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Optimal Pricing Policies For Deteriorating Items With Preservation Technology And Price Sensitive Demand

Nita H. Shah[1] & Arpan D. Shah[2]

[1,2]Department of Mathematics, Gujarat University, Ahmadabad – 380 009

Gujarat, India

[1]nitahshah@gmail.com, [2]arpan.math@gmail.com

ABSTRACT

This paper considers the problem of determining the price, cycle time and preservation technology cost strategies for deteriorating items. It is assumed that preservation technology investment and demand rate do follow the function of selling price. The objective is to maximize the total profit per unit time with determining the optimal selling price, length of replenishment cycle and preservation technology investment. We will be proving that the optimal cycle length and selling price are unique with respect to given preservation cost. Also, total profit per unit time will be a concave function as it will reach its optimum value for optimum value of selling price, cycle length and preservation technology cost. Numerical examples are also presented to demonstrate the solution process.

Keywords

Deteriorating items; Preservation technology investment; Price sensitive stock dependent demand

Academic Discipline And Sub-Disciplines

Mathematical modeling of Inventory

Mathematics Subject Classification

Mathematics,operation research

Type (Method/Approach)

Inventory modeling by using differential equations

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I. INTRODUCTION

In present scenario, deteriorating inventory problems are being studied by many researchers. As presented by Wee [10], deterioration was defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of commodities that result in decreasing usefulness. There are many examples like electronic goods, blood, food items , pharmaceutical, alcohol, volatile liquids which decrease during their storage period because of deterioration. As deterioration is playing a vital role, one can not neglect it during the research in this area. There are various models developed in this area of research. Shah and Jaiswal [31] developed an order level inventory model with a constant rate of deterioration. Goswami and Chaudhari [2] formulated an EOQ model with constant rate of deterioration and linearly changing demand rate over a fixed period. Chang and Dye [9] developed an inventory model with time-varying demand by considering constant deterioration rate. Teng et al. [11] extended this model for deteriorating items to consider the varying purchase cost and adjusting the opportunity cost. Balkhi [34] developed a model with assumption of production, demand and deteriorate rate as functions of time. Mandal and Pal [4] considered an inventory model for exponentially decaying items with shortages and time-varying demand. An extension of this model is provided by Wu and Ouyung [14] by considering two different replenishment policies with and without shortages. Then, amendment is provided by Deng et al. [23] for the models developed by Mandal and Pal. [4] and Wu and Oyung [14] by assuming demand rate as a ramp type function of time with constant on hand inventory deterioration rate. Later, Skouri et al. [15] extended the work of Deng et. al.[23] by considering demand rate as ramp type function, deterioration following a Weibull distribution and partial backlogging of unsatisfied demand. Hung [12] provided an extension of this model by considering the general type of demand rate and deterioration rate for partial backorder. Yao and Wang [18] studied an inventory control model with multiple deteriorating products considering the joint replenishment problem. Lot of inventory literature about deterioration items under various conditions is available such as Ouyung et al. [16], Alamri and Balkhi [1] , Chung and Huang [13] , Gupta et al. [20], Musa and Sani [3] and so on. A detailed review of deteriorating inventory literature is being given by Bakker et al. [17] since 2001.

In above mentioned papers, deterioration rate is considered as either constant or varying with time and viewed as uncontrollable. In recent times, corporations have started taking control measures to prevent and reduce deterioration by using better storage facilities, enhancing technology etc. This has taken attention of the researchers who have started taking this as an important factor for the development of the models. Lee [8] developed an investment model in preventive maintenance with an assumption of reduction in defective items. Uckan et al. [6] considered a supply chain to derive a model with the aim of maximizing profit with optimal investment. Hsu et al. [21] provided a deterioration inventory model with constant rate of demand and deterioration , and considered preservation technology investment. Hsieh and Dye [7] extended the model suggested by Hse et al. by considering time varying deterioration rate.

