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# AN EXTENSION OF SOME RESULTS DUE TO JARDEN

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## ABSTRACT

This paper defines some generalized Fibonacci and Lucas sequences which satisfy arbitrary order linear recurrence relations and which answer a problem posed by Jarden in 1966 about generalizing an elegant result for a connection between even and odd subscripted Fibonacci and Lucas numbers.

## Indexing terms/Keywords

product sums, shift operators, basic sequences, primordial sequences, fundamental sequences, Fibonacci sequences, Lucas sequences, characteristic equations, Cauchy calculus

## Academic Discipline And Sub-Disciplines

Mathematics: Number Theory

## SUBJECT CLASSIFICATION

11B75, 11Z05, 11B65

## TYPE (METHOD/APPROACH)

Properties of characteristic equations of arbitrary order difference equations are used to solve a long-standing question about generalizing a result which connects the second oreder recursive sequences whose elemenata are the Fibonacci and Lcas numbers

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Jarden [7, pp.87] extended the well-known relation between the second order Fibonacci and Lucas numbers, $F_{2n} = F_n L_n$, to the third order case. In doing so he showed "that third order sequences have another arithmetic than that of Lucas' sequences" [7, p.88]. This paper explores some arbitrary order extensions of Jarden's results.

In order to generalize the sequences of Horadam[14] and Jarden[7, pp.30,114] we define $r$ basic sequences of order $r$, $\{U_{s,n}^{(r)}\}$, by the recurrence relation which we define formally after the initial terms for $s$ = 1,2,...,$r$:

$$U_{s,n}^{(r)} = \sum_{j=1}^{r} U_{s,n-j}^{(r)}, \ n > r, \tag{1.1}$$

in which the $r$ initial terms, $n$ =0,1,2,...,$r$-1, are given by $U_{s,n}^{(r)} = \delta_{s,n}$, the Kronecker delta. We also consider $f(x)$ with zeros, $\alpha_i$, assumed distinct, to be the associated $r^{th}$ order characteristic polynomial. Examples of (1.1) appear in a later table. We also define the special sequence with $s$ = 0 $\{U_{0,n}^{(r)}\}$ [12] to satisfy (1.1) but with the $r$ initial terms defined by

$$U_{0,n}^{(r)} = \sum_{i=1}^{r} \alpha_i^n, \ 0 \le n < r \tag{1.2}$$

so that the first few values are, for example, $U_{0,0}^{(r)} = r, \ U_{0,1}^{(r)} = 1, \ U_{0,2}^{(r)} = 3$. The $\{U_{0,n}^{(r)}\}$ and $\{U_{r,n}^{(r)}\}$ correspond to Lucas' "primordial" and "fundamental" sequences respectively [14], while the other $\{U_{s,n}^{(r)}\}$ are "basic" sequences [8]. For notational convenience, $\{U_{s,n}^{(r)}\}$, $s \neq 0$, $U_{s,n}^{(r)}$=0, $n \le 0$.

## 2. A PROBLEM POSED BY JARDEN

Jarden[7, p.88] posed the problem to determine the values of $F_{2n} - F_n L_n = 0$ "for appropriate recurring sequences of higher order", or in the notation of this paper:

$$U_{s,2n}^{(r)} - U_{s,n}^{(r)} U_{0,n}^{(r)} \tag{2.1}$$

since $U_{2,2n}^{(2)} - U_{2,n}^{(2)} U_{0,n}^{(2)}$ "is of great importance for the arithmetic of Lucas' second order recurring sequences".

In order to do this we consider the sequence $\{U_{s,2n}^{(r)}\}$ as a bisection of the original sequence with auxiliary equation

$$h(x) = \prod_{i=1}^{r} \left(x - \alpha_i^2\right).$$

This can be illustrated with the bisection of the ordinary sequence of Fibonacci numbers with the auxiliary polynomial

$$\begin{aligned}\left(x - \alpha_1^2\right)\left(x - \alpha_2^2\right) &= x^2 - \left(\alpha_1^2 + \alpha_2^2\right)x + \left(\alpha_1 \alpha_2\right)^2 \\ &= x^2 - 3x + 1,\end{aligned}$$

which, in effect, is related to the second order recurrence relation

$$W_n^{(2)} = 3W_{n-1}^{(2)} - W_{n-2}^{(2)},$$

which, in turn, can generate the sequence {1,3,8,21,55,...}. Similarly, with the bisection of the Tribonacci sequence [2]: let $A = \left(19 + 3\sqrt{33}\right)^{1/3}, \ B = \left(19 - 3\sqrt{33}\right)^{1/3}$, so that

