# Fuzzy decision making in Business intelligence in the context of GilgitBaltistan 

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#### Abstract

The main purpose of this paper is to investigate and implement fuzzy decision based on unequal objectives and minimization of regret for the decision making in the business intelligence and to compare the weight of products while the minimization of regret that uses regression of products in Gilgit-Baltistan. Here we will convert the verbal expressions in to linguistic variables and use in fuzzy decision making model, which influences the main two factors, one is effect of the influence on the product and second its payoff for the most effectiveness on the products.


Keywords: Fuzzy set; Fuzzy logic.


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## Introduction:

Fuzzy decision making provide an important role in the business intelligence, the most useful aspects of fuzzy set theory is its ability to represent mathematically a class of decision problems called multiple objective decisions (MODs). This class of problems often involves many vague and ambiguous goals and constraints. The object of the fuzzy decision methodology is to obtain a decision, optimum in the sense that some set of goals is attained while observing, it also enables us to evaluate the ongoing practices of fuzzy decision, the core purpose of my thesis writing to express the business of fruits like cherry, dry fruit(almond) and vegetable(potato) in different levels because here we people do not know that the business of such mentioned products have an importance in getting high level business so by using two major algorithms one is unequal objectives where uses the Saaty's approach to compare the weight of products in approaching boom of the business, while the second algorithm is minimization of regret methods that uses regression of products.
Here in Gilgit-Baltistan we use the knowledge of verbal expression for the quality and demand of these products but we will use the linguistic variables for its weight, quality and demand. Where the linguistic variables is defined as a variable whose values are sentence in a natural or artificial language thus if good, very good, bad, satisfactory etc are values of quality are linguistic variable and The verbal expression of opinion of experts about these aspects is converted in fuzzy sets.

## Example 1.1:

Let in theorem the price of almond in the region of Gilgit Baltistan varies from place to place now compare the given price in Danyor and Gahkuch and we will see the direct effects of rise of price on the markets of both places.
$X=\{180,185,190,195,200\}$ when $X$ is a price of cherry.
$A=$ effects on the market while rise in price of cherry in Danyor.
$B=$ effects on the market while rise in price of cherry in Gahkuch.
Proof: Let
$\mathrm{A}=0.1 / 180+0.4 / 185+0.6 / 190+0.9 / 195+1 / 200$
$B=0.3 / 180+0.5 / 185+0.8 / 190+1 / 195+1 / 200$
$A \cup B=$ Effects in Gahkuch and Danyor
$A \cup B=\operatorname{Max}(0.1,0.3) / 180+\operatorname{Max}(0.4,0.5) / 185+\operatorname{Max}(0.6,0.8) / 190+\operatorname{Max}(0.9,1) / 195+$
$\operatorname{Max}(1,1) / 200$
$=0.3 / 180+0.5 / 185+0.8 / 190+1 / 195+1 / 200$

## Example 1.2:

Let price of almond with shells and without shells in Gahkuch at different quantity levels
$X=\{2,4,6,8\}$ when $X$ is the quantity in ponds in Gahkuch..
$A=$ price of almond with shells in Gahkuch.
$B=$ price of almond without shells in Gahkuch.
Proof: let
$\mathrm{A}=0.5 / 2+0.7 / 4+0.8 / 6+0.9 / 8$
$B=0.7 / 2+0.8 / 4+0.9 / 6+1 / 8$
$A \cup B=$ Prices with shells and without shells in Gahkuch.
$=\operatorname{Max}(0.5,0.7) / 2+\operatorname{Max}(0.7,0.8) / 4+\operatorname{Max}(0.8,0.9) / 6+\operatorname{Max}(0.9,1) / 8$
$=0.7 / 2+0.8 / 4+0.9 / 6+1 / 8$

