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COUPLED FIXED POINT THEOREMS WITH CLRG PROPERTY IN FUZZY METRIC SPACES.

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Abstract. In this paper, we prove common coupled fixed point theorems by using E.A. property and CLRg property for coupled mappings without exploiting the notion of continuity, completeness of the whole space or any of its range spaces. Our theorems generalize the result of [5] and [10-14]. We also find an affirmative answer in fuzzy metric space to the problem of Rhoades[2]. Illustrative examples supporting our results have also been cited.

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Introduction:

Recently ,Bhaskar and Lakshmikantham [3] introduced the concepts of coupled fixed points and mixed monotone property and illustrated these results by proving the existence and uniqueness of the solution for a periodic boundary value problem. Later on these results were extended and generalized by Sedghi et al. [7] , Fang [4] and Xin-Qi Hu [5] etc.

In the study of common fixed points of compatible mappings we often require assumption

on completeness of the space or continuity of mappings involved besides some contractive condition but the study of fixed points of noncomapatible mappings can be extend to the class of non expansive or Lipschitz type mapping pairs even without assuming the continuity of the mappings involved or completeness of the space. Aamri and El Moutawakil [1] generalized the concepts of noncomapatibility by defining the notion of (E.A) property and proved common fixed point theorems under strict contractive condition. Although E.A property is generalization of the concept of non compatible maps yet it requires either completeness of the whole space or any of the range space or continuity of maps. But on contrary , the new notion of CLR(g) property recently given by Sintunavarat and Kuman [8]does not impose such conditions. The importance of CLRg property ensures that one does not require the closeness of range subspaces.

The intent of this paper is to establish the concept of E.A. property and (CLRg) property for coupled mappings and an affirmative answer of question arised by Rhoades[2] whether, by using the concept of noncomapatibility or its generalized notion, can we find equally interesting results in fuzzy metric space also ?

So, our improvement in this paper is four fold as

(i) Relaxed continuity of maps completely

(ii) Completeness of the whole space or any of its range space removed.

(iii) Minimal type contractive condition used.

(iv) The condition $\lim_{t\to\infty} M(x,y,t)=1$ is not used.

(v) Weakened the concept of compatibility by a more general concept of weak compatible.

2. Definitions and Preliminaries

Definition 2.1[9]: A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.2 [9]: A binary operation * : [0,1] × [0,1]→ [0,1] is a continuous t-norm if ([0,1], *) is a topological abelian monoid with unit 1 s.t. a * b ≤ c * d whenever a ≤ c and

b ≤ d , ∀ a, b, c, d ∈ [0,1]. Some examples are below:

(i) *(a, b) = ab,

(ii) *(a, b) = min.(a, b).

Definition 2.3[5]: Let $\underset{0<t<1}{sup.}\Delta(t, t) = 1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at t = 1, where

$\Delta^1(t) = t$, $\Delta^{m+1}(t) = t\,\Delta\,(\Delta^m(t))$, m= 1, 2,….., t ∈ [0, 1]. A t-norm Δ is a H-type t-norm iff for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Delta^m(t) > (1-\lambda)$ for all m ∈ N, when $t > (1-\delta)$.

The t-norm Δ_M = min. is an example of t-norm of H-type.

Definition 2.4[9]: The 3-tuple (X, M, *) is called a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

(FM-1) M(x, y, 0) > 0,

(FM-2) M(x, y, t) = 1 iff x=y,

(FM-3) M(x, y, t) = M(y, x, t),

(FM-4) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s),

(FM-5) M(x, y, .) : (0,∞) → [0,1] is continuous, for all x, y, z ∈ X and s, t > 0.

We note that M(x, y, .) is non-decreasing for all x, y ∈ X.

Definition 2.5[6]: An element (x, y) ∈ X × X is called a

(i) coupled fixed point of the mapping f: X × X → X if

f(x, y) = x , f(y, x) = y.

(ii) coupled coincidence point of the mappings f: X × X → X and g: X → X if

f(x, y) = g(x), f(y, x) = g(y).

(iii) common coupled fixed point of the mappings f: X × X → X and g: X → X if

x = f(x, y) = g(x), y = f(y, x) = g(y).

