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FEASIBILTY ANALYSIS OF WALKING OF PASSIVE DYNAMIC BIPED ROBOT

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ABSTRACT

Passive dynamic walking is an essential development for the biped robots. So the focus of our work is a systematic analysis of the passive walk of a planar biped robot on an inclined slope. The dynamics of passive biped robot is only caused of gravity. The biped robot with two point masses at kneeless legs and a third point mass at the hip-joint is kinematically equivalent to a double pendulum. In this paper, we represent a general method for developing the equations of motion and impact equations for the study of multi-body systems, as in bipedal models. The solution of this system depends on the initial conditions. But it is difficult to find the proper initial conditions for which the system has solutions, in other words, the initial conditions for which the robot can walk. In this paper, we describe the cell mapping method which able to compute the feasible initial conditions for which the biped robot can move forward on the inclined ramp. The results of this method described the region of feasible initial conditions is small and bounded. Moreover, the results of cell mapping method give the fixed of Poincare map which explains the symmetric gait cycle of the robot and describe the orientation of legs of robot.

Keywords

Bipedal robot; passive walking; linearization; compass gait; Poincare map; phase space diagram; cell mapping method.

Academic Discipline And Sub-Disciplines

Mechanical Engineering and Robotics.

SUBJECT CLASSIFICATION

Mathematics Subject Classification: Analysis of ordinary differential equations

TYPE (METHOD/APPROACH)

Mathematical analytic approach to solve the problem.

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1. INTRODUCTION

In the recent years, the demand of legged robots is increase due to their capability of working in the hazardous atmosphere and on the rough terrain is better than wheeled vehicles. But the biped robots have higher mobility than other vehicles, mainly when walking on rough terrains, inclined slope and in location with obstacles.Hence, a huge number of researches have been done on the walking of bipedal robots.

The model of biped robots has been inspired by the most complicated and flexible biped recognized to human. For that reason, most of the models built up bear a strong similarity to the human body. The model of passive walking for biped robot was first introduced by MaGeer [1][2] , where a simple biped robot is designed. He observed that the biped robot can present a stable walking on a range of thin slopes. These robots are kinematically similar to double inverted pendulum[3]. Mochon and MacMahon [4] studied that the mathematical models for such biped robots are basically hybrid, consisting of ordinary differential equations to describe the swing phase of the walking motion, and a discrete map to model the impact when the leg touches the ground. Their concept is extended by Goswami [5]. One of the main problems of biped robot to a wider application of legged robot is their lack of stability. The stability of analysis walking of biped robot was done by Shah and Yeolekar[6][7] . Garcia, Chatterjee, Ruina, and Coleman [8] explained that the passive dynamic walker can have stable cyclic motion when walking on the inclined slope. They observed that even a very small disturbance may result in failure. This guides us to think that the range of allowable initial conditions for the cyclic solution. Therefore in this paper, we will examine feasibility of initial conditions for symmetric gait cycle using cell-mapping method [9][10].

2. MATHEMATICAL MODELING

This model is the simplest model of passive dynamic robots, consisting two rigid legs pivoted at the hip, three point-masses: one at the hip and other two at the centre of mass of legs. The feet have plastic (no-slip, no-bounce) collisions with the surface of inclined ramp, except for the period of forward swinging, as geometric interference is ignored. The mathematical modeling of biped robots is necessarily hybrid, consisting of ordinary differential equations to describe the swing phase of the walking motion, and a discrete map to model the impact when the leg touches the ground. Biped robots exhibit periodic behavior. Distinct events, such as contact with the ground, can act to trap the evolving system state within a constrained region of the state space. Therefore Limit cycles (periodic behavior) are often created in this way.

2.1 Differential Equation

The working model of a passive dynamic bipedal robot (PDBR) is shown in Figure 1 schematically. [6][7].

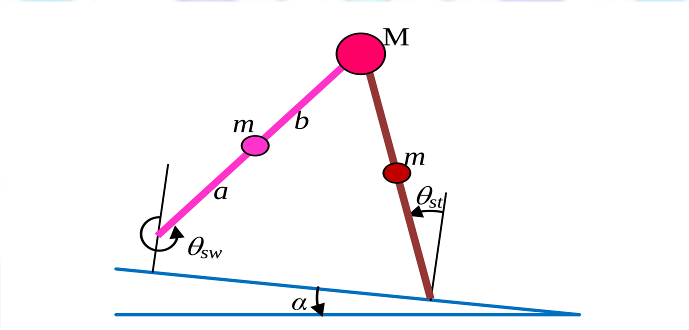

Fig 1: A Passive Dynamic Biped Robot

The following Table 1 lists physical parameters of this model.