## Fuzzy Relations

Fuzzy relations are similar to crisp relations and they are fuzzy subsets of the Cartesian product of two or more fuzzy sets. Thus fuzzy relation is defined on the Cartesian product. The relation between the fuzzy sets $S \subseteq R$ and $Y$ $\subseteq \mathrm{R}$ where R is a universal set is defined by;
$\tilde{R}=\left\{\left((x, y), \mu_{R}(x, y) /(x, y)\right\} \in \boldsymbol{S} \times \boldsymbol{Y}\right.$
This definition assumed that $U_{R}$ was a function from $S \times Y$ to $[0,1]$, which assigns to each pair a degree of membership within the interval $[0,1]$. On the other hand, it is useful to make the fuzzy relation that functions from fuzzy subsets in the universal sets into the unit interval. Rosenfeld [1975] has generalized this definition to the following one;
Let $\mathrm{S}, \mathrm{Y} \subseteq R$

$$
\tilde{A}=\left\{\left(x, \mu_{\tilde{A}}(x) / x \in S\right\}\right.
$$

and

$$
\tilde{B}=\left\{\left(y, \mu_{\tilde{B}}(y) / y \in Y\right\}\right. \text { As two fuzzy subsets }
$$

Then

$$
\left.\tilde{R}=\left\{\left[(x, y), \mu_{R}(x, y)\right] /(x, y)\right\} \in S \times Y\right\} \text { is a fuzzy relation on } \tilde{A} \text { and } \check{B} \text { if }
$$

$\mu_{R}(x, y) \leq \mu_{\overparen{A}}(x), \quad$ for all $(x, y) \in S \times Y$
and
$\mu_{R}(x, y) \leq \mu_{B}(x), \quad$ for all $(x, y) \in S \times Y$
The set-theoretic and algebraic operations can be represented by fuzzy relations as well as fuzzy sets, which we introduced above, furthermore, we will consider the union/intersection of two fuzzy relations $\widetilde{R}$ and $\breve{E}$ in the same product space, as;
$\mu_{\tilde{\mathrm{R}} \mathrm{n} \tilde{\mathrm{E}}}(x, y)=\min \left\{\mu_{\widetilde{R}^{( }}(x, y), \mu_{\check{E}}(x, y)\right\},(x, y) \in S \times Y$
$\mu_{\text {K̃U尺̃ }}(x, y)=\max \left\{\mu_{\overparen{R}}(x, y), \mu_{\check{E}}(x, y)\right\},(x, y) \in S \times Y$

### 3.1 Example \# 01

$\check{R}=$ " $x$ larger than $y "$

|  | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ | $y_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{1}$ | 0.8 | 0.6 | 0.1 | 0 | 0.4 |
| $x_{2}$ | 0.9 | 0.1 | 0.4 | 1 | 0.5 |
| $x_{3}$ | 0.3 | 0.7 | 0.1 | 0 | 0.7 |
|  |  |  |  |  |  |

## $\breve{E}=" y$ very close to $x^{n}$

|  | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ | $y_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{1}$ | 0.2 | 0 | 0.5 | 0.5 | 0.9 |
| $x_{2}$ | 0.6 | 0.2 | 0.2 | 0.1 | 0.5 |
| $x_{3}$ |  |  |  |  |  |
|  | 0.8 | 0.7 | 0.2 | 0 | 0.7 |
|  |  |  |  |  |  |

Then the intersection is stated as

|  | $y_{1}$ |  |  | $y_{2}$ | $y_{3}$ |  | $y_{4}$ | $y_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{1}$ | 0.2 | 0 | 0.1 | 0 | 0.4 |  |  |  |
| $x_{2}$ | 0.6 | 0.1 | 0.2 | 0.1 | 0.5 |  |  |  |
| $x_{3}$ | 0.3 | 0.7 | 0.1 | 0 | 0.7 |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

Then the is union is stated as

|  | $y_{1}$ |  | $y_{2}$ | $y_{3}$ | $y_{4}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y_{5}$ |  |  |  |  |  |
| $x_{1}$ | 0.8 | 0.6 | 0.1 | 0.5 | 0.9 |
| $x_{2}$ | 0.9 | 0.2 | 0.4 | 1 | 0.5 |
| $x_{3}$ | 0.8 | 0.7 | 0.2 | 0 | 0.7 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

### 3.2 Main Result: (Comparison of fuzzy relations)

It is a combination of fuzzy relations in different product spaces, and there are different types of compositions by their results and with respect to their mathematical properties. The most used one which becomes the best known is the max-min composition, often so-called max-product or max-average composition, and defined as follows;