Definition 2.6[5]: An element x ∈ X is called a common fixed point of the mappings f: X × X → X and g: X → X if

x = f(x, x) = g(x).

Definition 2.7: Let A,B : X × X → X and S , T : X → X be four mappings. Then, the pair of maps (B, S) and (A, T) are said to have Common Coupled Coincidence Point if there exist a, b in X such that

B(a, b) = S(a) = T(a) = A(a, b) and B(b, a) = S(b) = T(b) = A(b, a).

Definition 2.8[3]: The mappings f: X × X → X and g: X → X are said to be compatible if

$$\lim_{n\to\infty} M(gf(x_n,y_n), f(g(x_n),g(y_n)),t) = 1,$$

$$\lim_{n\to\infty} M(gf(y_n,x_n), f(g(y_n),g(x_n)),t) = 1,$$

for all t > 0 whenever {x_n} and {y_n} are sequences in X, such that

$\lim_{n\to\infty} f(x_n,y_n) = \lim_{n\to\infty} g(x_n)$ = x, $\lim_{n\to\infty} f(y_n,x_n) = \lim_{n\to\infty} g(y_n)$ = y, for all x, y ∈ X.

Now we introduce the followings:

Definition 2.9: The mappings f : X × X → X and g: X → X are called weakly compatible maps if

f(x, y) = g(x), f(y, x) = g(y) implies gf(x, y) = f(gx, gy), gf(y, x) = f(gy, gx), for all x, y in X.

Now, we fuzzify the newly defined concepts of E.A Property introduced by Aamri and Moutawakil [1] and (CLRg) property given by Sintunavarat and Kuman [8] for coupled maps as follows:

Definition 2.10: Let (X, M, *) be a FM space. Two maps f: X × X → X and g: X → X are said to satisfy E.A. property if there exist sequences {x_n} and {y_n} in X such that

$\lim_{n\to\infty}(f(x_n,y_n) = \lim_{n\to\infty} gx_n = x$ and $\lim_{n\to\infty}(f(y_n,x_n) = \lim_{n\to\infty} gy_n = y$, for some x, y in X..

Definition 2.11: Let (X, M, *) be a FM space .Two maps f: X × X → X and g: X → X are said to satisfy CLRg property if there exists sequences {x_n} and {y_n} in X such that

$\lim_{n\to\infty}(f(x_n,y_n) = \lim_{n\to\infty} gx_n = g(p)$ and $\lim_{n\to\infty}(f(y_n,x_n) = \lim_{n\to\infty} gy_n = g(q)$, for some p, q in X.

Example 2.1: Let (X, M, *) be a fuzzy metric space, * being a continuous norm with X = [0, 1] .Define M(x, y, t) = $\frac{t}{t+|x-y|}$ for all x, y in X and t > 0.

Also define the maps f: X × X → X and g: X → X by f(x, y) = $\frac{x^2}{2}+\frac{y^2}{2}$ and g(x) = $\frac{x}{2}$ respectively. Note that 0 is the points of coincidence of f and g. It is clear that the pair (f, g) is weakly compatible on X .We next show that the pair (f, g) is not compatible.

Consider the sequences {x_n} = {$\frac{1}{2}+\frac{1}{n}$} and {y_n} = {$\frac{1}{2}-\frac{1}{n}$}, then

$$f(x_n,y_n) = \frac{1}{4}+\frac{1}{n^2} = f(y_n,x_n), g(x_n) = \frac{1}{4}+\frac{1}{2n}, g(y_n) = \frac{1}{4}-\frac{1}{2n}$$

$$\lim_{n\to\infty} M(f(x_n,y_n),g(x_n),t) = \left[\frac{t}{t+\left|\frac{1}{n^2}-\frac{1}{2n}\right|}\right] \to 1 \neq g(p)$$, for any p .

$$\lim_{n\to\infty} M(f(y_n,x_n),g(y_n),t)=\left[\frac{t}{t+\left|\frac{1}{n^2}-\frac{1}{2n}\right|}\right]\to 1\neq g(q)$$, for any q.

M(f(gx$_n$, gy$_n$), gf(x$_n$, y$_n$), t) = $\frac{t}{t+|f(gx_n,gy_n)-gf(x_n,y_n)|}$ = $\frac{t}{t+\frac{1}{4}(\frac{1}{2}+\frac{2}{n^2})}$ $\nrightarrow$ 1 as n → ∞.