For success of many companies price is inevitable factor and must be taken into account as a part of inventory control policies. Abad [22] discussed pricing and lot-sizing problems for a product and assumed that demand could be backlogged and the selling price is constant throughout the cycle. Sana [27] considered an EOQ model for infinite time horizon with price dependent demand rate and partial backordering. Mai-Hami and Kamalabadi [25] developed an inventory system of joint pricing and inventory control policies where demand was a function of price, time and shortages allowed with partial backlogging. Shah et al. [19] solved a general inventory system with arbitrary holding cost rate and arbitrary deterioration rate with demand as a function of advertisement and selling price. So, various inventory models for deteriorating items have been developed by considering demand rate and selling price. Models are proposed by researchers like Mukhopadhyay et al. [26] , You [24] , Xu and Cai [30], Gupta et al. [20], Yang et al. [5] and so forth. Recent articles by researchers, Vinodkumar Mishra [29] , developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost and Vinodkumar Mishra et al. [28] provided an extension by considering partial backlogging. Yu-Ping Lee and Chung-yuan Dye [33] ,developed an inventory model for deteriorating items under stock dependent demand and controllable deterioration rate. Young Ha and Hongfu Huang [32] derived an optimizing inventory and pricing policies for seasonal deteriorating products with preservation technology investment. Hsu.ph et al. [21] provided a model for preservation technology investment for deteriorating inventory. ZhenyuBai et al. [35] suggested model showing optimal pricing policy for deteriorating items with preservation technology investment.

This paper proposes a model for deteriorating item in which demand is dependent on stock on display. Demand is considered as price sensitive and taken as a power function of selling price. Model does consider preservation technology investment to reduce the proportion of deteriorating items. This study doesn't allow any shortages during the cycle time. So, the objective of this study is to maximize the profit with respect to optimal selling price, optimal length of replenishment cycle and optimal preservation technology investment.

The rest of the paper is organized as follows. In section II, the notations and assumptions are presented. In section III, mathematical model to maximize the total profit per unit time is developed. In section IV, illustration of the model and the solution procedure are given. In section V numerical examples and conclusion is drawn in section VI.

II. Notations and Assumptions

- ***Notations:***

A	Replenishment cost per order
C	Purchase cost per unit

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P	Selling price per unit, $P > C$
h	Unit inventory holding cost per unit time
T	Cycle length
$I(t)$	Inventory level at time t
Q	Order quantity per cycle
$D(P)$	Demand rate
θ	Deterioration rate
u	Preservation technology investment per unit to reduce deterioration rate
f	Proportion reduced deterioration rate, $f(u) = 1 - e^{-mu}$, $0 \le f \le 1$
$\pi(P,T,u)$	Profit function

- ***Assumptions:***
  1. The inventory system involves a single deteriorating item over an infinite planning horizon.
  2. There is no repair or replacement during the period under consideration.
  3. Shortages are not allowed.
  4. The demand rate,
     $D(P) = \int_0^T (\alpha + \beta I(t))P^{-\eta}$, $\alpha>0, \beta>0, \eta>1$ is a non-negative power function of selling price.
  5. Existence of deterioration items assumed to be there with the initiation of production and the proportion reduced because of preservation technology are taken to be uniform.
  6. Replenishment occurs instantly.

III. Mathematical Modeling

We Consider a venture running a single item with a deterioration rate θ under conditions, price sensitive stock dependent demand with demand rate $D(P)$. After investing u in the preservation technology, venture is reducing deterioration rate θ. The proportion of reduced deterioration rate is denoted by $f(u)$. Inventory level decreases with respect to deterioration rate and demand in each cycle causing inventory level reaching to zero at the end of each replenishment cycle.