We have $\widetilde{R}_{1}(x, y),(x, y) \in S \times Y$ and $\widetilde{R}_{2}(y, z),(y, z) \in Y \times Z$ as two fuzzy relations, the max-min composition is a fuzzy relation stated by
$\widetilde{R_{1}} \circ \widetilde{R_{2}}=\left\{\left[(x, z), \max _{y}\left\{\min \left\{\mu_{R_{1}}(x, y), \mu_{R_{2}}(y, z)\right\}\right\}\right] / x \in S, y \in Y_{,} z \in Z\right.$
The more general definition of composition is a max*-composition which is
$\widetilde{R_{1}} \circ \widetilde{R_{2}}=\left\{\left[(x, z), \max _{y}\left(\mu_{R_{1}}(x, y) \star \mu_{R_{2}}(y, z)\right)\right] / x \in S, y \in \boldsymbol{Y}, z \in Z\right\}$
Where "*" is an associative operation that is monotonically non-decreasing in each argument.
3.2.1 Example \#01 Consider $\mu_{R_{1}}(x, y)$ and $\mu_{R_{z}}(y, z)$ as the following relations matrices $\widetilde{R_{1}}:$

|  | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
|  | 0.1 | 0.2 | 0 | 1 |
| $x_{1}$ | 0.1 |  |  |  |
|  | $x_{2}$ | 0.3 | 0.5 | 0 |
| 0 | 0.2 |  |  |  |
| $x_{3}$ | 0.8 | 0 | 1 | 0.4 |
|  |  |  |  |  |

$\widetilde{R_{2}}:$

|  | $z_{1}$ | $z_{2}$ | $z_{3}$ | $z_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $y_{1}$ | 0.9 | 0 | 0.3 |
| $y_{2}$ |  | 0.4 |  |  |
|  | 0.2 | 1 | 0.8 | 0 |
| $y_{2}$ |  |  |  |  |
|  | 0.8 | 0 | 0.7 | 1 |
|  |  |  |  |  |


$\boldsymbol{y}_{\mathbf{4}}$| 0.4 | 0.2 | 0.3 | 0 |
| :--- | :--- | :--- | :--- |

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=x_{1}, z=z_{1}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0.1,0.9\}=0.1 \\
& \operatorname{Min}\left\{\mu_{\mu}\left(\mathrm{x}_{1}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0.2,0.2\}=0.2 \\
& \operatorname{Min}\left\{\mathrm{u}_{\mu}\left(\mathrm{x}_{1}, \mathrm{y}_{3}\right), \mathrm{u} \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0,0.8\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{1,0.4\}=0.4
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{1}, z_{1}\right)=\left(\left(x_{1}, z_{1}\right), u_{\breve{\mu} \circ \widetilde{R_{2}}}\left(x_{1}, z_{1}\right)=\left(\left(x_{1}, z_{1}\right), \max \{0.1,0.2,0,0.4\}=\left(\left(x_{1}, z_{1}\right), 0.4\right)\right.\right.
$$

