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# INVENTORY MODEL OF DETERIORATING ITEMS FOR NONLINEAR HOLDING COST WITH TIME DEPENDENT DEMAND

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## ABSTRACT

The objective of this model is to investigate the inventory system for perishable items where time proportional deterioration rate is considered. The Economic order quantity is determined for minimizing the average total cost per unit time. Time dependent demand rate is used with finite time horizon. Nonlinear holding cost with shortage is considered. The result is illustrated with numerical example.

## Indexing terms/Keywords

Time dependent Demand; Optimal control; Inventory system; Shortage.

## Academic Discipline And Sub-Disciplines

Mathematics,Inventory Management

## SUBJECT CLASSIFICATION

$90B05$

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## INTRODUCTION

Deterioration is defined as decay or damage such that the item cannot be used for its original purposes. These items widely exist in our daily life, such as fresh products, fruits, vegetables, seafood, etc, which decrease in quantity or utility during their delivery and storage stage periods. To improve products quality and reduce deteriorating loss in the fashion goods supply chain, the emphasis is on the whole process life cycle management which includes production, storage, transportation, retailing etc. Accompany by the increasing of the variety and quantity of the deteriorating and fashion goods, consumers' appetite for high quality perishable items are continually upgrading rapidly, so the topic of deteriorating inventory system management has became popular in the field of research and business.

Several researchers have studied stock deterioration over the years. Ghare and Schrader [3] were among the first authors to consider the role of deterioration in inventory modeling. Other authors, such as Covert and Philip [1], Kang and Kim [4] and Raafat et.al. [5], assumed either instantaneous re-supply or a finite production rate with different assumptions on how deterioration occurred. Wee [7] studied a replenishment policy with price-dependent demand and a varying rate of deterioration.

In the early 1970s, Silver and Meal [6] proposed an approximate solution procedure for the general case of a deterministic, time-varying demand pattern. Donaldson [2] then considered an inventory model with a linear trend in demand. After Donaldson, numerous research works have been carried out incorporating time varying demand into inventory models under a variety of circumstances.

In this paper an attempt has been made to develop an inventory model for perishable items with time proportional deterioration rate and the Time dependent demand pattern is used over a finite planning horizon. Nature of the model is discussed for shortage state. Optimal solution for the proposed model is derived and the applications are investigated with the help of numerical example.

## ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposal model:

i. Single inventory will be used.
ii. Lead time is zero.
iii. Shortages are allowed and are completely backlogged.
iv. Time dependent Demand rate is considered.
v. Deterioration is considered to be Time proportional.
vi. Replenishment rate is infinite but size is finite.
vii. Time horizon is finite.
viii. There is no repair of deteriorated items occurring during the cycle.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time $t$, $t \geq 0$.

$R(t) = a\,e^{bt}$ is the time dependent demand rate at any time $t \geq 0, a \geq 0, 0 < b < 1.$

$\theta(t)$ = Instantaneous rate of deterioration of the on-hand inventory given by $\alpha \cdot t$ where $0 < \alpha < 1$ .

$I(0) = Q$ =Inventory at time $t = 0$.

$S$ = Inventory at time $t = T$.

$T$ = Duration of a cycle.

$A$ = The Ordering cost per order.

$c_p$ = The purchase cost per unit item.

$c_d$ = The deterioration cost per unit item.

$H(t)$ = The holding cost per unit item per unit time, $H(t) = l\,e^{mt}$..

$c_b =$ The shortage cost per unit item.

$\gamma\, c_p =$ The salvage value associated to the deteriorated units during the cycle time

where $0 \le \gamma < 1$.

$U$ =The total average cost of the system.

## FORMULATION

The amount of stock starts to deplete in the period $[0, t_1]$ due to the combined effect of demand and deterioration. By this process, the stock reaches zero at time $t_1$ . Hence, the inventory level at any instant of time during $[0, t_1]$ is described as follows.

If $I(t)$ be the on-hand inventory at time $t \ge 0$ , then at time $t + \Delta t$ , the on-hand inventory in the interval $[0, t_1]$ will be

$$I(t + \Delta t) \;= I(t) - \theta\, I(t)\, \Delta t - R(t)\, \Delta t$$

Dividing by $\Delta t$ and then taking as $\Delta t \to 0$ we get

**(1)** $$\frac{dI(t)}{dt} + \alpha\, t\, I(t) = -a\, e^{b\,t}; \quad 0 \le t \le t_1$$

The boundary conditions are $I(0) = Q,\, I(t_1) = 0.$

For the next interval $[t_1, T]$, we have

$$I(t + \Delta t) \;= I(t) - R(t)\, \Delta t$$

Dividing by $\Delta t$ and then taking as $\Delta t \to 0$ we get

**(2)** $$\frac{dI(t)}{dt} = -a\, e^{b\,t}; \quad t_1 \le t \le T$$

The boundary conditions are $I(t_1) = 0, I(T) = S.$

On solving equation **(1)** with boundary condition $I(t_1) = 0$ we have

**(3)** $$I(t) = \left(1 - \frac{\alpha\, t^2}{2}\right)\left[t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 - \left(t + \frac{\alpha}{6} t^3 + \frac{b}{2} t^2\right)\right]; \; 0 \le t \le t_1$$

