ISSN 2347-1921

Solving Some Kinds of Third Order Partial Differential Equations of Degree Four with Three Independent Variables

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ABSTRACT

In this paper, we find a solution for some kinds of nonlinear third order partial differential equations of homogeneous degree with three independent variables of the form

$$A Z_{xxx}^2 + B Z_{yyy}^2 + C Z_{ttt}^2 + D Z_{xxy}^2 + E Z_{xxt}^2 + F Z_{yyt}^2 + G Z_{xtt}^2 + H Z_{ytt}^2 + I Z_{xyt}^2 + J Z_{xyy}^2 +$$
$$K Z_{xx}^2 + L Z_{yy}^2 + M Z_{tt}^2 + N Z_{xy}^2 + O Z_{xt}^2 + P Z_{yt}^2 + Q Z_x^2 + R Z_y^2 + S Z_t^2 + T Z^2 = 0,$$

where *A, B, C, D, E, …* ,S and *T* are linear functions of dependent variable *Z* and partial derivatives of dependent variable with respect to the independent variables *x* , y and *t*, and that by using some of the assumptions .

Indexing terms/Keywords

Nonlinear Differential Equations; Partial Differential Equations; Third Order, Homogeneous Degree.

Academic Discipline And Sub-Disciplines

Applied Mathematics, Partial Differential Equations

SUBJECT CLASSIFICATION

Mathematics Subject Classification: 58J32; 49J52; 49L25; 35D05; 35J70

Council for Innovative Research

Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN MATHEMATICS

Vol. 9, No. 5

www.cirjam.com, editorjam@gmail.com

ISSN 2347-1921

INTRODUCTION

Since the world is full of nonlinear phenomenon, there has been much interest in recent years in studying the complete solution for certain types of nonlinear partial differential equations of different degrees and orders. Hani [6], and by assumed $Z(x,y,t)=e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$ found the solution to the linear second order partial differential equations with independent variables and of the form

$$A\,Z_{xx}+B\,Z_{xy}+C\,Z_{xt}+D\,Z_{yt}+E\,Z_{yy}+F\,Z_{tt}+G\,Z_{x}+H\,Z_{y}+I\,Z_{t}+J\,Z=0,$$

where *A,B,C,…,I* and *J* are arbitrary constants. Feng , Kao and Lewis [5], studies Convergent finite difference methods for one-dimensional fully nonlinear second order partial differential equations.

Mohsin [15], and by assumed $Z(x,y)=e^{\int u(x)dx+\int v(y)dy}$, $Z(x,y)=e^{\int\frac{u(x)}{x}dx+\int v(y)dy}$, $Z(x,y)=e^{\int u(x)dx+\int\frac{v(y)}{y}dy}$ and $Z(x,y)=e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy}$ found the complete solution the nonlinear second order partial differential equations, of homogeneous degree which have the general form

$$A\,Z_{xx}+B\,Z_{xy}+C\,Z_{yy}+D\,Z_{x}+E\,Z_{y}+F\,Z=0,$$

where *A, B, C, D, E* and *F* are linear functions of dependent variable *Z* and partial derivatives of dependent variable with respect to the independent variables *x* and *y.*

Mohammed , Mohsin and Hanoon [13], by assuming $Z(x,y)=e^{\int\frac{u(x)}{x}dx+\int v(y)dy}$, $Z(x,y)=e^{\int u(x)dx+\int\frac{v(y)}{y}dy}$ and $Z(x,y)=e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy}$ found the complete solution of special kinds of nonlinear second order partial differential equations with three independent variables of the form

$$A\,Z_{xx}+B\,Z_{xy}+C\,Z_{xt}+D\,Z_{yt}+E\,Z_{yy}+F\,Z_{tt}+G\,Z_{x}+H\,Z_{y}+I\,Z_{t}+J\,Z=0,$$

where *A , B , C , D , E , F , G , H , I* and *J* are functions of $x, y, t, Z, Z_x, Z_y, Z_t$, $Z_{xx}, Z_{xy}, Z_{xt}, Z_{yt}, Z_{yy}$ and Z_{tt} . And $Z, Z_x, Z_y, Z_t, Z_{xx}, Z_{xy}, Z_{xt}, Z_{yt},$ where Z_{yy} and Z_{tt} in these functions are of first degree and not multiplied with each other.

Ketap [8],studied the linear third order partial differential equations, with constant coefficients which have the form

$$A\,Z_{xxx}+B\,Z_{yyy}+C\,Z_{xxy}+D\,Z_{xyy}+E\,Z_{xx}+F\,Z_{yy}+G\,Z_{xy}+H\,Z_{x}+I\,Z_{y}+J\,Z=0,$$

Where *A, … I* and *J* are arbitrary constants , and used the assumption $Z(x,y)=\mathrm{e}^{\int u(x)\,dx+\int v(y)\,dy}$ to find the complete solution of it.

Liu, Ume, Anderson and Kang [11], studied singular nonlinear third-order differential equation

$$x'''(t)+\lambda\alpha(t)f(t,x(t))=0 \qquad \mathrm{a}\langle \mathrm{t}\langle b,$$
$$x(a)=x''(a)=x''(b)=0,$$

where $\lambda\rangle 0$ is a parameter, $\alpha\in\mathrm{C}((\mathrm{a},\mathrm{b}),\mathrm{R}^{+})$, $f\in C([a,b]\times(0,+\infty),R^{+})$, $\alpha(t)$ may be singular at $t=a,b$ and $f(t,s)$ may be singular at $\mathrm{s}=0$

Ademola , Ogundiran , Arawomo and Adesina [1], studied boundedness of solutions of the third nonlinear differential equation

$\dddot{x}+f(\ddot{x})+g(\dot{x})+h(x)=p(t,x,\dot{x},\ddot{x})$, or its equivalent system of differential equations

$$\dot{x}=y,\ \dot{y}=z,\ \dot{z}=p(t,x,y,z)-f(z)-g(y)-h(x)$$

where $f, g, h \in C(R,R), p \in C(R^+ \times R \times R \times R, R), R^+ = [0,\infty)$ and $R = (-\infty,\infty)$

Clarkson , Mansfield and Priestley [3], studies symmetry reductions of a class of nonlinear third-order partial differential equations

$u_t - \epsilon u_{xxt} + 2ku_x = uu_{xxx} + \alpha uu_x + \beta u_x u_{xx}$,

where ϵ, k, α *and* β *are arbitrary constants.*

This paper is devoted to solve the nonlinear third order partial differential equations of homogeneous degree with three independent variables of the general form

$$A Z_{xxx}^2 + B Z_{yyy}^2 + C Z_{ttt}^2 + D Z_{xxy}^2 + E Z_{xxt}^2 + F Z_{yyt}^2 + G Z_{xtt}^2 + H Z_{ytt}^2 + I Z_{xyt}^2 + J Z_{xyy}^2 +$$
$$K Z_{xx}^2 + L Z_{yy}^2 + M Z_{tt}^2 + N Z_{xy}^2 + O Z_{xt}^2 + P Z_{yt}^2 + Q Z_x^2 + R Z_y^2 + S Z_t^2 + T Z^2 = 0,$$

where *A, B, C, D, E*, … ,S and *T* are linear functions of dependent variable *Z* and partial derivatives of dependent variable with respect to the independent variables *x* , y and *t* .

By using the assumptions $Z(x,y,t) = e^{\int u(x)dx + \int v(y)dy + + \int w(t)dt}$, $Z(x,y,t) = e^{\int u(x)dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}$, $Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}$, $Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt}$ and $Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}$.

1. Solving Some Kinds of Third Order partial Differential Equations of omogeneous Degree with Three independent variables

Use The aim of this section is to solve the nonlinear third order of partial differential equations, of homogeneous degree with three independent variables which have the general form

$$A Z_{xxx}^2 + B Z_{yyy}^2 + C Z_{ttt}^2 + D Z_{xxy}^2 + E Z_{xxt}^2 + F Z_{yyt}^2 + G Z_{xtt}^2 + H Z_{ytt}^2 + I Z_{xyt}^2 + J Z_{xyy}^2 +$$
$$K Z_{xx}^2 + L Z_{yy}^2 + M Z_{tt}^2 + N Z_{xy}^2 + O Z_{xt}^2 + P Z_{yt}^2 + Q Z_x^2 + R Z_y^2 + S Z_t^2 + T Z^2 = 0,$$

where *A, B, C, D, E*, … ,S and *T* are linear functions of dependent variable *Z* and partial derivatives of dependent variable with respect to the independent variables *x* , y and *t* .

