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On Contra -wgrα-Continuous and Almost Contra-wgrα-Continuous Functions

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Abstract

In this paper a new class of function called contra-wgrα continuous function is introduced and its properties are studied. Further the notion of almost contra wgrα-continuous function is introduced.

Keywords: contra wgrα-continuous; almost contra wgrα-continuous; ap-wgrα-continuous; wgrα-regular graph; strongly contra wgrα-closed and contra wgrα-closed.

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1. Introduction

Many topologists studied various types of generalizations of continuity [4, 12, 13].In 1996, Dontchev[5] introduced contra-continuous functions. Jafari and Noiri[8] introduced contra-α continuous functions. A new weaker form of function called contra semi continuous function is introduced and investigated by Dontchev and Noiri[6]. Contra β-continuous and contra almost β-continuous, almost contra pre-continuous, contra πgb-continuous functions were introduced by Baker[1], E.Ekici [7], D.Sreeja and C.Janaki[19].

The aim of this paper is to study the notion of contra wgrα-continuous,almost contra wgrα-continuous and its various characterizations are discussed. Also we study the basic properties of approximately wgrα-continuous functions and wgrα-regular graph.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ),cl(A) and int(A) denote the closure of A and the interior of A respectively. (X,τ) will be replaced by X if there is no chance of confusion.

Definition:2.1

A subset A of a space (X,τ) is called

(i) α-open [14] if $A\subset$ int(cl(int(A))) and α-closed if cl(int(cl(A))) $\subset$ A.

(ii) a regular open[22] if A=int(cl(A)) and a regular closed set if A=cl(int(A)).

(iii) a regular α-open[23] if there is a regular open set U such that $U\subset A\subset \alpha cl(U)$.

(iv) a weak generalized regular α-closed (wgrα-closed)[9]if cl(int(A)) $\subset$ U whenever $A\subset U$ and U is regular α-open.

The family of all α-open subsets of a space (X,τ) is denoted by αO(X) and the collection of all α-open subsets of X containing of all α-open subsets of X containing a fixed point x is denoted by αO(X,x).

Definition:2.2

A function $f:X\rightarrow Y$ is called

(i) wgrα-continuous [9] if for every $f^{-1}(V)$ is wgrα-closed in (X,τ) for every closed set V of (Y,σ).

(ii) wgrα-irresolute [9] if for every $f^{-1}(V)$ is wgrα-closed in (X,τ) for every wgrα-closed set V of (Y,σ).

(iii) contra-continuous [5] if $f^{-1}(V)$ is closed in X for each open set V of Y.

(iv) regular set connected [3] if $f^{-1}(V)$ is clopen in X for every regular open set V of Y.

(v) contra-α-continuous [8] if $f^{-1}(V)$ is α-closed in X for each open set V of Y.

(vi) R-map[2] if $f^{-1}(V)$ is regular open in X for each regular open set V of Y.

(vii) wgrα-open(resp.wgrα-closed)[11] if f(U) is wgrα-open(resp.wgrα-closed) in Y for each wgrα-open set (resp.wgrα-closed)U of X.

(viii) almost continuous [16] if $f^{-1}(V)$ is open in X for every regular open set.

(ix) perfectly continuous [18]if $f^{-1}(V)$ is clopen in X for every open set V of Y.

Definition:2.3

A topological space X is

(i) wgrα-space[11] if every wgrα-closed set is closed.

(ii) wgrα-$T_{1\backslash 2}$ space[11] if every wgrα-closed set is α-closed.

(iii) strongly -S-closed[5] if every closed cover of X has a finite sub-cover.

(iv) mildly compact[20] if every clopen cover of X has a finite sub-cover.

(v) strongly-S-lindelof[5] if every closed cover of X has a countable sub-cover.

(vi) nearly Compact[17] if every regular open cover of X has a finite subcover.

(vii) nearly-Lindelof [17] if every cover of X by regular open sets has a countable subcover

(viii) weakly Hausdorff[18] if each element of X is an intersection of regular closed sets.

