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# (1,2) - Domination in the Total Graphs of $C_n$ , $P_n$ and $K_{1,n}$

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**Abstract:**

In this paper, we discuss the (1,2) - domination in the total graphs of $C_n$ , $P_n$ and $K_{1,n}$

**Key words:** Dominating set; domination number; (1,2)-dominating set; (1,2)-domination number; total graph.

**Mathematics Subject Classification:** 05C69

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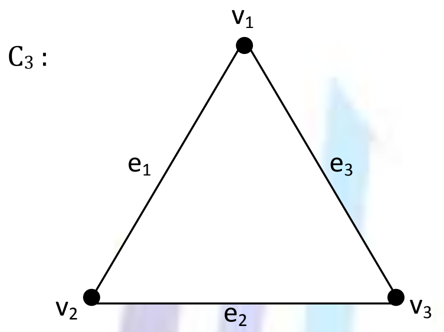
## 1. Introduction

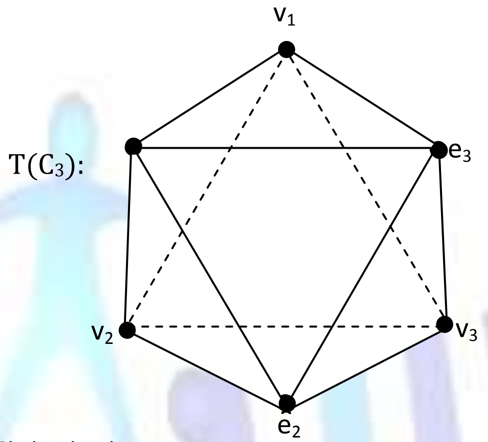
By a graph $G = (V,E)$, we mean a finite, undirected graph without loops or multiple edges. A subset $D$ of $V$ is a dominating set of $G$ if every vertex of $V$-$D$ is adjacent to a vertex of $D$. The domination number of $G$, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set. A *(1,2) dominating set* in a graph $G$ is a set $S$ having the property that for every vertex $v$ in $V$-$S$ there is at least one vertex in $S$ at distance 1 from $v$ and a second vertex in $S$ within distance 2 of $v$. The order of the smallest (1,2) - dominating set of $G$ is called the *(1,2) - domination number* of $G$ denoted by $\gamma_{(1,2)}(G)$.

For a given graph $G$,let $T(G)$ be a graph with vertex set $V(G)\cup E(G)$. Two vertices $x$, $y$ in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following holds (i) $x,y$ are in $V(G)$ and $x$ is adjacent to $y$ in $G$. (ii) $x,y$ are in $E(G)$ are $x,y$ are adjacent in $G$ (iii) $x$ is in $V(G)$, $y$ is in $E(G)$ and $x,y$ are incident in $G$. That is, the total graph $T(G)$ of a graph $G$ is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in $G$.

## 2. (1,2)- domination in Total graph of $C_n$

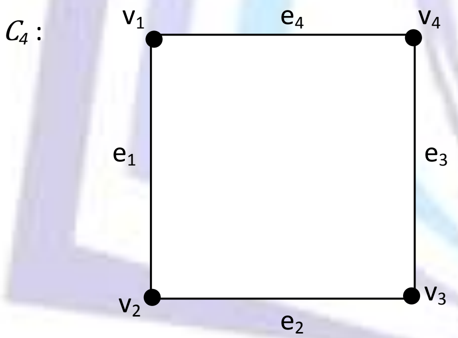
Consider the following examples

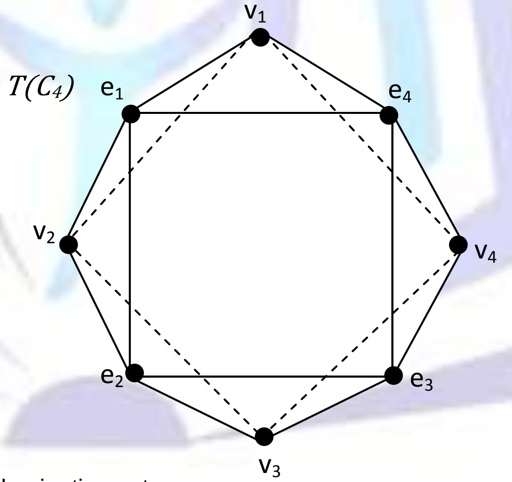



Hence in $T(C_3)$, $\{e_1, v_3\}$ is a dominating set and also (1,2) dominating set.

