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# SOME RESULTS OF LABELING ON BROOM GRAPH

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## ABSTRACT

A Broom Graph $B_{n,d}$ is a graph of n vertices, which have a path P with d vertices and (n-d) pendant vertices, all of these being adjacent to either the origin u or the terminus v of the path P. Here we consider various labeling on Broom graph such as Cordial labeling, Antimagic labeling and b-coloring.

## Keywords

Cordial labeling, Antimagic labeling, b-coloring.

## SUBJECT CLASSIFICATION

AMS Mathematics Subject Classification (2010):05C78

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## 1. INTRODUCTION

There are many special classes of graphs which have many interesting properties. One of these specific kind of graphs is the class of broom graphs (see [ 4, 9,10]) which are in fact one of the types of chemical trees [1]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

**Definition 1.1:** Let G=(V,E) be a graph. A mapping f:V(G)→ {0, 1} is called binary vertex labeling of G and f (v) is called the label of the vertex v of G under f.

For an edge e=uv, the induced edge labeling f*:E(G)→{0,1} is given by f*(e)=|f(u)-f(v)|. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_{f^*}(0)$,$e_{f^*}(1)$ be the number of edges having labels 0 and 1 respectively under f*.

According to Beineke and Hedge [9]graph labeling serves as a frontier between number theory[2] and structure of graphs[5]. The two best known labeling methods are called graceful and harmonious labeling. Cordial labeling is a variation of both graceful and harmonious labeling[7]. According to Graham and Sloane [11] the harmonious labelings are closely related to problems in error correcting codes while odd harmonious labeling is useful to solve undetermined equations as described by Liang and Bai[13]

**Definition 1.2:** A binary vertex labeling of a graph G is called a Cordial labeling if $|v_f(0)-v_f(1)|\leq 1$ and $|e_{f^*}(0)-e_{f^*}(1)|\leq 1$.

A graph G is Cordial if it admits cordial labeling.

Cahit proved some results in[6].

**Definition 1.3:** A graph with q edges is called antimagic if its edges can be labeled with 1, 2, 3…... q such that the sums of the labels of the edges incident to each vertex are distinct.

Vaidya and Vyas[12] proved some result on Antimagic labeling of some paths and cycle related graphs.

**Definition 1.4:** A b-coloring is a coloring of the vertices of a graph such that each color class contains a vertex that has a neighbor in all other classes. In other words, each color class contains a color dominating vertex (a vertex which has a neighbor in all the other classes).

For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian[8] and West[3] .

A Broom Graph $B_{n,d}$ is a graph of n vertices, which have a path P with d vertices and (n-d) pendant vertices, all of these being adjacent to either the origin u or the terminus v of the path P.

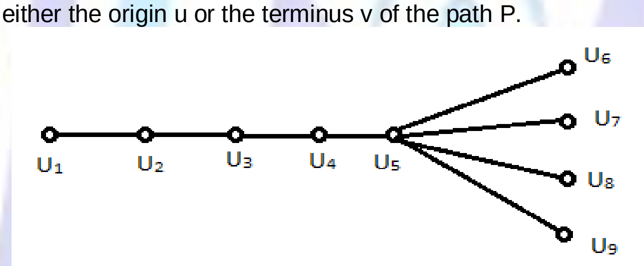


**Figure1: Broom Graph $B_{9,5}$**

## 2. CORDIAL LABELING OF BROOM GRAPH

Suppose $B_{n,d}$ is a Broom Graph. Then for various values of n and d there may exist 4 cases:-

- **(I)** n and d both even
- **(II)** n and d both odd
- **(III)** n odd and d even
- **(IV)** n even and d odd.

**Case I :-** Consider a Broom Graph where n and d both even number. Consider the map f:V→{0,1} by

$f(u_i)$= 1 if i≡0 mod(4)
=1 if i≡1 mod(4)
=0 if i≡2 mod(4)
=0 if i≡3 mod(4)

For this vertex labeling f and the corresponding edge labeling $f^*$, we will show that the Broom graph $B_{n,d}$ is Cordial Graph.

First we will consider the path $u_1u_2$…….$u_d$. Here number of vertices is d and number of edges is (d-1). From the above labeling we will get d/2 number of 1 and d/2 number of 0 for the vertices of the path. For the (n-d) pendant vertices we also get (n-d)/2 number of 1 and (n-d)/2 number of 0.