(ix) wgrα-connected [10] provided that X is not the union of two disjoint non-empty wgrα-open sets.

(x) wgrα-compact [14]if every wgrα-open cover of X has a finite subcover.

(xi) hyper connected [21] if every open set is dense.

3. Contra-wgrα-Continuous and Almost Contra-wgrα-Continuous

Definition:3.1

A function f: X→Y is called contra wgrα-continuous if $f^{-1}(V)$ is wgrα-closed set in X for every open set V of Y.

Definition:3.2

A function f: X→Y is called almost contra wgrα-continuous if $f^{-1}(V)$ is wgrα-closed set in X for every regular open set V of Y.

Definition:3.3

A function f:X→Y is called contra-rgα-continuous if $f^{-1}(V)$ is rgα-closed in X for each open set V of Y.

Theorem:3.4

Every contra continuous function is contra wgrα-continuous, but not conversely.

Proof:

It follows from the fact that every closed set is wgrα-closed.

Example:3.5

Let X=Y={a,b,c},τ={ϕ,X,{a},{c},{a,c}},σ={ϕ,X,{a,c}.f:X→Y defined by f(a)=b,f(b)=a and f(c)=c. Therefore f is contra wgrα-continuous, but it is not contra continuous.

Definition:3.6

A space (X,τ) is called wgrα-locally indiscrete if every wgrα-open set is closed.

Example:3.7

Let X={a,b},τ={ϕ,X,{a},{b}}.Here the space (X,τ) is wgrα-locally indiscrete space.

Theorem:3.8

If a function f: X→Y is wgrα-continuous and (X,τ) is wgrα-locally indiscrete, then f is contra continuous.

Proof:

Let V be an open set of (Y,σ).Then $f^{-1}(V)$ is wgrα-open in (X,τ) as f is wgrα-continuous. Since (X,τ) is wgrα-locally indiscrete, $f^{-1}(V)$ is closed in (X,τ). Hence f is contra continuous.

Theorem:3.9

Every contra wgrα-continuous function is almost contra wgrα-continuous, but not conversely.

Proof:

Since every regular open set is open, the proof follows.

Example:3.10

Let X=Y={a,b,c},τ={ϕ,X,{a},{b},{a,b}} and σ={ϕ,Y,{a},{a,b}}.Define the identity function f:X→Y. Therefore f is almost contra wgrα-continuous, but it is not contra wgrα-continuous.

Theorem:3.11

If a function f:X→Y is almost contra wgrα-continuous, almost continuous and X is $T_{wgr\alpha}$-space,then f is regular set connected.

Proof;

Let V be a regular open set in (Y,σ).Since f is almost contra wgrα-continuous and almost continuous,$f^{-1}(V)$ is wgrα-closed and open. Hence $f^{-1}(V)$ is clopen. Therefore f is regular set connected.

Theorem:3.12

For a function f:X→Y, the following properties are equivalent.

(i) f is almost contra wgrα-continuous

(ii) $f^{-1}(F)\in WGR\alpha O(X)$ for every $F\in RC(Y)$.

(iii)For each $x\in X$ and each regular closed set F in Y containing f(x), there exists a wgrα-open set U in X containing x such that $f(U)\subset F$.

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(iv) For each $x \in X$ and each regular open set V in Y containing f(x), there exists a wgrα-closed set K in X not containing x such that $f^{-1}(V) \subset K$.

(v) $f^{-1}(int(cl(G))) \in WGR\alpha C(X)$ for every open subset G of Y.

(vi) $f^{-1}(cl(int(F))) \in WGR\alpha O(X)$ for every closed subset F of Y.

Proof:

(i) $\Rightarrow$ (ii)

Let $F \in RC(Y)$.Then $Y-F \in RO(Y)$ and by (i),$f^{-1}(Y-F)=X-f^{-1}(F)$ is wgrα-closed in X. This implies that $f^{-1}(F)$ is wgrα-open set in X. Therefore (ii) holds.

