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Quasi-Steady State Solutions of MHD Oscillatory Flow between two Parallel Plates in a Rotating System

Dr.Ch. Baby Rani
Associate professor, Department of Mathematics
V. R. Siddhartha Engineering College
Vijayawada, Andhra Pradesh, India. Pin: 520 007.
Email: ranicbr1@yahoo.com

ABSTRACT

The physical situation considered is that of quasi-state hydro magnetic flow of a viscous, incompressible and electrically conducting fluid between two parallel plates distance ‘2L’ apart, when the lower plate is set in sinusoidal motion and the upper stationary .Neglecting the Magnetic Prandtl Number, Solution for Quasi-steady state when the lower plate moves with $U_0 \cos(\omega t)$ and the corresponding Skin –friction at the lower plate have been obtained. Discussion has been made of these two features for the two cases when the magnetic lines of force have been fixed relative to the fluid and the moving plate respectively.

Keywords: Quasi-state; Porous plates; Velocity Distribution; Skin friction.

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1. INTRODUCTION

Ramana Rao and Vidyanidhi (1970) studied the unsteady hydromagnetic flow between two parallel porous plates of which the lower is stationary and the upper moved with uniform velocity A t^n (n = 0, 1) for the two cases when (i) the magnetic lines of force are fixed relative to the fluid and (ii) the magnetic lines of force are fixed relative to the upper plate. They however assumed the magnetic Prandtl number $P_m \underline{\Omega}\ 0$ i.e, neglecting the induced magnetic field. Singh, Sacheti and Chandran (1994) brought out the transient effects of couette flow of an electrically conducting fluid subject to rotation and magnetic field, when one of the plates has been set into uniformly accelerated motion for the above two cases.

Sutton and Sherman (1965) studied the hydromagnetic transient couette flow when magnetic Prandtl number P_m is unity. They obtained exact solutions for the velocity and the induced magnetic field assuming the upper plate is in uniform motion and the lower plate is stationary. This work holds good when the magnetic lines of force are fixed relative to the fluid. Ramesh (1996) presented the work of Sutton and Sherman assuming that the magnetic lines of force are fixed relative to the upper plate.

Here we consider the hydromagnetic flow of a viscous, incompressible and electrically conducting fluid between two parallel plates, distant 2L apart, when the lower plate is set in sinusoidal motion and upper stationary .Neglecting the magnetic Prandtl number solution for quasi steady state when the lower plate moves with $U_0 \cos \omega t$ and the corresponding skin-friction at the lower plate have been obtained. Discussion has been made of these two features for the two cases when the Magnetic lines of force have been fixed relative to the fluid and the moving plate respectively.

2. MATHEMATICAL FORMULATION

The physical situation considered is that of quasi-state hydromagnetic flow of a viscous, incompressible and electrically conducting fluid bounded by two infinite parallel plates, distant 2L apart, when Z^1-axis is taken normal to the plates. It is assumed that the plates are electrically non conducting and an applied uniform magnetic field$H_0$is acting parallel to the Z^1-axis. When $t^1 > 0$, the lower plate $Z^1 = -L$ moves sinusoidallyi.e, $U_0 \cos(\omega^1 t^1)$ or$U_0 \sin(\omega^1 t^1)$. The upper plate $Z^1 = L$ is stationary for $t^1 > 0$. The governing equations of continuity and motion and Maxwell's equations are:

$$\nabla.\vec{V} = 0 \tag{1.1}$$

$$\frac{\partial \vec{V}}{\partial t^1} + (\vec{V}.\nabla)\vec{V} = (-\frac{1}{\rho})\nabla p + \nu \nabla^2 \vec{V} + \frac{\vec{J} \times \vec{B}}{\rho} \tag{1.2}$$

$$\nabla \times \vec{B} = \mu_0 J \tag{1.3}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t^1} \tag{1.4}$$

$$\nabla \cdot \vec{B} = 0 \tag{1.5}$$

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}) \tag{1.6}$$

Where $\vec{V} = (u^1, 0, 0)$ the velocity vector, p is the pressure, ρ is the density, ν is the kinematic viscosity, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic induction vector, $\vec{E}$ is the electric field vector, μ_0 is the magnetic permeability and σ is the electric conductivity. We assume the magnetic Reynolds number is so small, in others, P_m the magnetic Prandtl number is almost zero being the ratio of magnetic Reynolds number and the Reynolds number, that the induced magnetic field can be neglected in comparison with applied one (1962), so that

$$\vec{B} = (0, 0, B_0) \tag{1.7}$$

where B_0 is a constant. It is also assumed that no applied and polarization voltage exists (i. e, $\vec{E} = \vec{0}$).This then corresponds to the case when no energy is added or extracted from the fluid by the electric field. Science the plates are infinite in extent, all physical variables (except pressure) are functions of Z^1 and t^1 . Equation (1.1) is automatically satisfied. Now, the equation for the conservation of electric charge, $\nabla \cdot \vec{J} = 0$ leads to $J_z^1 =$ constant, where $J = (J_X^1, J_Y^1, J_Z^1)$. As in the case of vertical velocity, we immediately see that $J_Z^1 = 0$