The differential equation representing inventory status during interval $[0, T]$ is given by,

$$\frac{dI(t)}{dt} = -\theta(1 - f(u))I(t) - D(P), \; 0 \le t \le T \tag{1}$$

With initial boundary condition $I(T) = 0$

Solving the differential equation (1), we get the inventory level as follows:

$$I(t) = \frac{\alpha P^{-\eta}}{\theta(1-f) + \beta P^{-\eta}}\left[e^{(\theta(1-f)+\beta P^{-\eta})(T-t)} - 1\right] \tag{2}$$

Therefore, the ordering quantity over the replenishment cycle, which is the initial inventory on hand, can be determined as

$$Q = I(0) = \frac{\alpha P^{-\eta}}{\theta(1-f) + \beta P^{-\eta}}\left[e^{(\theta(1-f)+\beta P^{-\eta})T} - 1\right] \tag{3}$$

Also, preservation technology investment u during cycle time T can be given by uT. For convenience, $f(u)$ will be denoted by f .

- The replenishment cost is A
- The preservation technology investment is uT

- The inventory holding cost is given by,

$$HC = \int_0^T hI(t)dt \tag{4}$$

- The item's purchase cost is given by,

$$CQ = \frac{C\alpha P^{-\eta}}{\theta(1-f)+\beta P^{-\eta}}\left[e^{(\theta(1-f)+\beta P^{-\eta})T}-1\right] \tag{5}$$

- The sales revenue is $PD(I(t))$.

  By using all expressions from (1) to (5), profit function is given by,

$$\pi(P,T,u) = \frac{1}{T}\left\{\begin{array}{l}\text{Sales revenue} - \text{Purchase cost} - \text{Ordering cost} - \text{Preservation cost} \\ -\text{Holding cost}\end{array}\right.$$

$$= \frac{1}{T}\left\{\begin{array}{l}\dfrac{\alpha P^{-\eta}}{\theta(1-f)+\beta P^{-\eta}}\left(\begin{array}{l}PT(\theta(1-f)+\beta P^{-\eta}-1) \\ -C\left(e^{(\theta(1-f)+\beta P^{-\eta})T}-1\right) \\ +hT\end{array}\right)+\dfrac{\alpha P^{-\eta}}{(\theta(1-f)+\beta P^{-\eta})^2}\left[\begin{array}{l}\left(e^{(\theta(1-f)+\beta P^{-\eta})T}-1\right) \\ \left(\beta P^{-\eta}-h\right)\end{array}\right] \\ -uT-A\end{array}\right\} \tag{6}$$

IV. Solution Methodology

The objective of this study is to maximize the profit with respect to optimal selling price, optimal length of replenishment cycle and optimal preservation technology investment. To obtain the optimal value of (P,T,u), we will consider $\frac{\partial \pi}{\partial P}=0, \frac{\partial \pi}{\partial T}=0, \frac{\partial \pi}{\partial u}=0$ as an optimum condition. We will be solving generated equations by using mathematical software.

V. Numerical Examples

To illustrate the solution procedure, in this section we give the following examples:

Example: In this example we consider the reduced deterioration rate $f(u)=1-e^{-mu}, m>0$ and other parametric values are as follows: $\theta=0.3,\ \alpha=10000,\ \beta=0.2,\ h=1,\ m=0.05,\ C=10, A=100$ and $\eta=1.25$.Then by using this values we have $T^*=1.482,\ P^*=54.95$ and $u^*=41.75$. Also, $\pi(P^*,T^*,u^*)=2829.03$ with $Q^*=101.9$.

By using the software and model derived, we study the effects of changing the values of the system parameters $\pi^*, P^*, T^*, u^*, Q^*$.

Results are shown by using appropriate tabular and graphical representation.

[See table I] [See table II][See fig. I] [See table III] [See fig. II][See table IV]

Example: Deterioration versus preservation technology investment:

[See fig.III]

Above figure shows relation between preservation technology investment and deterioration rate. It suggests high investment with respect to high deterioration rate.

Example: Profit function versus preservation technology investment:

[See fig. IV]

Above figure shows, after certain value of preservation technology investment, increase in investment declines in profit function.