So, for this purpose we will search functions $u(x), v(y)$ and $w(t)$ such that the assumptions

$$Z(x,y,t) = e^{\int u(x)dx + \int v(y)dy + + \int w(t)dt}, \quad Z(x,y,t) = e^{\int u(x)dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}$$

$$Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}, \quad Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt} \quad \text{and}$$

$Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}$. To give the complete solution to the above equation we need to consider many cases:

<u>Case(1)</u> :

a) $A\, Z^2\, Z_{xxx}^2 = 0$

b) $B\, Z^2\, Z_{yyy}^2 = 0$

c) $C\, Z^2\, Z_{ttt}^2 = 0$

d) $D\, Z^2\, Z_{xxy}^2 + I\, Z^2\, Z_{xyt}^2 + J\, Z^2\, Z_{xyy}^2 = 0$

e) $E\, Z_y^2 Z_{xxt}^2 + F\, Z_x^2 Z_{yyt}^2 + G\, Z_y^2 Z_{xtt}^2 + H\, Z_x^2 Z_{ytt}^2 = 0$

Case(2):

a) $A\, y^2 Z_y^2 Z_{xxx}^2 + D\, y^2 t\, Z\, Z_t\, Z_{xxy}^2 + E\, y^2 t^2 Z_y^2 Z_{xxt}^2 + I\, y^2 t^2\, Z_x^2 Z_{xyt}^2 + J\, y^4 Z^2 Z_{xyy}^2 = 0$

b) $B\, x^2 t^2 Z_{xt}^2 Z_{yyy}^2 + F\, x^2 t^2 Z Z_{xx}\, Z_{yyt}^2 + G\, x^2 t^4\, Z_{yy}^2\, Z_{xtt}^2 + H x^2 t^4\, Z_{xy}^2 Z_{ytt}^2 = 0$

c) $C\, y^2 Z_{yt}^2 Z_{ttt}^2 + K\, x^4 y^2\, Z_y^2\, Z_{xx}^2 + L\, y^4\, Z_t^2 Z_{yy}^2 + M\, y^2 Z_y^2 Z_{tt}^2 = 0$

Case(3):

a) $N x^2 y^2 Z^2 Z_{xy}^2 + O x^2 y^4 t^2 Z_{yy}^2 Z_{xt}^2 + P\, x^4 y^2 t^2 Z_{xx}^2 Z_{yt}^2 = 0$

b) $D\, x^4 y^2 t^2 Z_t^2 Z_{xxy}^2 + E\, x^4 y^2 t^2 Z_y^2 Z_{xxt}^2 + F\, x^2 y^4 t^2 Z_x^2 Z_{yyt}^2 + G\, x^2 y^2 t^4 Z_y^2 Z_{xtt}^2 + H\, x^2 y^2 t^4 Z_x^2 Z_{ytt}^2 +$

$I\, x^2 y^2 Z^2 Z_{xyt}^2 + J\, x^2 y^4 t^2 Z_t^2 Z_{xyy}^2 + K\, x^4 y^2 t^2 Z_{yt}^2 Z_{xx}^2 + L\, x^2 y^4 t^2 Z_{xt}^2 Z_{yy}^2 + M\, x^2 y^2 t^4 Z_{xy}^2 Z_{tt}^2 +$

$N\, x^2 y^2 t^2 Z_t^2 Z_{xy}^2 + O\, x^2 y^2 t^2 Z_y^2 Z_{xt}^2 + P\, x^2 y^2 t^2 Z_x^2 Z_{yt}^2 = 0$

Where A,B,C,D, … , S and T are real constants.

Case(1)-a-: By using the assumption

$Z(x,y,t) = e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$, we get

$$Z_x = u(x)\, e^{\int u(x)dx+\int v(y)dy+\int w(t)dt} \Rightarrow Z_{xx} = (u'(x) + u^2(x))\, e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$\Rightarrow Z_{xxx} = (u''(x) + 3u(x)\, u'(x) + u^3(x))\, e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

So, the equation $A\, Z^2\, Z_{xxx}^2 = 0$. Transforms to the form

$$[A(u''(x) + 3u(x)\, u'(x) + u^3(x))^2]e^{4[\int u(x)dx+\int v(y)dy+\int w(t)dt]} = 0,$$

Since $e^{4[\int u(x)dx+\int v(y)dy+\int w(t)dt]} \neq 0$

So, $A[u''(x) + 3u(x)\, u'(x) + u^3(x)]^2 = 0 \quad \Rightarrow [u''(x) + 3u(x)\, u'(x) + u^3(x)]^2 = 0$

$$\Rightarrow (u''(x) + 3u(x)\, u'(x) + u^3(x)) = 0$$

This equation is called beloved equation [10],[17],[18].The beloved equation Possesses both Left Painleve Series (LPS) and Right Painleve Series (RPS) [9], it can be solved by Riccati transformation [12],[16]. And also it can be solved by using nonlocal symmetry [7] .

Mohammed and Ketap [14], he found the general solution of the beloved equation as follows:

$$u(x)=\frac{2(x+c_2)}{(x+c_2)^2+\frac{1}{c_1}}$$, then the complete solution is given by :

$$Z(x,y)=A_1Y_1(y)[(x+c_2)^2+c_3] \quad ; \ A_1=e^{a_1}, c_3=\frac{1}{c_1}, Y_1(y)=e^{h(y)}$$

where A_1 , c_2 and c_3 are arbitrary constants and $Y_1(y)$ is an arbitrary function of y .

Domain : $-\infty < x < \infty$, $-\infty < y < \infty$

So the complete solution is given by:

$$Z(x,y,t)=A_1Y_1(y)T_1(t)[(x+c_2)^2+c_3] \quad ; \ A_1=e^{a_1}, c_3=\frac{1}{c_1}, Y_1(y)=e^{h(y)}, T_1(t)=e^{k(t)}$$

where A_1 , c_2 and c_3 are arbitrary constants ,$Y_1(y)$ is an arbitrary function of y and $T_1(t)$ is an arbitrary function of t .

Domain : $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < t < \infty$.

<u>Case(1)-b-and</u> –c- By using the assumption $Z(x,y,t)=e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$ and by the same method as in case –a-, we get the complete solution for equations in case(1)-b- and –c

<u>Case(1)-d-</u>: By using the assumption

$Z(x,y,t)=e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$, we get

$$Z_{xxy} = v(y)\,(u'(x)+u^2(x))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$Z_{xy}=u(x)\,v(y)\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt} \Rightarrow Z_{xyt}=u(x)\,v(y)\,w(t)\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$Z_{xyy}=(u(x)\,(v'(y)+v^2(y))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

Then the equation $D\,Z^2\,Z^2_{xxy}+I\,Z^2\,Z^2_{xyt}+J\,Z^2\,Z^2_{xyy}=0$. Transforms to the form

$$[D(v^2(y)(u'(x)+u^2(x))^2)+I\,u^2(x)v^2(y)w^2(t)+J(u^2(x)(v'(y)+v^2(y))^2]e^{4[\int u(x)dx+\int v(y)dy+\int w(t)dt]}=0$$

Since $e^{4[\int u(x)dx+\int v(y)dy+\int w(t)dt]}\neq 0$

So, $D(v^2(y)(u'(x)+u^2(x))^2)+I\,u^2(x)v^2(y)w^2(t)+J(u^2(x)(v'(y)+v^2(y))^2=0$ ...(1)

This equation is variable separable equation [4] . $\Rightarrow D(\frac{u'(x)+u^2(x)}{u(x)})^2+J(\frac{v'(y)+v^2(y)}{v(y)})^2+Iw^2(t)=0$.

Let $D(\frac{u'(x)+u^2(x)}{u(x)})^2=\lambda_1^2$, $J(\frac{v'(y)+v^2(y)}{v(y)})^2=\lambda_2^2$

and $Iw^2(t)=-(\lambda_1^2+\lambda_2^2) \Rightarrow w(t)=\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\ i$

$$\Rightarrow u'(x)+u^2(x)\mp\frac{\lambda_1}{\sqrt{D}}u(x)=0 \qquad \ldots(2).$$

$$v'(y)+v^2(y)\mp\frac{\lambda_2}{\sqrt{J}}v(y)=0 \qquad \ldots(3).$$

The equations (2) and (3) are similar to Bernoulli equation [2], then the solution of them are given by :

$$u(x)=\frac{e^{\pm\frac{\lambda_1}{\sqrt{D}}x}}{\int e^{\pm\frac{\lambda_1}{\sqrt{D}}x}dx} \quad \text{and} \quad v(y)=\frac{e^{\pm\frac{\lambda_2}{\sqrt{J}}y}}{\int e^{\pm\frac{\lambda_2}{\sqrt{J}}y}dy}$$

So , Z(x, y, t) = $e^{\int\frac{e^{\pm\frac{\lambda_1}{\sqrt{D}}x}}{\int e^{\pm\frac{\lambda_1}{\sqrt{D}}x}dx}dx+\int\frac{e^{\pm\frac{\lambda_2}{\sqrt{J}}y}}{\int e^{\pm\frac{\lambda_2}{\sqrt{J}}y}dy}dy+\int\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,i\,dt}$

$$=e^{\ln(\int e^{\pm\frac{\lambda_1}{\sqrt{D}}x}dx)+\ln(\int e^{\pm\frac{\lambda_2}{\sqrt{J}}y}dy)\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,i\,t+c_1}$$

$$=c_2\left(\pm\frac{\sqrt{D}}{\lambda_1}e^{\pm\frac{\lambda_1}{\sqrt{D}}x}\right)\left(\pm\frac{\sqrt{J}}{\lambda_2}e^{\pm\frac{\lambda_2}{\sqrt{J}}y}\right)e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,it} \quad ;\ c_2=e^{c_1}\ ,\lambda_1 and\ \lambda_2\neq 0$$

$$=\mathrm{K}\,e^{\pm\frac{\lambda_1}{\sqrt{D}}x\pm\frac{\lambda_2}{\sqrt{J}}y\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,i\,t} \quad ;\ \mathrm{K}=\left(\pm\frac{\sqrt{D}}{\lambda_1}\right)\left(\pm\frac{\sqrt{J}}{\lambda_2}\right)c_2$$

So ,the complete solution of equation(1), is given by:

$$\left[Z(x,y,t)-\mathrm{K}\,e^{-\frac{\lambda_1}{\sqrt{D}}x-\frac{\lambda_2}{\sqrt{J}}y+\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,i\,t}\right]\left[Z(x,y,t)-\mathrm{K}\,e^{+\frac{\lambda_1}{\sqrt{D}}x+\frac{\lambda_2}{\sqrt{J}}y-\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,i\,t}\right]$$

$$\left[Z(x,y,t)-\mathrm{K}\,e^{-\frac{\lambda_1}{\sqrt{D}}x+\frac{\lambda_2}{\sqrt{J}}y+\sqrt{\frac{\lambda_1^2+\lambda_2^2}{I}}\,i\,t}\right]=0$$

where K , λ_1 and λ_2 are arbitrary constants.