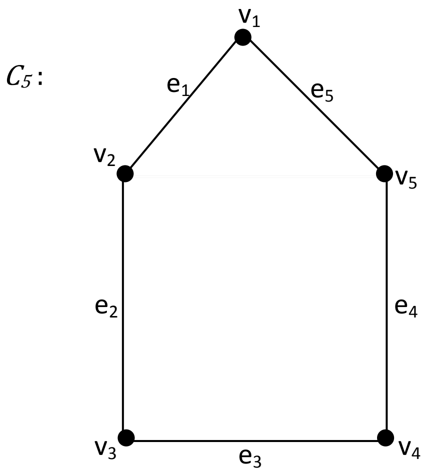
$\gamma[T(C_3)] = 2$ and $\gamma_{(1,2)}[T(C_3)] = 2$

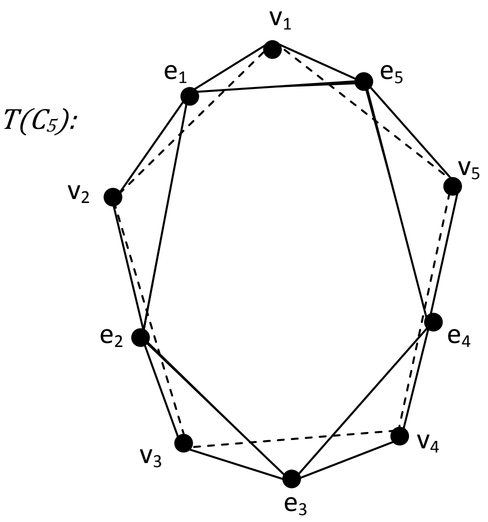



In $T(C_4)$, $\{e_1, v_3\}$ is a dominating set and also (1,2) dominating set.

$\gamma[T(C_4)] = 2$ and $\gamma_{(1,2)}[T(C_3)] = 2$.



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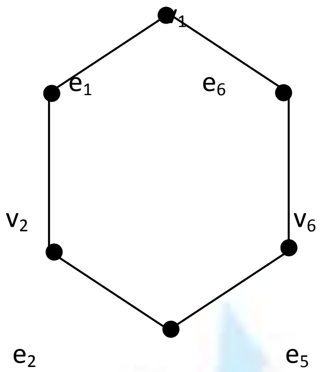
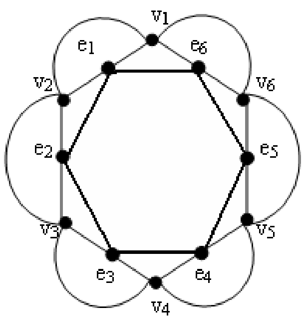
In $T(C_5)$, $\{e_1, v_4\}$ is a dominating set and also (1,2) dominating set.

$\gamma\,[T(C_5)] = 2$ and $\gamma_{(1,2)}\,[T(C_5)] = 2$

$C_6$ :

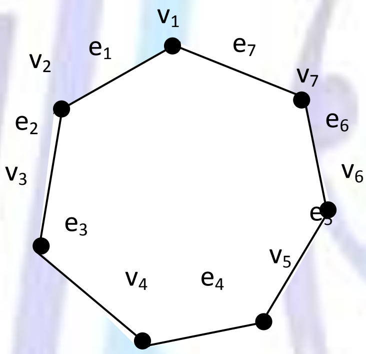



$T(C_6)$ :

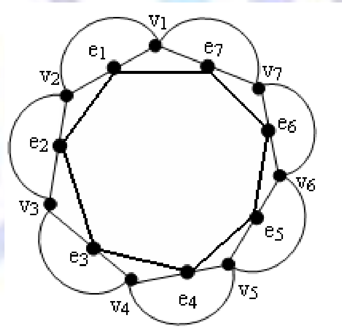



$\{e_1, v_4, v_6\}$ is a dominating set and also (1,2) dominating set.

$C_7$ :

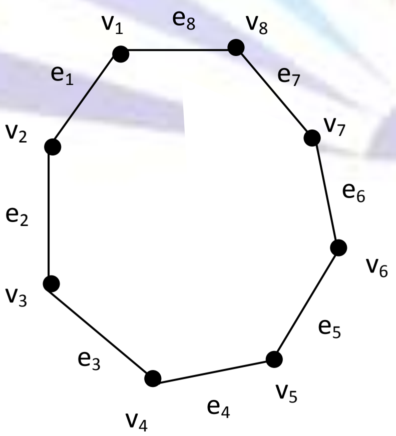



$T(C_7)$:

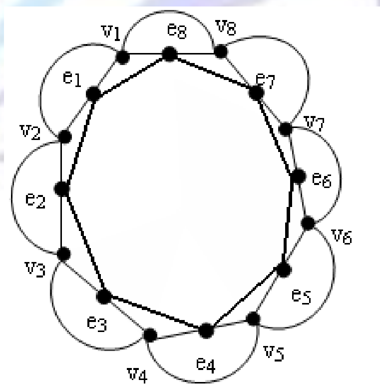



$\{e_1, v_4, v_6\}$ is a dominating set and also a (1,2) dominating set.

$C_8$ :

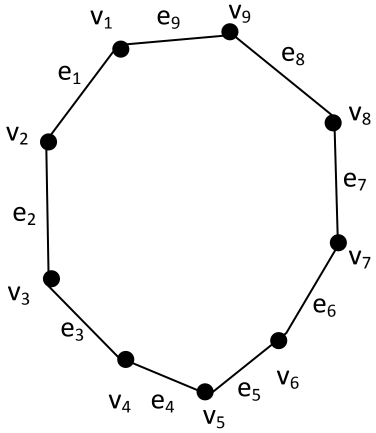



$T(C_8)$ :

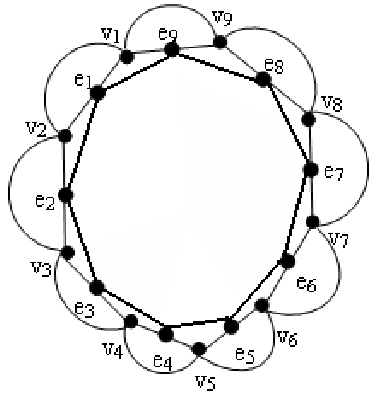



$\{e_1, v_4, v_6, v_8\}$ is a dominating set and also a (1,2) dominating set.