Hence the pair (f, g) is not compatible satisfying E.A property but not CLRg.

Example 2.2: Let (X, M, *) be a fuzzy metric space, * being a continuous norm with X = R. Define M(x, y, t) = $\frac{t}{t+|x-y|}$ for all x, y in X and t > 0. Define mappings

f: X × X → X and g: X → X by f(x, y) = x - y and g(x) = 2x for all x, y in X and consider the sequences $x_n=\{\frac{1}{n}\}$ and

$y_n=\{-\frac{1}{n}\}$, then $f(x_n,y_n)=\frac{2}{n}, g(x_n)=\frac{2}{n}\Rightarrow \lim_{n\to\infty} M(f(x_n,y_n),g(x_n),t)\to 1=g(\frac{1}{2})$

$$f(y_n,x_n)=-\frac{2}{n}, g(y_n)=-\frac{2}{n}\Rightarrow \lim_{n\to\infty} M(f(y_n,x_n),g(y_n),t)\to 1=g(\frac{1}{2})$$

therefore, f and g satisfy both the. properties E.A and CLRg

Remark 2.1: From above examples we can say that

(1) Weak compatibility does not imply compatibility

(2) E.A does not imply (CLRg).

(3) E.A and (CLRg) does not imply compatibility.

(4) In the next example, we show that the maps satisfying (CLRg) property need not be continuous, i.e. continuity is not the necessary condition for maps to satisfy (CLRg) property.

Example 2.3: Let (X, M, *) be a fuzzy metric space, * being a continuous norm with X = [0, ∞).Define M(x, y, t) = $\frac{t}{t+|x-y|}$ for all x, y in X and t > 0.Define mappings

f: X × X → X and g: X → X as follows

$$f(x,y)=\begin{cases} x+y & if\ x\in[0,1), y\in X \\ \frac{x+y}{2} & if\ x\in[1,\infty), y\in X\end{cases} \quad \text{and } g(x)=\begin{cases} 1+x & if\ x\in[0,1) \\ \frac{x}{2} & if\ x\in[1,\infty)\end{cases}$$

We consider the sequences {x$_n$} = {$\frac{1}{n}$} and {y$_n$} = {1 + $\frac{1}{n}$}.Then,

$$f(x_n,y_n)=f(\frac{1}{n},1+\frac{1}{n})=1+\frac{2}{n}, f(y_n,x_n)=f(1+\frac{1}{n},\frac{1}{n})=\frac{1}{2}+\frac{1}{n}$$

$$g(x_n)=g(\frac{1}{n})=1+\frac{1}{n}, g(y_n)=g(1+\frac{1}{n})=\frac{1}{2}+\frac{1}{2n}$$

$$\lim_{n\to\infty} M(f(x_n,y_n),g(x_n),t)\to 1=g(0) \text{ and } \lim_{n\to\infty} M(f(y_n,x_n),g(y_n),t)\to 1=g(0)$$

therefore, the maps f and g satisfy (CLRg) property but the maps are not continuous.

3. Main Results

Proposition 3.1: Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type with $a*b\geq ab, \forall a,b\in[0,1]$. Let A : X × X → X and S : X → X be compatible mappings such that $A(x,y)=Sx$ and $A(y,x)=Sy$ for some x,y in X , then

$$AA(x,y))=A(Sx,Sy)=SSx=SA(x,y) \text{ and } AA(y,x))=A(Sy,Sx)=SSy=SA(y,x)$$

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Proof : Let $\{x_n\}$ and $\{y_n\}$ be the sequences in X such that $\{x_n\} = x$ and $\{y_n\} = y, n = 1,2,3,............$ and $A(x,y) = Sx$, $A(y,x) = Sy$ so $A(x_n, y_n), Sx_n \to Sx$ and $A(y_n, x_n), Sy_n \to Sy$. Also the maps A,S are compatible , so

$\lim_{n\to\infty} M(A(Sx_n, Sy_n), SA(x_n, y_n), t) = 1$ and $\lim_{n\to\infty} M(A(Sy_n, Sx_n), SA(y_n, x_n), t) = 1$

$$M(A(Sx, Sy), SSx, t) = M(A(Sx, Sy), SA(x, y), t)$$

$$= \lim_{n\to\infty} M(A(Sx_n, Sy_n), SA(x_n, y_n), t) \to 1$$

Thus , $A(Sx, Sy) = SSx$, similarly $SA(x, y) = AA(x, y)$. But $A(x, y) = Sx$, hence

$AA(x, y)) = A(Sx, Sy) = SSx = SA(x, y)$,

Similarly we can have $AA(y, x)) = A(Sy, Sx) = SSy = SA(y, x)$

Now, we prove the following result for quadruple maps.

