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Combined resonance and vibration reduction of non linear dynamical system subject to tuned excitation

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ABSTRACT

The non-linear dynamical system subject to tuned excitation is consider, and studied . The system is represented by two degree of freedom differential equations of the system and absorber. The method of multiple scale is applied to solve the system up to 3rd order approximation. Effect of different parameters is studied numerically all resonance cases are studied numerically to obtain the worst case . Stability of the system is investigated using both phase plane and frequency response curves.

Keywords: vibration control; tuned excitation force; phase plane; frequency response curves.

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INTRODUCTION

Mechanical and structural systems are inherently are inherently non-linear due to many sources. Ultrasonic machining (USM) is of particular interest in the machining of conductive and non-conductive, brittle, complicated shape materials such as diamonds. Non-linearities necessarily introduce a whole range of phenomena that are not found in linear system [1], including jump phenomena, occurrence of multiple solutions, modulation, shift in natural frequencies, the generation of combination resonances, evidence of period multiplying bifurcations and chaotic motion [2-5]. In these systems the vibrations are needed to be controlled to minimize or eliminate the hazard of damage or destruction. There are two types for vibration control. They are active and passive control. One of the most effective tools of passive control is dynamic absorber or the neutralizer [6]. Nabergoj et al [7] Studied the stability of auto- parametric resonance in an externally excited system. Abdel Hafez and Eissa [8] studied the effects of non linear elastomeric torsion absorber to control the vibration of the crank shaft in internal combustion engines, when subject to external excitation torque. Mahmoud and Frghaly [9] investigated the steady-state analysis for a class of sliding mode controlled systems using describing function method. Eissa [10] has shown that to control the vibration of a system subjected to harmonic excitations, the fundamental or the first harmonic absorber is the most effective one.

Eissa et al. [11-13] investigated saturation phenomena in non-linear oscillating systems subject to multi-parametric and external excitation. Cao [14] studied primary resonate optimal control for homoclinic bifurcation in single degree of freedom non-linear oscillators. Jing and Wang [15] analyzed complex dynamics in Duffing system with two external forces. Eissa and sayed [16,17] presented tuned absorbers in both transversely and longitudinal directions of a simple pendulum which designed to control one frequency at primary resonance. El-Dib [18] investigated a theoretical analysis of parametric harmonic response of two resonate modes based on a cubic non-linear system. Eissa and Amer [19] investigated the vibration control of a cantilever beam under both external and parametric excitation using active control via cubic damping feedback. Amer [20] investigation the coupling of two non-linear oscillators of the main system and absorber representing ultrasonic cutting process subject to parametric excitation forces. Sayed and Hamed [21] studied the response of a two-degree-of –freedom system with quadratic coupling under parametric and harmonic excitations. Sayed and Kamel [22,23] investigated the effect of different controllers on the vibrating system and saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. Amer and Abd El salam [24] investigated the effect of a non-linear absorber to reduce vibrations due to dynamical system subjected to multi external forces. Kamel et al [25] studied the vibration suppression in ultrasonic machining described by non-linear differential equations via passive controller.

In this paper we studied the vibration control of a non-linear system under tuned excitation forces. the method of multiple scale is applied to obtained the approximate solution of the system. Vibration method is used to reduced the amplitude of vibration at the worst resonance case.The effect of different parameter are investigated.

2. MATHEMATICAL MODELING

A two – degree of freedom system composed of two weakly damped oscillators is considered. Here, x_1 and x_2 denote displacements of the main non- linear system and absorber, respectively. The following equations are obtained:

$$m_1\frac{d^2x_1}{dt^2}+c_1\frac{dx_1}{dt}+k_1x_1+c_2\left(\frac{dx_1}{dt}-\frac{dx_2}{dt}\right)+k_2\left(x_1-x_2\right)^3=F\cos\omega t\cos\Omega t \tag{1}$$

$$m_2\frac{d^2x_2}{dt^2}+c_2\left(\frac{dx_2}{dt}-\frac{dx_1}{dt}\right)+k_2\left(x_2-x_1\right)^3=0 \tag{2}$$

where m_1 and m_2 are the mass of the main system and absorber. c_1 and c_2 are damping coefficients of the main system and absorber. k_1 and k_2 are stiffness of the main system and absorber. F is excitation amplitude of tuned force. ω is frequency of the tuned force and Ω is excitation amplitued.Let $u=x_1$ and $v=x_2-x_1$ then, equations (1) and (2) can be written as:

$$\ddot{u}+\varepsilon\zeta_1\dot{u}+\omega_1^2u-\varepsilon\zeta_2\dot{v}-\varepsilon\omega_2^2v^3=\varepsilon f\cos\omega t\cos\Omega t \tag{3}$$

$$\ddot{v}+\omega_1^2v+\varepsilon\left\{(1+\delta)\zeta_2\dot{v}+(1+\delta)\omega_2^2v^3-\zeta_1\dot{u}-\delta\,\omega_1^2u-\delta\,\omega_1^2v\right\}=-\varepsilon f\cos\omega t\cos\Omega t \tag{4}$$

We can solve Eqs. (3) and (4) analytically using the multiple scale perturbation technique as follows:

$$u(t;\varepsilon)=u_0(T_0,T_1)+\varepsilon u_1(T_0,T_1)+\varepsilon^2u_2(T_0,T_1)+\varepsilon^3u_3(T_0,T_1) \tag{5}$$

$$v(t;\varepsilon)=v_0(T_0,T_1)+\varepsilon v_1(T_0,T_1)+\varepsilon^2v_2(T_0,T_1)+\varepsilon^3v_3(T_0,T_1) \tag{6}$$

And the time derivatives become

$$\frac{d}{dt}=D_0+\varepsilon D_1+\varepsilon^2 D_2+\varepsilon^3 D_3+\ldots \tag{7}$$

$$\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+\varepsilon^2\left(D_1^2+2D_0D_2\right)+2\varepsilon^3\left(D_0D_3+D_1D_2\right)+\ldots \tag{8}$$

where $T_n=\varepsilon^n t$, $D_n=\frac{\partial}{\partial T_n}$ and (n=0,1,2,3) , T_0 is the fast time scale and T_n is the slow time scale(n=1,2,3).