Domain : - ∞ < *x* < ∞ , - ∞ < *y* < ∞ , - ∞ < *t* < ∞ .

Case(1)-e-: By using the assumption

$Z(x,y,t)=e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$, we get

$$Z_{xxt} = w(t)\,(u'(x)+u^2(x))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$Z_{xt} = u(x)\,w(t)\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt} \Rightarrow Z_{xtt} = u(x)\,(w'(t)+w^2(t))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$Z_y = v(y)\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt} \Rightarrow Z_{yy} = (v'(y)+v^2(y))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$Z_{yyt} = w(t)(v'(y)+v^2(y))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

$$Z_{ytt} = v(y)\,(w'(t)+w^2(t))\,e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$

And by using Z_x from case(1)-a- , then the equation

So, the equation $\mathrm{E}\,Z_y^2 Z_{xxt}^2 + \mathrm{F}\,Z_x^2 Z_{yyt}^2 + \mathrm{G}\,Z_y^2 Z_{xtt}^2 + \mathrm{H}\,Z_x^2 Z_{ytt}^2 = 0$. Transforms to the form

$$[E\,v^2(y)w^2(t)(u'(x)+u^2(x))^2 + F\,u^2(x)w^2(t)(v'(y)+v^2(y))^2 + Gu^2(x)v^2(y)(w'(t)+w^2(t))^2 + Hu^2(x)v^2(y)(w'(t)+w^2(t))^2]\,e^{4[\int u(x)dx+\int v(y)dy+\int w(t)dt]} = 0$$

Since $e^{4[\int u(x)dx+\int v(y)dy+\int w(t)dt]} \neq 0$

So , $$E\,v^2(y)w^2(t)(u'(x)+u^2(x))^2 + F\,u^2(x)w^2(t)(v'(y)+v^2(y))^2 + Gu^2(x)v^2(y)(w'(t)+w^2(t))^2 + Hu^2(x)v^2(y)(w'(t)+w^2(t))^2 = 0 \quad ...(4)$$

This equation is variable separable equation [4] .

$$\Rightarrow E\left(\frac{u'(x)+u^2(x)}{u(x)}\right)^2 + F\left(\frac{v'(y)+v^2(y)}{v(y)}\right)^2 + (G+H)\left(\frac{w'(t)+w^2(t)}{w(t)}\right)^2 = 0$$

Let $E\left(\frac{u'(x)+u^2(x)}{u(x)}\right)^2 = \lambda_1^2$ and $F\left(\frac{v'(y)+v^2(y)}{v(y)}\right)^2 = \lambda_2^2$

Therefore $(G+H)\left(\frac{w'(t)+w^2(t)}{w(t)}\right)^2 = -(\lambda_1^2+\lambda_2^2)$

$$\Rightarrow u'(x)+u^2(x) \mp \frac{\lambda_1}{\sqrt{E}}u(x) = 0 \quad ...(5).$$

$$v'(y)+v^2(y) \mp \frac{\lambda_2}{\sqrt{F}}v(y) = 0 \quad ...(6).$$

$$w'(t)+w^2(t) \mp \sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,w(t) = 0 \quad ...(7).$$

The equations (5),(6) and (7) are similar to Bernoulli equation [2], then the solution of them are given by :

$$u(x)=\frac{e^{\pm\frac{\lambda_1}{\sqrt{E}}x}}{\int e^{\pm\frac{\lambda_1}{\sqrt{E}}x}dx}\quad,\quad v(y)=\frac{e^{\pm\frac{\lambda_2}{\sqrt{F}}y}}{\int e^{\pm\frac{\lambda_2}{\sqrt{F}}y}dy}\quad \text{and}\quad w(t)=\frac{e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}}{\int e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}dt}$$

So, $$Z(x,y,t)=e^{\int\frac{e^{\pm\frac{\lambda_1}{\sqrt{E}}x}}{\int e^{\pm\frac{\lambda_1}{\sqrt{E}}x}dx}dx+\int\frac{e^{\pm\frac{\lambda_2}{\sqrt{F}}y}}{\int e^{\pm\frac{\lambda_2}{\sqrt{F}}y}dy}dy+\int\frac{e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}}{\int e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}dt}dt}$$

$$=e^{\ln\left(\int e^{\pm\frac{\lambda_1}{\sqrt{E}}x}dx\right)+\ln\left(\int e^{\pm\frac{\lambda_2}{\sqrt{F}}y}dy\right)+\ln\left(\int e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}dt\right)+c_1}$$

$$=c_2\left(\pm\frac{\sqrt{E}}{\lambda_1}e^{\pm\frac{\lambda_1}{\sqrt{E}}x}\right)\left(\pm\frac{\sqrt{F}}{\lambda_2}e^{\pm\frac{\lambda_2}{\sqrt{F}}y}\right)\left(\pm\sqrt{\frac{G+H}{\lambda_1^2+\lambda_2^2}}\,\frac{1}{i}\,e^{\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}\right)\ ;\ c_2=e^{c_1},\ \lambda_1 \text{ and } \lambda_2\neq 0$$

$$=K\,e^{\pm\frac{\lambda_1}{\sqrt{E}}x\pm\frac{\lambda_2}{\sqrt{F}}y\pm\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}\quad ;\ K=c_2\left(\pm\frac{\sqrt{E}}{\lambda_1}\right)\left(\pm\frac{\sqrt{F}}{\lambda_2}\right)\left(\pm\sqrt{\frac{G+H}{\lambda_1^2+\lambda_2^2}}\,\frac{1}{i}\right)$$

So ,the complete solution of equation(4), is given by:

$$\left[Z(x,y,t)-K\,e^{+\frac{\lambda_1}{\sqrt{E}}x+\frac{\lambda_2}{\sqrt{F}}y+\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}\right]\left[Z(x,y,t)-K\,e^{-\frac{\lambda_1}{\sqrt{E}}x-\frac{\lambda_2}{\sqrt{F}}y-\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}\right]$$
$$\left[Z(x,y,t)-K\,e^{-\frac{\lambda_1}{\sqrt{E}}x+\frac{\lambda_2}{\sqrt{F}}y-\sqrt{\frac{\lambda_1^2+\lambda_2^2}{G+H}}\,i\,t}\right]=0$$

where K , λ_1 and λ_2 are arbitrary constants.

Domain : $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < t < \infty$.

Case(2)-a-: By using the assumption

$$Z(x,y,t)=e^{\int u(x)dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$, we get

$$Z_x=u(x)\,e^{\int u(x)dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}\Rightarrow Z_{xx}=(u'(x)+u^2(x))\,e^{\int u(x)dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$

$$\Rightarrow Z_{xxx} = (u''(x) + 3u(x)\,u'(x) + u^3(x))\, e^{\int u(x)dx + \int \frac{v(y)}{y} dy + \int \frac{w(t)}{t} dt}$$

$$Z_{xxy} = (\frac{v(y)}{y}(u'(x) + u^2(x)))e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt}$$

$$Z_{xxt} = (\frac{w(t)}{t}(u'(x) + u^2(x)))e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt}$$

$$Z_{xy} = \frac{v(y)}{y} u(x) e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt} \quad \Rightarrow Z_{xyt} = \frac{v(y)}{y}\frac{w(t)}{t} u(x) e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt}$$

$$\Rightarrow Z_{xyy} = u(x)\left(\frac{yv'(y) + v^2(y) - v(y)}{y^2}\right) e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt}$$

$$Z_t = \frac{w(t)}{t} e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt} \quad \text{and} \quad Z_y = \frac{v(y)}{y} e^{\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt}$$

So, the equation

$$\mathrm{A}y^2 Z_y^2 Z_{xxx}^2 + \mathrm{D}y^2 t\, Z\, Z_t\, Z_{xxy}^2 + \mathrm{E}y^2 t^2 Z_y^2 Z_{xxt}^2 + \mathrm{I}y^2 t^2\, Z_x^2 Z_{xyt}^2 + \mathrm{J}y^4 Z^2 Z_{xyy}^2 = 0$$ Transforms to the form

$$\Bigg[\mathrm{A}y^2\left(\frac{v^2(y)}{y^2}(u''(x) + 3u(x)\,u'(x) + u^3(x))^2\right) + \mathrm{D}y^2 t\left(\frac{v^2(y)}{y^2}\frac{w(t)}{t}(u'(x) + u^2(x))^2\right) +$$

$$\mathrm{E}y^2 t^2\left(\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}(u'(x) + u^2(x))^2\right) + \mathrm{I}y^2 t^2\left(u^4(x)\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\right) +$$

$$\mathrm{J}y^4\left(u^2(x)(\frac{yv'(y) + v^2(y) - v(y)}{y^2})^2\right)\Bigg] e^{4[\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt]} = 0$$