Theorem 3.2: Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type with $a * b \geq ab, \forall a,b \in [0,1]$. Let A, B : X × X → X and S , T : X → X be four mappings satisfying following conditions:

(3.1) The pairs (A, S) and (B, T) satisfy CLR(g) property

(3.2) $M(A(x,y), B(u,v), kt)) \geq \phi[Min\{M(Sx,Tu,t), M(A(x,y),Sx,t), M(B(u,v),Tu,t)\}]$

$\forall x, y, u, v \in X, t > 0, 0 < k < 1,$ where $\phi : [0,1] \to [0,1]$ is a continuous functions such that $\phi(1) = 1, \phi(t) > t$ for $0 < t < 1$. Then

(i) A and S have point of coincidence.

(ii) B and T have point of coincidence.

Moreover, if the pairs (A, S) and (B, T) are weakly compatible, then there exists unique x in X such that A(x, x) = T(x) = B(x, x) = S(x) = x.

Proof: Since the pairs (A, S) and (B, T) satisfy CLRg property, there exist sequences $\{x_n\}$, $\{y_n\}$ $\{x_n^{/}\}$ and $\{y_n^{/}\}$ in X such that $\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} S(x_n) = Sa$,

$\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} S(y_n) = Sb$ and $\lim_{n\to\infty} B(x_n^{/}, y_n^{/}) = \lim_{n\to\infty} T(x_n^{/}) = Ta^{/}$, $\lim_{n\to\infty} B(y_n^{/}, x_n^{/}) = \lim_{n\to\infty} T(y_n^{/}) = Tb^{/}$, for some a, b, $a^{/}$, $b^{/}$ in X

Step 1: We now show that the pairs (A, S) and (B, T) have common coupled coincidence point. We first show that $Sa = Ta^{/}$.Using (3.2), we have ,

$$M(A(x_n, y_n), B(x_n, y_n), kt)) \geq \phi[Min\{M(Sx_n, Tx_n, t), M(A(x_n, y_n), Sx_n, t), M(B(x_n, y_n), Tx_n, t)\}]$$

Taking $n \to \infty$, we get $M(Sa, Ta^{/}, kt) \geq \phi[Min\{M(Sa, Ta^{/}, t), 1, 1\}]$

i.e $M(Sa, Ta^{/}, kt) \geq M(Sa, Ta^{/}, t) \Rightarrow Sa = Ta^{/}$, similarly we can have $Sb = Tb^{/}$.

Also ,
$M(A(y_n, x_n), B(x^{/}_n, y^{/}_n), kt)) \geq \phi[Min\{M(Sy_n, Tx^{/}_n, t), M(A(y_n, x_n), Sy_n, t), M(B(x^{/}_n, y^{/}_n), Tx^{/}_n, t)\}]$ i.e
$M(Sb, Ta^{/}, kt) \geq M(Sb, Ta^{/}, t) \Rightarrow Sb = Ta^{/}$

Hence $Sb = Ta^{/} = Sa = Tb^{/}$. Now, for all t > 0, using condition (3.2), we have

$$M(A(x_n, y_n), B(a^{/}, b^{/}), kt)) \geq \phi[Min\{M(Sx_n, Ta^{/}, t), M(A(x_n, y_n), Sx_n, t), M(B(a^{/}, b^{/}), Ta^{/}, t)\}]$$

Taking $n \to \infty$, we get , $M(Sa, B(a^{/}, b^{/}), kt) \geq M(Sa, B(a^{/}, b^{/}), t)$

which implies that Sa = B($a^{/}, b^{/}$). Similarly, we can get that Sb = B($b^{/}, a^{/}$).