Substituting Eqs. (5) – (8) into Eqs. (3) and (4) and equating the coefficient of the same power of ϵ in the both sides, we obtain the following set of ordinary differential equations:

$$O\left(\varepsilon^0\right):\left(D_0^2+\omega_1^2\right)u_0=0 \tag{9}$$

$$\left(D_0^2+\omega_1^2\right)v_0=0 \tag{10}$$

$$O\left(\varepsilon\right):\left(D_0^2+\omega_1^2\right)u_1=-2D_0D_1u_0-\zeta_1 D_0u_0+\zeta_2 D_0v_0+\omega_2^2 v_0^3 +f\cos\omega t\cos\Omega t \tag{11}$$

$$\left(D_0^2+\omega_1^2\right)v_1=-2D_0D_1v_0-(1+\delta)\zeta_2 D_0v_0-(1+\delta)\omega_2^2v_0^3+\zeta_1 D_0u_0 +\delta\omega_1^2u_0+\delta\omega_1^2v_0-f\cos\omega t\cos\Omega t \tag{12}$$

$$O\left(\varepsilon^2\right):\left(D_0^2+\omega_1^2\right)u_2=-D_1^2u_0-2D_0D_1u_1-2D_0D_2u_0-\zeta_1 D_0u_1+\zeta_2 D_0v_1 +\zeta_2 D_1v_0+3\omega_2^2v_0^2v_1 \tag{13}$$

$$\left(D_0^2+\omega_1^2\right)v_2=-2D_0D_1v_1-D_1^2v_0-2D_0D_2v_0-(1+\delta)\zeta_2(D_1v_0 +D_0v_1)-3(1+\delta)\omega_2^2v_0^2v_1+\zeta_1(D_0u_1+D_1u_0) +\delta\omega_1^2(u_1+v_1) \tag{14}$$

$$O\left(\varepsilon^3\right):\left(D_0^2+\omega_1^2\right)u_3=-2D_0D_3u_0-D_1^2u_1-2D_1D_2u_0-2D_0D_1u_2 -2D_0D_2u_1-\zeta_1D_0u_2-\zeta_1D_2u_0-\zeta_1D_1u_1+\zeta_2D_0v_2 +3\omega_2^2\left(v_0v_1^2+v_2v_0^2\right) \tag{15}$$

$$\left(D_0^2+\omega_1^2\right)v_3=-2D_0D_3v_0-D_1^2v_1-2D_1D_2v_0-2D_0D_1v_2-2D_0D_2v_1 -(1+\delta)\zeta_2(D_0v_2+D_2v_0+D_1v_1)+\delta\omega_1^2(u_2+v_2) +\zeta_1(D_0u_2+D_1u_1+D_2u_0)-9\omega_2^2(1+\delta)\left(v_1^2v_0+v_0^2v_2\right) \tag{16}$$

The general solution of Eqs. (9) and (10) is given by

$$u_0(T_0,T_1)=A(T_1)e^{i\omega_1T_0}+\overline{A}(T_1)e^{-i\omega_1T_0} \tag{17}$$

$$v_0(T_0,T_1)=B(T_1)e^{i\omega_1T_0}+\overline{B}(T_1)e^{-i\omega_1T_0} \tag{18}$$

where A, B are unknown functions in T_1. Substituting Eqs. (17), (18) into Eqs. (11), (12) and eliminating the secular terms then, solve the resulting equations, yields:

$$u_1(T_0,T_1) = E_1 e^{3i\omega_1 T_0} + E_2 e^{i(\Omega+\omega)T_0} + E_3 e^{i(\Omega-\omega)T_0} + cc \tag{19}$$

$$v_1(T_0,T_1) = E_4 e^{3i\omega_1 T_0} + E_5 e^{i(\Omega+\omega)T_0} + E_6 e^{i(\Omega-\omega)T_0} + cc \tag{20}$$

where $E_j, (j=1,...,6)$ are complex functions of T_1. Substituting Eqs.(17)-(20) into Eqs.(13) and (14), hence solving the resulting equations, we obtain the following:

$$u_2(T_0,T_1) = H_1 e^{3i\omega_1 T_0} + H_2 e^{i(\Omega+\omega)T_0} + H_3 e^{i(\Omega-\omega)T_0} + H_4 e^{5i\omega_1 T_0} + H_5 e^{i(\Omega+\omega+2\omega_1)T_0}$$
$$+H_6 e^{i(\Omega-\omega+2\omega_1)T_0} + H_7 e^{i(\Omega+\omega-2\omega_1)T_0} + H_8 e^{i(\Omega-\omega-2\omega_1)T_0} + cc \tag{21}$$

$$v_2(T_0,T_1) = H_9 e^{3i\omega_1 T_0} + H_{10} e^{i(\Omega+\omega)T_0} + H_{11} e^{i(\Omega-\omega)T_0} + H_{12} e^{5i\omega_1 T_0} + H_{13} e^{i(\Omega+\omega+2\omega_1)T_0}$$
$$+H_{14} e^{i(\Omega-\omega+2\omega_1)T_0} + H_{15} e^{i(\Omega+\omega-2\omega_1)T_0} + H_{16} e^{i(\Omega-\omega-2\omega_1)T_0} + cc \tag{22}$$

where $H_j, (j=1,\ldots,16)$ are complex functions of T_1. Substituting Eqs. (17)-(22) into Eqs. (15) and (16), hence solving the resulting equations, we obtain the following:

$$u_3(T_0,T_1) = L_1 e^{3i\omega_1 T_0} + L_2 e^{4i\omega_1 T_0} + L_3 e^{5i\omega_1 T_0} + L_4 e^{6i\omega_1 T_0} + L_5 e^{7i\omega_1 T_0} + L_6 e^{i(\Omega+\omega)T_0}$$
$$+L_7 e^{i(\Omega-\omega)T_0} + L_8 e^{i(\Omega+\omega+2\omega_1)T_0} + L_9 e^{i(\Omega+\omega-2\omega_1)T_0} + L_{10} e^{i(\Omega-\omega+2\omega_1)T_0} + L_{11} e^{i(\Omega-\omega-2\omega_1)T_0}$$
$$+L_{12} e^{i(\Omega+\omega+\omega_1)T_0} + L_{13} e^{i(\Omega-\omega+\omega_1)T_0} + L_{14} e^{i(\Omega+\omega+3\omega_1)T_0} + L_{15} e^{i(\Omega+\omega-\omega_1)T_0} + L_{16} e^{i(\Omega-\omega+3\omega_1)T_0}$$
$$+L_{17} e^{i(\Omega-\omega-\omega_1)T_0} + L_{18} e^{i(\Omega+\omega-4\omega_1)T_0} + L_{19} e^{i(\Omega-\omega-4\omega_1)T_0} + L_{20} e^{i(2\Omega+2\omega+\omega_1)T_0} + L_{21} e^{i(2\Omega-2\omega+\omega_1)T_0}$$
$$+L_{22} e^{i(\Omega+\omega+4\omega_1)T_0} + L_{23} e^{i(\Omega-\omega+4\omega_1)T_0} + L_{24} e^{i(2\Omega+\omega)T_0} + L_{25} e^{i(2\Omega-\omega)T_0} + L_{26} e^{i(2\Omega+2\omega-\omega_1)T_0}$$
$$+L_{27} e^{i(2\Omega-2\omega-\omega_1)T_0} + cc \tag{23}$$