To compute the max-min composition $\widetilde{\mathrm{R}_{1}} \circ \widetilde{\mathrm{R}_{2}}$ we shall determine the minimum membership for each pair and we consider for $\mathrm{x}=\mathrm{x}_{1}, \mathrm{z}=\mathrm{z}_{2}$ and $\mathrm{y}=\mathrm{y}_{\mathrm{i}}$ where $\mathrm{i}=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.1,0\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.2,1\}=0.2 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0,0\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{1,0.2\}=0.2
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{1}, z_{2}\right)=\left(\left(x_{1}, z_{2}\right), \tilde{\mu}_{\tilde{R}_{1} \circ \widetilde{R_{2}}}\left(x_{1}, z_{2}\right)=\left(\left(x_{1}, z_{2}\right), \max \{0,0.2,0,0.2\}=\left(\left(x_{1}, z_{2}\right), 0.2\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=x_{1}, z=z_{3}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.1,0.3\}=0.1 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.2,0.8\}=0.2 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0,0.7\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{1,0.3\}=0.3
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{1}, z_{3}\right)=\left(\left(x_{1}, z_{3}\right), \mu_{\widetilde{R}_{1} \circ \widetilde{R_{2}}}\left(x_{1}, z_{3}\right)=\left(\left(x_{1}, z_{3}\right), \max \{0.1,0.2,0,0.3\}=\left(\left(x_{1}, z_{3}\right), 0.3\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=x_{1}, z=z_{4}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0.1,0.4\}=0.1 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0.2,0\}=0 \\
& \operatorname{Min}\left\{\mu_{\mu}\left(\mathrm{x}_{1}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0,1\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{1}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{1,0\}=0
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{1}, z_{4}\right)=\left(\left(x_{1}, z_{4}\right), \mu_{\widetilde{R}_{1} \circ \widetilde{R_{2}}}\left(x_{1}, z_{4}\right)=\left(\left(x_{1}, z_{4}\right), \max \{0.1,0,0,0\}=\left(\left(x_{1}, z_{4}\right), 0.1\right)\right. \text { To compute }\right.
$$ the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=x_{2}, z=z_{1}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0.3,0.9\}=0.3 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0.5,0.2\}=0.2 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0,0.8\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{1}\right)\right\}=\operatorname{Min}\{0.2,0.4\}=0.2
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{2}, z_{1}\right)=\left(\left(x_{2}, z_{1}\right), \mu_{\widetilde{R_{1}} \circ \stackrel{R_{2}}{ }}\left(x_{2}, z_{1}\right)=\left(\left(x_{2}, z_{1}\right), \max \{0.3,0.2 .0,0.2\}=\left(\left(x_{2}, z_{1}\right), 0.3\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{2}, z=z_{2}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mathrm{u}_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{1}\right), \mathrm{u}_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.3,0\}=0 \\
& \operatorname{Min}\left\{\mathrm{u}_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right), \mathrm{u}_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.5,1\}=0.5 \\
& \operatorname{Min}\left\{\mathrm{u}_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{3}\right), \mathrm{u}_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0,0\}=0 \\
& \operatorname{Min}\left\{\mathrm{u}_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{4}\right), \mathrm{u}_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.2,0.2\}=0.2
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{2}, z_{2}\right)=\left(\left(x_{2}, z_{2}\right), u_{\widetilde{R_{1}} \circ \widetilde{R_{2}}}\left(x_{2}, z_{2}\right)=\left(\left(x_{2}, z_{2}\right), \max \{0,0.5,0,0.2\}=\left(\left(x_{2}, z_{2}\right), 0.5\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{2}, z=z_{3}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given..

$$
\begin{aligned}
& \operatorname{Min}\left\{\uparrow \mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.3,0.3\}=0.3 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.5,0\}=0 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0,0.7\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.2,0.3\}=0.2
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{2}, z_{3}\right)=\left(\left(x_{2}, z_{3}\right), \mu_{\widetilde{R_{1}} \circ \stackrel{R_{2}}{ }\left(x_{2}, z_{3}\right)=\left(\left(x_{2}, z_{3}\right), \max \{0.3,0.5,0,0.2\}=\left(\left(x_{2}, z_{3}\right), 0.5\right)\right) .}\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{2}, z=z_{4}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0.3,0.4\}=0.3 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0.5,0\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0,1\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{2}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{4}\right)\right\}=\operatorname{Min}\{0.2,0\}=0
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{2}, z_{4}\right)=\left(\left(x_{2}, z_{4}\right), \mu_{\widetilde{R}_{1} \circ \widetilde{R}_{2}}\left(x_{2}, z_{4}\right)=\left(\left(x_{2}, z_{4}\right), \max \{0.3,0,0,0\}=\left(\left(x_{2}, z_{4}\right), 0.3\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{3}, z=z_{1}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{1}\right), \mu_{R_{2}}\left(y_{1}, z_{1}\right)\right\}=\operatorname{Min}\{0.8,0.9\}=0.8 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{2}\right), \mu_{R_{2}}\left(y_{2}, z_{1}\right)\right\}=\operatorname{Min}\{0,0.2\}=0 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{3}\right), \mu_{R_{2}}\left(y_{3}, z_{1}\right)\right\}=\operatorname{Min}\{1,0.8\}=0.8 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{4}\right), \mu_{R_{2}}\left(y_{4}, z_{1}\right)\right\}=\operatorname{Min}\{0.4,0.4\}=0.4
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{3}, z_{1}\right)=\left(\left(x_{3}, z_{1}\right), \mu_{\widetilde{R}_{1} \circ \widetilde{R}_{2}}\left(x_{3}, z_{1}\right)=\left(\left(x_{3}, z_{1}\right), \max \{0.8,0,0.8,0.4\}=\left(\left(x_{3}, z_{1}\right), 0.8\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{3}, z=z_{2}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.8,0\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{2}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0,1\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{1,0\}=0 \\
& \operatorname{Min}\left\{\mu_{\mathrm{R}_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{4}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{2}\right)\right\}=\operatorname{Min}\{0.4,0.2\}=0.2
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{3}, z_{2}\right)=\left(\left(x_{3}, z_{2}\right), \mu_{\widetilde{R}_{1} \circ \widetilde{R_{2}}}\left(x_{3}, z_{2}\right)=\left(\left(x_{3}, z_{2}\right), \max \{0,0,0,0.2\}=\left(\left(x_{3}, z_{2}\right), 0.2\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{3}, z=z_{3}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given.