(ii) $\Rightarrow$ (i)

Let G be a regular open set inY.Then Y−G is a regular-closed set in Y.By (ii),$f^{-1}(Y-G)=X-f^{-1}(G)$ is wgrα-open in X. which implies that $f^{-1}(G)$ is wgrα-closed set in X. Therefore f is almost contra wgrα-continuous.

(ii) $\Rightarrow$ (iii)

let F be any regular closed set in Y containing f(x).By(ii),$f^{-1}(F) \in WGR\alpha O(X,\tau)$and $x \in f^{-1}(F)$. Take $U=f^{-1}(F)$.Then $f(U) \subset F$.

(iii) $\Rightarrow$ (ii)

Let $F \in RC(Y,\sigma)$ and $x \in f^{-1}(F)$.From(iii),there exists a wgrα-open set U_x in X containing x such that $U_x \subset f^{-1}(F)$.We have $f^{-1}(F)=\bigcup\{U_x : x \in f^{-1}(F)\}$.Then $f^{-1}(F)$ is wgrα-open.

(iii) $\Rightarrow$ (iv)

Let V be any regular closed set in Y containing f(x).Then Y−V is a regular closed set containing f(x).By(iii),there exists a wgrα-openset U in X containing x suchthat $f(U) \subset Y-V$. Hence $U \subset f^{-1}(Y-V) \subset X-f^{-1}(V)$.Then $f^{-1}(V) \subset X-V$.Take K=X−U.We obtain a wgrα-closed set in X not containing x such that $f^{-1}(V) \subset K$.

(iv) $\Rightarrow$ (iii)

Let F be a regular closed set in Y containing f(x). Then Y−F is a regular open set in Y not containing f(x). By (iv) there exists a wgrα-closed set K in X not containing x such that $f^{-1}(Y-F) \subset K$. This implies $X-f^{-1}(F) \subset K$, which implies $f(X-K) \subset F$.Take U=X−K.Then U is a wgrα-open set in X containing x such that $f(U) \subset F$.

(ii) $\Rightarrow$ (v)

Let G be a open set in Y. Then int(cl(G)) is regular open set in Y.which implies that Y−int (cl(G)) is regular-closed in Y.By(ii), $f^{-1}(Y-int(cl(G)))$ is wgrα-open in X. Therefore $f^{-1}(int(cl(G)))$ is wgrα-closed in X.

(v) $\Rightarrow$ (vi)

Let F be closed in Y.Then cl(int(F)) is regular-closed in Y and Y−F is open. By(v),f^{-1} (int (cl(Y−F))) is wgrα-closed set in X.We have

$$f^{-1}(int(cl(Y-F)))=f^{-1}(int(Y-int(F)))$$
$$=f^{-1}(Y-cl(int(F)))$$
$$=X-f^{-1}(cl(int(F))).$$

Hence, we obtain that $f^{-1}(cl(int(F)))$ is wgrα-open in X.

(vi) $\Rightarrow$ (v)

Let V be open in Y, then Y−V is closed,which implies that cl(int(Y−V)) is regular closed. By (iv) f^{-1}(cl (int (Y−V))) is wgrα-open in X.Therefore $f^{-1}(int(cl(V)))$ is wgrα-closed in X.

Theorem:3.13

(i) If a function f: $(X,\tau)\rightarrow(Y,\sigma)$ is contra wgrα-continuous and (X,τ) is wgrα-$T_{1\backslash 2}$-space,then f is contra α-continuous.

(ii) If a function f: $(X,\tau)\rightarrow(Y,\sigma)$ is contra wgrα-continuous and (X,τ) is $T_{wgr\alpha}$-space, then f is contra continuous.

(iii) If a function f: $(X,\tau)\rightarrow(Y,\sigma)$ is contra wgrα-continuous and (X,τ) is $T_{wgr\alpha}$-space, then f is contra rgα-continuous.

Proof:

(i) Let V be open in (Y,σ).By hypothesis,$f^{-1}(V)$ is wgrα-closed in (X,τ).Since X is wgrα-$T_{1\backslash 2}$ space,$f^{-1}(V)$ is α-closed in X. Hence f is contra α-continuous.