$C_9$ :

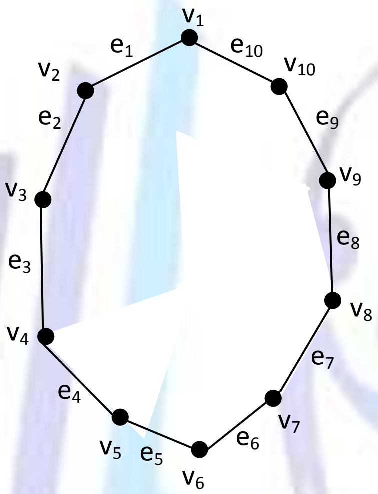



$T(C_9)$ :

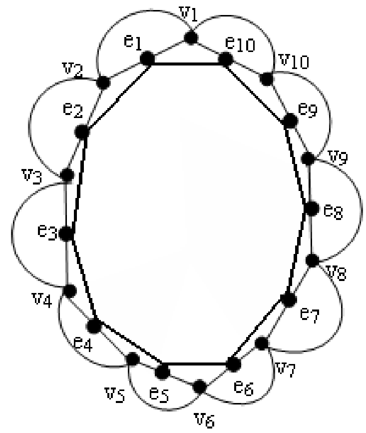



$\{e_1, v_4, v_6, v_8\}$ is a dominating set and also a (1,2) dominating set.

$C_{10}$ :




$T(C_{10})$ :




$\{e_1, v_4, v_6, v_8, v_{10}\}$ is a dominating set and also a (1,2) dominating set.

From the above examples, we have the following theorems.

**Theorem 2.1**

$$\gamma[T(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor.$$

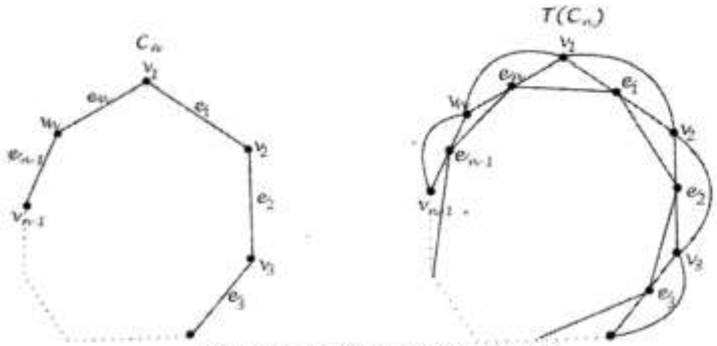



**Figure 1.** n-cycle and its Total Graph

Proof:

It can be easily observed that $\gamma[T(C_3)]=2$, $\gamma[T(C_4)]=2$, $\gamma[T(C_5)]=2$ and $\gamma[T(C_6)]=3$.

Let $V(C_n) = \{v_1, v_2,..., v_n\}$ and $E(C_n) = \{e_1, e_2,..., e_n\}$ in which $e_i = v_i v_{i+1}$. By the definition of total graph $V[T(C_n)] = \{v_1, v_2,..., v_n\} \cup \{e_1, e_2,..., e_n\}$ and

$E[T(C_n)] = \{e_i e_{i+1}/1 \le i \le n\} \cup e_n e_1 \cup \{v_i v_{i+1}/1 \le i \le n\} \cup v_n v_1 \cup \{e_i v_{i+1}/1 \le i \le n-1\} \cup \{e_n v_1\} \cup \{v_i e_i/1 \le i \le n\}$. The total number of vertices in $T(C_n)$ is $2n$ and each vertex is of degree 4. The cycles of $T(C_n)$ are $C_i = e_i v_{i+1} e_{i+1}$ $(1 \le i \le n-1)$, $C_n = e_n e_1 v_1$, $C'_i = v_i v_{i+1} e_i (1 \le i \le n-1)$, $C'_n = v_n v_1 e_n$. In $T(C_n)$, each $e_i$ is adjacent to $e_{i+1}$ and $e_1$ is adjacent to $e_n$. Also $v_i$ is adjacent to $v_{i+1}$ and $v_1$ is adjacent to $v_n$. Now we can find the dominating set of $T(C_n)$.

The minimum dominating set

$$D = \begin{cases} D_1 = \{e_1, v_4, v_6, ..., v_n\} \text{ if } n = 2k \\ D_2 = \{e_1, v_4, v_6, ..., v_{n-1}\} \text{ if } n = 2k+1 \quad \text{where } k = 3,4,5,.... \end{cases}$$

$$|D_1| = \frac{n-4}{2} + 1 + 1 = \frac{n}{2}$$

$$|D_2| = \frac{n-1-4}{2} + 1 + 1 = \frac{n-1}{2}$$

$$|D| = \begin{cases} \frac{n}{2} & \text{if } n = 2k \\ \frac{n-1}{2} & \text{if } n = 2k+1 \end{cases}$$

$$= \begin{cases} \frac{n}{2} & \text{if } n = 2k \\ \frac{n}{2} - \frac{1}{2} & \text{if } n = 2k+1 \end{cases}$$

Hence $\gamma[T(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor$ for all values of $C_n$.