$$\alpha_1 = \frac{1}{3}(A+B+1)$$

$$\alpha_2 = \frac{1}{6}\left(2-A-B+\sqrt{3}i(A-B)\right)$$

$$\alpha_3 = \frac{1}{6}\left(2-A-B-\sqrt{3}i(A-B)\right)$$

are the characteristic roots of $x^3 - x^2 - x - 1$ [15], and so

$$\begin{aligned}(x-\alpha_1^2)(x-\alpha_2^2)(x-\alpha_3^2) &= x^3 - (\alpha_1^2+\alpha_2^2+\alpha_3^2)x^2 + (\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_3^2\alpha_1^2)x - (\alpha_1\alpha_2\alpha_3)^2 \\ &= x^3 - 3x^2 - x - 1,\end{aligned}$$

which, in effect, is related to the third order recurrence relation

$$W_n^{(3)} = 3W_{n-1}^{(3)} + W_{n-2}^{(3)} + W_{n-3}^{(3)},$$

which, in turn, can generate the sequence {1,7,21,71,241,...}, a bisection of the Tribonacci sequence as seen in the table which follows. In more general terms, then

$$U_{s,n}^{(r)} = \sum_{i=1}^{r} A_{s,i}\alpha_i^n,\ s = 0,1,2,\ldots,r,$$

where the $A_{s,i}$ depend on the initial values, $U_{s,1}^{(r)}, U_{s,2}^{(r)}, \ldots, U_{s,r}^{(r)}$, ($A_{0,i}$=1). There are $r$ in each recurrence relation for each $U_{s,n}^{(r)}, s = 0,1,2,\ldots,r,$ so the number of terms in $U_{s,2n}^{(r)} - U_{s,n}^{(r)}U_{0,n}^{(r)}$ is $r^2$-$r$ or $r(r-1)$, of which $r$ can be paired as coefficients of each $\alpha_i\alpha_j, i \neq j$; that is, the number of terms in $U_{s,2n}^{(r)} - U_{s,n}^{(r)}U_{0,n}^{(r)}$ is ½$r(r-1)$ = $\binom{r}{2}$. Thus,

$$\begin{aligned}U_{s,2n}^{(r)} - U_{s,n}^{(r)}U_{0,n}^{(r)} &= \sum_{i=1}^{r} A_{s,i}\alpha_i^{2n} - \sum_{i=1}^{r} A_{s,i}\alpha_i^{n}\sum_{m=1}^{r}\alpha_m^n \\ &= \sum_{i=1}^{r} A_{s,i}\alpha_i^{2n} - \sum_{i=1}^{r} A_{s,i}\alpha_i^{2n} - \sum_{i<j}^{r}(A_{s,i}+A_{s,j})\alpha_i^n\alpha_j^n \\ &= -\sum_{\substack{i,j=1\\ i<j}}^{r}(A_{s,i}+A_{s,j})\alpha_i^n\alpha_j^n \\ &= \sum_k B_{s,k}\beta_k^n \\ &= Y_{s,n}^{\left(\binom{r}{2}\right)}.\end{aligned}$$

When $r$ = 2, the $Y$-sequence will be a first order sequence as expected, and when $r$ = 3, it will also be a 3rd order sequence. For example, consider

$$U_{0,n}^{(3)} = \alpha_1^n + \alpha_2^n + \alpha_3^n$$

$$U_{3,n}^{(3)} = A_{3,1}\alpha_1^n + A_{3,2}\alpha_2^n + A_{3,3}\alpha_3^n$$

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$$U_{3,2n}^{(3)} = A_{3,1}\alpha_1^{2n} + A_{3,2}\alpha_2^{2n} + A_{3,3}\alpha_3^{2n}$$

so that

$$\begin{aligned} U_{3,2n}^{(3)} - U_{3,n}^{(3)}U_{0,n}^{(3)} &= A_{3,1}\alpha_1^{2n} + A_{3,2}\alpha_2^{2n} + A_{3,3}\alpha_3^{2n} - (A_{3,1}\alpha_1^{2n} + A_{3,2}\alpha_2^{2n} + A_{3,3}\alpha_3^{2n} \\ &\quad + A_{3,2}\alpha_1^n\alpha_2^n + A_{3,3}\alpha_1^n\alpha_3^n + A_{3,1}\alpha_1^n\alpha_2^n + A_{3,3}\alpha_2^n\alpha_3^n + A_{3,1}\alpha_1^n\alpha_3^n + A_{3,2}\alpha_2^n\alpha_3^n) \\ &= -\left\{\left(A_{3,1} + A_{3,2}\right)\alpha_1^n\alpha_2^n + \left(A_{3,2} + A_{3,3}\right)\alpha_2^n\alpha_3^n + \left(A_{3,3} + A_{3,1}\right)\alpha_3^n\alpha_1^n\right\} \\ &= B_{3,1}\beta_1^n + B_{3,2}\beta_2^n + B_{3,3}\beta_3^n \\ &= Y_{3,n}^{\left(\binom{3}{2}\right)}. \end{aligned}$$