(ii) Let V be open in (Y,σ).By hypothesis,$f^{-1}(V)$ is wgrα-closed in (X,τ).Since X is $T_{wgr\alpha}$ -space, $f^{-1}(V)$ is closed in X. Hence f is contra continuous.

(iii) Let V be open in (Y,σ).By hypothesis,$f^{-1}(V)$ is wgrα-closed in (X,τ).Since X is $T_{wgr\alpha}$ -space,$f^{-1}(V)$ is rgα-closed in X. Hence f is contra rgα-continuous.

Theorem:3.14

Let f: X→Y be a function and let g:X→X×Y be the graph function of f, denoted by g(x)=(x,f(x)) for every $x\in X$. If g is almost contra wgrα-continuous function, then f is almost contra wgrα-continuous.

Proof:

Let V be a regular closed set in Y, then $X\times V=X\times cl(int(V))=cl(int(X))\times cl(int(V))=cl(int(X\times V))$. Therefore, $X\times V$ is regular closed in $X\times Y$. Since g is almost contra wgrα-continuous,then $f^{-1}(V)= g^{-1}(X\times V)$ is wgrα-open in X. Thus f is almost contra wgrα-continuous.

Theorem:3.15

For two functions f: X→Y and g: Y→Z, let $g\circ f$: X→Z is a composition function. Then the following properties hold:

(i) If f is almost contra-wgrα continuous and g is an R-map, then $g\circ f$ is almost contra wgrα-continuous.

(ii) If f is almost contra wgrα-continuous and g is perfectly continuous, then $g\circ f$ is contra wgrα-continuous.

(iii) If f is contra wgrα-continuous and g is almost continuous, then $g\circ f$ is almost contra wgrα-continuous.

Proof:

(i) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open. Since f is almost contra wgrα-continuous,

$f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is wgrα-closed set in X. Therefore $g\circ f$ is almost contra wgrα-continuous.

(ii) Let V be open in Z, since g is perfectly continuous,$g^{-1}(V)$ is clopen in Y. Since f is almost contra wgrα-continuous, $f^{-1}(g^{-1}(V)) =(g\circ f)^{-1}(V)$ is wgrα-closed set in X. Therefore $g\circ f$ is contra wgrα-continuous.

(iii) Let V be any regular open set in Z. Since g is almost continuous,$g^{-1}(V)$ is open in Y. Since f is contra wgrα-continuous,

$f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is wgrα-closed set in X. Therefore $g\circ f$ is almost contra wgrα-continuous.

Theorem:3.16

If f:X→Y is surjective wgrα-open(or wgrα-closed)and g:Y→Z is a function such that $g\circ f$:X→Z is almost contra wgrα-continuous,then g is almost contra wgrα-continuous.

Proof:

Let V be any regular closed (resp.regular open) set in Z.Since $g\circ f$ is almost contra wgrα-continuous,$(g\circ f)^{-1}(V)=f^{-1}(g^{-1}(V))$ is wgrα-open(resp.wgrα-closed),we have $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$ is wgrα-open. Therefore, g is almost contra wgrα-continuous.

Theorem:3.17

Let f:X→Y be a function and $x\in X$.If there exists $A\in WGR\alpha O(X)$ such that $x\in A$ and the restriction of f to A is almost contra wgrα-continuous at x, then f is almost contra wgrα-continuous at x.

Proof:

Suppose $f\in RC(Y)$ containing f(x),Since f |A is almost contra wgrα-continuous at x,there exists $V\in WGR\alpha O(A)$ containing x such that $f(V)=(f\ |A)(V)\subset F$.Since $A\in WGR\alpha O(X)$ containing x, we obtain that $V\in WGR\alpha O(X)$ containing x,by theorem 2.18.

Theorem:3.18

Suppose that wgrα-open sets are open under finite intersection. If f:X→Y is almost contra wgrα-continuous function and A is a wgrα open subset of X, then the restriction f |A :A→Y is almost contra wgrα-continuous

Proof:

Let $F\in RC(Y)$.Since f is almost contra wgrα-continuous,$f^{-1}(F)\in WGR\alpha O(X,\tau)$.Since A is wgrα-open in X. It follows that $(f|A)^{-1}(F)=A\cap f^{-1}(F)\in WGR\alpha O(A,\tau)$.Therefore f |A is almost contra wgrα-continuous.