### Theorem 2.2

$$\gamma_{(1,2)}[T(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor$$

Proof:

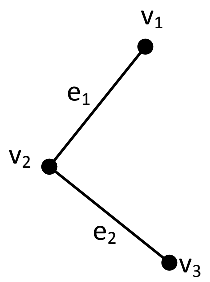
It can be easily observed that $\gamma_{(1,2)}[T(C_3)]=2$, $\gamma_{(1,2)}[T(C_4)]=2$, $\gamma_{(1,2)}[T(C_5)]=2$ and $\gamma_{(1,2)}[T(C_6)]=3$.Let $V(C_n) = \{v_1, v_2,..., v_n\}$ and $E(C_n) = \{e_1, e_2,..., e_n\}$ in which $e_i = v_i v_{i+1}$. By the definition of total graph $V[T(C_n)] = \{v_1, v_2,..., v_n\} \cup \{e_1, e_2,..., e_n\}$ and

$E[T(C_n)] = \{e_i e_{i+1}/1 \le i \le n\} \cup e_n e_1 \cup \{v_i v_{i+1}/1 \le i \le n\} \cup v_n v_1 \cup \{e_i v_{i+1}/1 \le i \le n-1\} \cup \{e_n v_1\} \cup \{v_i e_i/1 \le i \le n\}$. The total number of vertices in $T(C_n)$ is $2n$ and each vertex is of degree 4. The cycles of $T(C_n)$ are $C_i = e_i v_{i+1} e_{i+1}$ $(1 \le i \le n-1)$, $C_n = e_n e_1 v_1$, $C'_i = v_i v_{i+1} e_i (1 \le i \le n-1)$, $C'_n = v_n v_1 e_n$. Every minimum cardinality dominating set is also a (1,2) - dominating set in $T(C_n)$.Hence $\gamma_{(1,2)}[T(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor$.

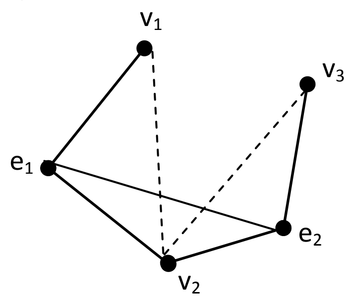
## 3. (1,2) domination in Total graph of $P_n$

Consider the following examples

$P_3$ :




$T(P_3)$:



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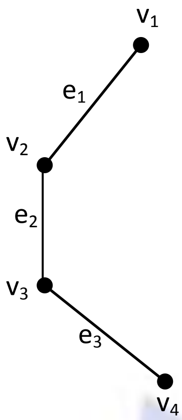
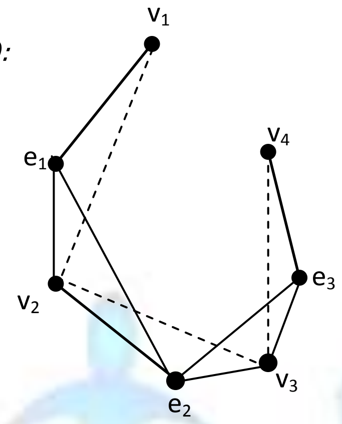
In $T(P_3)$, $\{v_2\}$ is a dominating set and $\{e_1, v_3\}$ is a (1,2) dominating set.

Therefore $\gamma[T(P_3)] = 1$ and $\gamma_{(1,2)}[T(P_3)] = 2$

*P₄ :*



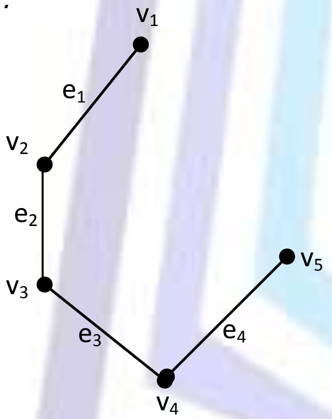

*T(P₄):*



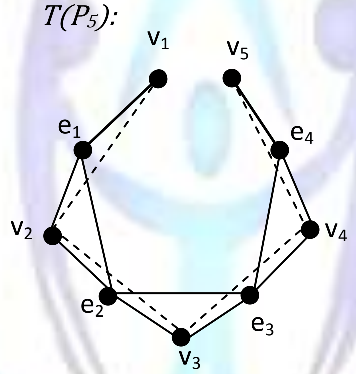

In $T(P_4)$, $\{v_2, e_3\}$ is a dominating set and $\{v_1, v_2, v_4\}$ is a (1,2) dominating set.