Table 1.List of parameters of PDBR model

Symbols	Parameters
θ_{st}	Angle of stance leg with vertical upward axis
θ_{sw}	Angle of swing leg with vertical upward axis
a	The length of rod from the CM of Stance (and Swing)leg to the foot
b	The length of rod from the CM of Stance (and Swing)leg to the Hip
l	Length of rod== $a + b$
m	Mass of leg
M	Mass of Hip
α	Slope of ramp
φ	Inter legs Angle

Using Euler-Lagrange equations, the dynamic equation of the robot can be derived as:

$$M(\theta)\ddot{\theta}+N(\theta,\dot{\theta})\dot{\theta}+G(\theta,\alpha)=0 \tag{1}$$

where $\theta=\left[\theta_{st} \quad \theta_{sw}\right]^T$, $M(\theta)$ is the inertia matrix, the matrix $N(\theta,\dot{\theta})$ contains terms of centrifugal and coriolis forces, $G(\theta,\alpha)$ is the gravity term as given below[6][7]:

$$M(\theta)=\begin{bmatrix} ma^2+Ml^2+ml^2 & -mlb\cos(\theta_{st}-\theta_{sw}) \\ -mlb\cos(\theta_{st}-\theta_{sw}) & mb^2 \end{bmatrix};$$

$$N(\theta,\dot{\theta})=\begin{bmatrix} 0 & -mlb\sin(\theta_{st}-\theta_{sw})\dot{\theta}_{sw} \\ mlb\sin(\theta_{st}-\theta_{sw})\dot{\theta}_{st} & 0 \end{bmatrix};$$

$$G(\theta,\alpha)=\begin{bmatrix} (-ma-Ml-ml)g\sin(\theta_{st}-\alpha) \\ mgb\sin(\theta_{sw}-\alpha) \end{bmatrix}$$

2.2 Linearized State Space Model

The above non-linear model can be linearized at the equilibrium point $0_e=\left[0.035 \;\; 0.035 \;\; 0 \;\; 0\right]^T$, the equation becomes

$$M_{0_e}\ddot{\theta}+G_{0_e}\theta=0$$

The linearized state space model can be written as

$$\dot{y}=Ay \tag{2}$$

where $y=x-0_e$, $x=\left[\theta_{st} \quad \theta_{sw} \quad \dot{\theta}_{st} \quad \dot{\theta}_{sw}\right]^T$ and the elements of matrix A are:

$$A=\begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -M_{\theta_e}^{-1}G_{\theta_e} & 0_{2\times2} \end{bmatrix} \text{ where } M_{0_e}^{-1}G_{0_e}=\begin{bmatrix} \dfrac{(-ma-Ml-ml)g}{(ma^2+Ml^2)} & \dfrac{mlg}{(ma^2+Ml^2)} \\ \dfrac{l(-ma-Ml-ml)g}{b(ma^2+Ml^2)} & \dfrac{(ma^2+Ml^2+ml^2)g}{b(ma^2+Ml^2)} \end{bmatrix}.$$

2.3 Heel strike and Impact equations

The heel strike occurs when the swing leg touches the ramp surface. We assumed that the heel strike to be inelastic and without slipping, and the stance-leg lifts from the ramp without interaction. This impact occurs when the geometric condition, the *y*-coordinate of a foot of the swing leg will become a zero is met, that is,

$$(\theta_{st}-\alpha)+(\theta_{sw}-\alpha)=0$$

and the angular velocities of the both are equal in the opposite direction at two leg stance point when heel strike occurred and so

$$\dot{\theta}_{st}+\dot{\theta}_{sw}=0$$

The angular momentums are conserved at the time of impact for the new stance-leg about the foot and the hip of PDBR (see Figure 2).[6][7]

