Existence of equilibria of maps for pair of generalized games

A.K.Dubey[1], Reena Shukla[2] ,R.P.Dubey[3]

Department of Mathematics, Bhilai Institute of Technology, Bhilai House, Durg 491001,Chhattisgarh, INDIA

Department of Mathematics, Dr. C.V.Raman University, Bilaspur 495113, Chhattisgarh, INDIA,

Abstract

In this paper, we prove some new common equilibrium existence theorems for generalized abstract economy pertaining to socio and techno economy with different types of correspondences.

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Key words: abstract economy; upper semicontinuous; locally convex Hausdorff topological vector space.

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1. Introduction and Preliminaries

The existence of equilibria is an abstract economy with compact strategy sets in $\mathbb{R}^n$ was proved by G. Debreu [4]. Since then many generalization of Debreu's theorem appeared in mathematical economics such as Borglin and Keiding [1], Shafer and Sonnenschein [14], Tulcea [15], Yannelis and Prabhakar [17] and the references therein. Borglin and Keiding [1] used for their existence results new concepts of K.F. correspondences and KF-majorized correspondences. Different types of majorized correspondences was introduced by Ding, Kim and Tan [5], Tulcea [15], Yannelis and Prabhakar [17], Yuan and Tarafdar [19]. In [19] Yuan and Tarafdar introduced the notion of U-majorized correspondences and proved several equilibrium theorems. Liu and Cai [12] proposed the notion of Q-majorized correspondences and proved a new fixed point theorem. As its application, they obtained some new existence theorems of an abstract economy.

In [3], Border established that the results appearing in economics about the existence of equilibria is indeed equivalent to some classical fixed point theorems coming from pure mathematics. It can also be explained that such results in pure mathematics have applications in other disciplines (eg. game theory, optimization theory and economics). In mathematical economics, it is possible to generalize the results containing several variables to generalized economic spaces. This attempt will help to understand the dimension of abstract economy related to other sector of social system. In this discussion the map P is related to abstract economy whereas the map M is related to social/Techno/government system etc. Thus, the new concept will be very helpful in dealing with socio-economic, techno-economic, government or any corporate sector related problems, in which the organizational constraints will begin to play their role and affecting the abstract economy. The maximal elements of $(P \cap M)(x) = \emptyset$ be very useful in economy and human relation problems.

In this paper, we propose an existence theorem of equilibria for pair of generalized games (abstract economies) in which the preference correspondences are U-majorized and constraint correspondences are upper semicontinuous with any set of players in locally convex Hausdorff topological vector space. Further we prove the existence theorem of equilibria for pairs of generalized games in which the preference correspondences are Q-majorized and constraint correspondences are lower semicontinuous. Our results improve and generalize some known results in literature [2,6,7,8,11,13,16].

Now we give some notations and definitions that are needed in the sequel.

Let E be a vector space and $A \subset E$. We shall denote coA the convex hull of A. If A is a subset of a topological space X, we denote by clA the closure of A in X. If A is a non-empty set, we denote by 2^A the family of all subsets of A. If A is a non-empty subset of a topological vector space E and $F, T: A \to 2^E$ are two correspondences, then $coT, clT, T \cap F: A \to 2^E$ are correspondences defined by $(coT)(x) = coT(x), (clT)(x) = clT(x)$ and $(T \cap F)(x) = T(x) \cap F(x)$ for each $x \in A$, respectively.

The graph of $T: X \to 2^Y$ is the set $Gr(T) = \{(x, y) \in X \times Y | y \in T(x)\}$.

The correspondence $\bar{T}$ is defined by

$\bar{T}(x) = \{y \in Y: (x, y) \in cl_{X \times Y} Gr(T)\}$

(the set $cl_{X \times Y} Gr(T)$ is called the adherence of the graph of T).

It is easy to see that $clT(x) \subset \bar{T}(x)$ for each $x \in X$.

Definition 1.1. Let X, Y be topological space and $T: X \to 2^Y$ be a correspondence.

1. T is said to be upper semicontinuous if for each $x \in X$ and each open set V in Y with $T(x) \subset V$, there exists an open neighborhood U of x in X such that $T(y) \subset V$ for each $y \in U$.

2. T is said to be lower semicontinuous if for each $x \in X$ and each open set V in Y with $T(x) \cap V \neq \emptyset$, there exists an open neighborhood U of x in X such that $T(y) \cap V \neq \emptyset$ for each $y \in U$.