Equation (1.6) thus yields $J_X^1 = 0$, $J_Y^1 = -\sigma\beta_0(\mathrm{u}^1 - U_0 e^{i\omega^1 t^1})$ (1.8)

Noting that when the magnetic lines of force are fixed relative to the lower plate, moving sinusoidally equation (1.6) is replaced by

$$\vec{J} = \sigma(\vec{E} + (\vec{V} - u^1 - U_0 e^{i\omega^1 t^1}\hat{i}) \times \vec{B} \qquad (1.9)$$

where $\hat{i}$ is the unit vector in X^1 - direction

In view of the above considerations equation (1.2) can be written in the component form as in (1986).

$$\frac{\partial u^1}{\partial t^1} = \nu \frac{\partial^2 u^1}{\partial z^{1^2}} - m(u^1 - KU_0 e^{i\omega^1 t^1}) \qquad (1.10)$$

Where $m = \frac{\sigma B_0^2}{\rho}$

$$K = \begin{cases} 0 & if\ B_0\ is\ fixed\ relative\ to\ the\ fluid \\ 1 & if\ B_0\ is\ fixed\ relative\ to\ the\ sinusoidally\ moving\ lower\ plate \end{cases}$$

Since we seek the quasi-state solution for u^1 the boundary conditions are

$u^1 = U_0 e^{i\omega^1 t^1}$ at $Z^1 = -L$ and $u^1 = 0$ at $Z^1 = L$ for $t^1 > 0$ (1.11)

The real and imaginary parts of u^1 , when obtained corresponds to the motion of the lower plate either with $U_0 \cos\omega^1 t^1$ or $U_0 \sin\omega^1 t^1$ respectively.

3. METHOD OF SOLUTION

Substituting: $u^1 = U_0 e^{i\omega^1 t^1} f(Z^1)$ (1.12)

In equation (1.10) we get

$$f^{11} - \left\{\frac{m + i\omega^1}{\nu}\right\} f = -\frac{mk}{\nu} \qquad (1.13)$$

which is to be solved subject to the conditions of equation (1.11) namely

$$f(-L) = 1 \ , \ f(L) = 0 \qquad (1.14)$$

And dash denotes the differentiation with respect to Z^1

In terms of the non-dimensional quantities

$$V = \frac{u^1}{U_0}, \ \alpha = \sqrt{\frac{m}{2\nu}}L, \ \sigma = \sqrt{\frac{\omega}{2\nu}}L, \ Z = \frac{z^1}{L}, \ T = \omega^1 t^1 \qquad (1.15)$$

$$P = (\sqrt{\alpha^4 + \sigma^4} + \alpha^2)^{\frac{1}{2}} , \ Q = (\sqrt{\alpha^4 + \sigma^4} - \alpha^2)^{\frac{1}{2}} \qquad (1.16)$$

We get

$$f(Z)=\left[\frac{1}{Ch^2P\cos^2Q+Sh^2P\sin^2Q}\right]$$

$$\left[\begin{array}{l}(\frac{1}{2}-\frac{K\alpha^4}{\alpha^4+\sigma^4})(\mathrm{Ch}\,\overline{PZ}\cos\overline{QZ}ChP\cos Q+Sh\overline{PZ}\sin\overline{QZ}ShP\sin Q)-\\ \frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}(\mathrm{Sh}\,\overline{PZ}\sin\overline{QZ}ChP\cos Q-Ch\overline{PZ}\cos\overline{QZ}ShP\sin Q)+\\ i\{\frac{1}{2}-\frac{K\alpha^4}{\alpha^4+\sigma^4}(\mathrm{Sh}\,\overline{PZ}\sin\overline{QZ}chPQ\cos Q-Ch\overline{P}\cos\overline{QZ}ShP\sin Q)+\\ \frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}(Ch\overline{PZ}\cos\overline{QZ}ChP\cos Q+Sh\overline{PZ}\sin\overline{QZ}ShP\sin Q)\}\end{array}\right]-$$