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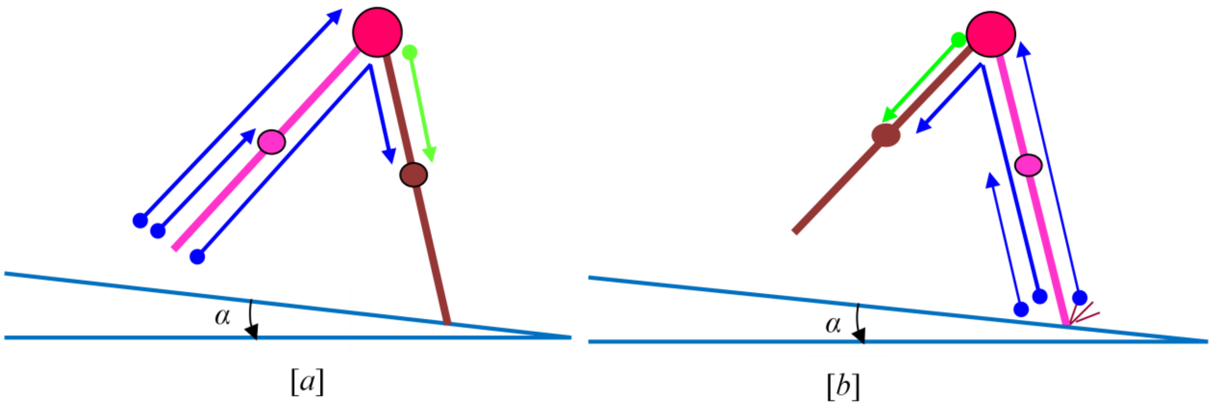


Fig 2:[*a*] Directions of angular momentums about the swing leg's foot and the hip before the heel strike [*b*] Directions of angular momentums about the new stance leg's foot and after the heel strike.

The conservation law of the angular momentum gives to the following compressed equation between the pre- and post-impact angular velocities after the heel strike:

$$\dot{\theta}^{+} = K(\varphi)\dot{\theta}^{-}$$

where $\theta_{sw} - \theta_{st} = \varphi$ and $K(\varphi) = \left[V^{+}(\varphi)\right]^{-1} V^{-}(\varphi)$

where

$$V^{-}(\varphi) = \begin{bmatrix} -mab + \left(2mla + Ml^{2}\right)\cos\varphi & -mab \\ -mab & 0 \end{bmatrix}$$

$$V^{+}(\varphi) = \begin{bmatrix} ma^{2} + Ml^{2} + ml^{2} - mlb\cos\varphi & mb^{2} - mlb\cos\varphi \\ -mlb\cos\varphi & mb^{2} \end{bmatrix}$$

After the heel strike the swing leg will become the new stance and the stance leg will be the new swing leg that change can be computed using the following equation:

$$\theta^{+} = J\theta^{-} \text{ where } J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Therefore the transition matrix of the state-space system can be written as:

$$y^{+} = T(\varphi)y^{-} \text{ where } T(\varphi) = \begin{pmatrix} J & 0 \\ 0 & K(\varphi) \end{pmatrix} \quad (3)$$

2.4 The complete PDBR model

The complete biped passive dynamic robot can be described as follows [6][7]:

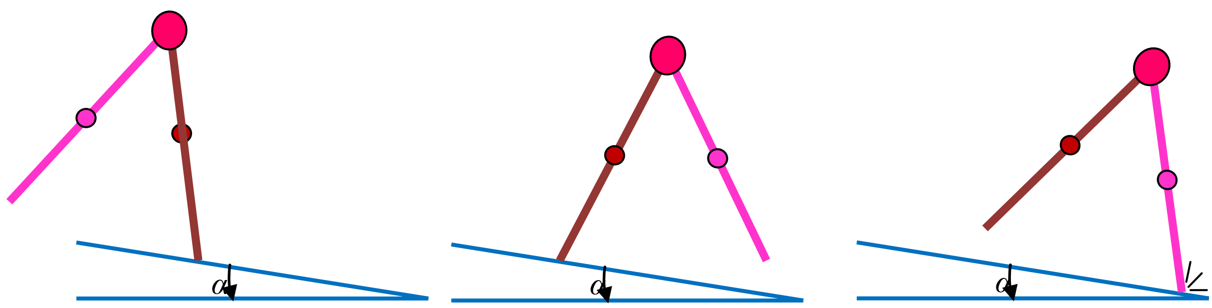


Fig 3: Walking steps for PDBR

Step-I Step-II Step-III

Impact Equation: $y^{+} = T(\varphi) y^{-}$ when $(\theta_{st} + \alpha) + (\theta_{sw} + \alpha) = 0$ and $\dot{\theta}_{st} + \dot{\theta}_{sw} = 0$.