Since $e^{4[\int u(x)dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt]} \neq 0$

$$\text{So}\ \mathrm{A}\left(v^2(y)(u''(x) + 3u(x)\,u'(x) + u^3(x))^2\right) + \mathrm{D}\left(v^2(y)w(t)(u'(x) + u^2(x))^2\right) +$$

$$\mathrm{E}\left(v^2(y)w^2(t)(u'(x) + u^2(x))^2\right) + \mathrm{I}\,u^4(x)v^2(y)w^2(t) + \mathrm{J}u^2(x)\left(yv'(y) + v^2(y) - v(y)\right)^2 = 0 \qquad \ldots(8)$$

Here we can't separate the variables in this equation , so we suppose that $u(x) = \lambda_1$

where λ is an arbitrary constant, then the last equation becomes

$$\mathrm{A}\lambda_1^6 v^2(y) + \mathrm{D}\lambda_1^4 v^2(y)w(t) + \mathrm{E}\lambda_1^4 v^2(y)w^2(t) + \mathrm{I}\lambda_1^4 v^2(y)w^2(t) + \mathrm{J}\lambda_1^2\left(yv'(y) + v^2(y) - v(y)\right)^2 = 0$$

This equation is variable separable equation [4].

$$\Rightarrow A\lambda_1^4 + D\,\lambda_1^2 w(t) + (E+I)\lambda_1^2 w^2(t) + J\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2 = 0$$

Let $(E+I)\lambda_1^2 w^2(t) + D\lambda_1^2 w(t) + A\,\lambda_1^4 = -J\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2 = -\lambda_2^2$

$$\Rightarrow (E+I)w^2(t) + D\,w(t) + A\,\lambda_1^2 + \frac{\lambda_2^2}{\lambda_1^2} = 0 \qquad \Rightarrow w(t) = \frac{-D \pm \sqrt{D^2 - 4(E+I)(A\lambda_1^2 + \frac{\lambda_2^2}{\lambda_1^2})}}{2(E+I)}$$

And $yv'(y) + v^2(y) - (\pm\frac{\lambda_2}{\sqrt{J}} + 1)v(y) = 0$

Let $A_1 = \pm\frac{\lambda_2}{\sqrt{J}} + 1$ Then $yv'(y) + v^2(y) - A_1 v(y) = 0$ …(9).

The equation(9), is variable separable equation [4], we can solve it as follows :

$$-\frac{dv}{b^2 - \left(v(y) - \frac{A_1}{2}\right)^2} + \frac{dy}{y} = 0 \qquad ; \qquad b^2 = \frac{A_1^2}{4}$$

$$-\frac{1}{b}\tanh^{-1}\left(\frac{v(y) - \frac{A_1}{2}}{b}\right) = -\ln(cy) \qquad \Rightarrow v(y) = b\tanh(b\ln(cy)) + \frac{A_1}{2}$$

So, $$Z(x,y,t) = e^{\int \lambda_1\,dx + \int \frac{b\tanh(b\ln(cy)) + \frac{A_1}{2}}{y}dy + \int \frac{-D \pm \sqrt{D^2 - 4(E+I)(A\lambda_1^2 + \frac{\lambda_2^2}{\lambda_1^2})}}{2(E+I)t}dt} \quad ; \quad cy > 0$$

$$= e^{\lambda_1 x + \ln|\cosh(b\ln(cy))| + \frac{A_1}{2}\ln y + \frac{-D \pm \sqrt{D^2 - 4(E+I)(A\lambda_1^2 + \frac{\lambda_2^2}{\lambda_1^2})}}{2(E+I)}\ln t + g} \quad ; \quad cy > 0$$

$$= K\,y^{\frac{A_1}{2}}\, t^{\frac{-D \pm \sqrt{D^2 - 4(E+I)(A\lambda_1^2 + \frac{\lambda_2^2}{\lambda_1^2})}}{2(E+I)}}\, e^{\lambda_1 x}\,(\cosh(b\ln c\,y)) \quad ; \; K = e^g,\; cy > 0$$

$$= K\,y^{\pm\frac{\lambda_2}{2\sqrt{J}} + \frac{1}{2}}\, t^{\frac{-D \pm \sqrt{D^2 - 4(E+I)(A\lambda_1^2 + \frac{\lambda_2^2}{\lambda_1^2})}}{2(E+I)}}\, e^{\lambda_1 x}\left(\cosh((\pm\frac{\lambda_2}{2\sqrt{J}} + \frac{1}{2})\ln c\,y)\right) \quad ; \; cy > 0$$

Then the complete solution of the equation(8), is given by :

$$\left[Z(x,y,t) - \mathrm{K}\, y^{+\frac{\lambda_2}{2\sqrt{J}}+\frac{1}{2}}\, t^{\frac{-\mathrm{D}+\sqrt{\mathrm{D}^2-4(\mathrm{E}+\mathrm{I})(\mathrm{A}\lambda_1^2+\frac{\lambda_2^2}{\lambda_1^2})}}{2(\mathrm{E}+\mathrm{I})}}\, e^{\lambda_1 x} \left(\cosh((+\frac{\lambda_2}{2\sqrt{J}}+\frac{1}{2})\ln c\, y) \right) \right]$$

$$\left[Z(x,y,t) - \mathrm{K}\, y^{-\frac{\lambda_2}{2\sqrt{J}}+\frac{1}{2}}\, t^{\frac{-\mathrm{D}-\sqrt{\mathrm{D}^2-4(\mathrm{E}+\mathrm{I})(\mathrm{A}\lambda_1^2+\frac{\lambda_2^2}{\lambda_1^2})}}{2(\mathrm{E}+\mathrm{I})}}\, e^{\lambda_1 x} \left(\cosh((-\frac{\lambda_2}{2\sqrt{J}}+\frac{1}{2})\ln c\, y) \right) \right]$$

$$\left[Z(x,y,t) - \mathrm{K}\, y^{+\frac{\lambda_2}{2\sqrt{J}}+\frac{1}{2}}\, t^{\frac{-\mathrm{D}-\sqrt{\mathrm{D}^2-4(\mathrm{E}+\mathrm{I})(\mathrm{A}\lambda_1^2+\frac{\lambda_2^2}{\lambda_1^2})}}{2(\mathrm{E}+\mathrm{I})}}\, e^{\lambda_1 x} \left(\cosh((+\frac{\lambda_2}{2\sqrt{J}}+\frac{1}{2})\ln c\, y) \right) \right] = 0$$

Where $\mathrm{K}, \lambda_1, \lambda_2$ and c are arbitrary constants.

Domain : $-\infty < x < \infty$, $y > 0$, $-\infty < t < \infty$.

Case(2)-b-: By using the assumption

$Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}$, we get

$$Z_y = v(y)\, e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt} \Rightarrow Z_{yy} = (v'(y) + v^2(y))\, e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}$$

$$\Rightarrow Z_{yyy} = (v''(y) + 3v(y)\, v'(y) + v^3(y))\, e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}$$

$$\Rightarrow Z_{yyt} = \frac{w(t)}{t}(v'(y) + v^2(y))\, e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}$$

$$Z_x = \frac{u(x)}{x}\, e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt} \Rightarrow Z_{xx} = (\frac{xu'(x) + u^2(x) - u(x)}{x^2})\, e^{\int \frac{u(x)}{x}dx + \int v(y)dy + \int \frac{w(t)}{t}dt}$$

$$Z_{xy}=\frac{u(x)}{x}v(y)\,e^{\int\frac{u(x)}{x}dx+\int v(y)dy+\int\frac{w(t)}{t}dt}$$

$$Z_{xt}=\frac{u(x)}{x}\frac{w(t)}{t}e^{\int\frac{u(x)}{x}dx+\int v(y)dy+\int\frac{w(t)}{t}dt}\Rightarrow Z_{xtt}=\frac{u(x)}{x}\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)e^{\int\frac{u(x)}{x}dx+\int v(y)dy+\int\frac{w(t)}{t}dt}$$

$$\Rightarrow Z_{ytt}=v(y)\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)e^{\int\frac{u(x)}{x}dx+\int v(y)dy+\int\frac{w(t)}{t}dt}$$

So, the equation $\mathrm{B}\,x^2t^2Z_{xt}^2Z_{yyy}^2+\mathrm{F}\,x^2t^2ZZ_{xx}\,Z_{yyt}^2+\mathrm{G}\,x^2t^4\,Z_{yy}^2Z_{xtt}^2+Hx^2t^4\,Z_{xy}^2Z_{ytt}^2=0$.

