ISSN 2347-1921

Mathematical modeling of infectious disease and designing vaccination law for control of this diseases

Javad Khaligh *, Aghileh Heydari **, Aliakbar Heydari***, Siroos Nazari*,

*Department of mathematics, PNU,Tehran ,Iran

khaligh@phd.pnu.ir

nazari@phd.pnu.ir

**Department of mathematics, Payame Noor University, Mashhad , Iran

a_heidari@pnu.ac.ir,

*** Department of Medical Sciences, Mashad University of Medical Sciences, Iran

heydariaa@mums.ac.ir

ABSTRACT

In this paper, we propose the concept of partial stability instead of that of global stability to deal with the stability issues of epidemic models. The partial stability is able to provide a more meaningful analysis of the problem since it only focuses on the behavior of some of the variables (infected and infectious) instead of the complete population. It has been shown that the vaccination free SEIR model can still be partially stable even when a globally stability property does not hold, for two types of nonlinear incidence rates. By introducing the concept of partial stability and by designing a control vaccination based on it. Guarantee the eradication of an epidemic disease without requiring the global stability of the epidemic model.

Keywords

sliding mode control, SEIR disease model, vaccination strategy, infectious disease, Control signal...

Academic Discipline And Sub-Disciplines

determining appropriate suppliers for supply chain

SUBJECT CLASSIFICATION

68T05, 68T40, 68T45, 68N20, 68N30, 68N99, 68U07, 968U20 , 68U35, 68P10

TYPE (METHOD/APPROACH)

determining appropriate suppliers for supply chain with using genetic algorithm

Council for Innovative Research

Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN MATHEMATICS

Vol. 9, No. 5

www.cirjam.com, editorjam@gmail.com

ISSN 2347-1921

INTRODUCTION

In the 18th and 19th centuries infectious epidemics increased due to the creation of large population centers. The trend reversed in the 20th century due to the administration of vaccines. There exists a wide variety of mathematical models, e.g., deterministic vs. stochastic or distributed vs. lumped. However, infectious diseases are still important factors in causing souring and mortality in developing countries. Deferent types of vaccination laws based on Control Theory have appeared in the literature during the last years. In addition, pulse vaccination has also gained much attention during the last decade, [1], mainly for its successful application in the eradication of poliomyelitis and measles across Central and South America, [3]. Nevertheless, all these works assume the parameters dining the epidemics, such as the nativity and mortality rates or the incidence rate to be known. This is a rather unrealistic situation since the estimation of such parameters from experimental data may lead to highly inaccurate values. As a consequence, the vaccination law is miscalculated and its application to the actual system may prevent the illness to be eradicated, as desired. Hence, the objective of this paper is to design a robust vaccination law capable of overcoming the potential mismatch between the nominal values used to design the control law and the actual parameters of the system. [3]

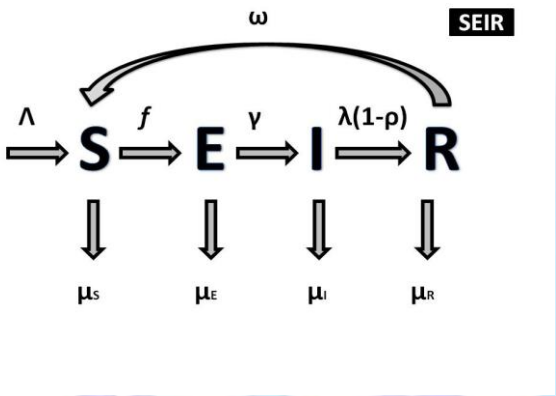

Figure 1: SEIR model

:

Problem Formulation

Consider the SEIR epidemic model given by:

$$\begin{cases} \dot{S}(t) = -\mu S(t) + \omega R(t) - \varphi(S,E,I,R) + \nu N(t) - N(t)V(t) \\ \dot{E}(t) = \varphi(S,E,I,R) - (\mu+\sigma)E(t) \\ \dot{I}(t) = -(\mu+\gamma)I(t) + \sigma E(t) \\ \dot{R}(t) = -(\mu+\omega)R(t) + \gamma I(t) + N(t)V(t) \end{cases} \quad (1)$$

The dynamics of the total population is given by:

$$\dot{N}(t) = (\nu - \mu)N(t)$$

When $\nu = \mu$ the total population is constant and if $\nu < \mu$ the total population decreases and if $\nu > \mu$ the total population increases.