The table below illustrates the foregoing, for the *U*-sequences $r$ = 3, $s$ = 0, (see Fielder [3]) and for $r$ =3, $s$ = 1,2,3 (see Emerson [1]). Our choice of the initial conditions, while it emphasises the “basic” nature of these sequences, shifts the elements of the sequences from their normal subscript identifiers as we can see in the table. Thus, for instance, the usual form of the Fibonacci sequence is obtained in our notation from

$$\left\{U_n^{(2)}\right\} \equiv \left\{\left\{U_{1,n}^{(2)}\right\} + \left\{U_{2,n}^{(2)}\right\}\right\},$$

and the original form of the Tribonacci sequence was [3]

$$\left\{U_n^{(3)}\right\} \equiv \left\{\left\{U_{1,n}^{(3)}\right\} + \left\{U_{2,n}^{(3)}\right\} + \left\{U_{3,n}^{(3)}\right\}\right\}$$

which is similar to Jarden’s third order sequences, $\{U_n\}$ and $\{V_n\}$, with unit coefficients and initial terms $U_0 = U_1 = 0, U_2 = 1, V_0 = 3, V_1 = 1, V_2 = 3,$ respectively [7, p.86]. Note that Jarden’s subscripts also differ by one from those which we use. This is why we have introduced the more familiar $\left\{U_n^{(r)}\right\}, r = 2,3,$ at this stage for notational convenience though they generally obscure the roles of the underlying basic sequences.

**Table 1: Second and third order examples**

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $U_{0,n}^{(2)}$ | 1 | 3 | 4 | 7 | 11 | 18 | 29 | 47 | 76 | 123 | 199 | 322 |
| $U_{1,n}^{(2)}$ | 1 | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 |
| $U_{2,n}^{(2)}$ | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 |
| $U_n^{(2)}$ | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 |
| $Y_{2,n}^{(1)}$ | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| $U_{0,n}^{(3)}$ | 1 | 3 | 7 | 11 | 21 | 39 | 71 | 131 | 241 | 443 | 815 | 1,499 |
| $U_{1,n}^{(3)}$ | 1 | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 13 | 24 | 44 | 81 |
| $U_{2,n}^{(3)}$ | 0 | 1 | 0 | 1 | 2 | 3 | 6 | 11 | 20 | 37 | 68 | 125 |
| $U_{3,n}^{(3)}$ | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 13 | 24 | 44 | 81 | 149 |
| $U_n^{(3)}$ | 1 | 1 | 1 | 3 | 5 | 9 | 17 | 31 | 57 | 105 | 193 | 355 |
| $Y_{3,n}^{(3)}$ | 0 | 1 | -3 | 2 | 2 | -7 | 7 | 2 | -16 | 21 | -3 | -34 |

For the *Y*-sequences, each term is the homogeneous product sum of weight *n* of the roots of the auxiliary equation [9]; that is, the *Y*-sequences in the table satisfy the 1st and 3rd order recurrence relations respectively:

$$Y_{0,n}^{(1)} = -Y_{0,n-1}^{(1)},\ n > 1, \tag{2.2}$$

and

$$Y_{0,n}^{(3)} = -Y_{0,n-1}^{(3)} - Y_{0,n-2}^{(3)} + Y_{0,n-3}^{(3)},\ n > 3\,. \tag{2.3}$$