Example: Behavior of profit function with respect to two variables by taking optimum value of third variable

1. Optimum "u" (Preservation technology investment)
   [See fig.V]
2. Optimum "P"(Selling price)
   [See fig.VI]
3. Optimum "T"(Replenishment cycle time)

[See fig. VII]

The above figures show concavity of profit function with respect to two variables by taking the optimum value of the third variable.

VI. Conclusion

The main objective of this paper to maximize the profit with respect to optimum selling price, replenishment cycle time and preservation technology investment for deterioration items. Optimum values of P, T, u are derived by using mathematical software. Change in different parameters suggests change in values in profit function, replenishment cycle time and preservation technology investment. Graphs and results shows, profit function is positively affected by scale demand (α) and negatively affected by price elasticity (η)and purchase cost (C). Also, Profit function versus preservation technology investment gives a concave function which suggests after certain limit of preservation technology investment, increase in preservation technology investment gives decrease in profit value. Hence, existence of optimum value is obtained. From the graph preservation technology versus deterioration, we conclude that if the value of deterioration is too small then preservation technology investment is not needed. Hence, preservation technology investment should be made only if items are highly deteriorating in nature.

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Table I : Effect of changing all parameters

Table 1		u	P	T	π	Q
	-20	37.3	54.95	1.48	2833.5	101.9
	-10	37.6	54.95	1.48	2831.1	101.9
θ	0	41.8	54.95	1.48	2829	101.9
	10	43.8	54.95	1.48	2827	101.9
	20	45.4	54.95	1.48	2825.4	101.9
	-20	39.2	55.69	1.63	2242.3	88.9
	-10	40.5	55.29	1.55	2535.4	95.6
α	0	41.8	54.95	1.48	2829	101.9
	10	42.9	54.67	1.42	3123	108
	20	43.9	54.43	1.37	3417.4	113.8
	-20	41.6	54.98	1.48	2828.4	101.3
	-10	41.7	54.97	1.48	2828.7	101.6
β	0	41.8	54.95	1.48	2829	101.9
	10	41.8	54.94	1.49	2829.3	102.2
	20	41.9	54.92	1.49	2829.6	102.5
	-20	45.6	55.2	1.42	2819.3	97.8
	-10	43.6	55	1.45	2824.5	100.1
m	0	41.8	54.95	1.48	2829	101.9
	10	39.9	54.9	1.51	2832.9	103.5
	20	38.3	54.8	1.53	2836.3	104.8
	-20	40.6	44.22	1.33	2986.2	119.4

	-10	41.2	49.59	1.41	2902.3	109.8
C	0	41.8	54.95	1.48	2829	101.9
	10	42.3	60.3	1.55	2764.1	95.2
	20	42.7	65.65	1.62	2706	89.3
	-20	39.1	54.5	1.29	2843.4	89.9
	-10	40.5	54.7	1.39	2835.9	96.1
A	0	41.8	54.9	1.48	2829	101.9
	10	42.9	55.2	1.57	2822.5	107.5
	20	43.9	55.4	1.65	2816.3	112.7
	-20					
	-10	42.3	59.7	1.41	4898	94.3
η	0	41.8	54.95	1.48	2829	101.9
	10	40.7	40.5	1.52	1708	96.4
	20	38.7	33.5	1.62	1051	86.6
	-20	43.7	54.5	1.63	2839.6	113.1
	-10	42.7	54.7	1.55	2834.2	107.1
h	0	41.7	54.95	1.48	2829	101.9
	10	40.9	55.2	1.42	2824	97.4
	20	40.1	55.4	1.37	2819	93.5