Therefore $\gamma[T(P_4)] = 2$ and $\gamma_{(1,2)}[T(P_4)] = 3$

*P₅ :*



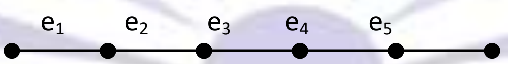

*T(P₅):*



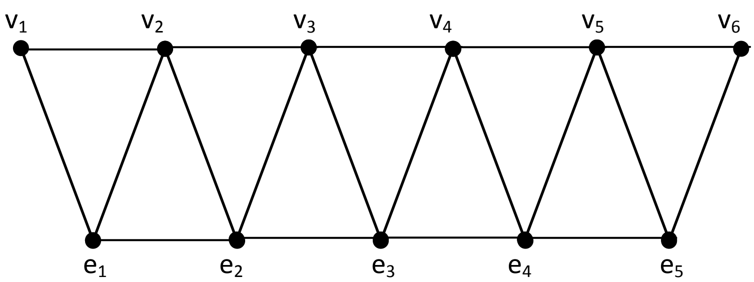

In $T(P_5)$, $\{v_2, v_4\}$ is a dominating set and $\{v_1, v_3, v_5\}$ is a (1,2) dominating set.

Therefore $\gamma[T(P_5)] = 2$ and $\gamma_{(1,2)}[T(P_5)] = 3$

*P₆ :*

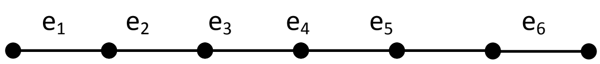



*T(P₆):*

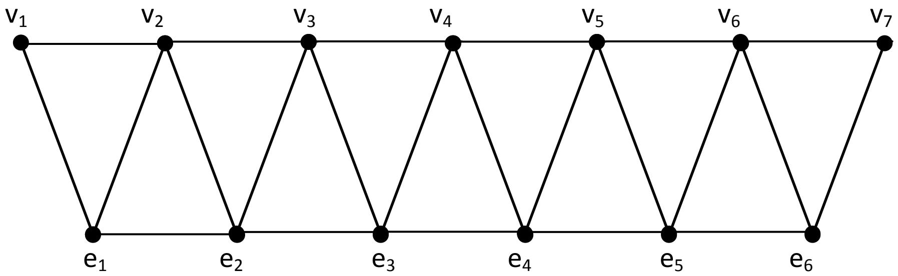



$\{v_2, v_4, v_6\}$ is a dominating set and $\{v_1, v_3, v_5\}$ is a (1,2) dominating set.

*P₇ :*

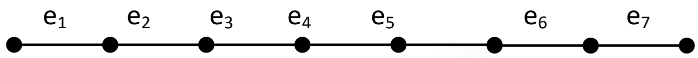


*T(P₇):*

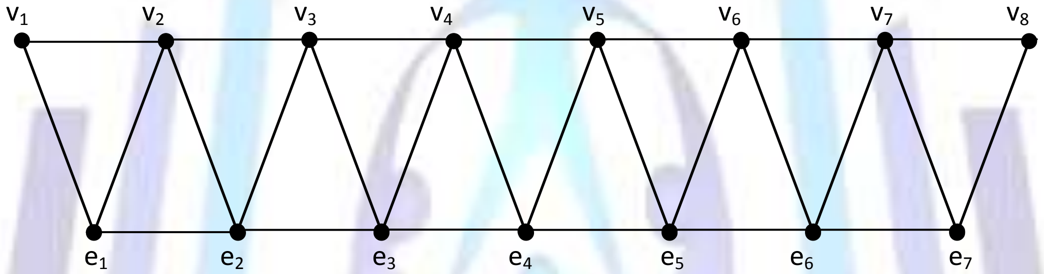



$\{v_2, v_4, v_6\}$ is a dominating set and $\{v_1, v_3, v_5, v_7\}$ is a (1,2) dominating set.

*$P_8$ :*

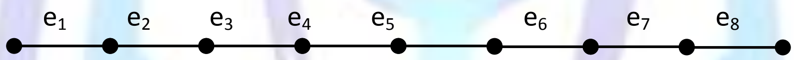



*$T(P_8)$:*

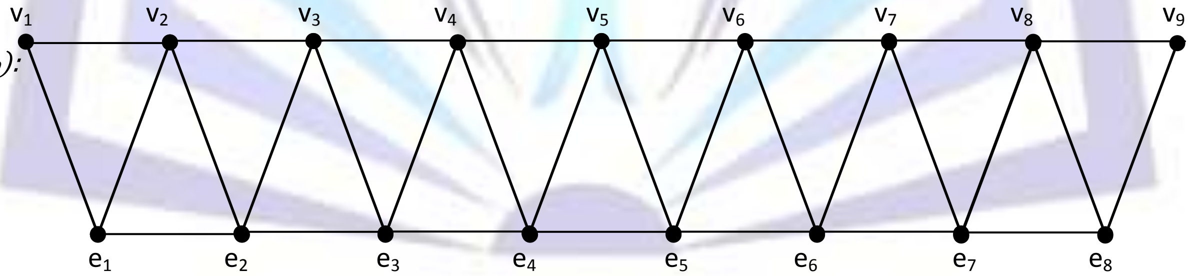



$\{v_2, v_4, v_6, v_8\}$ is a dominating set and $\{v_1, v_3, v_5, v_7\}$ is a (1,2) dominating set.