Transforms to the form

$$\Bigg[\mathrm{B}x^2t^2\left(\frac{u^2(x)}{x^2}\frac{w^2(t)}{t^2}(v''(y)+3v(y)\,v'(y)+v^3(y))^2\right)+\mathrm{F}x^2t^2\left(\frac{w^2(t)}{t^2}(v'(y)+v^2(y))^2\right.$$
$$\left.\left(\frac{xu'(x)+u^2(x)-u(x)}{x^2}\right)\right)+\mathrm{G}x^2t^4\left(\frac{u^2(x)}{x^2}(v'(y)+v^2(y))^2\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)^2\right)+$$
$$H\,x^2t^4\left(v^4(y)\frac{u^2(x)}{x^2}\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)^2\right)\Bigg]e^{4[\int\frac{u(x)}{x}dx+\int v(y)dy+\int\frac{w(t)}{t}dt]}=0$$

Since $e^{4[\int\frac{u(x)}{x}dx+\int v(y)dy+\int\frac{w(t)}{t}dt]}\neq 0$

$$\text{So }\ \mathrm{B}\left(u^2(x)w^2(t)(v''(y)+3v(y)\,v'(y)+v^3(y))^2\right)+$$
$$\mathrm{F}\left(w^2(t)(v'(y)+v^2(y))^2(xu'(x)+u^2(x)-u(x))\right)+\mathrm{G}\left(u^2(x)(v'(y)+v^2(y))^2\right.$$
$$\left.(tw'(t)+w^2(t)-w(t))^2\right)+H\left(v^4(y)u^2(x)(tw'(t)+w^2(t)-w(t))^2\right)=0 \qquad \text{...(10)}$$

Here we can't separate the variables in this equation, so we suppose that $v(y)=\lambda_1$ where λ_1 is arbitrary constant, then the last equation becomes

$$\mathrm{B}\,\lambda_1^6u^2(x)w^2(t)+\mathrm{F}\,\lambda_1^4w^2(t)(xu'(x)+u^2(x)-u(x))+$$
$$G\lambda_1^4u^2(x)(tw'(t)+w^2(t)-w(t))^2+H\lambda_1^4u^2(x)(tw'(t)+w^2(t)-w(t))^2=0$$

This equation is variable separable equation [4].

$$\Rightarrow \mathrm{B}\,\lambda_1^6+\mathrm{F}\,\lambda_1^4\frac{(xu'(x)+u^2(x)-u(x))}{u^2(x)}+(G+H)\lambda_1^4\left(\frac{tw'(t)+w^2(t)-w(t)}{w(t)}\right)^2=0$$

$$\text{Let }\mathrm{B}\,\lambda_1^2+\mathrm{F}\frac{(xu'(x)+u^2(x)-u(x))}{u^2(x)}=-(G+H)\left(\frac{tw'(t)+w^2(t)-w(t)}{w(t)}\right)^2=-\lambda_2^2$$

$$\Rightarrow xu'(x) + (\frac{B\lambda_1^2 + \lambda_2^2}{F} + 1)u^2(x) - u(x) = 0$$

$$\Rightarrow (\frac{F}{B\lambda_1^2 + \lambda_2^2 + F})xu'(x) + u^2(x) - (\frac{F}{B\lambda_1^2 + \lambda_2^2 + F})u(x) = 0$$

Let $A_1 = \frac{F}{B\lambda_1^2 + \lambda_2^2 + F}$ Then $A_1 xu'(x) + u^2(x) - A_1 u(x) = 0$ …(11)

And $tw'(t) + w^2(t) - (\pm\frac{\lambda_2}{\sqrt{G+H}} + 1)w(t) = 0$ …(12)

The equation (11), is variable separable equation [4], we can solve it as follows :

$$-\frac{A_1 du}{d^2 - \left(u(x) - \frac{A_1}{2}\right)^2} + \frac{dx}{x} = 0 \quad ; \quad d^2 = \frac{A_1^2}{4}$$

$$-\frac{A_1}{d}\tanh^{-1}\left(\frac{u(x) - \frac{A_1}{2}}{d}\right) = -\ln(c_1 x) \;\; ; \; c_1 x > 0 \;\; \Rightarrow u(x) = d\tanh(\frac{d}{A_1}\ln(c_1 x)) + \frac{A_1}{2}$$

The equation (12) is similar to equation [9], then the same method the solution of it is given by :

$$w(t) = \frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2 t)) + \frac{A_2}{2} \quad ; \quad A_2 = \pm\frac{\lambda_2}{\sqrt{G+H}} + 1 \;, c_2 t > 0$$

So, $$Z(x,y) = e^{\int \frac{(d\tanh(\frac{d}{A_1}\ln c_1 x) + \frac{A_1}{2})}{x}dx + \int \lambda_1 dy + \int \frac{\frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2 t)) + \frac{A_2}{2}}{t}dt} \quad ; c_1 x \text{ and } c_2 t > 0$$

$$= e^{A_1 \ln\left|\cosh(\frac{d}{A_1}\ln(c_1 x))\right| + \frac{A_1}{2}\ln x + \lambda_1 y + \ln\left|\cosh(\frac{A_2}{2}\ln(c_2 t)\right| + \frac{A_2}{2}\ln t + g}$$

$$= K\, x^{\frac{A_1}{2}}\, t^{\frac{A_2}{2}}\, e^{\lambda_1 y}\, (\cosh(\frac{d}{A_1}\ln c_1 x))^{A_1} (\cosh(\frac{A_2}{2}\ln(c_2 t)) \quad ; \; K = e^g \;, c_1 x \text{ and } c_2 t > 0$$

$$= K\, x^{\frac{F}{2(B\lambda_1^2 + \lambda_2^2 + F)}}\, t^{\pm\frac{\lambda_2}{2\sqrt{G+H}} + \frac{1}{2}}\, e^{\lambda_1 y}\, (\cosh\frac{1}{2}\ln(c_1 x))^{\frac{F}{B\lambda_1^2 + \lambda_2^2 + F}}$$

$$(\cosh((\pm\frac{\lambda_2}{2\sqrt{G+H}} + \frac{1}{2})\ln(c_2 t))$$

Then the complete solution of the equation(11), is given by :

$$\left[Z(x,y,t) - K\, x^{\frac{F}{2(B\lambda_1^2+\lambda_2^2+F)}}\, t^{\frac{\lambda_2}{2\sqrt{G+H}}+\frac{1}{2}}\, e^{\lambda_1 y}\, (\cosh\frac{1}{2}\ln(c_1 x))^{\frac{F}{B\lambda_1^2+\lambda_2^2+F}} (\cosh((\frac{\lambda_2}{2\sqrt{G+H}}+\frac{1}{2})\ln(c_2 t)) \right]$$

$$\left[Z(x,y,t) - K\, x^{\frac{F}{2(B\lambda_1^2+\lambda_2^2+F)}}\, t^{-\frac{\lambda_2}{2\sqrt{G+H}}+\frac{1}{2}}\, e^{\lambda_1 y}\, (\cosh\frac{1}{2}\ln(c_1 x))^{\frac{F}{B\lambda_1^2+\lambda_2^2+F}} (\cosh((\frac{-\lambda_2}{2\sqrt{G+H}}+\frac{1}{2})\ln(c_2 t)) \right] = 0$$

Where K, λ_1, λ_2 , c_1 and c_2 are arbitrary constants.

Domain : $x > 0$, $-\infty < y < \infty$, $t > 0$.

Case(2)-c-: By using the assumption

$Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)\,dt}$, we get

$$Z_t = w(t)e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)\,dt} \Rightarrow Z_{tt} = (w'(t)+w^2(t))e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)\,dt}$$

$$\Rightarrow Z_{ttt} = (w''(t) + 3w(t)\,w'(t) + w^3(t))\,e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt}$$

$$Z_y = \frac{v(y)}{y}e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)\,dt} \Rightarrow Z_{yy} = (\frac{yv'(y)+v^2(y)-v(y)}{y^2})\,e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt}$$

$$Z_{yt} = \frac{v(y)}{y}w(t)e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)\,dt} \;,\; Z_{xx} = (\frac{xu'(x)+u^2(x)-u(x)}{x^2})\,e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt}$$

So, the equation $C\,y^2 Z_{yt}^2 Z_{ttt}^2 + K\,x^4 y^2\, Z_y^2\, Z_{xx}^2 + L\,y^4\, Z_t^2 Z_{yy}^2 + M\,y^2 Z_y^2 Z_{tt}^2 = 0$ .Transforms to the form

$$\left[C\,y^2\left(\frac{v^2(y)}{y^2}w^2(t)(w''(t)+3w(t)\,w'(t)+w^3(t))^2\right) + Kx^4y^2\left(\frac{v^2(y)}{y^2}(\frac{xu'(x)+u^2(x)-u(x)}{x^2})^2\right) + \right.$$

S

$$\left. Ly^4\left(w^2(t)(\frac{yv'(y)+v^2(y)-v(y)}{y^2})^2\right) + My^2\left((\frac{v^2(y)}{y^2})(w'(t)+w^2(t))^2\right)\right] e^{4[\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt]} = 0$$

ince $e^{4[\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int w(t)dt]} \neq 0$

$$\text{So } C\left(v^2(y)w^2(t)(w''(t)+3w(t)\,w'(t)+w^3(t))^2\right)+K\left(v^2(y)(xu'(x)+u^2(x)-u(x))^2\right)+$$
$$L\left(w^2(t)(yv'(y)+v^2(y)-v(y))^2\right)+M\left(v^2(y)(w'(t)+w^2(t))^2\right)=0 \qquad ...(13)$$