$$\left[\frac{1}{2(Sh^2P\cos^2Q+Ch^2P\sin^2Q}\right]$$

$$\left[\begin{array}{l}Sh\overline{PZ}\cos\overline{QZ}ShP\cos Q+Ch\overline{PZ}\sin\overline{QZ}ChP\sin Q+\\ i\left\{Ch\overline{PZ}\sin\overline{QZ}ShP\cos Q-Sh\overline{PZ}cos\overline{QZ}ChP\sin Q\right\}\end{array}\right]+ \quad (1.17)$$

$$\frac{K\alpha^4}{\alpha^4+\sigma^4}-i\frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}$$

The boundary condition $u^1=U_0\cos\omega^1t^1$ of equation (1.11) gives the real part of $u^1=U_0e^{i\omega^1t^1}f(Z^1)$ i.e,

$$V=\left[\frac{1}{Ch^2P\cos^2Q+Sh^2P\sin^2Q}\right]$$

$$\left[\begin{array}{l}\{(\frac{1}{2}-\frac{K\alpha^4}{\alpha^4+\sigma^4})(\mathrm{Ch}\,\overline{ZP}\cos\overline{ZQ}ChP\cos Q+Sh\overline{ZP}\sin\overline{ZQ}ShP\sin Q)-\\ \frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}(\mathrm{Sh}\,\overline{ZP}\sin\overline{ZQ}ChP\cos Q-Ch\overline{ZP}\cos\overline{ZQ}ShP\sin Q)\}\cos\mathrm{T}-\\ \{\frac{1}{2}-\frac{K\alpha^4}{\alpha^4+\sigma^4}(\mathrm{Sh}\,\overline{ZP}\sin\overline{ZQ}chP\cos Q-Ch\overline{ZP}\cos\overline{ZQ}ShP\sin Q)+\\ \frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}(Ch\overline{ZP}\cos\overline{ZQ}ChP\cos Q+Sh\overline{ZP}\sin\overline{ZQ}ShP\sin Q)\}\sin\mathrm{T}\end{array}\right]-$$

$$\left[\frac{1}{2(Sh^2P\cos^2Q+Ch^2P\sin^2Q}\right]$$

$$\left[\begin{array}{l}(Sh\overline{ZP}\cos\overline{ZQ}ShP\cos Q+Ch\overline{ZP}\sin\overline{ZQ}ChP\sin Q)\cos T-\\ \left\{Ch\overline{ZP}\sin\overline{ZQ}ShP\cos Q-Sh\overline{ZP}cos\overline{ZQ}ChP\sin Q\right\}\sin T\end{array}\right]+$$

$$\frac{K\alpha^4}{\alpha^4+\sigma^4}\cos T+\frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}\sin T \quad (1.18)$$

Where in equations (1.17-1.18) Ch and Sh stands for cos hyperbolic and sine hyperbolic respectively.

The corresponding skin-friction τ at the lower plate $\left[\frac{-\partial V}{\partial Z}\right]_{z=-1}$ is given by

$$\tau=\left[\frac{1}{2(Ch^2P\cos^2Q+Sh^2P\sin^2Q)}\right]$$
$$\left[\begin{array}{l}\{(\frac{1}{2}-\frac{K\alpha^4}{\alpha^4+\sigma^4})(-\mathrm{PSh}\,\overline{2P}+Q\sin\overline{2Q})+\frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}(P\sin\overline{2Q}+QSh\overline{2P})\}\cos\mathrm{T}+\\ \{(\frac{1}{2}-\frac{K\alpha^4}{\alpha^4+\sigma^4})(P\sin\overline{2Q}+QSh\overline{2P})-\frac{K\alpha^2\sigma^2}{\alpha^4+\sigma^4}(-Psh\overline{2P}+Q\sin\overline{2Q})\}\sin T\end{array}\right]- \qquad (1.19)$$
$$\left[\frac{1}{4(Sh^2P\cos^2Q+Ch^2P\sin^2Q)}\right]$$
$$\left[(PSh\overline{2P}+Q\sin\overline{2Q})\cos\mathrm{T}-(-\mathrm{P}\sin\overline{2Q}+QSh\overline{2P})\sin T\right]$$

The boundary condition $U^1=U_0\cos\omega^1t^1$ of equation (1.11) gives the imaginary part of $U^1=U_0e^{i\omega^1t^1}f(Z^1)$ and its corresponding skin- friction at the lower plate can be determined but these two expressions are not obtained here.