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{R_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{1}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{1}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.8,0.3\}=0.3 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{2}\right), \mu_{R_{2}}\left(\mathrm{y}_{2}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0,0.8\}=0 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right), \mu_{\mathrm{R}_{2}}\left(\mathrm{y}_{3}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{1,0.7\}=0.7 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(\mathrm{x}_{3}, \mathrm{y}_{4}\right), \mu_{R_{2}}\left(\mathrm{y}_{4}, \mathrm{z}_{3}\right)\right\}=\operatorname{Min}\{0.4,0.3\}=0.3
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{3}, z_{3}\right)=\left(\left(x_{3}, z_{3}\right), \mu_{\overparen{R_{1}} \circ \stackrel{R_{2}}{ }}\left(x_{3}, z_{3}\right)=\left(\left(x_{3}, z_{3}\right), \max \{0.3,0,0.7,0.3\}=\left(\left(x_{3}, z_{3}\right), 0.7\right)\right.\right.
$$

To compute the max-min composition $\widetilde{R_{1}} \circ \widetilde{R_{2}}$ we shall determine the minimum membership for each pair and we consider for $x=, x_{3}, z=z_{4}$ and $y=y_{i}$ where $i=1,2,3, \ldots 5$ as given..

$$
\begin{aligned}
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{1}\right), \mu_{R_{2}}\left(y_{1}, z_{4}\right)\right\}=\operatorname{Min}\{0.8,0.4\}=0.4 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{2}\right), \mu_{R_{2}}\left(y_{2}, z_{4}\right)\right\}=\operatorname{Min}\{0,0\}=0 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{3}\right), \mu_{R_{2}}\left(y_{3}, z_{4}\right)\right\}=\operatorname{Min}\{1,1\}=1 \\
& \operatorname{Min}\left\{\mu_{R_{1}}\left(x_{3}, y_{4}\right), \mu_{R_{2}}\left(y_{4}, z_{4}\right)\right\}=\operatorname{Min}\{0.4,0\}=0
\end{aligned}
$$

Therefore,

$$
\widetilde{R_{1}} \circ \widetilde{R_{2}}\left(x_{3}, z_{4}\right)=\left(\left(x_{3}, z_{4}\right), \mu_{\widetilde{R_{1}} \circ \stackrel{R_{2}}{ }}\left(x_{3}, z_{4}\right)=\left(\left(x_{3}, z_{4}\right), \max \{0.4,0,1,0\}=\left(\left(x_{3}, z_{4}\right),\right.\right.\right.
$$

By using the above formula and so on to calculate for all $x$ and $z$ at the end we get the following relation matrix;
$\overline{R_{1}} \circ \widetilde{R_{2}}$

|  | $z_{1}$ | $z_{2}$ | $z_{3}$ | $z_{4}$ |
| :--- | :---: | :---: | :---: | :---: |
| $x_{1}$ | 0.4 | 0.3 | 0.2 | 0.1 |
| $x_{2}$ | 0.3 | 0.5 | 0.5 | 0.3 |
| $x_{3}$ | 0.8 | 0.2 | 0.7 | 1 |
|  |  |  |  |  |

By this example we conclude this chapter by making short description of main point about the fuzzy sets, some of their basic operation, and composition which is most useful operation on fuzzy set-theoretic. In next chapter we will concentrate on the technique of making decision and its steps to formulate it under fuzzy environment.

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