Here we can't separate the variables in this equation, so we suppose that $w(t)=\lambda_1$ where λ_1 is arbitrary constant, then the last equation becomes

$$C\lambda_1^8 v^2(y)+Kv^2(y)(xu'(x)+u^2(x)-u(x))^2+L\lambda_1^2\,(yv'(y)+v^2(y)-v(y))^2+M\lambda_1^4\,v^2(y)=0$$

This equation is variable separable equation [4],we get

$$C\lambda_1^8+M\lambda_1^4+K(xu'(x)+u^2(x)-u(x))^2+L\lambda_1^2\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2=0$$

Let $C\lambda_1^8+M\lambda_1^4+K(xu'(x)+u^2(x)-u(x))^2=-L\lambda_1^2\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2=-\lambda_2^2$

$$\Rightarrow xu`(x)+u^2(x)-u(x)\mp\sqrt{\frac{C\lambda_1^8+M\lambda_1^4+\lambda_2^2}{K}}\,i=0$$

Let $A_1=\mp\sqrt{\frac{C\lambda_1^8+M\lambda_1^4+\lambda_2^2}{K}}\,i$, then the last equation becomes:

$$xu'(x)+u^2(x)-u(x)+A_1=0 \qquad ...(14)$$

$$\text{And } yv'(y)+v^2(y)-\left(\pm\frac{\lambda_2}{\sqrt{L}}+1\right)v(y)=0$$

$$\text{Let } A_2=\pm\frac{\lambda_2}{\sqrt{L}}+1 \quad \text{Then} \quad yv'(y)+v^2(y)-A_2v(y)=0 \qquad ...(15)$$

The equation(14), is variable separable equation [3], we can solve it as follows :

$$\frac{du}{(u(x)-\frac{1}{2})^2+d^2}+\frac{dx}{x}=0 \quad ;\; d=\sqrt{A_1-\frac{1}{4}}$$

$$\text{Since } A_1\neq\frac{1}{4} \;\Rightarrow\; \frac{1}{d}\tan^{-1}\left(\frac{u(x)-\frac{1}{2}}{d}\right)=-\ln(c_1x) \quad ;\; c_1x>0$$

$$\Rightarrow u(x)=-d\tan(d\ln(c_1x))+\frac{1}{2} \qquad ;\; c_1x>0$$

The equation (15) is similar to equation [9], then the same method the solution of it is given by :

$$v(y)=\frac{A_2}{2}\tanh\left(\frac{A_2}{2}\ln(c_2y)\right)+\frac{A_2}{2} \quad ;\; A_2=\pm\frac{\lambda_2}{\sqrt{L}}+1$$

ISSN 2347-1921

So, $Z(x,y,t) = e^{\int \frac{(-d\tan(d\ln(c_1 x)) + \frac{1}{2})}{x} dx + \int \frac{\frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2 y)) + \frac{A_2}{2}}{y} dy + \int \lambda_1 dt}$; $c_1 x, c_2 y > 0$

$= e^{\frac{1}{2}\ln x + \ln|\cos(d\ln(c_1 x))| + \frac{A_2}{2}\ln y + \ln|\cosh(\frac{A_2}{2}\ln(c_2 y))| + \lambda_1 t + g}$; $c_1 x, c_2 y > 0$

$= K\, x^{\frac{1}{2}} y^{\frac{A_2}{2}} e^{\lambda_1 t} (\cos(d\ln(c_1 x)))(\cosh(\frac{A_2}{2}\ln(c_2 y)))$; $K = e^g$ and $c_1 x, c_2 y > 0$

$$\Rightarrow Z(x,y,t) = K\, x^{\frac{1}{2}}\, y^{\pm\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}} e^{\lambda_1 t}\left(\cos\left(\sqrt{\mp\sqrt{\frac{C\lambda_1^8+M\lambda_1^4+\lambda_2^2}{K}}\, i - \frac{1}{4}}\ \ln(c_1 x)\right)\right)$$

$$\left(\cosh\left(\left(\pm\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}\right)\ln(c_2 y)\right)\right)$$

Then the complete solution of the equation(13), is given by :

$$\left[Z(x,y,t) - K\, x^{\frac{1}{2}} y^{\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}} e^{\lambda_1 t}\left(\cos\left(\sqrt{-\sqrt{\frac{C\lambda_1^8+M\lambda_1^4+\lambda_2^2}{K}}\, i - \frac{1}{4}}\ \ln(c_1 x)\right)\right)\left(\cosh\left(\left(\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}\right)\ln(c_2 y)\right)\right)\right]$$

$$\left[Z(x,y,t) - K\, x^{\frac{1}{2}} y^{-\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}} e^{\lambda_1 t}\left(\cos\left(\sqrt{\sqrt{\frac{C\lambda_1^8+M\lambda_1^4+\lambda_2^2}{K}}\, i - \frac{1}{4}}\ \ln(c_1 x)\right)\right)\left(\cosh\left(\left(\frac{-\lambda_2}{2\sqrt{L}}+\frac{1}{2}\right)\ln(c_2 y)\right)\right)\right]$$

$$\left[Z(x,y,t) - K\, x^{\frac{1}{2}} y^{\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}} e^{\lambda_1 t}\left(\cos\left(\sqrt{\sqrt{\frac{C\lambda_1^8+M\lambda_1^4+\lambda_2^2}{K}}\, i - \frac{1}{4}}\ \ln(c_1 x)\right)\right)\left(\cosh\left(\left(\frac{\lambda_2}{2\sqrt{L}}+\frac{1}{2}\right)\ln(c_2 y)\right)\right)\right] = 0$$

Where $K, \lambda_1, \lambda_2, c_1$ and c_2 are arbitrary constants.

Domain: $x > 0$, $y > 0$, $-\infty < t < \infty$.

<u>Case(3)-a-</u>: By using the assumption

$Z(x,y,t) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}$, we get

$$Z_{xy} = \frac{u(x)}{x}\frac{v(y)}{y} e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}, \quad Z_{yt} = \frac{v(y)}{y}\frac{w(t)}{t} e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy + \int \frac{w(t)}{t}dt}$$

And using Z_{xt}, Z_{xx}, Z_{yy} from case(2)-b-,-c- , then the equation

$Nx^2y^2Z^2Z_{xy}^2 + Ox^2y^4t^2Z_{yy}^2Z_{xt}^2 + P\,x^4y^2t^2Z_{xx}^2Z_{yt}^2 = 0$. Transforms to the form

$$\left[Nx^2y^2\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\right)+Ox^2y^4t^2\left(\frac{u^2(x)}{x^2}\frac{w^2(t)}{t^2}\left(\frac{yv'(y)+v^2(y)-v(y)}{y^2}\right)^2\right)+\right.$$

$$\left.P\,x^4y^2t^2\left(\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\left(\frac{xu'(x)+u^2(x)-u(x)}{x^2}\right)^2\right)\right]\;e^{4[\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt]}=0$$

Since $e^{4[\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt]}\neq 0$

So, $Nu^2(x)v^2(y)+Ou^2(x)w^2(t)(yv'(y)+v^2(y)-v(y))^2+$
$Pv^2(y)w^2(t)(xu'(x)+u^2(x)-u(x))^2=0$ ...(16)

Here we can't separate the variables in this equation, so we suppose that $w(t)=\lambda_1$ where λ_1 is arbitrary constant, then the last equation becomes

$$Nu^2(x)v^2(y)+O\lambda_1^2u^2(x)(yv'(y)+v^2(y)-v(y))^2+P\lambda_1^2v^2(y)(xu'(x)+u^2(x)-u(x))^2=0$$

This equation is variable separable equation [4],we get

$$N+O\lambda_1^2\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2+P\lambda_1^2\left(\frac{xu'(x)+u^2(x)-u(x)}{u(x)}\right)^2=0$$

Let $N+O\lambda_1^2\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2=-P\lambda_1^2\left(\frac{xu'(x)+u^2(x)-u(x)}{u(x)}\right)^2=-\lambda_2^2$

$$\Rightarrow yv'(y)+v^2(y)-\left(\pm\frac{\sqrt{\lambda_2^2+N}}{\sqrt{O}\,\lambda_1}i+1\right)v(y)=0$$

Let $A_1=\pm\frac{\sqrt{\lambda_2^2+N}}{\sqrt{O}\,\lambda_1}i+1$ Then $yv'(y)+v^2(y)-A_1v(y)=0$ ...(17)

And $xu'(x)+u^2(x)-\left(\pm\frac{\lambda_2}{\sqrt{P}\,\lambda_1}+1\right)u(x)=0$

Let $A_2=\pm\frac{\lambda_2}{\sqrt{P}\,\lambda_1}+1$ Then $xu'(x)+u^2(x)-A_2u(x)=0$ ...(18)

The equations (17) and (18), are similar to equation [9], then the same method the solutions of them are given by :

$$v(y)=\frac{A_1}{2}\tanh\left(\frac{A_1}{2}\ln(c_1y)\right)+\frac{A_1}{2}\quad;\quad A_1=\pm\frac{\sqrt{\lambda_2^2+N}}{\sqrt{O}\,\lambda_1}i+1$$