ISSN 2347-1921

Parameter	Definition
$S(t)$	Susceptible
$E(t)$	Infectious
$I(t)$	Infective
$R(t)$	Recovery
μ	The rate of deaths from causes unrelated to the infection
ν	The natality rate
ω	The rate of losing immunity
$g(S,E,I,R)$	The disease incidence rate
σ^{-1}	The average durations of the latent periods
γ^{-1}	The average durations of infective periods
$V(t)$,	The vaccination function

Also

$$g_1(S,E,I,R) = \beta \frac{S(t)\,I(t)}{N(t)}$$

$$g_2(S,E,I,R) = \beta \frac{S(t)\,I(t)}{1+\alpha\, S(t)}$$

Thus, if $\nu > \mu$ the global stability property of the epidemic model will not hold. However, from an epidemic point of view, it is not needed all the populations to be bounded, just the infected, E(t), and infectious, I(t). Hence, in this paper we propose to use the concept of partial stability to study the stability of the SEIR epidemic model instead of that of global stability.

Partial stability of systems

Consider a nonlinear dynamic system $\dot{x}(t) = f(x)$ with state vector $x(t)$ decomposed in the form:

$$x(t)^T = [x(t)^T \; z(t)^T]$$

Consider also that the origin $x(t)^T = 0 = [0 \;\; 0]$ is an equilibrium point. Then, the concept of partial stability reads:

Definition.

An equilibrium position x = 0 of the above system is:

i) locally y-stable if for any $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ such that $\|x_0\| < \delta$ implies $\|y(t)\| < \varepsilon$ for all $t \geq t_0$ with x_0 denoting the initial condition,

ii) locally asymptotically y-stable if it is locally y-stable and further more $y(t) \to 0$ as $t \to \infty$

iii) globally asymptotically y-stable if the asymptotically y-stability holds for any bounded initial condition $\|x_0\|$

In this paper to study the partial stability of the vaccination-free SEIR model with respect to E(t) and I(t). For this, consider $y^T = [E\ I]$ and $z^T = [S\ R]$. This property will be referred to as (E; I)-stability. To design a feedback-type vaccination control law guaranteeing the partial (E; I)-stability of the epidemic model and the convergence of the infected and infectious to zero, eradicating the illness. To perform these goals, the following theorem will be used:

ISSN 2347-1921

Theorem 1. For the system

$$\dot{x}(t) = f(x)$$

assume that there exists a continuous function L with continuous partial derivatives satisfying:

$$L(0) = 0$$

$$L(x) \geq a(\|y\|)$$

$$\dot{L}(x) \leq 0$$

Where $a(.)$ denotes any continuous strictly increasing function with argument $\|y(t)\|$. Then, the equilibrium position $x = 0$ of the nonlinear system is y-stable.

Partial stability of the vaccination-free model

Theorem 2. The nonlinear SEIR epidemic model is (E; I)-stable forany set of positive parameters with $\alpha \geq 1$and incidence rate $g_1$or g_2 provided that $(\mu + \sigma)(\mu + \gamma) - \beta\sigma \geq 0$