Table II: Variation in profit with respect to change in all parameters

variation in profit								
% change	θ	α	β	m	C	A	η	h
-20	2833.5	2242.3	2828.4	2819.3	2986.2	2843.4	-	2839.6
-10	2831.1	2535.4	2828.7	2824.5	2902.3	2835.9	4880.4	2834.2
0	2829	2829	2829	2829	2829	2829	2829	2829
10	2827.1	3123	2829.3	2832.9	2764	2822.5	1708	2824
20	2825.4	3417.4	2829.6	2836.3	2706	2816.3	1051	2819.4

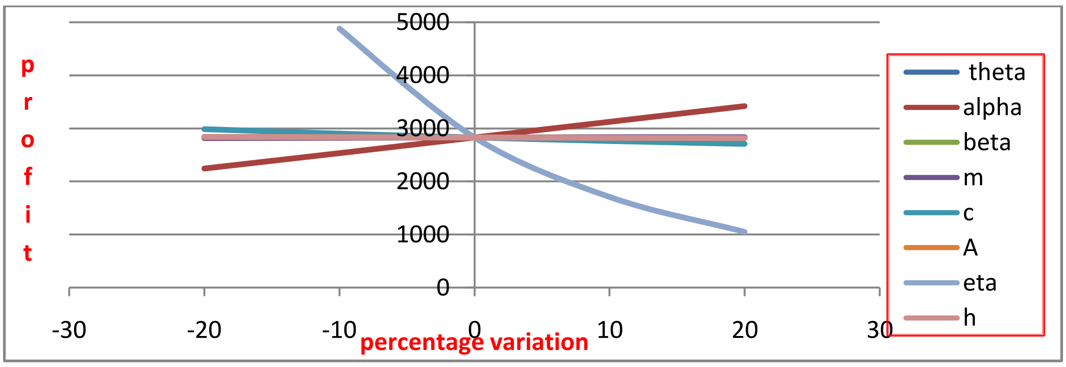

Fig. I: change in profit with respect to all parameters

Table III: Change in quantity with respect to all parameters

variation in quantity								
% change	θ	α	β	m	C	A	η	h
-20	101.92	88.9	101.3	97.8	119.4	89.9	-	113.1
-10	101.92	95.6	101.6	100.1	109.8	96.1	111	107.1
0	101.92	101.92	101.92	101.9	101.9	101.92	101.92	101.92
10	101.92	107.96	102.21	103.5	95.15	107.5	96.36	97.4
20	101.92	113.76	102.5	104.8	89.3	112.7	86.6	93.5