Hence , $v_f(1)$= Total number of vertices with label 1=d/2+(n-d)/2=n/2

And $v_f(0)$= Total number of vertices with label 0=d/2+(n-d)/2=n/2.

Thus,$|v_f(1)-v_f(0)|$=|n/2-n/2| =0.

From the corresponding edge labeling, we have alternatively 1 and 0 starting from 1 for the path $u_1u_2$……$u_d$. As the number of edges of the graph is (d-1) so there are d/2 number of 1 and (d-2)/2 of 0.

Next consider the (n-d) pendant edges. There we will get the same labeling according as $u_{d+1}$, $u_{d+2}$,......., $u_n$, if $u_d$ is labeled as 0, i.e, if d is a multiple of 2 but not 4. And the opposite labeling with $u_{d+1}$, $u_{d+2}$,......., $u_n$, when $u_d$ is labeled as 1, i.e, if d is a multiple of 4. So, in both the cases we will have (n-d)/2 number of 1 and (n-d)/2 number of 0.

Hence, $e_{f^*}(1)$= Total number of edges with label 1=d/2+(n-d)/2=n/2

And $e_{f^*}(0)$= Total number of edges with label 0= (d-2)/2+(n-d)/2=(n-2)/2.

Thus, $|e_{f^*}(1)-e_{f^*}(0)|=|n/2-(n-2)/2|=1$.

Hence, for the case I, the graph is Cordial.

**Illustration:- (i)**Consider $B_{n,d}$ when d is a multiple of 2 but not 4. Consider $B_{10,6}$..

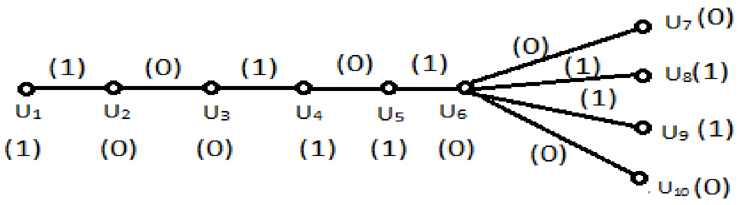


**Figure2: Broom Graph $B_{10,6}$**

Here n=10 and d=6.

$v_f(1)=n/2=5$ and $v_f(0)=n/2=5$.

$|v_f(1)-v_f(0)|=0$

And $e_{f^*}(1)=(n/2=5$ and $e_{f^*}(0)=(n-2)/2=4$.

$|e_{f^*}(1)-e_{f^*}(0)|=1$.

Hence the graph is Cordial.

(ii) Consider $B_{n,d}$ when d is multiple of 4. Consider, $B_{12,8}$

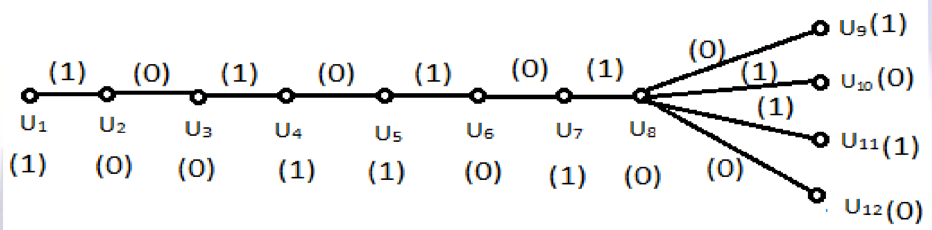


**Figure3: Broom Graph $B_{12,8}$**

Here n=12 and d=8.

$v_f(1)=n/2=6$ and $v_f(0)=n/2=6$

$|v_f(1)-v_f(0)|=0$

And $e_{f^*}(1)=n/2=6$ and $e_{f^*}(0)=(n-2)/2=5$

$|e_{f^*}(1)-e_{f^*}(0)|=1$.

Hence the graph is Cordial.

**Case II: -** Consider a Broom Graph $B_{n,d}$ where n and d both odd number. Consider the map $f:V\rightarrow\{0,1\}$ by

$f(u_i)=0$ if $i\equiv 0 \bmod(4)$

$=1$ if $i\equiv 1 \bmod(4)$

$=1$ if $i\equiv 2 \bmod(4)$

$=0$ if $i\equiv 3 \bmod(4)$

First we will consider the path $u_1u_2.....u_d$. Here number of vertices is d. From the above labeling we will get (d+1)/2 number of 1 and (d-1)/2 number of 0. For the (n-d) pendant vertices we also get (n-d)/2 number of 1 and (n-d)/2 number of 0.