We have computed the expressions for the velocity and skin-friction as given by equations (1.18) and (1.19). The velocity distribution has been entered in Table I-VI and skin-friction in table VII.

Figures 1 and 2 show tables I, III and IV respectively.

4. DISCUSSION

In the non-magnetic case i.e., α = 0, shown in Fig .1the effect of increasing T(w^1t^1) is to reduce the velocity at any point of the channel independent of K(K does not rise) for fixed σ, the penetration depth. In the magnetic case shown in Fig.2, the effect of increasing T also reduces the velocity at any point of the channel for fixed σ in both the cases when the magnetic lines of force are fixed relative to the fluid K = 0 shown by ______and the lower plate K = 1 shown by-------. The effect of increasing K as shown in Fig.2 is to increase the velocity at any point of the channel, its width taken as unity. From Table II, in non-magnetic case, for fixed T the effect of increasing σ is to increase the velocity but very feebly. This also seen in the magnetic case from tables V and VI, corresponding to K=0 and 1 respectively. From these tables, as K increases the velocity increases. From Table VII ,it observed that the skin-friction at the lower plate (i) when α = 0, i.e, in the non-magnetic case is the same independent of K(K does not arise) , (ii) in magnetic case decreases as K increases , (iii) decreases as both T and σ increase in both non-magnetic and magnetic cases, (iv) increases as α increases when K = 0 and decreases when K = 1.

TABLE-I

Z \ T	0.5	1.0	5.0
-1.0	.87758260	.54030230	.28366220
-0.8	.79399850	.49382050	.24649410
-0.6	.70905900	.44493130	.21209510
-0.4	.62293530	.39391520	.18016350
-0.2	.53579140	.34104820	.15039520
0	.44778500	.28660250	.12248450
0.2	.35906820	.23084650	.09612405
0.4	.26978870	.17404580	.07100530
0.6	.18009040	.11646370	.04681866
0.8	.09011438	.05836169	.02325379
1.0	0	0	0

Velocity distribution when α = 0, σ = 0.2, K = 0 and 1

TABLE-II

Z \ σ	0.2	0.4	0.6
-1.0	.87758260	.87758260	.87758260
-0.8	.79399850	.80403510	.81233530
-0.6	.70905900	.72538010	.73679440
-0.4	.62293530	.64252720	.65386150
-0.2	.53579140	.55626710	.56582870
0	.44778500	.46728540	.47444590
0.2	.35906820	.37617670	.38099470
0.4	.26978870	.28345900	.28636310
0.6	.18009040	.18958880	.19112330
0.8	.09011438	.09497619	.09561023
1.0	0	0	0

Velocity distribution when α = 0, T= 0.5, K = 0 and 1

TABLE-III

Z \ T	0.5	1.0	5.0
-1.0	.87758260	.54030230	.28366220
-0.8	.66153110	.40951220	.20928790
-0.6	.49759850	.30965790	.15410920
-0.4	.37288020	.23321390	.11309990
-0.2	.27755530	.17441380	.08251508
0	.20411540	.12881780	.05955396
0.2	.14677330	.09298015	.04210772
0.4	.10100920	.06418966	.02856892
0.6	.06321429	.04026641	.01768596
0.8	.03040743	.01939742	.00844937
1.0	0	0	0

Velocity distribution when α = 1, σ = 0.2, K = 0

TABLE-IV

Z \ T	0.5	1.0	5.0
-1.0	.87758260	.54030230	.28366220
-0.8	.85117060	.52804790	.26695550
-0.6	.82116040	.51220150	.25189280
-0.4	.78520610	.49159570	.23715110
-0.2	.74048600	.46469190	.22141350
0	.68348220	.42946620	.20325330
0.2	.60970400	.38325820	.18100620
0.4	.51333510	.32257140	.15262010
0.6	.38677640	.24281010	.11546960
0.8	.22004700	.13793330	.06611700
1.0	0	0	0

Velocity distribution when α = 1, σ = 0.2, K = 1

TABLE-V

Z \ σ	0.2	0.4	0.6
-1.0	.87758260	.87758260	.87758260
-0.8	.66153110	.66679070	.67387630
-0.6	.49759850	.50520470	.51485320
-0.4	.37288020	.38107450	.39082780
-0.2	.27755530	.28532100	.29397460
0	.20411540	.21090040	.21797770
0.2	.14677330	.15230640	.15772380
0.4	.10100920	.10517840	.10903610
0.6	.06321429	.06598935	.06844366
0.8	.03040743	.03179109	.03297934
1.0	0	0	0