$$v_3(T_0,T_1) = L_{28} e^{3i\omega_1 T_0} + L_{29} e^{4i\omega_1 T_0} + L_{30} e^{5i\omega_1 T_0} + L_{31} e^{6i\omega_1 T_0} + L_{32} e^{7i\omega_1 T_0} + L_{33} e^{i(\Omega+\omega)T_0}$$
$$+L_{34} e^{i(\Omega-\omega)T_0} + L_{35} e^{i(\Omega+\omega+2\omega_1)T_0} + L_{36} e^{i(\Omega+\omega-2\omega_1)T_0} + L_{37} e^{i(\Omega-\omega+2\omega_1)T_0} + L_{38} e^{i(\Omega-\omega-2\omega_1)T_0}$$
$$+L_{39} e^{i(\Omega+\omega+\omega_1)T_0} + L_{40} e^{i(\Omega-\omega+\omega_1)T_0} + L_{41} e^{i(\Omega+\omega+3\omega_1)T_0} + L_{42} e^{i(\Omega+\omega-\omega_1)T_0} + L_{43} e^{i(\Omega-\omega+3\omega_1)T_0}$$
$$+L_{44} e^{i(\Omega-\omega-\omega_1)T_0} + L_{45} e^{i(\Omega+\omega-4\omega_1)T_0} + L_{46} e^{i(\Omega-\omega-4\omega_1)T_0} + L_{47} e^{i(2\Omega+2\omega+\omega_1)T_0} + L_{48} e^{i(2\Omega-2\omega+\omega_1)T_0}$$
$$+L_{49} e^{i(\Omega+\omega+4\omega_1)T_0} + L_{50} e^{i(\Omega-\omega+4\omega_1)T_0} + L_{51} e^{i(2\Omega+\omega)T_0} + L_{52} e^{i(2\Omega-\omega)T_0} + L_{53} e^{i(2\Omega+2\omega-\omega_1)T_0}$$
$$+L_{54} e^{i(2\Omega-2\omega-\omega_1)T_0} + cc \tag{24}$$

where $L_j, (j=1,...,54)$ are complex functions of T_1.

Resonance cases:

From the above derived solutions, the reported resonance cases are:-

(i) $\omega_1 \cong (\Omega+\omega)$ (ii) $\omega_1 \cong \pm(\Omega-\omega)$ (iii) $\omega_1 \cong \frac{1}{2}(\Omega+\omega)$ (iv) $\omega_1 \cong \pm\frac{1}{2}(\Omega-\omega)$

(v) $\omega_1 \cong \frac{1}{3}(\Omega+\omega)$ (vi) $\omega_1 \cong \pm\frac{1}{3}(\Omega-\omega)$ (vii) $\omega_1 = \frac{1}{5}(\Omega\pm\omega)$

3. STABILTY ANALYSIS

We study the different resonance numerically to see the worst resonance, one of the worst cases has been chosen to study the system stability .The selected resonance case $\omega_1 \cong \Omega - \omega$ .In this case we introduce the detuning parameter σ according to

$$\omega_1 = \Omega - \omega + \varepsilon \sigma \qquad (25)$$

Substituting Eq. (25) into Eqs. (11) and (12) and eliminating the secular and small divisor terms from u_1 and v_1 ,we get the following :

$$-2i\,\omega_1 D_1 A - i\,\zeta_1 \omega_1 A + i\,\zeta_2 \omega_1 B + 3\omega_2^2 B^2 \overline{B} + \frac{f}{4} e^{-i\sigma T_1} = 0 \qquad (26)$$

$$-2i\,\omega_1 D_1 B + i\,\zeta_1 \omega_1 A - i\,\zeta_2 \omega_1 (1+\delta) B + 3\omega_2^2 (1+\delta) B^2 \overline{B} + \delta\,\omega_1^2 A + \delta\,\omega_1^2 B - \frac{f}{4} e^{-i\sigma T_1} = 0 \qquad (27)$$

To analyze the solution of equations (26) and (27) it is convenient to express A and B in the polar form:

$$\mathrm{A}(T_1) = \frac{1}{2} a(T_1) e^{i\gamma_1(T_1)} \text{ and } B(T_1) = \frac{1}{2} b(T_1) e^{i\gamma_2(T_1)} \qquad (28)$$

where a, b, γ_1 and γ_2 are unknown real-valued functions. Inserting Eq. (28) into Eqs. (26) and (27) and separating real and imaginary parts, we have

$$a' = -\frac{1}{2}\zeta_1 a + \frac{1}{2}\zeta_2 b \cos\varphi_1 + \frac{3\omega_2^2}{8\omega_1} b^3 \sin\varphi_1 - \frac{f}{4\omega_1}\sin\varphi_2 \qquad (29)$$

$$a\,\gamma_1' = \frac{1}{2}\zeta_2 b \sin\varphi_1 - \frac{3\omega_2^2}{8\omega_1} b^3 \cos\varphi_1 - \frac{f}{4\omega_1}\cos\varphi_2 \qquad (30)$$

$$b' = \frac{1}{2}\zeta_2 (1+\delta) b + \frac{1}{2}\zeta_1 a \cos\varphi_1 - \frac{\delta}{2}\omega_1 a \sin\varphi_1 + \frac{f}{4\omega_1}\sin\varphi_3 \qquad (31)$$

$$b\,\gamma_2' = \frac{3\omega_2^2}{8\omega_1}(1+\delta) b^3 - \frac{\delta}{2}\omega_1 b - \frac{1}{2}\zeta_1 a \sin\varphi_1 - \frac{\delta}{2}\omega_1 a \cos\varphi_1 + \frac{f}{4\omega_1}\cos\varphi_3 \qquad (32)$$

where $\varphi_1 = \gamma_2 - \gamma_1$, $\varphi_2 = \gamma_1 + \sigma T_1$ and $\varphi_3 = \gamma_2 + \sigma T_1$.