*$P_9$ :*



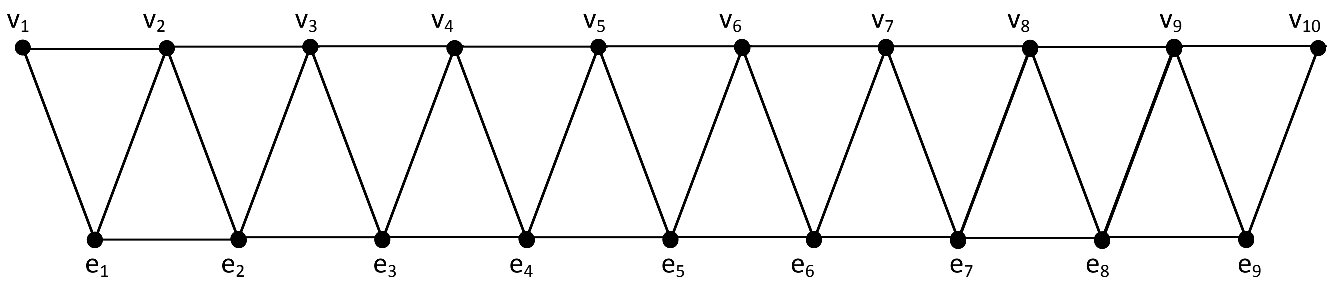

*$T(P_9)$:*




$\{v_2, v_4, v_6, v_8\}$ is a dominating set and $\{v_1, v_3, v_5, v_7, v_9\}$ is a (1,2) dominating set.

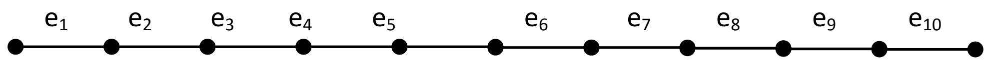
*$P_{10}$ :*


*$T(P_{10})$:*

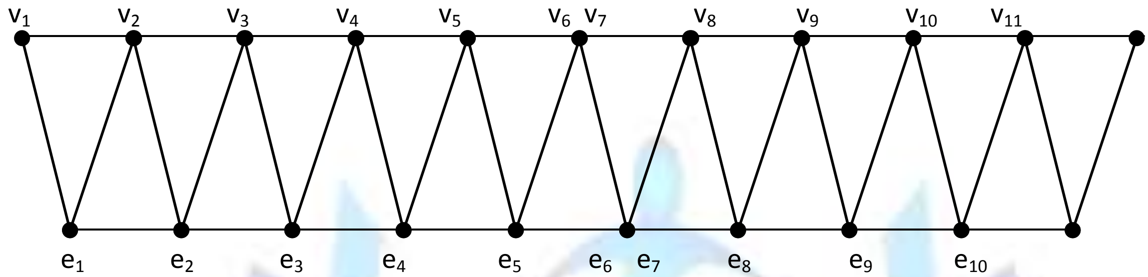


{$v_2$, $v_4$, $v_6$, $v_8$, $v_{10}$} is a dominating set and { $v_1$, $v_3$,$v_5$,$v_7$,$v_9$} is a (1,2) dominating set.

$P_{11}$ :




$T(P_{11})$:




{$v_2$, $v_4$, $v_6$, $v_8$, $v_{10}$} is a dominating set and { $v_1$, $v_3$,$v_5$,$v_7$,$v_9$,$v_{11}$} is a (1,2) dominating set.

From the above examples, we have the following theorems.

## Theorem 3.1

$$\gamma[T(P_n)] = \left\lfloor \frac{n}{2} \right\rfloor$$

Proof : It is easy to observe that $\gamma[T(P3)] = 1$, $\gamma[T(P_4) = 2$, $\gamma[T(P_5)] = 3$

Let $P_n$ be a path on n vertices where $n > 5$.