And $u(x)=\frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2 x))+\frac{A_2}{2}$; $A_2=\pm\frac{\lambda_2}{\sqrt{P}\,\lambda_1}+1$

$$\text{So, } Z(x,y,t)=e^{\int\frac{\frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2x))+\frac{A_2}{2}}{x}dx+\int\frac{\frac{A_1}{2}\tanh(\frac{A_1}{2}\ln(c_1y))+\frac{A_1}{2}}{y}dy+\int\frac{\lambda_1}{t}dt} \quad ; c_2x, c_1y>0$$

$$=e^{\frac{A_2}{2}\ln x+\ln\left|\cosh(\frac{A_2}{2}\ln(c_2x))\right|+\frac{A_1}{2}\ln y+\ln\left|\cosh(\frac{A_1}{2}\ln(c_1y))\right|+\lambda_1\ln t+g} \quad ; c_2x, c_1y>0$$

$$=K\,x^{\frac{A_2}{2}}\,y^{\frac{A_1}{2}}\,t^{\lambda_1}(\cosh(\frac{A_2}{2}\ln(c_2x)))(\cosh(\frac{A_1}{2}\ln(c_1y))) \quad ; K=e^{g} \text{ and } c_2x, c_1y>0$$

$$=K\,x^{\pm\frac{\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2}}\,y^{\pm\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2}}\,t^{\lambda_1}\left(\cosh((\pm\frac{\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2})\ln(c_2x))\right)\left(\cosh((\pm\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2})\ln(c_1y))\right)$$

Then the complete solution of the equation(16), is given by:

$$\left[Z(x,y,t)-K\,x^{\frac{\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2}}\,y^{\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2}}\,t^{\lambda_1}\left(\cosh((\frac{\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2})\ln(c_2x))\right)\left(\cosh((\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2})\ln(c_1y))\right)\right]$$
$$\left[Z(x,y,t)-K\,x^{-\frac{\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2}}\,y^{-\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2}}\,t^{\lambda_1}\left(\cosh((\frac{-\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2})\ln(c_2x))\right)\left(\cosh((\frac{-\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2})\ln(c_1y))\right)\right]$$
$$\left[Z(x,y,t)-K\,x^{\frac{-\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2}}\,y^{\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2}}\,t^{\lambda_1}\left(\cosh((\frac{-\lambda_2}{2\sqrt{P}\,\lambda_1}+\frac{1}{2})\ln(c_2x))\right)\left(\cosh((\frac{\sqrt{\lambda_2^2+N}}{2\sqrt{O}\,\lambda_1}i+\frac{1}{2})\ln(c_1y))\right)\right]=0$$

Where $K, \lambda_1, \lambda_2, c_1$ and c_2 are arbitrary constants.

Domain : $x>0 \quad y>0, \quad -\infty<t<\infty$.

<u>Case(3)-b-</u>: By using the assumption

$$Z(x,y,t)=e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$, we get

$$Z_x=\frac{u(x)}{x}e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt} \Rightarrow Z_{xxy}=\frac{v(y)}{y}\left(\frac{xu'(x)+u^2(x)-u(x)}{x^2}\right)e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$

ISSN 2347-1921

$$Z_{yyt}=\frac{w(t)}{t}\left(\frac{yv'(y)+v^2(y)-v(y)}{y^2}\right)e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$

$$Z_{ytt}=\frac{v(y)}{y}\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$

$$Z_{xyt}=\frac{u(x)}{x}\frac{v(y)}{y}\frac{w(t)}{t}e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$

$$Z_{xyy}=\frac{u(x)}{x}\left(\frac{yv'(y)+v^2(y)-v(y)}{y^2}\right)e^{\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt}$$

And using $Z_t, Z_y, Z_{xt}, Z_{xx}, Z_{yy}, Z_{yt}, Z_{xy}$ and Z_{xtt} from case(2)-a-,-b- ,-c- and case(3)-a-then the equation

$$\mathrm{D}\,x^4y^2t^2Z_t^2Z_{xxy}^2+\mathrm{E}\,x^4y^2t^2Z_y^2Z_{xxt}^2+\mathrm{F}\,x^2y^4t^2Z_x^2Z_{yyt}^2+\mathrm{G}\,x^2y^2t^4Z_y^2Z_{xtt}^2+\mathrm{H}\,x^2y^2t^4Z_x^2Z_{ytt}^2+$$
$$\mathrm{I}\,x^2y^2t^2Z^2Z_{xyt}^2+\mathrm{J}\,x^2y^4t^2Z_t^2Z_{xyy}^2+\mathrm{K}\,x^4y^2t^2Z_{yt}^2Z_{xx}^2+\mathrm{L}\,x^2y^4t^2Z_{xt}^2Z_{yy}^2+\mathrm{M}\,x^2y^2t^4Z_{xy}^2Z_{tt}^2+$$
$$\mathrm{N}\,x^2y^2t^2Z_t^2Z_{xy}^2+\mathrm{O}\,x^2y^2t^2Z_y^2Z_{xt}^2+\mathrm{P}\,x^2y^2t^2Z_x^2Z_{yt}^2=0$$

Transforms to the form

$$\Bigg[\mathrm{D}x^4y^2t^2\left(\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\left(\frac{xu'(x)+u^2(x)-u(x)}{x^2}\right)^2\right)+\mathrm{E}x^4y^2t^2\left(\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\left(\frac{xu'(x)+u^2(x)-u(x)}{x^2}\right)^2\right)+$$
$$\mathrm{F}\,x^2y^4t^2\left(\frac{u^2(x)}{x^2}\frac{w^2(t)}{t^2}\left(\frac{yv'(y)+v^2(y)-v(y)}{y^2}\right)^2\right)+\mathrm{G}x^2y^2t^4\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)^2\right)+$$
$$\mathrm{H}\,x^2y^2t^4\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)^2\right)+\mathrm{I}x^2y^2t^2\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\right)+$$
$$\mathrm{J}x^2y^4t^2\left(\frac{u^2(x)}{x^2}\frac{w^2(t)}{t^2}\left(\frac{yv'(y)+v^2(y)-v(y)}{y^2}\right)^2\right)+\mathrm{K}x^4y^2t^2\left(\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\left(\frac{xu'(x)+u^2(x)-u(x)}{x^2}\right)^2\right)+$$
$$\mathrm{L}x^2y^4t^2\left(\frac{u^2(x)}{x^2}\frac{w^2(t)}{t^2}\left(\frac{yv'(y)+v^2(y)-v(y)}{y^2}\right)^2\right)+Mx^2y^2t^4\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\left(\frac{tw'(t)+w^2(t)-w(t)}{t^2}\right)^2\right)+$$
$$Nx^2y^2t^2\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\right)+Ox^2y^2t^2\left(\frac{v^2(y)}{y^2}\frac{u^2(x)}{x^2}\frac{w^2(t)}{t^2}\right)+$$
$$Px^2y^2t^2\left(\frac{u^2(x)}{x^2}\frac{v^2(y)}{y^2}\frac{w^2(t)}{t^2}\right)\Bigg]\,e^{4[\int\frac{u(x)}{x}dx+\int\frac{v(y)}{y}dy+\int\frac{w(t)}{t}dt]}=0$$

Since $e^{4[\int \frac{u(x)}{x}dx+\int \frac{v(y)}{y}dy+\int \frac{w(t)}{t}dt]} \neq 0$

$$\begin{aligned}\text{So, } & \mathrm{D}\left(v^2(y)w^2(t)(xu'(x)+u^2(x)-u(x))^2\right)+\mathrm{E}\left(v^2(y)w^2(t)(xu'(x)+u^2(x)-u(x))^2\right)+\\ & \mathrm{F}\left(u^2(x)w^2(t)(yv'(y)+v^2(y)-v(y))^2\right)+\mathrm{G}\left(u^2(x)v^2(y)(tw'(t)+w^2(t)-w(t))^2\right)+\\ & \mathrm{H}\left(u^2(x)v^2(y)(tw'(t)+w^2(t)-w(t))^2\right)+\mathrm{I}\left(u^2(x)v^2(y)w^2(t)\right)+\\ & \mathrm{J}\left(u^2(x)w^2(t)(yv'(y)+v^2(y)-v(y))^2\right)+\mathrm{K}\left(v^2(y)w^2(t)(xu'(x)+u^2(x)-u(x))^2\right)+\\ & \mathrm{L}\left(u^2(x)w^2(t)(yv'(y)+v^2(y)-v(y))^2\right)+M\left(u^2(x)v^2(y)(tw'(t)+w^2(t)-w(t))^2\right)+\\ & \mathrm{N}\left(u^2(x)v^2(y)w^2(t)\right)+\mathrm{O}\left(u^2(x)v^2(y)w^2(t)\right)+\mathrm{P}\left(u^2(x)v^2(y)w^2(t)\right)=0 \qquad \text{...(19)}\end{aligned}$$

This equation is variable separable equation [4],we get

$$(\mathrm{D}+\mathrm{E}+\mathrm{K})\left(\frac{xu'(x)+u^2(x)-u(x)}{u(x)}\right)^2+(\mathrm{F}+\mathrm{J}+\mathrm{L})\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2+$$
$$(\mathrm{G}+\mathrm{H}+M)\left(\frac{tw'(t)+w^2(t)-w(t)}{w(t)}\right)^2+(\mathrm{I}+\mathrm{N}+\mathrm{O}+\mathrm{P})=0$$