For steady state solutions, $a' = b' = 0$ and $\varphi_n' = 0$, (n=1,2,3) into Eqs.(29)- (32) we obtained

$$-\frac{1}{2}\zeta_1 a + \frac{1}{2}\zeta_2 b \cos\varphi_1 + \frac{3\omega_2^2}{8\omega_1} b^3 \sin\varphi_1 - \frac{f}{4\omega_1}\sin\varphi_2 = 0 \qquad (33)$$

$$-a\,\sigma = \frac{1}{2}\zeta_2 b \sin\varphi_1 - \frac{3\omega_2^2}{8\omega_1} b^3 \cos\varphi_1 - \frac{f}{4\omega_1}\cos\varphi_2 \qquad (34)$$

$$\frac{1}{2}\zeta_2 (1+\delta) b + \frac{1}{2}\zeta_1 a \cos\varphi_1 - \frac{\delta}{2}\omega_1 a \sin\varphi_1 + \frac{f}{4\omega_1}\sin\varphi_3 = 0 \qquad (35)$$

$$-b\,\sigma = \frac{3\omega_2^2}{8\omega_1}(1+\delta) b^3 - \frac{\delta}{2}\omega_1 b - \frac{1}{2}\zeta_1 a \sin\varphi_1 - \frac{\delta}{2}\omega_1 a \cos\varphi_1 + \frac{f}{4\omega_1}\cos\varphi_3 \qquad (36)$$

Solving the resulting algebraic equations yields two possibilities for the fixed points for each case.

Case(1): the controller is deactivated, and $a \neq 0, b = 0$, the frequency response equations can be obtained the form:

$$a = \frac{f}{2\omega_1\sqrt{\zeta_1^2 + 4\sigma^2}} \tag{37}$$

Case (2): the controller is activated, and $a, b \neq 0$ and from Eqs. (33) – (36), the resulting two equations are:

$$\frac{1}{4}\zeta_1^2 a^2 + \sigma^2 a^2 - \frac{1}{4}\zeta_2^2 b^2 - \frac{9\omega_2^4}{64\omega_1^2}b^6 - \frac{f^2}{16\omega_1^2} + \frac{\zeta_2 f}{4\omega_1}b\sin(\varphi_1+\varphi_2)$$

$$-\frac{3\omega_2^2 f}{16\omega_1^2}b^3\cos(\varphi_1+\varphi_2) = 0 \tag{38}$$

$$\frac{1}{4}\zeta_1^2 a^2 + \frac{\delta^2}{4}\omega_1^2 a^2 - \frac{\zeta_2^2}{4}(1+\delta)^2 b^2 - \left(\left(\frac{\delta\omega_1}{2} - \sigma\right)b - \frac{3\omega_2^2}{8\omega_1}(1+\delta)b^3\right)^2 + \frac{f^2}{16\omega_1^2}$$

$$+\frac{\zeta_1 f}{4\omega_1}a\sin(\varphi_3-\varphi_1) - \frac{\delta f}{4}a\cos(\varphi_3-\varphi_1) = 0 \tag{39}$$

3.1 Linear solution

Now, to study the stability of the linear solution of the obtained fixed let us consider A and B in the forms:

$$A(T_1) = \frac{1}{2}(p_1 - iq_1)e^{i\delta_1 T_1}, \quad B(T_1) = \frac{1}{2}(P_2 - iq_2)e^{i\delta_2 T_1} \tag{40}$$

where p_1, q_1, p_2 *and* q_2 are real values and considering $\delta_1 = \delta_2 = -\sigma$.

Substituting from Eq. (40) into the linear parts of Eqs. (26), (27) and separating real and imaginary parts, the following system of equations are obtained:

1- For the solution $(a \neq 0, b = 0)$, we have

$$p_1' + \frac{\zeta_1}{2}p_1 - \sigma q_1 = 0 \tag{41}$$

$$q_1' + \sigma p_1 + \frac{\zeta_1}{2}q_1 + \frac{f}{4} = 0 \tag{42}$$

The stability of the linear solution is obtained from the zero characteristic equation:

$$\begin{vmatrix} -\left(\lambda + \frac{\zeta_1}{2}\right) & \sigma \\ -\sigma & -\left(\lambda + \frac{\zeta_1}{2}\right) \end{vmatrix} = 0 \tag{43}$$

where, $$\lambda_{1,2} = -\frac{\zeta_1}{2} \pm i\sigma \tag{44}$$

The linear solution is stable in this case if and only if $\zeta_1 > 0$, ad otherwise it is unstable.

2- For the practical solution $(a \neq 0, b \neq 0)$, we have

$$p_1' + \frac{\zeta_1}{2}p_1 - \sigma q_1 - \frac{\zeta_2}{2}p_2 = 0 \tag{45}$$

$$q_1' + \sigma p_1 + \frac{\zeta_1}{2}q_1 - \frac{\zeta_2}{2}q_2 - \frac{f}{4\omega_1} = 0 \tag{46}$$

$$p_2' - \frac{\zeta_1}{2} p_1 + \frac{\delta}{2}\omega_1 q_1 + \frac{\zeta_2}{2}(1+\delta) p_2 - \left(\sigma - \frac{\delta}{2}\omega_1\right) q_2 = 0 \qquad (47)$$

$$q_2' - \frac{\delta}{2}\omega_1 p_1 - \frac{\zeta_1}{2} q_1 + \left(\sigma - \frac{\delta}{2}\omega_1\right) p_2 + \frac{\zeta_2}{2}(1+\delta) q_2 + \frac{f}{4\omega_1} = 0 \qquad (48)$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

$$\begin{vmatrix} -(\lambda + \frac{\zeta_1}{2}) & \sigma & \frac{\zeta_2}{2} & 0 \\ -\sigma & -(\lambda + \frac{\zeta_1}{2}) & 0 & \frac{\zeta_2}{2} \\ \frac{\zeta_1}{2} & -\frac{\delta\omega_1}{2} & -(\lambda + \frac{\zeta_2}{2}(1+\delta)) & (\sigma - \frac{\delta}{2}\omega_1) \\ \frac{\delta\omega_1}{2} & \frac{\zeta_1}{2} & -(\sigma - \frac{\delta}{2}\omega_1) & -(\lambda + \frac{\zeta_2}{2}(1+\delta)) \end{vmatrix} = 0 \qquad (49)$$

After extract we obtain that:

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0 \qquad (50)$$

where r_1, r_2, r_3 and r_4 are defined in Appendix.