Hence, $v_f(1)$= Total number of vertices with label 1=(d+1)/2+(n-d)/2=(n+1)/2

And $v_f(0)$= Total number of vertices with label 0=(d-1)/2+(n-d)/2=(n-1)/2

Thus $|v_f(1)-v_f(0)|=|(n+1)/2-(n-1)/2|=1$.

From the corresponding edge labeling, we have alternatively 0 and 1 starting from 0 for the path $u_1u_2.....u_d$. As the number of edges of the path is (d-1),which is an even number so there is (d-1)/2 number of 1 and (d-1)/2 number of 0.

Next, consider the (n-d) number of pendant edges. There we will get the same labeling according as $u_{d+1}$, $u_{d+2}$,......$u_n$, if $u_d$ is labeled as 0 and the opposite labeling according as $u_{d+1}$, $u_{d+2}$,......$u_n$, if $u_d$ is labeled as 1. But, in both the cases we will have (n-d)/2 number of 1 and (n-d)/2 number of 0.

So, $e_{f^*}(1)$= Total number of edges with label 1=(d-1)/2+(n-d)/2=(n-1)/2

And $e_{f^*}(0)$=Total number of edges with label 0=(d-1)/2+(n-d)/2=(n-1)/2.

Thus $|e_{f^*}(1)-e_{f^*}(0)|=0$.

Hence, for case II the graph is also Cordial Graph.

**Illustration:-** Consider $B_{11,7}$.

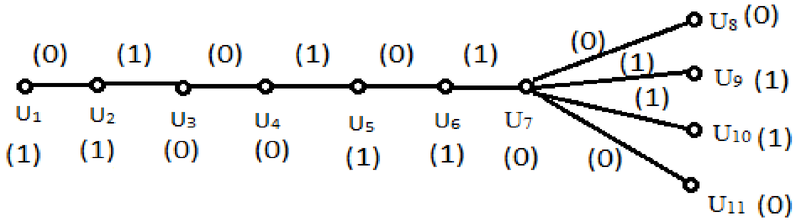


**Figure 4: Broom Graph $B_{11,7}$**

Here n=11 and d=7.
vf(1)=(n+1)/2=6 and vf(0)=(n-1)/2=5.
|vf(1)-vf(0)|=1
And ef*(1)=(n-1)/2=5 and ef*(0)=(n-1)/2=5.
|ef*(1)-ef*(0)|=0.
Hence the graph is Cordial.

**Case III:-** Consider a Broom Graph $B_{n,d}$ where n is odd and d is even. Consider the map f:V→{0,1} by
$f(u_i)$=0 if i≡0 mod(4)
=1 if i≡1 mod(4)
=1 if i≡2 mod(4)
=0 if i≡3 mod(4)
For the vertices of the path there may exists two subcases; i.e,
Subcase1: If d is divisible by 2 but not 4 then
number of vertices with label 1=(d+2)/2 and number of vertices with label 0=(d-2)/2
Subcase2 : If d is divisible by 4 then
number of vertices with label 1 =number of vertices with label 0=d/2
Similarly for the pendant vertices there may also exists two subcases; i.e,
Subcase1: If n is odd and d is divisible by 2 but not 4 then
number of vertices with label 1=(n-d-1)/2 and number of vertices with label 0=(n-d+1)/2
Subcase2: If n is odd and d is divisible by 4,then
number of vertices with label 1=(n-d+1)/2 and number of vertices with label 0=(n-d-1)/2.
For subcase1, considering all the vertices of the graph of case III, we have
$v_f(1)$= (d+2)/2+(n-d-1)/2
= (n+1)/2
And $v_f(0)$= (d-2)/2+(n-d+1)/2
=(n-1)/2
Similarly for subcase2, considering all the vertices of the graph of case III, we have
$v_f(1)$= d/2+(n-d+1)/2
=(n+1)/2
And $v_f(0)$=d/2+(n-d-1)/2
= (n-1)/2.
Hence for both the cases we have $|v_f(1)-v_f(0)|$=|(n+1)/2-(n-1)/2|=1
Now for the edges of the path, we can have the above two subcases. But in both the cases we will get number of edges with label 1=(d-2)/2 and number of edges with label 0=d/2.
For the pendant edges two cases are (i) if $f(u_d)$=0 and (ii) if $f(u_d)$=1. But in both the cases we will get number of edges with label 1=(n-d+1)/2 and number of edges with label 0=(n-d-1)/2.
So, $e_{f^*}(1)$= (d-2)/2+(n-d+1)/2
=(n-1)/2
And $e_{f^*}(0)$=d/2+(n-d-1)/2
=(n-1)/2
Thus, $|e_f(1)-e_f(0)|$=|(n-1)/2-(n-1)/2|=0.
So, for case III the graph is a Cordial Graph.

