ISSN 2347-1921

Anti (Q, L)-Fuzzy Subhemirings of a Hemiring

M.Latha, Dr.N.Anitha
Department of Mathematics,Karpagam University,Coimbatore-641021
Department of Mathematics,Periyar University,PG Extension Centre, Dharmapuri-636 705

ABSTRACT:

In this paper, an attempt has been made to study the algebraic nature of an anti (Q, L)-fuzzy subhemirings of a hemi ring.

Indexing terms/Keywords

L-fuzzy set, anti (Q, L)-fuzzy subhemiring, pseudo anti (Q, L)-fuzzy coset.

Academic Discipline And Sub-Disciplines

Mathematics;

SUBJECT CLASSIFICATION

Mathematics; 2000 AMS Subject Classification: 03F55, 06D72, 08A72.

Council for Innovative Research
Peer Review Research Publishing System
Journal: JOURNAL OF ADVANCES IN MATHEMATICS
Vol .11, No.4
www.cirjam.com , editorjam@gmail.com

ISSN 2347-1921

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R; +; .). Some of them, in particular, about near rings and several kinds of semirings have been proved very useful. Semirings (also called half rings) are algebras (R; + ;.) which share the same properties as a ring excepting that (R; +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications the theory of automata and formal languages. An algebra (R; +;.) is said to be a semi ring (R; +) and (R; .) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemi ring if it is additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh [12], several researchers explored the generalization of the concept of fuzzy sets. The notion of anti left h-ideals in hemi ring was introduced by Akram.M and K.H.Dar [1].The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [6].Osman Kanzanci, Sultan Yamark and Serife Yilmaz in [13] introduced the notion of intuitionistic Q-fuzzification of N-subgroups (sub near-rings) in a near–ring and investigated some related properties. A.Solairaju and R.Nagarajan have given a new structure in the construction of Q-fuzzy groups and subgroups [14] and [15]. In this paper are to be introduced some theorems in (Q,L)-fuzzy subhemirings of a hemiring.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set and L= (L,≤) be a lattice with least element 0 and greastest element 1.

1.2 Definition: Let X be a non-empty set and Q be a non-empty set. A (Q, L)--fuzzy subset A of X is function $A: X \times Q \rightarrow L$

1.3 Definition:Let (R,+, .) be a hemiring. A (Q,L)-fuzzy subset A of R is said to be an anti (Q,L)-fuzzy subhemiring of R if it satisfies the following conditions:

(i) $\mu_A(x+y,q) \leq \left((\mu_A(x,q) \vee \mu_A(y,q)\right)$

(ii) $\mu_A(xy,q) \leq \left((\mu_A(x,q) \vee \mu_A(y,q)\right)$, for all x and y in R, and q in Q.

1.4 Definition: Let A and B be (Q, L)-fuzzy subsets of sets G and H respectively. The anti-product of A and B denoted by AxB is defined as AxB=$\{\langle(x,y),q),\ \mu_{AXB}(x,y),q)\rangle /$ for all x in G and y in H & q in Q$\}$, where $\mu_{AXB}(x,y),q) = \{(\mu_A(x,q) \vee \mu_B(y,q)\}$.

1.5 Definition: Let A be a (Q,L)-fuzzy subset in a set S, the anti-strongest relation (Q,L)-fuzzy relation on S, that is a (Q,L)- fuzzy relation on A is V given by $\mu_V((x,y),q) = \{(\mu_A(x,q) \vee \mu_B(y,q)\}$ for all x and y in S and q in Q.

1.6 Definition: Let (R, +,) and $(R',+,\ .)$ be any two hemirings. Let $f: R \rightarrow R'$ be any function and A be an anti (Q, L)-fuzzy subhemiring in R, V be an anti (Q, L)-fuzzy subhemiring in $f(R) = R'$, defined by $\mu_V(y,q) = \inf_{x \in f^{-1}(y)} \mu_A(x,q)$ for all x in R and y in R' and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be an anti (Q, L)-fuzzy subhemiring of a hemiring (R, +, .) and a in R then the pseudo anti (Q, L)-fuzzy coset $(aA)^p$ is defined by$\left(\left(a\mu_A\right)^p\right)(x,q) = p^{(a)}\mu_A(x,q)$, for every x in R, q in Q and for some p in P.

2. PROPERTIES OF ANTI (Q, L)-FUZZY SUBHEMIRINGS OF A HEMIRING

2.1 Theorem: Union of any two anti (Q,L)-fuzzy subhemiring of a hemiring R is an anti (Q,L)-fuzzy subhemiring of R.

Proof: Let A and B be any two anti (Q, L)-fuzzy subhemirings of a hemiring R and x and y in R. Let A $=\{\langle(x,q),\mu_A(x,q)\rangle / x \in R \ \& q \in Q\}$ and B $=\{\langle(x,q),\mu_B(x,q)\rangle / x \in R \ \& q \in Q\}$ and also Let $C = AUB = \{\langle(x,q),\mu_C(x,q)\rangle \ / \ x \in R \ \& \ q \in Q\}$,where $\{(\mu_A(x,q) \vee \mu_B(x,q)\} = \mu_C(x,q)$, Now,

$\mu_C(x+y,q) \leq \{(\mu_A(x,q) \vee \mu_A(y,q) \vee ((\mu_B(x,q) \vee \mu_B(y,q))\} \leq \left(\mu_C(x,q) \vee \mu_C(y,q)\right)$.Therefore,

$\mu_C(x+y,q) \leq \left(\mu_C(x,q) \vee \mu_C(y,q)\right)$, for all x and y in R and q in Q. And. $\mu_C(xy,q) \leq$

$\left\{\left(\mu_A(x,q) \vee \mu_A(y,q)\right) \vee \left((\mu_B(x,q) \vee \mu_B(y,q)\right)\right\} \leq \left(\mu_C(x,q) \vee \mu_C(y,q)\right)$Therefore, $\mu_C(xy,q) \leq \left(\mu_C(x,q) \vee \mu_C(y,q)\right)$, for all x and y in R and q in Q. Therefore C is an anti (Q, L)-fuzzy subhemiring of a hemiring R.