3. T is said to have open lower sections if $T^{-1}(y) := \{x \in X: y \in T(x)\}$ is open in X for each $y \in Y$.

Lemma 1.1.[18] Let X and Y be two topological spaces and let A be a closed (resp. open) subset of X. Suppose $F_1: X \to 2^Y, F_2: X \to 2^Y$ are lower (resp. upper) semicontinuous such that $F_2(x) \subset F_1(x)$ for all $x \in A$. Then the correspondence $F_2: X \to 2^Y$ defined by

$$F(x) = \begin{cases} F_1(x), if\ x \notin A \\ F_2(x), if\ x \in A. \end{cases}$$

is also lower (resp. upper) semicontinuous.

Definition 1.2.[12] Let X be a topological space and Y be a non-empty subset of a vector space $E, \theta: X \to E$ be a mapping and $T: X \to 2^Y$ be a correspondence.

1. T is said to be of class $Q_\theta (or\ Q)$ if

   (a) for each $x \in X, \theta(x) \notin clT(x)$ and

   (b) T is lower semicontinuous with open and convex values in Y;

2. A correspondence T_x is said to be a Q_θ-majorant of T at x if there exists an open neighborhood $N(x)$ of x such that $T_x: N(x) \to 2^Y$ and

(a) for each $z \in N(x), T(z) \subset T_x(z) and\ \theta(z) \notin cl\ T_x(z)$ and

(b) T_x is lower semicontinuous with open and convex values;

3. T is said to be Q_θ-majorized if for each $x \in X$ with $T(x) \neq \emptyset$ there exists a Q_θ-majorant T_x of T at x.

Lemma 1.2.[12] Let X be a paracompact topological space and Y be a non-empty subset of a vector space E. Let $\theta: X \to E$ be a single valued function and $P: X \to 2^Y \backslash \{\emptyset\}$ be Q-majorized. Then there exists a correspondence $S: X \to 2^Y$ of class Q such that $P(x) \subset S(x)$ for each $x \in X$.

Definition 1.3.[19] Let X be a topological space and Y be a non-empty subset of a vector space $E, \theta: X \to E$ be a mapping and $T: X \to 2^Y$ be a correspondence.

1. T is said to be of class $U_\theta (or\ U)$ if

   (a) for each $x \in X, \theta(x) \notin T(x)$ and

   (b) T is upper semicontinuous with closed and convex values in Y;

2. A correspondence $T_x: X \to 2^Y$ is said to be a U_θ-majorant of T at x if there exists an open neighborhood $N(x) of\ x$ such that

   (a) for each $z \in N(x), T(z) \subset T_x(z) and\ \theta(z) \notin T_x(z)$ and

   (b) T_x is upper semicontinuous with closed and convex values;

3. T is said to be U_θ-majorized if for each $x \in X$ with $T(x) \neq \emptyset$ there exists an U-majorant T_x of T at x.

Lemma 1.3.[19] Let X be a paracompact normal space and Y be a non-empty subset of a vector space E. Let $\theta: X \to E$ be a single valued function and $P: X \to 2^Y \backslash \{\emptyset\}$ be U_θ-majorized. Then there exists a correspondence $S: X \to 2^Y$ of class U_θ such that $P(x) \subset S(x)$ for each $x \in X$.

Definition 1.4. Let X, Y be topological spaces and $T: X \to 2^Y$ be a correspondence. An element $x \in X$ is called maximal element for T if $T(x) = \emptyset$.

An abstract and socio (or Techno) economy are a family on quadruples $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1})_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; P_{2i+2})_{i\in I}$ respectively, where I is a (finite or infinite) set of players (agents) such that for each $i \in I, X_i$ is a non-empty subset of a topological vector space and $A_i, B_i: X = \Pi_{j\in I} X_j \to 2^{X_i}$ are constraint correspondences and $P_{2i+1}, P_{2i+2}: X \to 2^{X_i}$ are preference correspondences.In this paper we take a model of abstract economy in which the preference correspondence is also split in two parts P_i and F_i and describe the equilibrium pair and also the constraint mapping has been split into two parts A and B in the sense of Yuan [18].

2. Existence of Equilibria for Abstract Economies

In this section, we give some new equilibrium existence theorems for abstract economies. Theorem 2.2 is a common equilibrium theorem for pair of abstract economies with U-majorized correspondences P_{2i+1}, P_{2i+2} and upper semicontinuous correspondences B_i. To prove this theorem, we shall need the following Theorem.