Velocity distribution when α = 1, T = 0.K = 0

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TABLE-VI

Z \ σ	0.2	0.4	0.6
-1.0	.87758260	.87758260	.87758260
-0.8	.85117060	.86034430	.87178670
-0.6	.82116040	.83608260	.85381200
-0.4	.78520610	.80329900	.82390430
-0.2	.74048600	.75975080	.78094540
0	.68348220	.70230730	.72249130
0.2	.60970400	.62673620	.64469460
0.4	.51333510	.52740290	.54211260
0.6	.38677640	.39686730	.40740250
0.8	.22004700	.22534470	.23088980
1.0	0	0	0

Velocity distribution when α = 1, T = 0.5, K = 1

TABLE-VII

α	σ	T	K	τ
0	0.2	0.5	0	0.41423600
			1	0.41423600
		1	0	0.22591600
			1	0.22591600
0.4	0.5	0	0	0.35329120
			1	0.35329120
1	0.2	0.5	0	1.23688100
			1	0.12656620
		1	0	0.74656890
	0.4		1	0.05386135
		0.5	0	1.20067900
			1	0.07028925

Values of skin-friction τ at the lower plate

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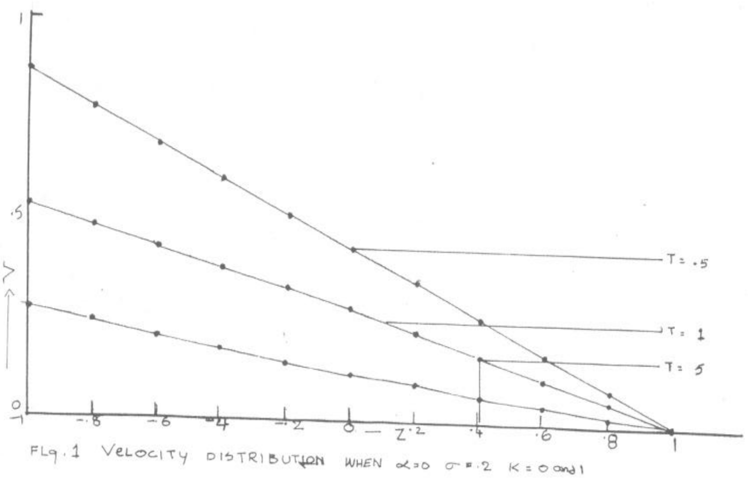

FIg. 1 VELOCITY DISTRIBUTION WHEN $\alpha = 0$ $\sigma = .2$ K = 0 and 1

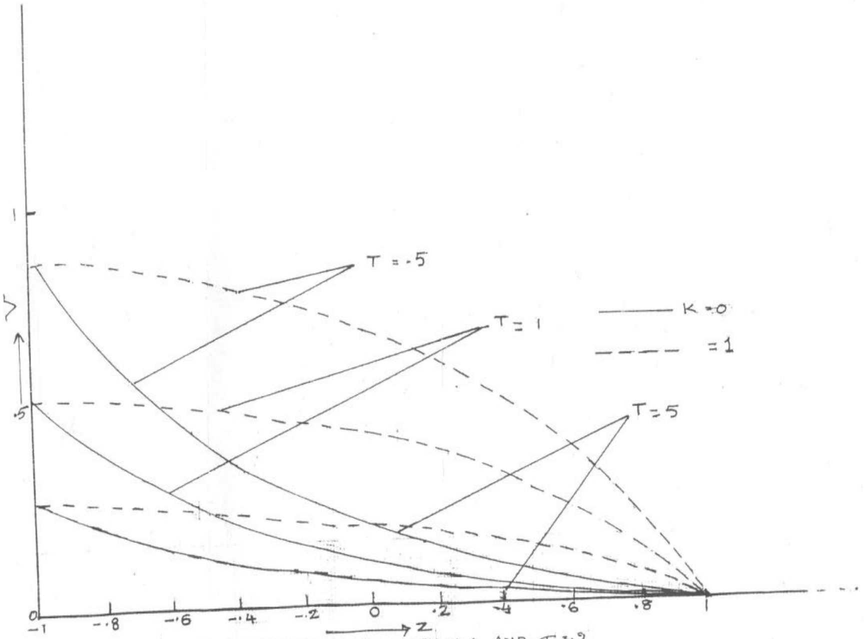

FIg. 2 VELOCITY DISTRIBUTION WHEN $\alpha = 1$ AND $\sigma = .2$

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