3. ANALYTICAL PROCEDURE

The system of PDBR is hybrid of two discrete events: motion equation (2) and transition function (3) [6]. The analytical solution of the differential equation of motion (2) is given by the explicit equation

$$y(t) = e^{At} y_0$$

wherey_0 is the initial state-space position of PDBR. If τ denotes the time of heel strike when the foot of swing leg touch the ramp-surface, then the state-vector y at the time of heel strike, can be expressed as

$$y(\tau) = e^{A\tau} y_0 .$$

At the time of impact, the legs will change their positions, that is, the swing leg will be the stance leg and the stance will become swing leg for the next step. This change will be done by the transition equation (3) and the state-space position of PDBR for the next step can be worked out by

$$y^{+}(\tau) = T(\varphi) y(\tau) .$$

Considering the state-vector $y^{+}(\tau)$ as the initial position vector for the next step and repeating the above process with it.

4. CELL MAPPING METHOD

Cell mapping method is a new approach for the study of dynamic behavior of a nonlinear system (Hsu, 1987). The cell mapping method is based on the cells which are the rectangular boxes in state space. Considering the dynamic system with Euclidean system R^n($n \geq 2$) and the domain of interest Ω is restricted to a bounded subset of the state space which contains feasible initial conditions for the dynamic system. Let $X = [x_1 \quad x_2 \quad \ldots \quad x_n]$ be the state space vector. The cell of state space is generated by dividing each coordinate axis (x_i) of the domain of interest (Ω) into a large number (n_i) of equal intervals with respect to lower (x_i^l) and upper boundaries(x_i^u). So the size h_i of each interval is $h_i = \dfrac{x_i^u - x_i^l}{n_i}$, $i = 1, 2, \ldots, n$. In this way, Ω is divided into $N = \prod_{i=1}^{n} n_i$ number of cells. The region excluding Ω of R^n is called the sink cell and Ω is called regular cell region. The mapping C: M→M$\left(M = \{n_1, n_2, \ldots, n_N\}\right)$ maps a set of positive integer to a set of positive integer where each positive integer represents the cell number. This map C is called cell mapping which gives image of each cell with respect to the dynamical system. The notation C(i) = j shows that the evolution of i^{th} cell is the j^{th} cell under the map C and $C^m(i) = k$ showsthat the evolution of i^{th} cell after m times application of a map C on the i^{th} cell. A cell i is called fixed cell if C(i)=i and if $C^P(i)=i$, then it is called P^{th} periodic cell. The cells which are not fixed or periodic are transient cells. If transient cell reach to fixed cell or periodic cell after some steps, then those cells generate the basin of attraction for fixed cell or periodic cell. The cell mapping method is as accurate as discretization.

5. RESULTS OF CELL MAPPING METHOD

The cell mapping method is applied to the robot with the following values of parameters:

Table 2.List of values of parameters of biped robot

Notation	Parameters	Values
a	The length of rod from the CM of Stance (and Swing)leg to the foot	0.5 m
b	The length of rod from the CM of Stance (and Swing)leg to the Hip	0.5 m
l	Length of rod= $a + b$	1 m
m	Mass of leg	5 kg
M	Mass of Hip	10 kg
α	Slope of ramp	2^{o}deg

5.1 Feasible initial conditions

By the physical laws of conservation, the state space has only two independent variables θ and $\dot{\theta}$ after the first heel-strike. So the Poincare section is of dimension 2. Furthermore, the passive dynamic biped robot is walking on the inclined ramp, so it walks forward only when $\theta > 0$ and $\dot{\theta} < 0$. Appling cell mapping method on the state space by considering the above conditions, the state space plane is divided into three regions; first is a region of feasible conditions when $-5.8 < \frac{\dot{\theta}}{\theta} < -4.63$ for which robot can walk, second is a region of forward falling when $\dot{\theta} < (-5.8)\theta$ because after passing the mid-stance the swing foot does not go up from the ground level and third is the region of backward falling when $\dot{\theta} > (-4.63)\theta$ because the robot does not have enough initial velocity to begin the walk on the ramp. Moreover, the analysis also suggested that the feasible initial conditions for which the biped robot can walk are found only when $\theta < \frac{\pi}{6}$ for the ramp slope α=0.0349 rad.(2°deg.). The general behavior of the robot is shown the Figure 4.