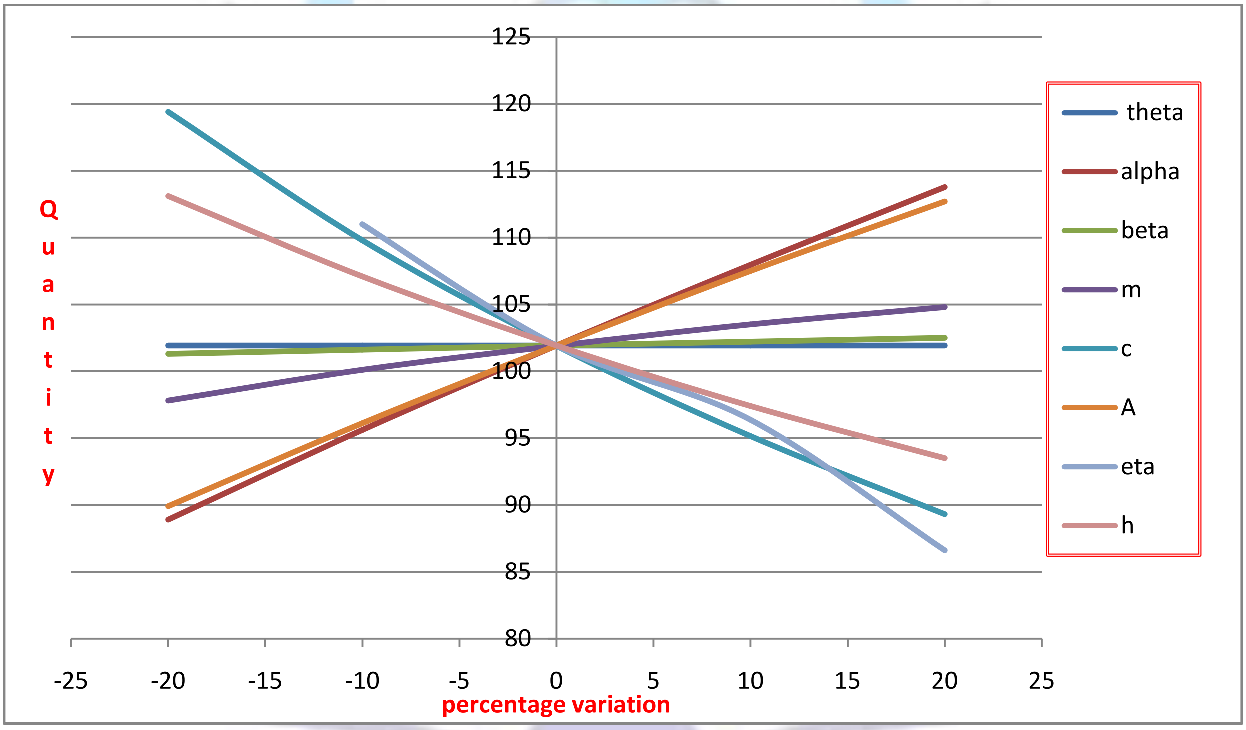

Fig. II: Change in quantity with respect to change in all parameters

Table-IV: Effectiveness of different parameters on Profit or Quantity

Decision variable	Sensitively positive	Sensitively negative	Ineffective	Negligible
π	α	η, C	NA	θ, β, m, A, h
Q	α, A	h, η, C	θ	β, m

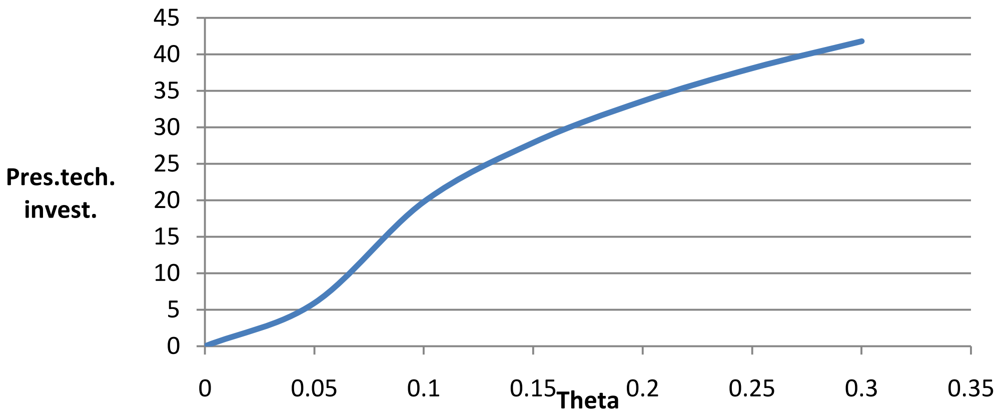

Fig. III: Change in preservation technology investment with respect to deterioration rate