Let $(\mathrm{D}+\mathrm{E}+\mathrm{K})\left(\frac{xu'(x)+u^2(x)-u(x)}{u(x)}\right)^2=\lambda_1^2$ and $(\mathrm{F}+\mathrm{J}+\mathrm{L})\left(\frac{yv'(y)+v^2(y)-v(y)}{v(y)}\right)^2=\lambda_2^2$

Therefore $(\mathrm{G}+\mathrm{H}+M)\left(\frac{tw'(t)+w^2(t)-w(t)}{w(t)}\right)^2+(\mathrm{I}+\mathrm{N}+\mathrm{O}+\mathrm{P})=-(\lambda_1^2+\lambda_2^2)$

$$\Rightarrow xu'(x)+u^2(x)-\left(\pm\frac{\lambda_1}{\sqrt{\mathrm{D}+\mathrm{E}+\mathrm{K}}}+1\right)u(x)=0$$

$$yv'(y)+v^2(y)-\left(\pm\frac{\lambda_2}{\sqrt{\mathrm{F}+\mathrm{J}+\mathrm{L}}}+1\right)v(y)=0$$

$$tw'(t)+w^2(t)-\left(\pm\sqrt{\frac{\mathrm{I}+\mathrm{N}+\mathrm{O}+\mathrm{P}+\lambda_1^2+\lambda_2^2}{\mathrm{G}+\mathrm{H}+M}}\,i+1\right)w(t)=0$$

Let $\mathrm{A}_1=\pm\frac{\lambda_1}{\sqrt{\mathrm{D}+\mathrm{E}+\mathrm{K}}}+1 \quad \Rightarrow xu'(x)+u^2(x)-\mathrm{A}_1u(x)=0$ ...(20)

And $A_2 = \pm\frac{\lambda_2}{\sqrt{F+J+L}} + 1 \quad \Rightarrow yv'(y) + v^2(y) - A_2 v(y) = 0$ …(21)

Also $A_3 = \pm\sqrt{\frac{I+N+O+P+\lambda_1^2+\lambda_2^2}{G+H+M}}\; i+1 \quad \Rightarrow tw'(t) + w^2(t) - A_3 w(t) = 0$ …(22)

The equations (20) , (21) and (22) are similar to equation [9], then the same method the solutions of them are given by :

$$u(x) = \frac{A_1}{2}\tanh(\frac{A_1}{2}\ln(c_1 x)) + \frac{A_1}{2} \quad ; \; A_1 = \pm\frac{\lambda_1}{\sqrt{D+E+K}} + 1 \qquad ; \; c_1 x > 0$$

$$v(y) = \frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2 y)) + \frac{A_2}{2} \quad ; \; A_2 = \pm\frac{\lambda_2}{\sqrt{F+J+L}} + 1 \qquad ; \; c_2 y > 0$$

And $w(t)=\frac{A_3}{2}\tanh(\frac{A_3}{2}\ln(c_3 t))+\frac{A_3}{2}$; $A_3=\pm\sqrt{\frac{I+N+O+P+\lambda_1^2+\lambda_2^2}{G+H+M}\,i+1}$; $c_3 t>0$

So, $$Z(x,y,t)=e^{\int\frac{\frac{A_1}{2}\tanh(\frac{A_1}{2}\ln(c_1x))+\frac{A_1}{2}}{x}dx+\int\frac{\frac{A_2}{2}\tanh(\frac{A_2}{2}\ln(c_2y))+\frac{A_2}{2}}{y}dy+\int\frac{\frac{A_3}{2}\tanh(\frac{A_3}{2}\ln(c_3t))+\frac{A_3}{2}}{t}dt}$$; $c_1x, c_2y, c_3t>0$

$$=e^{\frac{A_1}{2}\ln x+\ln\left|\cosh(\frac{A_1}{2}\ln(c_1x))\right|+\frac{A_2}{2}\ln y+\ln\left|\cosh(\frac{A_2}{2}\ln(c_2y))\right|+\frac{A_3}{2}\ln t+\ln\left|\cosh(\frac{A_3}{2}\ln(c_3t))\right|+g}$$; $c_1x, c_2y, c_3t>0$

$$=K\,x^{\frac{A_1}{2}}\,y^{\frac{A_2}{2}}\,t^{\frac{A_3}{2}}\,(\cosh(\frac{A_1}{2}\ln(c_1x)))(\cosh(\frac{A_2}{2}\ln(c_2y)))$$
$$(\cosh(\frac{A_3}{2}\ln(c_3t)))$$; $K=e^g$ and $c_1x, c_2y, c_3t>0$

$$=K\,x^{\pm\frac{\lambda_1}{2\sqrt{D+E+K}}+\frac{1}{2}}\,y^{\pm\frac{\lambda_2}{2\sqrt{F+J+L}}+\frac{1}{2}}\,t^{\pm\sqrt{\frac{I+N+O+P+\lambda_1^2+\lambda_2^2}{4(G+H+M)}\,i+\frac{1}{2}}}$$
$$\left(\cosh(\pm\frac{\lambda_1}{2\sqrt{D+E+K}}+\frac{1}{2})\ln(c_1x))\right)\left(\cosh((\pm\frac{\lambda_2}{2\sqrt{F+J+L}}+\frac{1}{2})\ln(c_2y))\right)$$
$$\left(\cosh((\pm\sqrt{\frac{I+N+O+P+\lambda_1^2+\lambda_2^2}{4(G+H+M)}\,i+\frac{1}{2}})\ln(c_3t))\right)$$; $K=e^g$ and $c_1x, c_2y, c_3t>0$

Then the complete solution of the equation(19), is given by:

$$\left[Z(x,y,t)-\mathrm{K}\,x^{\frac{\lambda_1}{2\sqrt{\mathrm{D+E+K}}}+\frac{1}{2}}\,y^{\frac{\lambda_2}{2\sqrt{\mathrm{F+J+L}}}+\frac{1}{2}}\,t^{\sqrt{\frac{\mathrm{I+N+O+P}+\lambda_1^2+\lambda_2^2}{4(\mathrm{G+H}+M)}}\,i+\frac{1}{2}}\left(\cosh(\frac{\lambda_1}{2\sqrt{\mathrm{D+E+K}}}+\frac{1}{2})\ln(\mathrm{c}_1 x))\right)\right.$$

$$\left.\left(\cosh((\frac{\lambda_2}{2\sqrt{\mathrm{F+J+L}}}+\frac{1}{2})\ln(\mathrm{c}_2 y))\right)\left(\cosh((\sqrt{\frac{\mathrm{I+N+O+P}+\lambda_1^2+\lambda_2^2}{4(\mathrm{G+H}+M)}}\,i+\frac{1}{2})\ln(\mathrm{c}_3 t))\right)\right]$$

$$\left[Z(x,y,t)-\mathrm{K}\,x^{-\frac{\lambda_1}{2\sqrt{\mathrm{D+E+K}}}+\frac{1}{2}}\,y^{-\frac{\lambda_2}{2\sqrt{\mathrm{F+J+L}}}+\frac{1}{2}}\,t^{-\sqrt{\frac{\mathrm{I+N+O+P}+\lambda_1^2+\lambda_2^2}{4(\mathrm{G+H}+M)}}\,i+\frac{1}{2}}\left(\cosh(\frac{-\lambda_1}{2\sqrt{\mathrm{D+E+K}}}+\frac{1}{2})\ln(\mathrm{c}_1 x))\right)\right.$$

$$\left.\left(\cosh((\frac{-\lambda_2}{2\sqrt{\mathrm{F+J+L}}}+\frac{1}{2})\ln(\mathrm{c}_2 y))\right)\left(\cosh((-\sqrt{\frac{\mathrm{I+N+O+P}+\lambda_1^2+\lambda_2^2}{4(\mathrm{G+H}+M)}}\,i+\frac{1}{2})\ln(\mathrm{c}_3 t))\right)\right]$$

$$\left[Z(x,y,t)-\mathrm{K}\,x^{\frac{\lambda_1}{2\sqrt{\mathrm{D+E+K}}}+\frac{1}{2}}\,y^{\frac{-\lambda_2}{2\sqrt{\mathrm{F+J+L}}}+\frac{1}{2}}\,t^{\sqrt{\frac{\mathrm{I+N+O+P}+\lambda_1^2+\lambda_2^2}{4(\mathrm{G+H}+M)}}\,i+\frac{1}{2}}\left(\cosh(\frac{\lambda_1}{2\sqrt{\mathrm{D+E+K}}}+\frac{1}{2})\ln(\mathrm{c}_1 x))\right)\right.$$

$$\left.\left(\cosh((\frac{-\lambda_2}{2\sqrt{\mathrm{F+J+L}}}+\frac{1}{2})\ln(\mathrm{c}_2 y))\right)\left(\cosh((\sqrt{\frac{\mathrm{I+N+O+P}+\lambda_1^2+\lambda_2^2}{4(\mathrm{G+H}+M)}}\,i+\frac{1}{2})\ln(\mathrm{c}_3 t))\right)\right]=0$$

Where $\mathrm{K}, \lambda_1, \lambda_2, \mathrm{c}_1, \mathrm{c}_2$ and c_3 are arbitrary constants.

Domain : $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < t < \infty$.

ACKNOWLEDGMENTS

The authors would like to thank the referee for his useful suggestions which helped us improve the exposition. Also, they wish to thank University of Kufa for supporting the scientific research.

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