Theorem 2.1.[19] Let X be a non-empty convex subset of a Hausdorff locally convex topological vector space E and let D be a non-empty compact subset of X. Let $P: X \to 2^D$ be $U_\theta -$ majorized. Then there exists a point $\bar{x} \in coD$ such that $P(\bar{x}) = \emptyset$.

Theorem 2.2. Let $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1}, F_i)_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; P_{2i+2}, F_i)_{i\in I}$ be a pair of generalized games (abstract economy), where I be any index set such that for each $i \in I$:

(1) X_i be a non-empty compact and convex subset of a locally convex Hausdorff topological vector space E;

(2) $P_{2i+1}, P_{2i+2}: X = \Pi_{i\in I} X_i \to 2^{X_i}$ have non-empty values and are $U_{\Pi_i} - majorized$ on X_i;

(3) $A_i, B_i: X \to 2^{X_i}$ are such that B_i is upper semicontinuous, each $B_i(x)$ is closed convex subset of X_i, A_i has non-empty closed convex values and $A_i(x) \subset B_i(x)$ for each $x \in X$;

(4) $F_i: X \to 2^{X_i}$ is such that each $F_i(x)$ is a non-empty open convex subset of X_i and $P_{2i+1}(x) \subset F_i(x), P_{2i+2}(x) \subset F_i(x)$ for each $x \in X$.

Then Γ_1 and Γ_2 have a common equilibria pair i.e. ther exists an equilibrium pair $(\bar{x}, \bar{y}) \in X \times X$ such that for each $i \in I, \bar{x}_i \in B_i(\bar{x}), \bar{y}_i \in F_i(\bar{x})$ with $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ and $A_i(\bar{x}) \cap P_{2i+2}(\bar{y}) = \emptyset$.

Proof: For each $i \in I$, B_i is upper semicontinuous and it has non-empty, convex and closed values. We define $B: X \to 2^X$, by $B(x) = \Pi_{i\in I} B_i(x)$. Then B is upper semicontinuous with non-empty, convex and closed values. By Fan's fixed point theorem [9], there exists $\bar{x} \in X$ a fixed point for B i.e. $\bar{x} \in B(\bar{x})$, i.e. $\bar{x}_i \in B_i(\bar{x})$ for each $i \in I$. It remains to show that there exists a point $\bar{y} \in X$ such that $\bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ for each $i \in I$.

Since X is paracompact and P_{2i+1} is U_{Π_i}-majorized, by Lemma 1.3 there exists a correspondence $r_i: X \to 2^{X_i}$ of class U_{Π_i} such that $P_{2i+1}(x) \subset r_i(x)$ for each $x \in X$. Then, r_i is upper semicontinuous with non-empty closed, convex values and $x_i \notin r_i(x)$ for $x \in X$.

For each $i \in I$ define $T_i: X \to 2^{X_i}$,

$$T_i(y) = \begin{cases} A_i(\bar{x}) \cap r_i(y), & if\ y_i \in F_i(\bar{x}); \\ r_i(y), & if\ y_i \notin F_i(\bar{x}). \end{cases}$$

By Lemma 1.1., it follows that T_i is upper semicontinuous on X, it has convex closed values and $y_i \notin T_i(y)$. Define $T: X \to 2^X, T(y) = \Pi_{i\in I} T_i(y)$. T is upper semicontinuous on X and it has convex closed values and $y \notin T(y)$. Therefore, it is U-majorized.

By Theorem 2.1 of existence of maximal elements, there exists $\bar{y} \in X$ such that $T(\bar{y}) = \emptyset$, i.e. $T_i(\bar{y}) = \emptyset$ for each $i \in I$.

For each $y_i \notin F_i(\bar{x}), r_i(y)$ is a non-empty subset of X_i.We have $\bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap r_i(\bar{y}) = \emptyset$. Since $P_{2i+1}(\bar{y}) \subset r_i(\bar{y})$ we have that $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$. Hence, $\bar{x}_i \in B_i(\bar{x}), \bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ for each $i \in I$. Similarly, it can be established that for $i \in I$, $\bar{x}_i \in B_i(\bar{x})$, $\bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap P_{2i+2}(\bar{y}) = \emptyset$, and then $(\bar{x}, \bar{y})$ is a common equilibrium pair for Γ_1 *and* Γ_2.