Theorem:3.19

Let f:X→Y and g:Y→Z be function.Then the following properties hold.

(i) If f is almost contra wgrα-continuous and g is regular set connected, then $g\circ f$:X→Z is almost contra wgrα-continuous and almost wgrα-continuous.

(ii) If f is contra wgrα-continuous and g is regular set connected, then $g\circ f$:X→Z is almost contra wgrα-continuous and almost wgrα-continuous.

Proof:

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(i) Let $V \in RO(Z)$.Since g is regular set connected, $g^{-1}(V)$ is clopoen in Y. Since f is almost contra wgrα-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is wgrα-open and wgrα-closed. Therefore $g \circ f$ is almost contra wgrα-continuous and almost wgrα-continuous.

(ii) Let $V \in RO(Z)$.Since g is regular set connected, $g^{-1}(V)$ is clopoen in Y. Since f is contra wgrα-continuous, $f^{-1}(g^{-1}(V))=$

$(g \circ f)^{-1}(V)$ is wgrα-open and wgrα-closed. Therefore $g \circ f$ is almost contra wgrα-continuous and almost wgrα-continuous.

Theorem:3.20

If a function f:X→Y is almost contra wgrα-continuous ,almost continuous and X is $T_{wgr\alpha}$-space,then f is regular set connected.

Proof:

Let V be a regular open set in Y.Since f is almost contra wgrα-continuous and almost continuous,$f^{-1}(V)$ is wgrα-closed and open. Since X is $T_{wgr\alpha}$-space,$f^{-1}(V)$ is clopen.Hence f is regular set connected.

Theorem:3.21

Let f:X→Y be a function and $x \in X$.If there exists $U \in WGR\alpha O(X)$ such that $x \in U$ and the restriction of f to U is almost contra wgrα-continuous at x, then f is almost contra wgrα-continuous at x.

Proof:

Suppose that $F \in RC(Y)$ containing f(x).Since f|U is almost contra wgrα-continuous at x,there exists $V \in WGR\alpha O(U)$ containing x such that $f(V)=(f|U)(V) \subset F$.Since $U \in WGR\alpha O(X)$ containing x. It follows that $V \in WGR\alpha O(X)$ containing x. Hence f is almost contra wgrα-continuous at x.

Definition:3.22

A space X is said to be wgrα -T_1 if for each pair of distinct points x and y in X, there exists a wgrα-open sets U and V containing x and y respectively of X such that $y \notin U$ and $x \notin V$.

Definition:3.23

A space X is said to be wgrα -Hausdorff if for each pair of distinct points x and y in X, there exists a wgrα-open sets U and V containing x and y respectively of X such that $U \cap V = \phi$.

Theorem:3.24

If f:X→Y is an almost contra wgrα- continuous injection and Y is weakly Hausdorff, then X is wgrα-T_1.

Proof:

Suppose that Y is weakly Hausdorff. For any distinct points x and y in Y,there exists $V,W \in RC(Y)$ such that $f(x) \in V$, $f(y) \in W$, $f(x) \notin W$, $f(y) \notin V$. Since f is almost wgrα-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are wgrα-open subsets of X such that $x \in f^{-1}(V)$ and $y \in g^{-1}(W)$,$y \notin f^{-1}(V)$,$x \notin f^{-1}(W)$.This shows that X is wgrα-T_1.

Definition:3.25

A topological space X is called wgrα-ultra connected if every two non-void wgrα-closed subsets of X intersect.

Theorem:3.26

If X is wgrα-ultra connected and f:X→Y is almost contra wgrα-continuous and surjective,then Y is hyperconnected.