According to Routh-Huriwitz criterion, the above linear solution is stable if the following are satisfied:

$$r_1 > 0, r_1 r_2 - r_3 > 0, r_3(r_1 r_2 - r_3) - r_1^2 r_4 > 0, r_4 > 0 \qquad (51)$$

3.2 Non-linear solution

To determine the stability of the fixed points, one lets

$$a = a_{10} + a_{11}, b = b_{10} + b_{11}, \varphi_m = \varphi_{m0} + \varphi_{m1} \left(m = 1,2,3\right) \qquad (52)$$

Where a_{10}, b_{10} and φ_{m0} are solutions of Eqs. (33)- (36) and $a_{11}, b_{11}, \varphi_{m1}$ are perturbations which are assumed to be small comparing to a_{10}, b_{10} and φ_{m0} .substituting Eq.(52) into Eqs.(29)-(32) using Eqs. (33)- (36) and keeping only the linear terms in we obtain:

1- For the solution $\left(a \neq 0, b = 0\right)$, we have

$$a_{11}' = \left[-\frac{\zeta_1}{2}\right] a_{11} - \left[\frac{f}{4\omega_1}\cos\varphi_{20}\right]\varphi_{21} \qquad (53)$$

$$\varphi_{21}' = \left[\frac{\sigma}{a_{10}}\right] a_{11} - \left[\frac{f}{4a_{10}\omega_1}\sin\varphi_{20}\right]\varphi_{21} \qquad (54)$$

The stability of a given fixed point to a disturbance proportional to exp (λ t) is determined by the roots of:

$$\begin{vmatrix} -\frac{\zeta_1}{2} - \lambda & -\frac{f}{4\omega_1}\cos\varphi_{20} \\ \frac{\sigma}{a_{10}} & -\frac{f}{4a_{10}\omega_1}\sin\varphi_{20} - \lambda \end{vmatrix} = 0 \qquad (55)$$

Consequently, a non-trivial solution is stable if and only if the real parts of both eigen values of the coefficient matrix (55) are less than zero.

2- For the practical solution $\left(a \neq 0, b \neq 0\right)$, we have

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$$a_{11}^{'} = \left[-\frac{\zeta_1}{2}\right]a_{11} + \left[\frac{\zeta_2}{2}\cos\varphi_{10} + \frac{9\omega_2^2}{8\omega_1}b_{10}^2\sin\varphi_{10}\right]b_{11}$$

$$-\left[\frac{\zeta_2}{2}b_{10}\sin\varphi_{10} - \frac{3\omega_2^2}{8\omega_1}b_{10}^3\cos\varphi_{10}\right]\varphi_{11} - \left[\frac{f}{4\omega_1}\cos\varphi_{20}\right]\varphi_{21} \qquad (56)$$

$$\varphi_{21}^{'} = \left[\frac{\sigma}{a_{10}}\right]a_{11} + \left[\frac{\zeta_2}{2a_{10}}\sin\varphi_{10} - \frac{9\,\omega_2^2}{8\,\omega_1}b_{10}^2\cos\varphi_{10}\right]b_{11}$$

$$-\left[\frac{\zeta_2}{2a_{10}}b_{10}\cos\varphi_{10} + \frac{3\omega_2^2}{8\omega_1}b_{10}^3\sin\varphi_{10}\right]\varphi_{11} + \left[\frac{f}{4\omega_1}\sin\varphi_{20}\right]\varphi_{21} \qquad (57)$$

$$b_{11}^{'} = \left[\frac{\zeta_2}{2}\cos\varphi_{10} - \frac{\sigma}{2}\omega_1\sin\varphi_{10}\right]a_{11} + \left[\frac{\zeta_1}{2}(1+\delta)\right]b_{11}$$

$$-\left[\frac{\zeta_1}{2}\sin\varphi_{10} + \frac{\delta}{2}\omega_1 a_{10}\cos\varphi_{10}\right]\varphi_{11} + \left[\frac{f}{4\omega_1}\cos\varphi_{30}\right]\varphi_{31} \qquad (58)$$

$$\varphi_{31}^{'} = -\left[\frac{\zeta_1}{2b_{10}}a_{10}\sin\varphi_{10} + \frac{\delta}{2b_{10}}\omega_1\cos\varphi_{10}\right]a_{11}$$

$$+\left[\frac{\delta}{b_{10}} + \frac{9\omega_2^2}{8\omega_1}(1+\delta)b_{10} + \frac{\delta}{2b_{10}}\omega_1\right]b_{11}$$

$$-\left[\frac{\zeta_1}{2b_{10}}a_{10}\cos\varphi_{10} - \frac{\delta}{2b_{10}}\omega_1 a_{10}\sin\varphi_{10}\right]\varphi_{11} - \left[\frac{f}{4\omega_1}\sin\varphi_{30}\right]\varphi_{31} \qquad (59)$$

The stability of a particular fixed point with respect to perturbations proportional to exp (λ t) depends on the real parts of the roots of the matrix .thus, a fixed point given by Eqs. (56)-(59) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

4. NUMERICAL RESULTS

The non linear dynamical system without absorber is solved numerically using Maple, at non resonance case (basic case) as shown in Fig.1, we can see that the steady state amplitude is about 0.002 (0.0014 times of the excitation amplitude f) and the system is stable. All resonance cases obtained the worst cases as shown in Fig.2, the resonance case $\omega_1 = \Omega + \omega$ the steady state amplitude is increased to 15 times of the basic case shown in Fig.2 while the case $\omega_1 = \Omega - \omega$ the steady state is increased to 30 times of the basic case. Shown in Fig.2 so, this case is considered to study the stability of the system if the control is active. Frequency response equation (37) is non linear algebraic equation of the amplitude a against the detuning parameter σ, when the absorber is deactivated $(a \neq 0, b = 0)$, this equation is solved numerically as shown in Fig. 4, from this figure we see that the amplitude of the main system is monotonic increasing function on the excitation amplitude f as shown in Fig. 4a, but the amplitude of the main system is monotonic decreasing function on natural frequency ω_1 and damping coefficient ζ_1 as shown in Fig. 4b and 4c. Frequency response equation (38) and (39) is solved numerically as shown in Fig.5 and Fig.6, we can obtained that the steady state amplitude of the main system is monotonic decreasing function of the natural frequency ω_1 and damping coefficient ζ_1 as shown in Figs. 5a, 5c, 6a and 6c, and the amplitude of the main system is monotonic increasing function of the excitation force amplitude f as shown in Fig. 5d and Fig.6e. From Figs. 5b, 5d, 6b and 6d, the steady state amplitude of the absorber is monotonic increasing in the natural frequency ω_1 and damping coefficient ζ_1, which is opposite to the main system. The steady state amplitude of the absorber is monotonic increasing function of the excitation force amplitude f as shown in Fig. 6f.