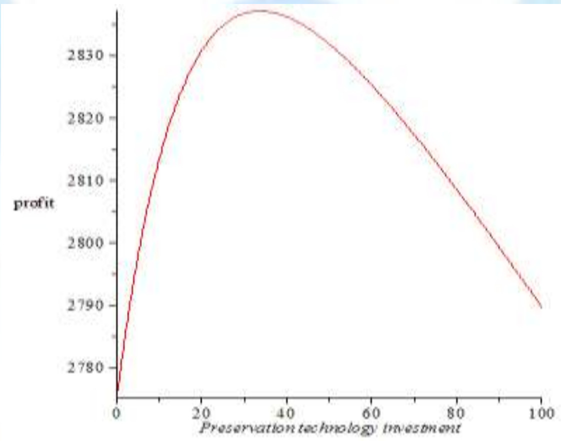

Fig. IV: Profit function with respect to preservation technology investment

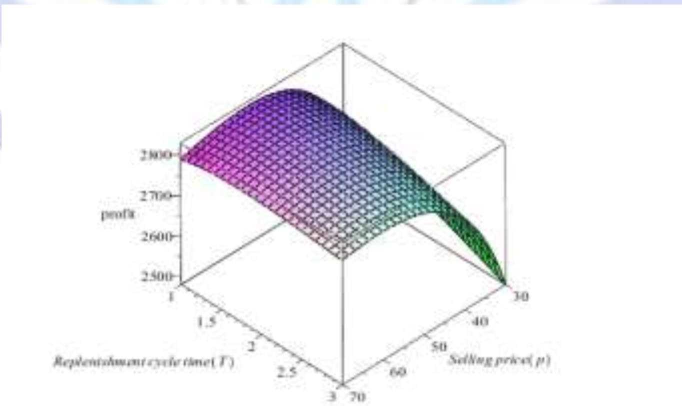

Fig. V: Concavity of profit function with respect to T and P

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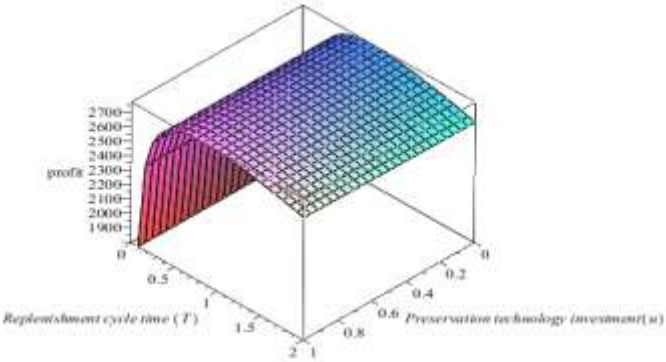

Fig. VI: Concavity of profit function with respect to T and u

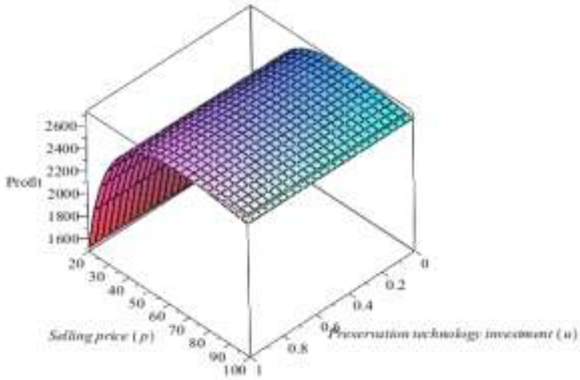

Fig. VII: Concavity of profit function with respect to u and P

Author' biography

Nita H.Shah is a professor in the Department of Mathematics at Gujarat University, Ahmedabad, Gujarat ,India. She received her phD in Inventory Control Management, Operation Research. Currently, she is engaged in research areas of inventory control and management, supply chain management, forecasting and information technology and information systems, neural networks, sensors and image processing. She has more than 275 papers published in international and national journals. She is the author of four books. She is serving as a member of the editorial board of Investigation Operational , Journal of Social Science and Mangement International Journal of Industrial Engineering and Computations and Mathematics Today.

Arpan D. Shah is a research student in the Department of Mathematics , Gujarat University, Ahmedabad, Gujarat, India. His research interest is Mathematical Modeling in Inventory and supply chain management. He has published articles in Mexican Journal of Operation Research, Opsearch,International Journal of Inventory Control and Management, International Journal of Operation Research,International Journal of Engineering and Applied Sciences and Journal of Sci-Tech Research.