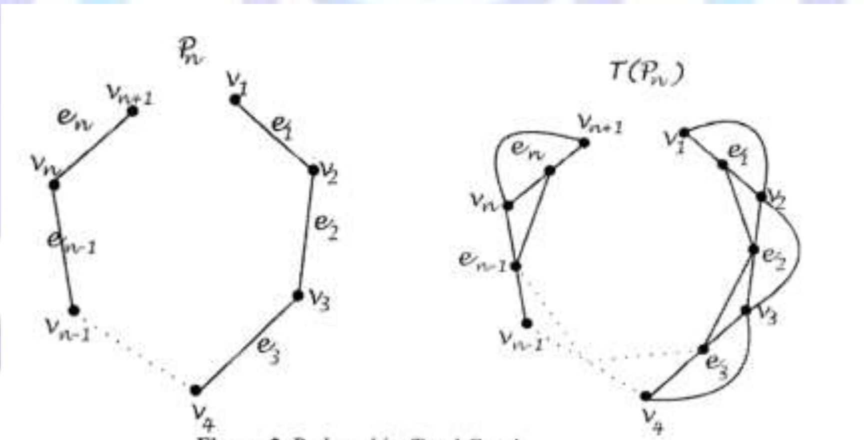


**Figure 2.** Path and its Total Graph

Let $V(P_n) = \{v_1, v_2,..., v_n\}$ and $E(P_n) = \{e_1, e_2,..., e_n\}$. By the definition of total graph, $V[T(P_n)] = V(P_n) \cup E(P_n)$, $E[T(P_n)] = \{v_i e_i/1\leq i\leq n\} \cup \{e_i v_{i+1}/1\leq i \leq n-1) \cup \{v_i v_{i+1}/1\leq i \leq n \} \cup \{e_i e_{i+1}/1\leq i \leq n-1 \}$. The cycles of $T(P_n)$ are $C_i = v_i v_{i+1} e_i$ $(1\leq i \leq n)$, and $C'_i = e_i e_{i+1} v_{i+1} (1\leq i \leq n-1)$. In $T(P_n)$ $v_1$ and $v_n$ are non- adjacent. Also $e_1$ and $e_{n-1}$ are non adjacent. Each vertex $v_i$, $2 \leq i \leq n-1$, and $e_i$, $2 \leq i \leq n-2$ are of degree 4. Now we can find the minimum dominating set. For $n = 2k$, $\{v_2, v_4, ... , v_n\}$ is a dominating set and for $n = 2k+1$, $\{v_2, v_4, ..., v_{n-1}\}$ is a dominating set. Therefore the minimum dominating set

$$D = \begin{cases} D_1 = \{v_2, v_4, ..., v_n\} & if \quad n = 2k \\ D_2 = \{v_2, v_4, ..., v_{n-1}\} & if \quad n = 2k+1 \end{cases}$$

$$| D_1| = \frac{n-2}{2} + 1 = \frac{n}{2}$$

$| D_2| = \frac{n-1-2}{2}+1=\frac{n-1}{2}$

Hence $|D| = \begin{cases} \frac{n}{2} & if\ n=2k \\ \frac{n}{2}-\frac{1}{2} & if\ n=2k+1 \end{cases}$

Hence $\gamma[T(P_n)] = \left\lfloor \frac{n}{2} \right\rfloor$ for all values of $P_n$.

### Theorem 3.2

$\gamma_{(1,2)}[T(P_n)] = \left\lceil \frac{n}{2} \right\rceil$

Proof

It is easy to observe that $\gamma_{(1,2)}[T(P_3)] = 2$, $\gamma_{(1,2)}[T(P_4)] = 3$, $\gamma_{(1,2)}[T(P_5)] = 3$.

Let $P_n$ be a path on $n$ vertices where $n$ >5.

Let $V(P_n) = \{v_1, v_2,..., v_n\}$ and $E(P_n) = \{e_1, e_2,..., e_n\}$. By the definition of total graph, $V[T(P_n)] = V(P_n) \cup E(P_n)$, $E[T(P_n)] = \{v_i e_i /1\le i\le n\} \cup \{e_i v_{i+1}/1\le i \le n-1)$

$\cup \{v_i v_{i+1}/1\le i \le n\} \cup \{e_i e_{i+1}/1\le i \le n-1\}$. The cycles of $T(P_n)$ are $C_i = v_i v_{i+1} e_i$ $(1\le i \le n)$, and $C'_i = e_i e_{i+1} v_{i+1} (1\le i \le n-1)$. In $T(P_n)$ $v_1$ and $v_n$ are non- adjacent. Also $e_1$ and $e_{n-1}$ are non adjacent.

Now we can find the (1,2)- dominating set.

For $n= 2k$, $\{v_1, v_3, ..., v_{n-1}\}$ will form a (1,2)- dominating set and for $n = 2k+1$, $\{v_1, v_3, ..., v_n\}$ will form a (1,2) dominating set for $T(P_n)$.

Therefore the minimum dominating set

$S = \begin{cases} S_1 = \{v_1, v_3, ..., v_{n-1}\} & if \quad n=2k \\ S_2 = \{v_1, v_3, ..., v_n\} & if \quad n=2k+1 \end{cases}$

$| S| = \begin{cases} | S_1 | & if\ n=2k \\ | S_2 | & if\ n=2k+1 \end{cases}$

$| S_1| = \frac{n-1-1}{2}+1=\frac{n}{2}$

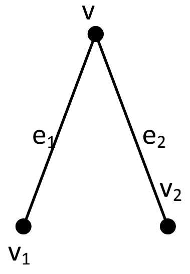
$| S_2| = \frac{n-1}{2}+1=\frac{n-1+2}{2}=\frac{n+1}{2}$

$| S| = \begin{cases} \frac{n}{2} & if\ n=2k \\ \frac{n+1}{2} & if\ n=2k+1 \end{cases}$

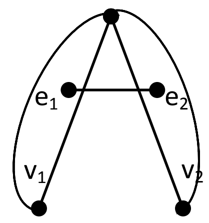
$\gamma_{(1,2)}[T(P_n)] = \left\lceil \frac{n}{2} \right\rceil$

## 4. (1,2)- domination in Total graph of $K_{1,n}$

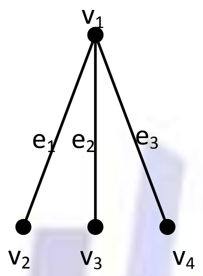
$K_{1,2}$ :



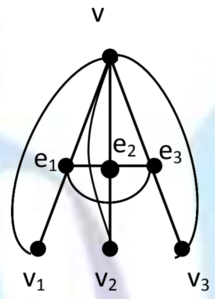

$T(K_{1,2})$ :