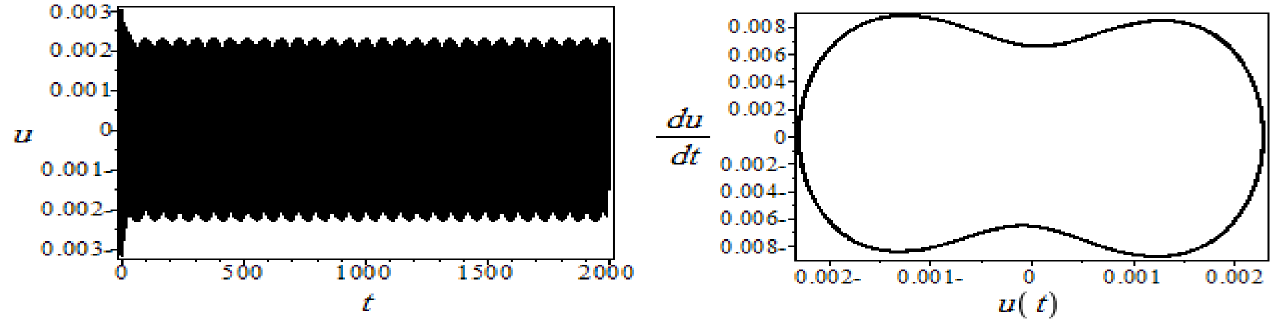
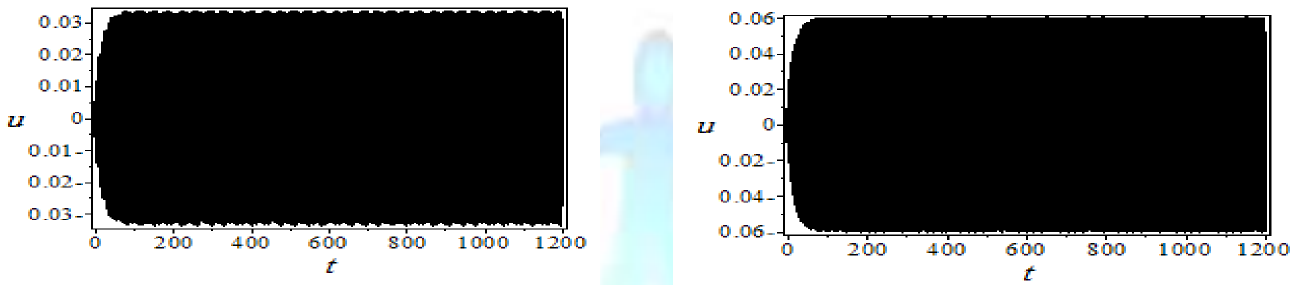


Fig. 1 the system behavior without absorber at non resonance case (basic case)



a) $\omega_1 = \Omega + \omega$ b) $\omega_1 = \Omega - \omega$

Fig. 2. Some of selected resonance cases of the system without absorber.

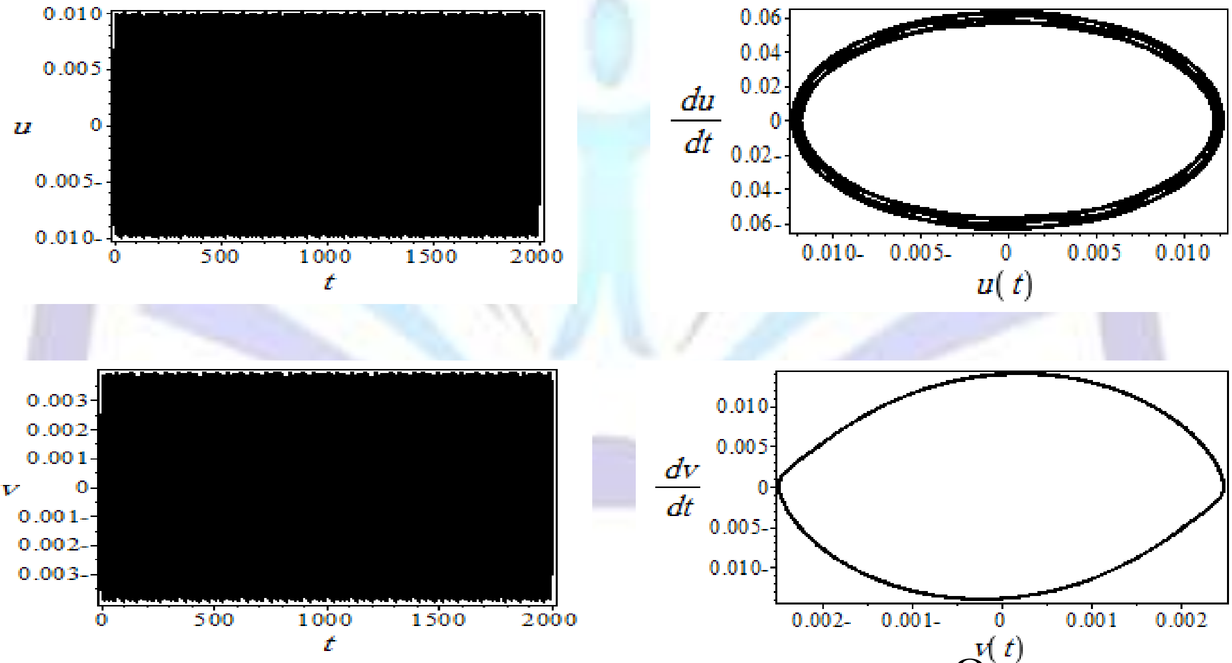
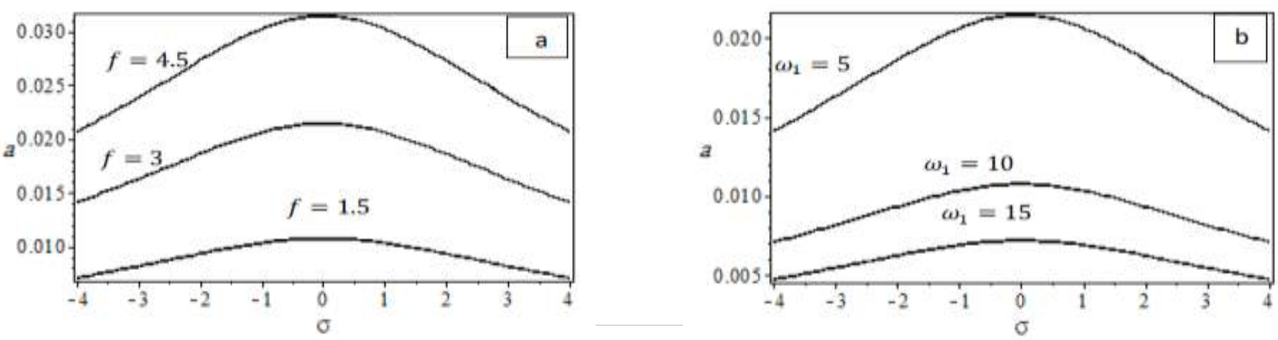