Proof:

Assume that Y is not hyper connected. There exists an open set V such that V is not dense in Y. Then there exists disjoint non-empty regular open subsets B_1 and B_2 in Y namely B_1 =int(cl(V)) and B_2=Y−cl(V). Since f is almost contra-wgrα continuous and surjective.$f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint non-empty wgrα-closed subsets of X. Which is a contradiction to the fact that X is wgrα-ultra connected. Hence Y is hyper connected.

Theorem:3.27

If f:X→Y is almost contra wgrα-continuous surjection and X is wgrα-connected,then Y is connected.

Proof

Suppose that Y is not connected. Then there exists non-empty disjoint open sets V_1 and V_2 such that $Y=V_1 \cup V_2$.Here V_1 and V_2 are clopen in Y. Since f is almost contra wgrα-continuous,$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are wgrα-open in X. Moreover $f^{-1}(V_1)$ and $f^{-1}(V_2)$ disjoint and $X=f^{-1}(V_1) \cup f^{-1}(V_2)$, which is a contradiction to the fact that X is wgrα-connected. Hence Y is connected.

Definition:3.28

A space X is said to be

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(i) wgrα-closed if every wgrα-closed cover of X has a finite subcover.

(ii) countable wgrα-closed if every countable wgrα-closed cover of X by wgrα-closed sets has a finite subcover.

(iii) wgrα-Lindelof if every cover of X by wgrα-closed sets has a countable cover.

Theorem:3.29

Let f:X→Y be an almost contra wgrα-continuous surjection. Then the following statements hold.

(i) If X is wgrα-closed compact then Y is nearly compact .

(ii) If X is wgrα-lindelof then Y is nearly lindelof .

(iii) If X is Countably wgrα-closed compact, then Y is nearly countably compact.

Proof:

(i) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra wgrα-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a wgrα-closed cover of X.Since X is wgrα-closed compact there exists a finite subset I_0 of I such that $X=\bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$.Therefore, we have $Y=\bigcup\{V_\alpha:\alpha \in I_0\}$.Hence Y is nearly compact.

(ii) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra wgrα-continuous,$\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a wgrα-closed cover of X. Since X is wgrα-lindelof, there exists a countable subset I_0 of I such that $X= \bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$,since f is surjective, $Y=\bigcup\{V_\alpha: \alpha \in I_0\}$ is finite subcover for Y. Therefore Y is nearly lindelof.

(iii) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y. Since f is almost contra wgrα-continuous,then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable wgrα-closed cover of X .Since X is countably wgrα-closed compact, there exists a finite subset I_0 of I such that $X= \bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$. Since f is surjective,$Y=\bigcup\{V_\alpha:\alpha \in I_0\}$ is finite subcover of Y.Therefore, Y is nearly countably compact.

Theorem:3.30

Let f:X→Y be an almost contra wgrα-continuous and almost wgrα-continuous surjection. Then the following statements hold.

(i) If X is mildly wgrα-closed compact, then Y is nearly compact .

(ii) If X is mildly countably wgrα-compact, then Y is nearly countably compact .

(iii) If X is mildly wgrα-lindelof, then Y is nearly compact.

Proof:

(i) Let $V \in RO(Y)$.Since f is almost contra wgrα-continuous and almost wgrα-continuous,$f^{-1}(V)$ is wgrα-closed and wgrα-open in X respectively.Then $f^{-1}(V)$ is wgrα-clopen in X.Let$\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y.Then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a wgrα-clopen in X.Since X is mildly WGRα-compact, there exists a finite subset I_0 of I such that $X=\bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$.Since X is surjective,we obtain $Y=\bigcup\{V_\alpha:\alpha \in I_0\}$. Hence Y is nearly compact.

(ii) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y. Since f is almost contra wgrα-continuous and almost wgrα-continuous surjection,,$\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable wgrα-closed cover of X. Since X is mildly countably wgrα-compact, there exists a finite subset I_0 of I such that $X=\bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$.Since f is surjective, $Y=\bigcup\{V_\alpha: \alpha \in I_0\}$ is finite subcover for Y. Therefore Y is nearly Compact.