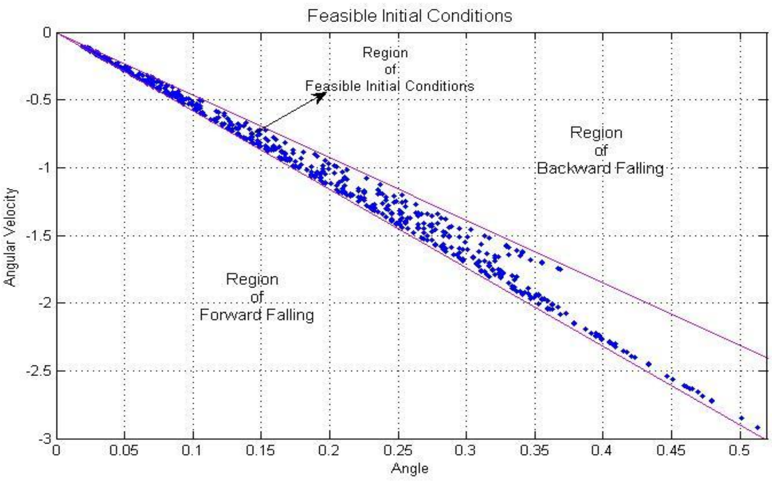


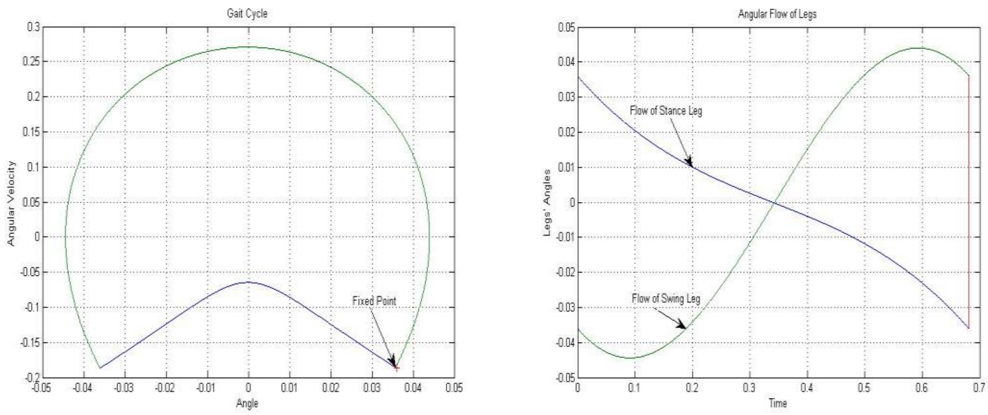
Fig 4: Feasible Initial conditions $(\theta, \dot{\theta})$ for the passive dynamic biped robot with respect tostance leg angle θ and stance leg angular velocity $\dot{\theta}$ jointly with region of forward falling and backward following at a ramp slope of α = 0:0349 rad. (2°deg).

5.2 Fixed Point Analysis

The robot can walk if it begins to start walk within the region of feasible initial conditions but the walking of biped robot is stable if it has the symmetric gait. The symmetric gait means repetitive motion of the robot. It can be calculated by finding the fixed point of Poincare map

$$y_{i+1} = P(y_i) = e^{A\tau}T(\varphi)y_i$$

Applying the cell mapping method on the region of feasible initial conditions, we got fixed point (0.0360, –0.1871). The symmetric gait cycle of the robot is shown in the Figure 5(a) and the angular flow of stance leg and swing leg are shown in the Figure 5(b) with respect to a fixed point.[6]



(a) (b)

Fig 5: (a) Symmetric gait of fixed point (0.0360,-0.1871) (b) Angular Flow of Stance leg and Swing leg during walking of passive biped robot on the inclined ramp at α=0.0349 for the fixed point.

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6. CONCLUSION AND DIRECTION FOR THE FUTURE

A passive dynamic biped robot without keen is an unactuated system. In this paper, we have described the theory of cell mapping method which can be used to find the regions of state space where robot can walk and also where fails to walk. Moreover, it can also compute the fixed point of Poincare map which gives the symmetric gait cycle for biped robot. In the future study, we will use the method to estimate the stability of symmetric gait cycle of linear and non linear model of biped robot.

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Author' biography with Photo

Dr. Nita H. Shahis a professor in the Department of Mathematics, Gujarat University, Ahmedabad, India. She received her Ph.D. in inventory control management, operations research. Currently, she is engaged in research in inventory control and management, supply chain management, forecasting and information technology and information systems, neural networks, sensors and image processing. She has more than 275 papers published in international and national journals. She is author of four books. She is serving as a member of the editorial board of *Investigation Operational*, *Journal of Social Science* and *Management*, *International Journal of Industrial Engineering and Computations* and *Mathematics Today*.

Mahesh A. Yeolekar got the M.Phil degree in Mathematics from the Gujarat University in 2005. He published two books on engineering mathematics. He is presently doing research under the guidance of Dr. Nita H. Shah. His main research interests are in mathematical modeling, dynamical systems, stability.