Since N = S+E+I +R, then $\frac{S}{N} \leq 1$ and $g_1(S, E, I, R) = \beta I \frac{S}{N} \leq \beta I$. In addition, since $\alpha \geq 1$ then $1 + \alpha S > S$ and $\frac{S}{1+\alpha S} < 1$. Hence,

$$g_2(S, E, I, R) = \beta I \frac{S}{1 + \alpha S} \leq \beta I$$

Thus, regardless the incidence rate, we have:

$$\dot{E}(t) \leq \beta I(t) - (\mu + \sigma)E(t)$$

Now, we can analyze the stability of the system given by:

$$\dot{E}(t) \leq \beta I(t) - (\mu + \sigma)E(t)$$

$$\dot{I}(t) = -(\mu + \sigma)I(t) + \sigma E(t)$$

Its stability can be stated by analyzing the stability of the dynamics matrix:

$$A = \begin{bmatrix} -(\mu + \sigma) & \beta \\ \sigma & -(\mu + \gamma) \end{bmatrix}$$

whose characteristic equation is:

$$\det(sI - A) = s^2$$

$$+(2\mu + \sigma + \gamma)s + (\mu + \sigma)(\mu + \gamma) - \beta\sigma$$

According to the Routh-Hurwitz criterion, all the coe_cients must be non-negative in order to make the dynamics matrix stable. $(2\mu + \sigma + \gamma)$ is trivially positive and we only need to require:

$$(\mu + \sigma)(\mu + \gamma) - \beta\sigma \geq 0$$

Notice that the system may still be (E; I)-stable despite $\nu > \mu$, which implies that some of the other variables diverge. The above SEIR system is (E; I)-stable for both incidence rates provided that all the parameters are positive,$\alpha \geq 1$ and $\mu + \gamma \geq \beta$. We have stated some conditions under which the (E; I)-stability of the vaccination-free model holds. The next step is to design a vaccination (control) law able to (E; I)-stabilize the SEIR model under any condition and to eradicate the infective and infectious.

ISSN 2347-1921

Vaccination Law

Theorem The vaccination law given by:

$$V(t) = 1 + \frac{\omega}{\upsilon N(t)} R(t)$$

(E; I)-stabilizes the system for any set of positive parameters and any incidence rate. Furthermore, $S(t), E(t), I(t) \to 0$ as $t \to \infty$

However, before proving this theorem, we need a preliminary result guaranteeing the positivity of all the variables under the above vaccination law.

Theorem (Positivity of the closed-loop)

The solution of the SEIR model under the above presented vaccination law satisfies , $S(t), E(t), I(t), R(t) \geq 0$ for all $t \geq 0$ for any incidence rate and any set of positive parameters provided that , $S(0), E(0), I(0), R(0) \geq 0$

Once we have proved the positivity of all the variables under the presented control law, we are able to proof the stability theorem.

proof stability theorem.)

Consider the partially positive function:

$$L(t) = S(t) + E(t) + I(t) \geq 0$$

Its time derivative is calculated as:

$$\dot{L}(t) = \dot{S}(t) + \dot{E}(t) + \dot{I}(t)$$
$$= -\mu S(t) + \omega R(t) + \upsilon N(t)(1 - V(t))$$
$$-\mu E(t) - (\mu + \gamma) I(t)$$

If we introduce

$$V(t) = 1 + \frac{\omega}{\upsilon N(t)} R(t)$$

in the above equation we obtain:

$$\dot{L}(t) = -\mu S(t) - \mu E(t) - (\mu + \gamma) I(t) \leq 0$$

Therefore, $S(t), E(t)$ and $I(t)$ are bounded for all time. Furthermore, while any of the variables, S;E; I is positive,

$$\dot{L}(t) < 0$$

, implying that L decreases continually until it arrives to

$$, S = E = I = 0$$

simulation

These parameters have been chosen so as to illustrate the theoretical results in a short-time simulation The total simulation period is 50 days. The initial conditions are given by

$$S(0) = 400, E(0) = 150 , I(0) = 250, R(0) = 200$$

The actual parameters of the model are given by:

$$\sigma^{-1} = 2.2\, days^{-1}, \quad \mu^{-1} = 200\ days^{-1}$$

$$\omega^{-1} = 15\, days^{-1}, \quad \gamma = \sigma$$

ISSN 2347-1921

$$\nu^{-1} = 150\ days^{-1},\ \beta = 1.66\ days^{-1}$$

$$g = g_1 = 1.66\ days^{-1}$$

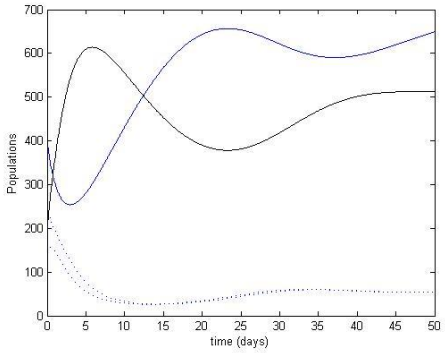

Figure 1: Dynamics of the system without vaccination.

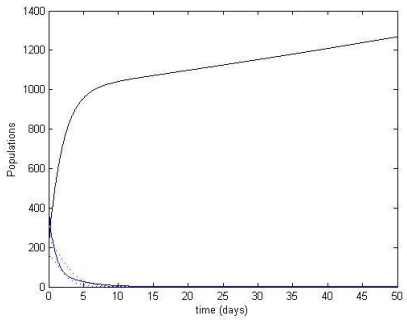

Figure 2: Dynamics of the system with the proposed controller.

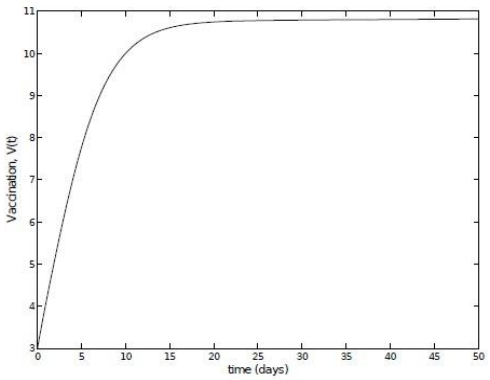

Figure 3: vaccination function

Conclusions

In this paper proposed to use the concept of partial stability instead of that of global stability to deal with the stability issues of epidemic models. The partial stability is able to provide a more meaningful analysis of the problem since it only focuses on the behavior of some of the variables (infected and infectious) instead of the complete population. It has been shown that the vaccination-free SEIR model can still be partially stable even when a globally stability property does not hold, for two typesof nonlinear incidence rates. A feedback-type vaccination control law has been designed from the concept of partial stability through adapted Lyapunov-type methods.

References

[1] X. Liu, and P. Stechlinski , Infectious disease models with timevarying parameters and general nonlinear incidence rate. Applied Mathematical Modelling , 36 (2012), 19741994 .

[3] Z. Jin, M. Haque, Q. Liu, Pulse vaccination in the periodic infection rate SIR epidemic model Int. J. Biomath,. Vol.54 (2006).

[4] X. Meng, L. Chen, " The dynamics of a new sir epidemic model concerning pulse vaccination strategy, Appl. Math. Comput.,. Vol.192 (2008)58

[5] Wang, C.C., Su, J.P.: A new adaptive variable structure control for chaotic synchronization and secure communication, Chaos, Solitons & Fractals. 20, 967–77 (2004)

[6] Li, Z., Shi, S.: Robust adaptive synchronization of Rossler and Chen chaotic systems via slide technique. Phys. Lett. A. 311, 389–95 (2003)

[7] Li, Y. Yung, Z. , Xu, Y.: Chaos synchronization and parameter estimation of single-degree of freedom oscillators via adaptive control. J. Sound. Vib. 329, 973–9 (2010)

[8]Li, W., Liu, Z., Miao, J.: Adaptive synchronization for a unified chaotic system with uncertainty. Commun Nonlinear. Sci Numer. Simul. 15, 2015–2021 (2010)

[9] Mu, X., Pei,L.: Synchronization of the near-identical chaotic systems with the unknown parameters. Appl. Math. Model. 34, 1788–97 (2010)

[10] Nazari, S., Heydari, A., Khaligh, J.: Modified modeling of the heart by applying nonlinear oscillators and designing proper control signal. Applied Mathematics. 4, 972-978 (2013)