Corollary 2.1. Let I be any index set and $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1}, F_i)_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; P_{2i+2}, F_i)_{i\in I}$ be a pair of generalized games (abstract economy) such that for each $i \in I$:

(1) X_i be a non-empty compact and convex subset of a locally convex Hausdorff topological vector space E ;

(2) $P_{2i+1}, P_{2i+2}: X = \Pi_{i\in I} X_i \to 2^{X_i}$ are upper semicontinuous on X, have non-empty convex values and $x_i \notin P_{2i+1}(x)$ and $x_i \notin P_{2i+2}(x)$ for each $x \in X$;

(3) $A_i, B_i: X \to 2^{X_i}$ are such that B_i is upper semicontinuous, each $B_i(x)$ is a closed and convex subset of X_i, A_i has non-empty closed convex values and $A_i(x) \subset B_i(x)$ for each $x \in X$;

(4) $F_i: X \to 2^{X_i}$ is such that each $F_i(x)$ is a non-empty open convex subset of X_i and $P_{2i+1}(x) \subset F_i(x)$ *and* $P_{2i+2}(x) \subset F_i(x)$ for each $x \in X$.

Then there exists a common equilibrium pair $(\bar{x}, \bar{y}) \in X \times X$ such that for each $i \in I, \bar{x}_i \in B_i(\bar{x}), \bar{y}_i \in F_i(\bar{x})$ with $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ and $A_i(\bar{x}) \cap P_{2i+2}(\bar{y}) = \emptyset$.

Theorem 2.5 is a common equilibrium theorem for pair of abstract economies with Q-majorized correspondences P_{2i+1}, P_{2i+2} and lower semicontinuous correspondences B_i.To prove this theorem, we shall need the following Theorems.

Theorem 2.3.[16] Let $\Gamma = (X_i, P_i)_{i\in I}$ be a qualitative game where I is an index set such that for each $i \in I$, the following conditions hold:

(1) X_i is a non-empty convex compact metrizable subset of a Hausdorff locally convex topological vector space E and $X := \Pi_{i\in I} X_i$;

(2) $P_i: X \to 2^{X_i}$ is lower semicontinuous;

(3) for each $x \in X$, $x_i \notin clcoP_i(x)$.

Then there exists a point $\bar{x} \in X$ such that $P_i(\bar{x}) = \emptyset$ for all $i \in I$ i.e. $\bar{x}$ is a maximal element of Γ.

Theorem 2.4.[16] Let I be an index set. For each $i \in I$, let X_i be a non-empty convex subset of a Hausdorff locally convex topological space E_i, D_i a non-empty compact metrizable subset of X_i and $S_i, T_i: X = \Pi_{i\in I} X_i \to 2^{D_i}$ be correspondences such that :

(1) $S_i(x)$ is non-empty and $clcoS_i(x) \subset T_i(x)$ for each $x \in X$;

(2) S_i is lower semicontinuous.

Then there exists $\bar{x} \in D := \Pi_{i\in I} D_i$ such that $\bar{x}_i \in T_i(\bar{x})$ for each $i \in I$.

Theorem 2.5. Let $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1}, F_i)_{i\in I}$ and $\Gamma_2 = (X_i; A_i, B_i; P_{2i+2}, F_i)_{i\in I}$ be a pair of generalized games (abstract economy), where I be any index set such that for each $i \in I$:

(1) X_i be a non-empty compact convex metrizable subset of a locally convex Hausdorff topological vector space E;

(2) $P_{2i+1}, P_{2i+2}: X = \Pi_{i\in I} X_i \to 2^{X_i}$ are Q_{Π_i}- majorized on X and have non-empty values;

(3) $A_i, B_i: X \to 2^{X_i}$ are such that B_i is lower semicontinuous, each $B_i(x)$ is a closed convex subset of X_i, $A_i(x)$ is non-empty convex and $A_i(x) \subset B_i(x)$ for each $x \in X$;

(4) $F_i: X \to 2^{X_i}$ is such that each $F_i(x)$ is a non-empty closed subset of X_i and $P_{2i+1}(x) \subset F_i(x), P_{2i+2}(x) \subset F_i(x)$ for each $x \in X$.