(iii) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra wgrα-continuous and almost wgrα-continuous surjection, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is wgrα-closed cover of X . Since X is mildly-lindelof, there exists a finite subset I_0 of I such that $X= \bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$.Since f is surjective, $Y=\bigcup\{V_\alpha:\alpha \in I_0\}$ is finite subcover of Y. Therefore, Y is nearly lindelof.

Theorem:3.31

Let f:X→Y be an almost contra wgrα-continuous surjection, then the following properties hold:

(i) If X is wgrα-compact, then Y is S-closed.

(ii) If X is countably wgrα-closed, then Y is countably S-closed.

(iii) If X is wgrα-lindelof, then Y is S-lindelof.

Proof:

(i) Let $\{V_\alpha:\alpha \in I\}$ be any regular-closed cover of Y.Since f is almost contra wgrα-continuous, $\{f^{-1}(V_\alpha:\alpha \in I\}$ is wgrα-open cover of X. Since X is wgrα-compact, there exists a finite subset I_o of I such that $X=\bigcup\{f^{-1}(V_\alpha): \alpha \in I_0\}$.Since f is surjective, $Y= \bigcup\{V_\alpha:\alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is S-closed.

(ii) Let $\{V_\alpha:\alpha\in I\}$ be any countable regular closed cover of Y. Since f is almost contra wgrα-continuous,$\{f^{-1}(V_\alpha:\alpha\in I\}$ is countable wgrα-open cover of X. Since X is countably wgrα-compact, there exists a finite subset I_o of I such that X= $\bigcup\{f^{-1}(V_\alpha) : \alpha\in I_0\}$. Since f is surjective, Y=$\bigcup\{V_\alpha:\alpha\in I_0\}$ is finite subcover for Y. Therefore, Y is S-closed.

(iii)Let $\{V_\alpha:\alpha\in I\}$ be any regular-closed cover of Y. Since f is almost contra wgrα-continuous,$\{f^{-1}(V_\alpha:\alpha\in I\}$ is wgrα-open cover of X. Since X is wgrα-lindelof, there exists a countable subset I_o of I such that X=$\bigcup\{f^{-1}(V_\alpha): \alpha\in I_0\}$. Since f is surjective, Y= $\bigcup\{V_\alpha:\alpha\in I_0\}$ is finite subcover for Y.Therefore, Y is S-lindelof.

4. Approximately wgrα-Continuous Function

Definition:4.1

A map f:X→Y is said to be approximately wgrα-continuous(ap-wgrα-continuous) if $cl(int(F))\subset f^{-1}(U)$ whenever U is an open subset of Y and F is a wgrα-closed subset of X such that $F\subset f^{-1}(U)$.

Definition:4.2

A map f:X→Y is said to be approximately wgrα-closed(ap-wgrα-closed) if $f(F)\subset int(cl(V))$ whenever V is a wgrα-open subset of Y, F is a wgrα-closed subset of X and $f(F)\subset V$.

Definition:4.3

A map f:X→Y is said to be approximately wgrα-open(ap-wgrα-open) if $cl(int(F))\subset f(U)$ whenever U is an open subset of Y, F is a wgrα-closed subset of Y and $F\subset f(U)$.

Definition:4.4

A map f:X→Y is said to be contra wgrα-closed(resp.contra wgrα-open) if f(U) is wgrα-open(resp.wgrα-closed) in Y for each closed(resp.open) set U of X.

Theorem:4.5

Let f:X→Y be a function,then

(i) If f is contra α-continuous, then f is an ap-wgrα-continuous

(ii) If f is contra α-closed, then f is an ap-wgrα-closed.

(ii) If f is contra α-open, then f is ap-wgrα-open.

Proof:

(i) Let $F\subset f^{-1}(U)$, where U is a open subset in Y and F is a wgrα-closed subset of X.Then $cl(int(F))\subset cl(int(f^{-1}(U)))$. Since f is contra α-continuous,$cl(int(F))\subset cl(int(f^{-1}(U))=f^{-1}(U)$. This shows that f is ap-wgrα-continuous.

(ii) Let $f(F)\subset V$. Where F is closed subset of X and V is a wgrα-open subset of Y. Therefore f(F)=int(cl(F)) .Thus f is ap-wgrα-closed.