In a similar fashion , we can have Ta/ = A(a, b) and Tb/ = A(b, a).

Thus, B($a^{/}, b^{/}$) = Sa = Ta/ = A(a, b) and B($b^{/}, a^{/}$) = Sb = Tb/ = A(b, a). Thus the pairs (A,S) and (B,T) have common coincidence points.

Let Sa = A(a, b) = B($a^{/}, b^{/}$) = Ta/ = x and Sb = A(b, a) = B($b^{/}, a^{/}$) = Tb/ = y. Since (A,S) and (B,T) are weakly compatible , so

$Sx = SA(a,b) = A(Sa, Sb) = A(x, y)$ and $Sy = SA(b,a) = A(Sb, Sa) = A(y, x)$.

$Tx = TB(a^{/}, b^{/}) = B(Ta^{/}, Tb^{/}) = B(x, y)$ and $Ty = TB(b^{/}, a^{/}) = B(Tb^{/}, Ta^{/}) = B(y, x)$.

Step 2: We next show that x = y . From (3.2) ,

$$M(x, y, kt) = M(A(a,b), B(a^{/}, b^{/}), kt)$$

$$= \phi\left[Min\{M(Sa, Ta^{/}, t), M(A(a,b), Ta, t), M(B(a^{/}, b^{/}), Ta^{/}, t)\} = 1 \right]$$

Thus , x = y .

Step 3: Now , we prove that Sx = Tx, using (3.2) again

$$M(Sx, Tx, kt) = M(Sx, Ty, kt) = M(A(x, y), B(y, x), kt)$$

$$\geq \phi\left[Min\{M(Sx, Ty, t), M(A(x, y), Sx, t), M(B(y, x)Ty, t)\} \right]$$

$$= \phi\left[Min\{M(Sx, Tx, t), M(A(x, y), Sx, t), M(B(y, x)Ty, t)\} \right]$$

i.e $M(Sx, Tx, kt) \geq M(Sx, Ty, t) \Rightarrow Sx = Tx = Ty$.

Step 4: Lastly , we prove that Sx = x

$$M(Sx, x, kt) = M(Sx, y, kt) = M(A(x, y), B(x, y), kt)$$

$$\geq \phi\left[Min\{M(Sx, Tx, t), M(A(x, y), Sx, t), M(B(x, y), Tx, t)\} \right]$$

Hence $x = Sx = Tx = A(x, x) = B(x, x)$. This shows that A, B, S, T have a common fixed point and uniqueness of x follows easily from (3.2).

Corollary 3.1 : Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type with $a * b \geq ab, \forall a, b \in [0,1]$. Let A , B : X × X → X and S , T : X → X be four mappings satisfying following conditions:

(3.3) The pairs (A, S) and (B, T) satisfy CLR(g) property

$$M(A(x, y), B(u, v), kt)) \geq \phi\left[Min\{M(Sy, Tv, t), M(A(y, x), Sy, t), M(B(v, u), Tv, t)\} \right]$$

$\forall x, y, u, v \in X, t > 0, 0 < k < 1,$ where $\phi : [0,1] \to [0,1]$ is a continuous functions such that $\phi(1) = 1, \phi(t) > t$ for $0 < t < 1$. Then

(iii) A and S have point of coincidence.

(iv) B and T have point of coincidence.

Moreover if the pairs (A, S) and (B, T) are compatible, then there exists unique x in X such that A(x, x) = T(x) = B(x, x) = S(x) = x.

Corollary 3.2: Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type with $a*b \geq ab, \forall a,b \in [0,1]$. Let A , B : X × X → X and S , T : X → X be four mappings satisfying following conditions:

(3.4) The pairs (A, S) and (B, T) satisfy CLR(g) property

(3.5) $M(A(x,y),B(u,v),kt)) \geq \{M(Sx,Tu,t)*M(A(x,y),Sx,t)*M(B(u,v),Tu,t)\}$

$\forall x,y,u,v \in X, t>0, 0<k<1,$ where $\phi:[0,1]\to[0,1]$ is a continuous functions such that $\phi(1)=1, \phi(t)>t$ for $0<t<1$. Then

(v) A and S have point of coincidence.

(vi) B and T have point of coincidence.