Jarden [7, p.87, Equation (20)] found in his notation that

$$U_{2n+1} - U_{n+1}V_n = a^n U_{-n+1} \tag{2.4}$$

which, because of the different initial values, is in our notation

$$U_{3,2n-1}^{(3)} - U_{3,n-1}^{(3)}U_{0,n-1}^{(3)} = U_{3,-n+1}^{(3)} \tag{2.5}$$

with his recurrence relation coefficients *a* = *b* = *c* = 1 in his search for “the counterpart of the formula $U_{2n} - U_nV_n = 0$”. Jarden’s (2.4) can be re-written as

$$U_{3,2n-1}^{(3)} - U_{3,n-1}^{(3)}U_{0,n-1}^{(3)} = Y_n^{(3)} + U_{3,-n+1}^{(3)} \tag{2.6}$$

which reduces to our

$$U_{3,2n}^{(3)} - U_{3,n}^{(3)}U_{0,n}^{(3)} = Y_n^{(3)} \tag{2.7}$$

because

$$U_{-n} - U_{3,-n}^{(3)} = Y_n^{(3)},$$

and (2.7) is a particular case of the previous result, namely,

$$U_{2,2n}^{(r)} - U_{2,n}^{(r)}U_{0,n}^{(r)} = Y_{2,n}^{\left(\binom{r}{2}\right)},$$

when *s* = *r* = 3. When *r* = *s* = 2, it can be similarly confirmed from the table that

$$\begin{aligned} U_{2,2n}^{(2)}U_{0,n}^{(2)} &= \tfrac{1}{\sqrt{5}}\left(\alpha_1^{n-1} - \alpha_2^{n-1}\right)\left(\alpha_1^n + \alpha_2^n\right) \\ &= \tfrac{1}{\sqrt{5}}\left(\alpha_1^{2n-1} - \alpha_2^{2n-1} + \alpha_1^{n-1}\alpha_2^n - \alpha_1^n\alpha_2^{n-1}\right) \\ &= \tfrac{1}{\sqrt{5}}\left(\alpha_1^{2n-1} - \alpha_2^{2n-1} + (-1)^{n-1}\alpha_2 - (-1)^{n-1}\alpha_1\right) \\ &= U_{2,2n}^{(2)} - (-1)^{n-1}; \end{aligned}$$

that is,

$$U_{s,2n}^{(2)} - U_{s,n}^{(2)} U_{0,n}^{(2)} = Y_{s,n}^{(1)}$$

as in Table 1.

There are various other inter-relationships among these sequences in the table which the interested reader might like to unravel and then extend to the arbitrary order analogs. For example, the following capture the initial terms of the sequences:

$$U_{0,n}^{(2)} = U_{1,n}^{(2)} + 3U_{2,n}^{(2)}$$

and

$$U_{0,n}^{(3)} = U_{1,n}^{(3)} + 3U_{2,n}^{(3)} + 7U_{3,n}^{(3)}.$$

## 3. EXTENSIONS

We next show that the ordinary generating function for the sequence when $s = r$, namely $\{U_{r,n}^{(r)}\}$, can be expressed formally as an exponential by

$$\sum_{n=1}^{\infty} U_{r,n}^{(r)} x^n = x^r \exp\left(\sum_{m=1}^{\infty} U_{0,m}^{(r)} \frac{x^m}{m}\right). \tag{3.1}$$

*Proof:*

$$\sum_{n=1}^{\infty} U_{r,n}^{(r)} x^{n-r} = \prod_{j=1}^{r} (1-\alpha_j x)^{-1}$$

$$\log\left(\sum_{n=1}^{\infty} U_{r,n}^{(r)} x^{n-r}\right) = \log \prod_{j=1}^{r} (1-\alpha_j x)^{-1} \qquad |\alpha_j x| < 1$$

$$= -\log \prod_{j=1}^{r} (1-\alpha_j x)$$

$$= -\sum_{j=1}^{r} \log(1-\alpha_j x)$$

$$= \sum_{j=1}^{r} \sum_{m=1}^{\infty} \alpha_j^m \frac{x^m}{m}$$

$$= \sum_{m=1}^{\infty} \left(\sum_{j=1}^{r} \alpha_j^m\right) \frac{x^m}{m}$$

$$= \sum_{m=1}^{\infty} U_{0,m}^{(r)} \frac{x^m}{m}$$

$$\sum_{n=1}^{\infty} U_{r,n}^{(r)} x^{n-r} = \exp\left(\sum_{m=1}^{\infty} U_{0,m}^{(r)} \frac{x^m}{m}\right).$$