On solving equation **(2)** with boundary condition $I(t_1) = 0.$ we have

**(4)** $$I(t) = \frac{a}{b}\left\{e^{b\,t_1} - e^{b\,t}\right\} \qquad ; \quad t_1 \le t \le T$$

Using $I(T) = S$ in equation **(4)** we have

**(5)** $$S = \frac{a}{b}\left\{e^{b\,t_1} - e^{b\,T}\right\}$$

Using $I(0) = Q$ in equation **(3)** we have

**(6)** $$Q = \left[t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2\right]$$

The number of deteriorated units during one cycle time is given by.

**(7)** $$D(T) = Q - \int_0^T R(t)\,dt = t_1 + \frac{\alpha}{6}t_1^3 + \frac{b}{2}t_1^2 - \frac{a}{b}\left(e^{bT} - 1\right).$$

From of equation **(4)** amount of shortage during the time interval $[t_1, T]$ is

**(8)** $$\int_{t_1}^{T} I\,dt = \frac{a}{b}\left\{ e^{bt_1}(T - t_1) - \frac{1}{b}(e^{bT} - e^{bt_1}) \right\}$$

Using the above equations into consideration the different costs will be as follows.

1. **Purchasing cost per cycle**

**(9)** $$C_p\, I(0) = C_p\left[ t_1 + \frac{\alpha}{6}t_1^3 + \frac{b}{2}t_1^2 \right]$$

2. **Holding cost per cycle**

**(10)** $$\int_0^{t_1} H(t)\, I(t) dt = l\left[ t_1 - \frac{1}{2}t_1^2 + \left(\frac{m-b}{6}\right)t_1^3 + \left(\frac{2\alpha + bm}{8}\right)t_1^4 + \left(\frac{16\, b\,\alpha - 13\, m\,\alpha}{120}\right)t_1^5 - \frac{m\, b\,\alpha}{48}\, t_1^6 \right]$$

3. **Deterioration cost per cycle**

**(11)** $$C_d\ D(T) = C_d\left\{ t_1 + \frac{\alpha}{6}t_1^3 + \frac{b}{2}t_1^2 - \frac{a}{b}\left(e^{bT} - 1\right) \right\}$$

**4.Shortage cost per cycle**

**(12)** $$-C_b\int_{t_1}^{T} I\,dt = -\frac{a\,C_b}{b}\left\{ e^{bt_1}(T - t_1) - \frac{1}{b}(e^{bT} - e^{bt_1}) \right\}$$

5. **Ordering cost**

**(13)** $OC = A$

6. **Salvage value of deteriorated items**

**(14)** $$SV = \gamma\, C_p\left\{ t_1 + \frac{\alpha}{6}t_1^3 + \frac{b}{2}t_1^2 - \frac{a}{b}\left(e^{bT} - 1\right) \right\}$$

The average total cost per unit time of the model will be

$$U(t_1) = \frac{1}{T}\Bigg[ C_p\left\{ t_1 + \frac{\alpha}{6}t_1^3 + \frac{b}{2}t_1^2 \right\} + \left(C_d - \gamma\, C_p\right)\left\{ t_1 + \frac{\alpha}{6}t_1^3 + \frac{b}{2}t_1^2 - \frac{a}{b}\left(e^{bT} - 1\right) \right\}$$

$$+\, l\left\{ t_1 - \frac{1}{2}t_1^2 + \left(\frac{m-b}{6}\right)t_1^3 + \left(\frac{2\alpha + bm}{8}\right)t_1^4 + \left(\frac{16\, b\,\alpha - 13\, m\,\alpha}{120}\right)t_1^5 - \frac{m\, b\,\alpha}{48}\, t_1^6 \right\} + A$$

$$-\frac{a\,C_b}{b}\left\{ e^{bt_1}(T - t_1) - \frac{1}{b}(e^{bT} - e^{bt_1}) \right\}\Bigg]$$

Now equation **(14)** can be minimized but as it is difficult to solve the problem by deriving a closed equation of the solution of equation **(14),** Matlab Software has been used to determine optimal $t_1^*$ and hence the optimal cost $U(t_1^*)$ can be evaluated.

## EXAMPLE

The values of the parameters are considered as follows:

$c_p = \$20/unit, c_b = \$12/unit/year, c_d = \$8/unit, \alpha = 0.1, a = 100, b = 0.2, \gamma = 0.1,$

$A = 50, l = 2, m = 0.7, T = 1\,Year.$

According to equation (3.14), we obtain the optimal $t_1^* = 0.4875$. In addition, the optimal $I^*(0) = 0.513$ units. Moreover, from equation (3.14), we have the minimum average total cost per unit time as $U^* = 291.663$

## CONCLUSION

Here an EOQ model is derived for perishable items with time dependent demand pattern. Time proportional deterioration rate is used. The model is studied for minimization of total average cost. Numerical example is used to illustrate the result.

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