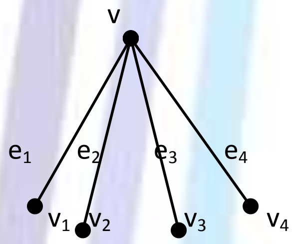

$K_{1,3}$



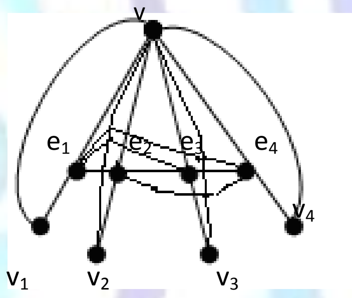

$T(K_{1,3})$:



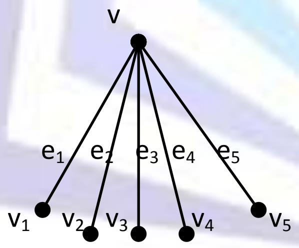

$K_{1,4}$



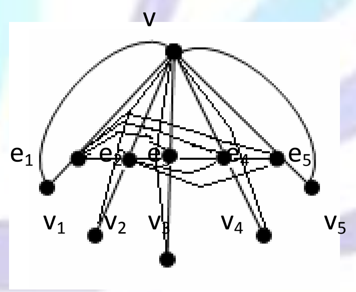

$T(K_{1,4})$:




$K_{1,5}$




$T(K_{1,5})$:




In the above examples the central vertex {*v*} is a dominating set and {*v*, $e_1$} is a(1,2) - dominating set.

### Theorem 4.1

$\gamma\,[T(K_{1,n})] = 1$

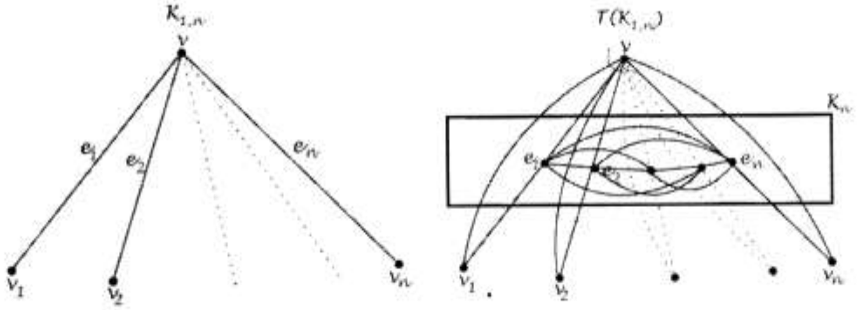



**Figure 3.** Star graph and its Total Graph

**Proof :**

Let $V(K_{1,n}) = \{v, v_1, v_2,..., v_n\}$ and $E(K_{1,n}) = \{e_1, e_2,..., e_n\}$. By the definition of total graph $V[T(K_{1,n})] = \{v\} \cup \{e_i / 1 \le i \le n\} \cup \{v_i / 1 \le i \le n\}$in which the vertices. $e_1, e_2, ..., e_n$ induces a clique of order $n$. Also the vertex $v$ is adjacent with $v_i$ $(1 \le i \le n)$. In $T(K_{1,n})$, the central certex $\{v\}$ dominates all other vertices. Hence $\gamma[T(K_{1,n})] = 1$

## Theorem 4.2

$\gamma_{(1,2)}[T(K_{1,n})] = 2$

Proof:

Let $V(K_{1,n}) = \{v, v_1, v_2,..., v_n\}$ and $E(K_{1,n}) = \{e_1, e_2,..., e_n\}$. By the definition of total graph $V[T(K_{1,n})] = \{v\} \cup \{e_i / 1 \le i \le n\} \cup \{v_i / 1 \le i \le n\}$in which the vertices. $e_1, e_2, ..., e_n$ induces a clique of order $n$. Also the vertex $v$ is adjacent with $v_i$ $(1 \le i \le n)$. In $T(K_{1,n})$, the central certex $\{v\}$ dominates all other vertices. Hence $\gamma[T(K_{1,n})] = 1$. So we can form a (1,2) - dominating set by selecting the central vertex v and any one of $e_i, (1 \le i \le n)$. Hence $\gamma_{(1,2)}[T(K_{1,n})] = 2$.

## 5. Relation between domination number and (1,2) domination number in the total graph of $C_n$, $P_n$ and $K_{1,n}$.

## Theorem 5.1

In $T(C_n)$, domination number equals (1,2) - domination number.

Proof :

By theorem 2.1 and 2.2, we have this result.

## Theorem 5.2

In $T(P_n)$, domination number is less than or equal to (1,2) - domination number.

Proof:

This result is obvious from theorem 3.1 and 3.2.

## Theorem 5.3

In $T(K_{1,n})$, domination number is less than (1,2) - domination number.

Proof :

This is clear from theorem 4.1 and 4.2.

## 6. Conclusion

In the paper, we found the domination number and (1,2) domination number of total graphs of $C_n$, $P_n$ and $K_{1,n}$. It can be seen that domination number is less than or equal to the (1,2)- domination number in all cases which conincides with the result of [5].

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