Moreover if the pairs (A, S) and (B, T) are compatible, then there exists unique x in X such that A(x, x) = T(x) = B(x, x) = S(x) = x.

4. An Application

Corollary 3.3; Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type with $a*b \geq ab, \forall a,b \in [0,1]$. Let A , B : X × X → X and S , T : X → X be four mappings satisfying following conditions:

(3.6) The pairs (A, S) and (B, T) satisfy (E.A) property

(3.7) A(X × X) ⊆ T(X), B(X × X) ⊆ S(X),

(3.8) S(X) and T(X) are closed subsets of X.

(3.9) $M(A(x,y),B(u,v),kt)) \geq \{M(Sx,Tu,t)*M(A(x,y),Sx,t)*M(B(u,v),Tu,t)\}$

$\forall x,y,u,v \in X, t>0, 0<k<1,$ where $\phi:[0,1]\to[0,1]$ is a continuous functions such that $\phi(1)=1, \phi(t)>t$ for $0<t<1$. Then

(vii) A and S have point of coincidence.

(viii) B and T have point of coincidence.

Moreover if the pairs (A, S) and (B, T) are compatible, then there exists unique x in X such that A(x, x) = T(x) = B(x, x) = S(x) = x.

Next, we give an example in support of our theorem 3.2

.**Example 3.1:** Let X = [-2, 2], a * b = ab for all a, b ϵ [0, 1] and $M(x,y,t) = \left\{\begin{matrix} \frac{t}{t+|x-y|}, t\neq 0 \\ 0, t=0 \end{matrix}\right\}$. Then (X, M, *) is a Fuzzy Metric space. Define the mappings

$A,B: X\times X \to X$ and $S,T: X \to X$ as follows

$A(x,y) = \left\{\begin{matrix} x+y, x\in[0,2], y\in X \\ 1, otherwise \end{matrix}\right\}$ and $B(x,y) = \left\{\begin{matrix} x+y, x\in[0,2], y\in X \\ 2, otherwise \end{matrix}\right\}$

$S(x) = \left\{\begin{matrix} x, x\in[0,2], y\in X \\ 1, otherwise \end{matrix}\right\}$ and $T(x) = \left\{\left\{\begin{matrix} 2x, x\in[0,2], y\in X \\ 2, otherwise \end{matrix}\right\}\right\}$ Consider the sequences $x_n = \left\{\frac{1}{n}\right\}, y_n = \left\{-\frac{1}{n}\right\}, x'_n = \left\{1+\frac{1}{n}\right\}, y'_n = \left\{1-\frac{1}{n}\right\}, n\in N$, then the pairs (A,S) and (B,T) are weakly compatible and we see

$$\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} A(\frac{1}{n}, -\frac{1}{n}) = \lim_{n\to\infty} S(\frac{1}{n}) = 0$$

$$\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} A(-\frac{1}{n}, \frac{1}{n}, t) = \lim_{n\to\infty} S(-\frac{1}{n}) = 0$$

$$\lim_{n\to\infty} B(x'_n, y'_n) = \lim_{n\to\infty} B(1+\frac{1}{n}, 1-\frac{1}{n}) = \lim_{n\to\infty} T(1+\frac{1}{n}) = 2$$

$$\lim_{n\to\infty} M(B(y'_n, x'_n) = \lim_{n\to\infty} B(1-\frac{1}{n}, 1+\frac{1}{n}) = \lim_{n\to\infty} T(1-\frac{1}{n}) = 2$$

So, all the conditions of our theorem are satisfied . Thus A,B,S and T have a unique common coupled fixed point in X. Indeed, x = 0 is the unique common fixed point which is also a point of discontinuity.

Remark: Our work sets analogues, unifies, generalizes, extends and improves several well known results existing in literature as the notion of weak compatible is more general than commuting, weakly commuting and compatible maps. Our theorems 3.1, 3.2 and 3.3 have been proved by assuming much weaker condition than in analogous results. The results concerning commuting, weakly commuting and compatible maps being extendable in the spirit of our theorems, can be extended verbatim by simply using wider class of weak compatibly in place of commuting, weakly commuting and compatibility maps. Moreover, our results don't need the maps to be continuous and hence provides an affirmative answer of Rhoade's problem.

Conflict of interest: All the author declare that they have no conflict of interest.

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