Then Γ_1 and Γ_2 have a common equilibria pair i.e. there exists an equilibrium pair $(\bar{x}, \bar{y}) \in X \times X$ such that for each $i \in I, \bar{x}_i \in B_i(\bar{x}), \bar{y}_i \in F_i(\bar{x})$ with $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ and $A_i(\bar{x}) \cap P_{2i+2}(\bar{y}) = \emptyset$.

Proof: For each $i \in I$, B_i is lower semicontinuous and it has non-empty, convex and closed values. By Theorem 2.4, there exists $\bar{x} \in X$ with $\bar{x}_i \in B_i(\bar{x})$ for each $i \in I$. It remains to show that there exists a point $\bar{y} \in X$ such that $\bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ for each $i \in I$.

Since X is paracompact and P_{2i+1} is Q_{Π_i}-majorized, by Lemma 1.2 there exists a correspondence $r_i: X \to 2^{X_i}$ of class Q_{Π_i} such that $P_{2i+1}(x) \subset r_i(x)$ for each $x \in X$. Then, r_i is lower semicontinuous with non-empty open convex values and $x_i \notin clr_i(x)$ for $x \in X$.

For each $i \in I$ define $T_i: X \to 2^{X_i}$,

$$T_i(y) = \begin{cases} A_i(\bar{x}) \cap r_i(y), & if\ y_i \in F_i(\bar{x}); \\ r_i(y), & if\ y_i \notin F_i(\bar{x}). \end{cases}$$

By Lemma 1.1., it follows that T_i is lower semicontinuous on X. Then $cl\, T_i$ is lower semi- continuous, it has convex values and $x_i \notin cl\, T_i(x)$.

By Theorem 2.3 of existence of maximal elements, there exists $\bar{y} \in X$ such that $cl\, T_i\,(\bar{y}) = \emptyset$ for each $i \in I$.

For each $y_i \notin F_i(\bar{x})$, $r_i(y)$ is a non-empty subset of X_i.We have $\bar{y}_i \in F_i(\bar{x})$ and cl($A_i(\bar{x}) \cap r_i(\bar{y})) = \emptyset$. It follows that $A_i(\bar{x}) \cap r_i\,(\bar{y}) = \emptyset$.Since $P_{2i+1}(\bar{y}) \subset r_i(\bar{y})$ we have that $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$. Hence, $\bar{x}_i \in B_i(\bar{x})$, $\bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ for each $i \in I$. Similarly, it can be established that for $i \in I, \bar{x}_i \in B_i(\bar{x})$, $\bar{y}_i \in F_i(\bar{x})$ and $A_i(\bar{x}) \cap P_{2i+2}(\bar{y}) = \emptyset$, and then $(\bar{x}, \bar{y})$ is a common equilibrium pair for Γ_1 *and* Γ_2.

Corollary 2.2. Let I be any index set and $\Gamma_1 = (X_i; A_i, B_i; P_{2i+1}, F_i)_{i \in I}$ and $\Gamma_2 = (X_i; A_i, B_i; P_{2i+2}, F_i)_{i \in I}$ be a pair of generalized games (abstract economy) such that for each $i \in I$:

(1) X_i be a non-empty compact convex metrizable subset of a locally convex Hausdorff topological vector space E;

(2) $P_{2i+1}, P_{2i+2}: X = \Pi_{i \in I} X_i \to 2^{X_i}$ are lower semicontinuous on X, have non-empty convex values and are $x_i \notin P_{2i+1}(x)$ and $x_i \notin P_{2i+2}(x)$ for each $x \in X$;

(3) $A_i, B_i: X \to 2^{X_i}$ are such that B_i is lower semicontinuous, each $B_i(x)$ is a closed convex subset of X_i, $A_i(x)$ is non-empty convex and $A_i(x) \subset B_i(x)$ for each $x \in X$;

(4) $F_i: X \to 2^{X_i}$ is such that each $F_i(x)$ is a non-empty closed and convex subset of X_i and $P_{2i+1}(x) \subset F_i(x)$ *and* $P_{2i+2}(x) \subset F_i(x)$ for each $x \in X$.

Then there exists a common equilibrium pair $(\bar{x}, \bar{y}) \in X \times X$ such that for each $i \in I, \bar{x}_i \in B_i(\bar{x})$, $\bar{y}_i \in F_i(\bar{x})$ with $A_i(\bar{x}) \cap P_{2i+1}(\bar{y}) = \emptyset$ and $A_i(\bar{x}) \cap P_{2i+2}(\bar{y}) = \emptyset$.

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