**Illustration:- (i)** Consider $B_{n,d}$ when d is a multiple of 2 but not 4. Consider $B_{11,6}$.

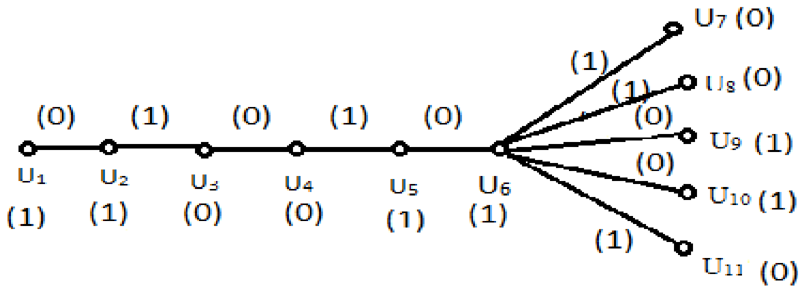


**Figure 5: Broom Graph $B_{11,6}$**

Here n=11 and d=6.
$v_f(1)=(n+1)/2=6$ and $v_f(0)=(n-1)/2=5$.
$|v_f(1)-v_f(0)|=1$
And $e_{f^*}(1)=(n-1)/2=5$ and $e_{f^*}(0)=(n-1)/2=5$.
$|e_{f^*}(1)-e_{f^*}(0)|=0$.
Hence the graph is Cordial.

**(ii)** Consider $B_{n,d}$ when d is a multiple of 4. Consider $B_{13,8}$.

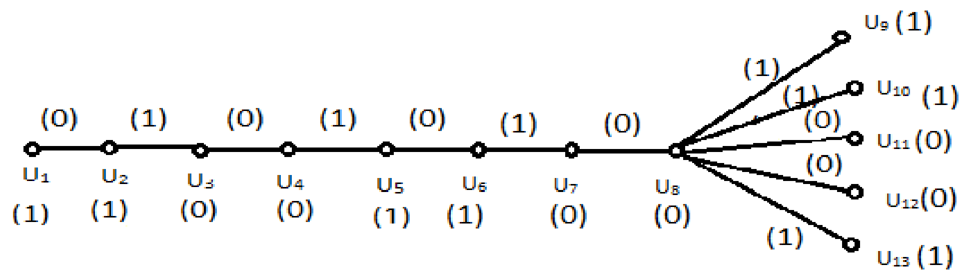


**Figure 6: Broom Graph $B_{13,,8}$**

Here n=13 and d=8.
$v_f(1)=(n+1)/2=7$ and $v_f(0)=(n-1)/2=6$.
$|v_f(1)-v_f(0)|=1$
And $e_{f^*}(1)=(n-1)/2=6$ and $e_{f^*}(0)=(n-1)/2=6$.
$|e_{f^*}(1)-e_{f^*}(0)|=0$.
Hence the graph is Cordial.

**Case IV**:- Consider a Broom Graph $B_{n,d}$ where n is an even number and d is an odd number. Consider the map $f:V\rightarrow\{0,1\}$ by
$f(u_i)=1$ if $i\equiv0$ mod(4)
=1 if $i\equiv1$ mod(4)
=0 if $i\equiv2$ mod(4)
=0 if $i\equiv3$ mod(4)

