap B$ is also surely convex set but not possibly convex set in general.

Proof: Let A and B are two surely convex sets .Then, for all $x, y \underline{\in} A \cap B \rightarrow x, y \underline{\in} A$ and $x, y \underline{\in} B \rightarrow \gamma_{xy} \subseteq_s A$ and $\gamma_{xy} \subseteq_s B \rightarrow \underline{R}(\gamma_{xy}) \subseteq \underline{R}(A)$ and $\underline{R}(\gamma_{xy}) \subseteq \underline{R}(B) \rightarrow \underline{R}(\gamma_{xy}) \subseteq \underline{R}(A) \cap \underline{R}(B) = \underline{R}(A \cap B) \rightarrow \underline{R}(\gamma_{xy}) \subseteq \underline{R}(A \cap B) \rightarrow \gamma_{xy} \subseteq_s A \cap B.$ Hence $A \cap B$ is also surely convex set. Similarly, let A and B are two possibly convex sets. Then, for all $x, y \overline{\in} A \cap B \rightarrow x, y \overline{\in} A$ and $x, y \overline{\in} B \rightarrow \gamma_{xy} \subseteq_p A$ and $\gamma_{xy} \subseteq_p B \rightarrow \overline{R}(\gamma_{xy}) \subseteq \overline{R}(A)$ and $\overline{R}(\gamma_{xy}) \subseteq \overline{R}(B) \rightarrow \overline{R}(\gamma_{xy}) \subseteq \overline{R}(A) \cap \overline{R}(B) \supseteq \overline{R}(A \cap B) \rightarrow \overline{R}(\gamma_{xy}) \supseteq \overline{R}(A \cap B).$ Hence $A \cap B$ is not possibly convex set.

Definition 2.6 Let A be a set and $x, y \in A.$ We say that x sees y surely (possibly) through A iff the segment γ_{xy} from x to y is surely (possibly) subset of $A.$ Also , we say that x sees y roughly through A iff the segment γ_{xy} from x to y is roughly included in $A.$

Definition 2.7 The set A is called roughly star shaped set w.r. to x iff $x \in A$ and x sees y roughly through A for all $y \in A.$

Definition 2.8 The set A is called roughly convex set iff the segment γ_{xy} from x to y is roughly included in A , for all $x, y \in A.$

Remark 2.4 It is easy to see that:

1) The intersection (the union) of any two roughly star shaped sets is not roughly star shaped set in general.
2) The intersection of two roughly convex sets is also roughly convex set but the union is not in general.
3) Ker A is roughly convex set.
4) The set A is roughly convex set iff $A = KerA.$
5) Every roughly convex set is roughly star shaped set, but the converse is not true in general.

Now, let us introduce a short note on a new special kind of relation for opening new point of research.

Definition 2.9 The relation $R \subseteq AXA$ is said to be star shaped relation w.r. to x iff $x \in A$ and xRy , for all $y \in A.$

Definition 2.10 The set of all points of A which the relation R is star shaped w.r.to them is called the kernel of R and denoted by Ker R. Thus, $KerR = \{x \in A : xRy,$ for all $y \in A\}.$

Definition 2.11 The relation $R \subseteq AXA$ is said to be convex relation iff xRy , for all $x, y \in A.$

Proposition2.6 Every convex relation is an equivalence relation

Proof: Let $R \subseteq AXA$ be a convex relation, then xRy for all $x, y \in A$. This means that $xRx,$ for all x and xRy and yRx are exist. Also, we have xRy, yRz , xRz for all $x, y, z \in A$ are also exist . Hence $R \subseteq AXA$ is an equivalence relation.

Remark 2.5 It is easy to see that the converse of the above proposition is not true in general as shown in the following example.

Example2.1 Let $A = \{a.b, c\}$ and $R = \{(a, a), (b, b), (c, c)\}$. It is easy to see that R is an equivalence relation but it is not a convex relation.

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