(iii) Let $F\subset f(U)$.Where F is wgrα-closed subset of Y and U is an open subset of X. Since f is contra-α-open. f(U) is α-closed in Y for each open set U of X. $cl(int(F))\subset cl(int(f(U)))=f(U)$. Thus f is ap-wgrα-open.

5.wgrα-Regular Graph and Strongly Contra wgrα-Closed Graphs

Definition:5.1

A graph G(f) of a function f:X→Y is said to be WGRα-regular if for each $(x,y)\in(X\times Y) - G(f)$, there exists a wgrα-closed set U in X containing x and regular open set V of Y containing y such that $(U\times V)\bigcap G(f)=\phi$.

Definition:5.2

A graph G(f) of a function f:X→Y is said to be strongly contra wgrα-closed if for each $(x,y)\in(X\times Y) - G(f)$, there exists a wgrα-open set U in X containing x and regular closed set V of Y containing y such that $(U\times V)\bigcap G(f)=\phi$.

Theorem:5.3

Let f:X→Y be a function and let g:X→X×Y be the graph function f, defined by g(x)=(x,f(x)) for every $x\in X$. If g is almost contra wgrα-continuous function, then f is an almost contra wgrα-continuous.

Proof:

Let $V\in RC(Y)$, then X×V=X×cl(int(V))=cl(int(X)) ×cl(int(V))=cl(int(X×V)). Therefore, $X\times V\in RC(X\times Y)$. Since g is almost contra wgrα-continuous, $f^{-1}(V)=g^{-1}(X\times V)\in WGR\alpha O(X)$.Thus, f is an almost contra wgrα-continuous.

Definition:5.4

The graph G(f) of a function f:X→Y is said to be contra wgrα-closed if for each $(x,y) \in (X \times Y) - G(f)$, there exists $U \in WGR\alpha O(X,x)$ and $V \in C(Y,y)$ such that $(U\times V) \bigcap G(f)= \phi$.

Theorem:5.5

If f:X→Y is almost contra wgrα-continuous and Y is T_2, then G(f) is wgrα-regular graph in X ×Y.

Proof:

Let $(x,y) \in X \times Y - G(f)$, it follows that f(x)≠y. Since Y is T_2, there exists open sets V and W containing f(x) and y respectively such that V∩W=ϕ, we have int(cl(V))∩int(cl(W))=ϕ. Since f is almost contra wgrα-continuous, f^{-1} (int(cl(V))) is wgrα-closed in X containing x. Take $U=f^{-1}$(int (cl(V)).Then $f(U) \subset int(cl(V))$. Therefore $f(U)\cap int(cl(W))= \phi$. Hence G(f) is wgrα-regular.

Theorem:5.6

Let f:X→Y have a wgrα-regular graph G(f), if f is injective, then X is wgrα-T_1.

Proof:

Let x and y be any two distinct points of X.Then we have $(x,f(y)) \in X\times Y-G(f)$.By definition of wgrα-regular graph, there exists a wgrα-closed set U of X and $V \in RO(Y)$ such that $(x,f(y)) \in U\times V$ and $f(U)\cap V= \phi$. Hence $U\cap f^{-1}(V)= \phi$. Therefore we have $y \in X-U$ and $x \notin X-U$. $X-U \in WGR\alpha O(X)$ implies X is wgrα-T_1.

Theorem:5.7

Let f:X→Y have a wgrα-regular graph G(f), if f is surjective, then Y is weakly-T_2.

Proof:

Let y_1 and y_2 be two distinct points of Y. Since f is surjective $f(x)=y_1$ for some $x \in X$ and $(x,y_2) \in X\times Y-G(f)$. By the above lemma, there exists a wgrα-closed set U of X and $F \in RO(Y)$ such that $(x,y_2) \in U\times F$ and $f(U)\cap F= \phi$. Hence $y_1 \notin F$. Then $y_2 \notin Y-F \in R(Y)$ and $y_1 \in Y-F$. Which implies that Y is weakly T_2.

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