Fig. 3. System behavior with controller at the resonance case $\omega_1 = \Omega - \omega$



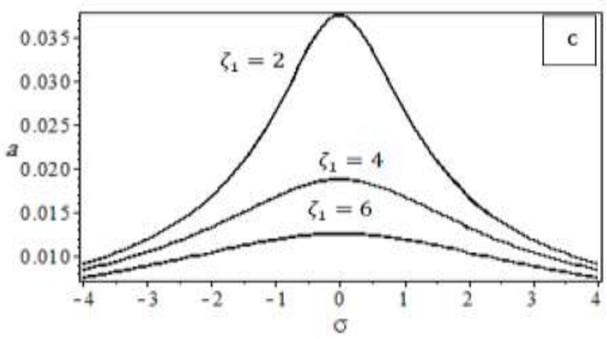


Fig. 4. Frequency response curves $\left(a \neq 0 and\;\; b = 0\right)$

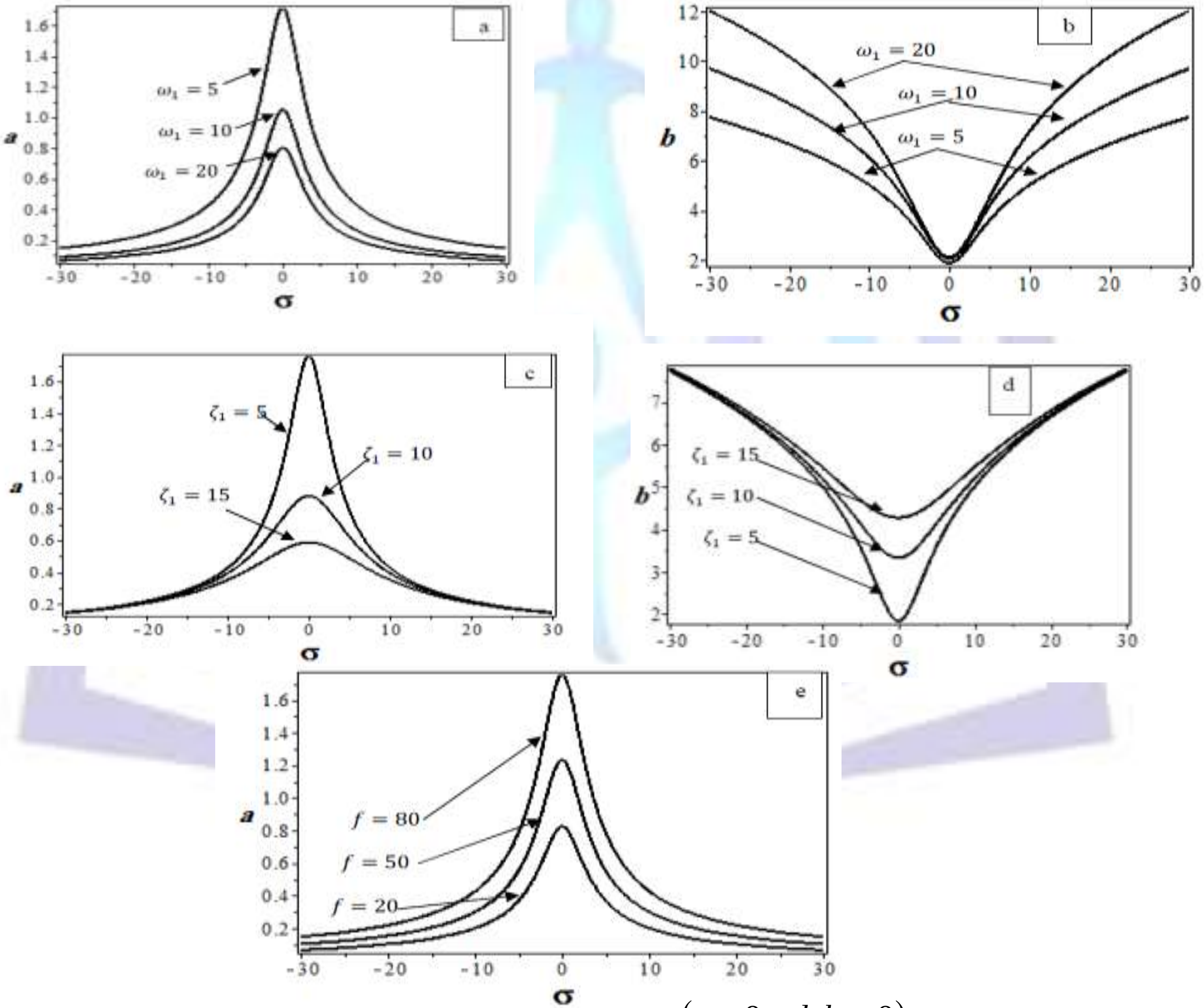


Fig. 5. Response curves of equation (38) $\left(a \neq 0 and\;\; b \neq 0\right)$.