First we will consider the path $u_1u_2\ldots u_d$. Here number of vertices is d. From the above labeling we will get (d+1)/2 number of 1 and (d-1)/2 number of 0. For the (n-d) pendant vertices we also get (n-d-1)/2 number of 1 and (n-d+1)/2 number of 0.
Hence, $v_f(1)$= Total number of vertices with label 1=(d+1)/2+(n-d-1)/2=n/2
And $v_f(0)$= Total number of vertices with label 0=(d-1)/2+(n-d+1)/2=n/2
Thus $|v_f(1)-v_f(0)|=|n/2-n/2|=0$.
From the corresponding edge labeling, we have alternatively 0 and 1 starting from 1 for the path $u_1u_2\ldots u_d$. As the number of edges of the path is (d-1),which is an even number so there are (d-1)/2 number of 1 and (d-1)/2 number of 0.
Next, consider the (n-d) number of pendant edges, when $f(u_d)=0$. Then number of edges with label 1=(n-d-1)/2 and number of edges with label 0=(n-d+1)/2.
So in this case, $e_{f^*}(1)$= total number of edges with label 1=(d-1)/2+(n-d-1)/2=(n-2)/2
and $e_{f^*}(0)$= total number of edges with label 0=(d-1)/2+(n-d+1)=n/2.
Hence $|e_{f^*}(1)-e_{f^*}(0)|=|(n-2)/2-n/2|=1$.
Now consider the pendant edges when $f(u_d)=1$. Then number of edges with label 1=(n-d+1)/2 and number of edges with label 0=(n-d-1)/2.
So in this case, $e_{f^*}(1)$= total number of edges with label 1=(d-1)/2+(n-d+1)/2=n/2
and $e_{f^*}(0)$= total number of edges with label 0=(d-1)/2+(n-d-1)=(n-2)/2.
Hence $|e_{f^*}(1)-e_{f^*}(0)|=|n-/2-(n-2)/2|=1$.
So, for case IV the graph is a Cordial Graph.

**Illustration:- (i)** When $f(u_d)=1$. Consider $B_{10,5}$.

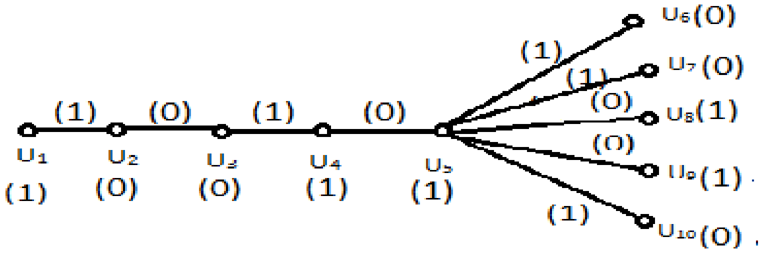


**Figure 7: Broom Graph $B_{10,5}$**

Here n=10 and d=5.
$v_f(1)$=n/2=5 and $v_f(0)$=n/2=5.
$|v_f(1)-v_f(0)|$=0.
And $e_{f^*}(1)$=n/2=5 and $e_{f^*}(0)$=(n-1)/2=4.
$|e_{f^*}(1)-e_{f^*}(0)|$=1.
Hence the graph is Cordial.
**(ii)** When $f(u_d)$=0. Consider $B_{12,7}$.

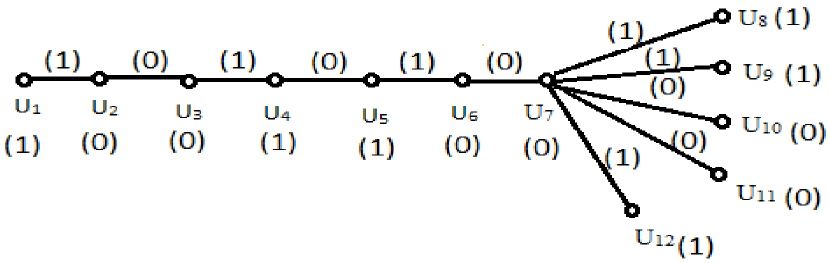


**Figure 8: Broom Graph $B_{12,7}$**

Here n=12 and d=7.
$v_f(1)$=n/2=6 and $v_f(0)$=n/2=6.
$|v_f(1)-v_f(0)|$=0.
And $e_{f^*}(1)$=n/2=6 and $e_{f^*}(0)$=(n-1)/2=5.
$|e_{f^*}(1)-e_{f^*}(0)|$=1.
Hence the graph is Cordial.

## 3. ANTIMAGIC LABELING OF BROOM GRAPH:-

From the definition of Antimagic Labeling and the structure of Broom graph we can easily say that, if the q edges of a Broom graph are labeled with 1,2……,q then the sums of the labels of the edges incident to each vertex must be distinct. So, Broom graph is always Antimagic Graph.

## 4. b-COLORING OF BROOM GRAPH:-

Similarly by choosing any 2 color we can color a Broom graph which will satisfy all the condition of b-coloring. Thus b-chromatic number of any Broom graph is always 2.

## 5. CONCLUTION:-

Here we have some new results by investigating Cordial Labeling, Antimagic Labeling and b-coloring for Broom Graph. More exploration is possible for other graph families and in the context of different graph labeling problems. Fuzzy labeling of these types of graphs is also an open area of research.

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