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If we take the derivatives of each side of (3.1) with respect to *x,* and use the double summation $\sum_m^\infty \sum_n^\infty A(n,m) = \sum_m^\infty \sum_n^m A(n,m-n)$ rewritten as $\sum_m^\infty \sum_j^\infty A(j,n-j)$ [10], we get

$$\frac{d}{dx}\left(\sum_{n=1}^{\infty} U_{r,n}^{(r)} x^{n-r} = \exp\left(\sum_{m=1}^{\infty} U_{0,m}^{(r)} \frac{x^m}{m}\right)\right)$$

so that

$$\sum_{n=r+1}^{\infty} (n-r) U_{r,n}^{(r)} x^{n-r-1} = \exp\left(\sum_{m=1}^{\infty} U_{0,m}^{(r)} \frac{x^m}{m}\right)\left(\sum_{n=1}^{\infty} U_{0,n}^{(r)} x^{n-1}\right)$$

and, since $U_{s,n}^{(r)} = 0$, $n < 0$ ($s \neq 0$),

$$\begin{aligned}\sum_{n=1}^{\infty} (n-1) U_{r,n+r-1}^{(r)} x^n &= \left(\sum_{m=1}^{\infty} U_{r,m}^{(r)} x^m\right)\left(\sum_{n=1}^{\infty} U_{0,n}^{(r)} x^{n-1}\right)\\ &= \left(U_{r,1}^{(r)} x + U_{r,2}^{(r)} x^2 + U_{r,3}^{(r)} x^3 + \ldots\right)\times\\ &\qquad \left(U_{0,1}^{(r)} + U_{0,2}^{(r)} x + U_{0,3}^{(r)} x^2 + \ldots\right)\\ &= U_{0,1}^{(r)} U_{r,1}^{(r)} x + \left(U_{0,1}^{(r)} U_{r,2}^{(r)} + U_{0,2}^{(r)} U_{r,1}^{(r)}\right) x^2 + \ldots\\ &= \sum_{n=1}^{\infty} \sum_{j=0}^{n} U_{0,j+1}^{(r)} U_{r,n-j}^{(r)} x^n \end{aligned} \qquad (3.2)$$

in which we have also used the fact that $U_{r,0}^{(r)} = U_{r,1}^{(r)} = 0$ (which we use again in what follows). On equating coefficients of *x*, (3.2) becomes

$$(n-1) U_{r,n+r-1}^{(r)} = \sum_{j=0}^{n} U_{0,j+1}^{(r)} U_{r,n-j}^{(r)} \qquad (3.3)$$

as in [13]. When $r = 2$, and after replacing $n$ by $(n + 1)$, this can be written as

$$n U_{2,n+2}^{(2)} = \sum_{j=0}^{n-1} U_{0,j+1}^{(2)} U_{2,n-j+1}^{(2)}$$

which, from the table, in turn can be reduced to the known result [6]:

$$nF_{n+1} = \sum_{j=0}^{n-1} L_{j+1} F_{n-j}. \qquad (3.4)$$

For example, when $n = 6$, $nF_{n+1} = 6 \times 13$, and

$$\begin{aligned}\sum_{j=0}^{n-1} L_{j+1} F_{n-j} &= 8+15+12+14+11+18\\ &= 78.\end{aligned}$$

## 4. CONCLUDING COMMENTS

The algebra associated with the ordinary generating function is sometimes known as the Cauchy calculus, whereas the algebra associated with the exponential generating function is known variously as the Blissard or umbral or symbolic calculus, depending on the context [4]. Further related research can similarly be built upon the relation in (4.1) between the ordinary and exponential generating functions for these arbitrary order sequences:

$$\sum_{n=0}^{\infty} U_{r,n}^{(r)} t^n = \int_0^{\infty} e^{-z} \sum_{n=0}^{\infty} U_{r,n}^{(r)} \frac{t^n}{n!} z^n dz, \ if \left|\alpha_j t\right| < 1. \qquad (4.1)$$

The work can also be extended to generalizations of recurrence relations associated with some of the special functions [5,11].

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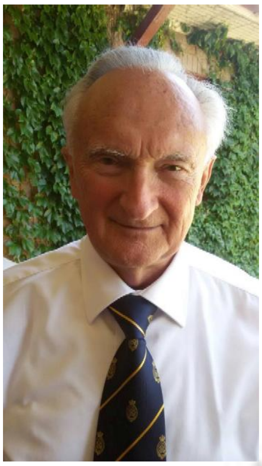
## Authors' biographies with Photos



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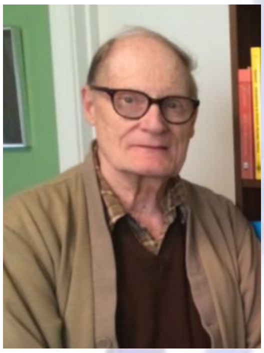
Professor Shannon is a Fellow of several professional societies. He is presently Registrar of Campion College, a liberal arts degree granting institution in Sydney. In June 1987 he was appointed a Member of the Order of Australia (AM) for services to education. He enjoys reading, walking, theatre, number theory, and thoroughbred racing.



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