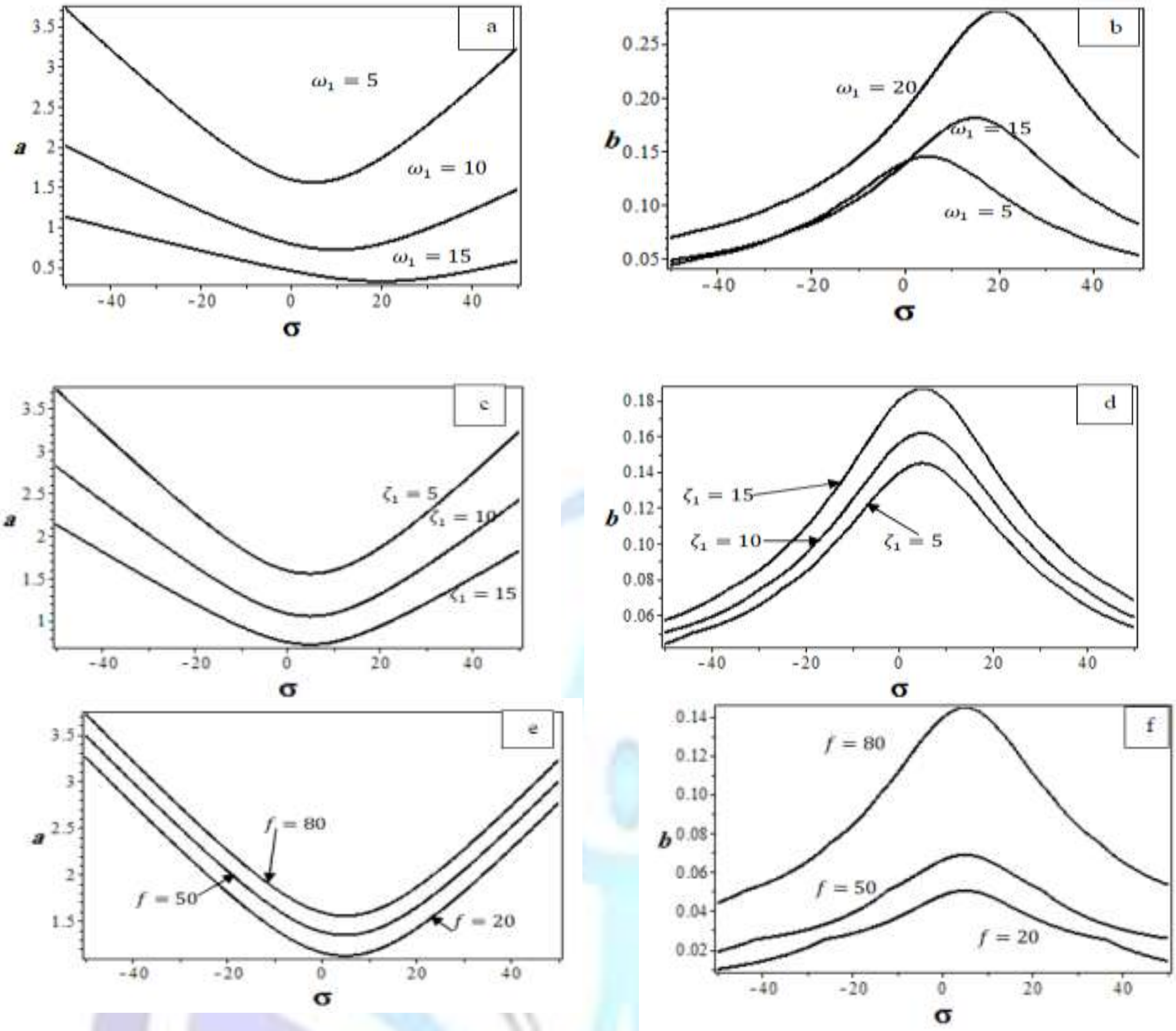


Fig. 6. Response curves of equation (39) $(a \neq 0 and\ b \neq 0)$.

5.CONCLUSION

The vibration of non- linear dynamical system subjected to tuned excitation force is studied, the worst resonance case is $(\omega_1 = \Omega - \omega)$ .Hence the stabilty of the system and absorber are studied using the frequency response functions from the above study, the following results are conclloded:

1- The steady state of the system without absorber is about 0.002 which consider as basic case.

2- The worest resonanance case is $\omega_1 = \Omega - \omega$ the steady state is increased to 30 times of the basic case.

3- The amplitude of the main system is monotonic decreasing function on natural frequency ω_1 and damping coefficient ζ_1.

4- The amplitude of the main system is monotonic increasing function on the excitation amplitude f .

5- The effectiveness of the controller is Ea is about 6.

Appendix:

Coefficients of equations (3) and (4):-

$$\epsilon = \frac{m_2}{m_1}, \zeta_1 = \frac{c_1}{m_2}, \zeta_2 = \frac{c_2}{m_2}, \omega_1^2 = \frac{k_1}{m_1}, \omega_2^2 = \frac{k_2}{m_2}, f = \frac{F}{\epsilon m_1}, \delta = \frac{1}{\epsilon}$$

Coefficients of equation (50):-

$$r_1 = \zeta_1 + (1+\delta)\zeta_2, r_2 = \frac{\zeta_1^2}{4} + \zeta_1\zeta_2(1+\delta) + \frac{\zeta_2^2}{4}(1+\delta)^2 + \sigma^2 + \left(\sigma - \frac{\delta}{2}\omega_1\right)^2$$

$$r_3 = \frac{\zeta_1^2\zeta_2}{4}(1+\delta) + \frac{\zeta_2^2\zeta_1}{4}(1+\delta)^2 + \zeta_1\left(\sigma - \frac{\delta}{2}\omega_1\right)^2 + \sigma^2\zeta_2(1+\delta) - \frac{\delta\sigma\zeta_2}{4}\omega_1$$

$$r_4 = \frac{\zeta_1^2\zeta_2^2}{16}(1+\delta) + \frac{\zeta_1^2}{4}\left(\sigma - \frac{\delta}{2}\omega_1\right)^2 + \sigma^2\zeta_2^2(1+\delta)^2 + \sigma^2\left(\sigma - \frac{\delta}{2}\omega_1\right)^2 -$$

$$\frac{\delta\sigma}{4}\zeta_2^2\,\omega_1(1+\delta) + \frac{\sigma}{4}\zeta_1\zeta_2\left(\sigma - \frac{\delta}{2}\omega_1\right)^2$$

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