2.2 Theorem: The Union of a family of anti (Q,L)-fuzzy subhemiring of hemiring R is an anti (Q,L)-fuzzy subhemiring of R.

Proof: It is trivial.

2.3 Theorem: If A and B are two anti (Q,L)-fuzzy subhemirings of the hemirings R_1 and R_2 respectively,then anti product AXB is an anti (Q,L)-fuzzy subhemiring of $R_1 X R_2$.

Proof: Let A and B be two anti (Q,L)- fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then $((x_1, y_1), q)$ and $((x_2, y_2),q)$ are in $R_1 X R_2$.Now,

$$\mu_{AXB}[(x_1,y_1),q) + (x_2,y_2),q)] \leq \{((\mu_A(x_1,q) \vee \mu_A(x_2,q)) \vee (\mu_B(y_1,q) \vee \mu_B(y_2,q))\} \leq (\mu_{AXB}(x_1,y_1),q) \vee \mu_{AXB}(x_2,y_2),q)).$$

Therefore, $\mu_{AXB}[(x_1,y_1),q) + (x_2,y_2),q)] \leq (\mu_{AXB}(x_1,y_1),q) \vee \mu_{AXB}(x_2,y_2),q))$. Also,

$$\mu_{AXB}[(x_1,y_1),q)(x_2, y_2),q)] \leq \{(\mu_A(x_1,q) \vee \mu_A(x_2,q)\} \vee ((\mu_B(y_1,q) \vee \mu_B(y_2,q))\} \leq (\mu_{AXB}(x_1,y_1),q) \vee \mu_{AXB}(x_2,y_2),q)).$$

Therefore,$\mu_{AXB}[(x_1,y_1),q)(x_2,y_2),q)] \leq (\mu_{AXB}(x_1,y_1),q) \vee \mu_{AXB}(x_2,y_2),q))$.Hence AXB is an anti (Q,L)-fuzzy subhemiring of a hemiring $R_1 X R_2$.

2.4 Theorem: Let A be a (Q, L)-fuzzy subset of a hemiring R and V be the anti-strongest fuzzy relation of R. Then A is an anti (Q, L)-fuzzy subhemiring of R if and only if V is an anti (Q, L)-fuzzy subhemiring of RxR.

Proof: Suppose that A is an anti (Q,L)-fuzzy subhemiring of a hemiring R. Then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ are in R x R . We have$\mu_V(XY,q) \leq \{(\mu_A(x_1y_1,q) \vee \mu_A(x_2y_2,q)) \leq$

$$\{(\mu_A(x_1,q) \vee \mu_A(y_1,q)) \vee ((\mu_A(x_2,q) \vee \mu_A(y_2,q))\} \leq (\mu_V((x_1,x_2),q) \vee \mu_V((y_1,y_2),q) \leq (\mu_V(X,q) \vee \mu_V(Y,q)$$

for all X and Y in R x R and q in Q. Therefore, $\mu_V(XY,q) \leq (\mu_V(X,q) \vee \mu_V(Y,q))$, for all X and Y in R x R and q in Q. This proves that V is an anti (Q, L)-fuzzy subhemiring of a hemiring of R x R. Conversely assume that V is an anti (Q, L) - fuzzy subhemiring of a hemiring of R x R, then for any $X = (x_1,x_2)$ and $Y = (y_1,y_2)$ are in R x R we have $\{(\mu_A(x_1+y_1,q) \vee \mu_A(x_2+y_2,q)\} = \mu_V(X+Y,q) \leq (\mu_V(X,q) \vee \mu_V(Y,q)) = (\mu_V((x_1,x_2),q) \vee \mu_V((y_1,y_2),q) =$

$(\mu_A(x_1,q) \vee \mu_A(y_1,q)) \vee ((\mu_A(x_2,q) \vee \mu_A(y_2,q))$ If $x_2 = 0, y_2 = 0$ we get $\mu_A(x_1 + y_1,q) \leq (\mu_A(x_1,q) \vee \mu_A(y_1,q))$

for all x_1 and y_1 in R .And, $\{(\mu_A(x_1y_1,q) \vee \mu_A(x_2y_2,q)\} = \mu_V(XY,q) \leq (\mu_V(X,q) \vee \mu_V(Y,q)) =$ $(\mu_V((x_1,x_2),q)) \vee \mu_V((y_1,y_2),q) = (\{\mu_A(x_1,q) \vee \mu_A(x_2,q)\} \vee \{\mu_A(y_1,q) \vee \mu_A(y_2,q)\})$. If $x_2 = 0, y_2 = 0$ we get $\mu_A(x_1y_1,q) \leq (\mu_A(x_1,q) \vee \mu_A(y_1,q))$ for all x_1 and y_1 in R.Therefore A is an anti(Q,L)-fuzzy subhemiring of R.

2.5 Theorem: If A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .) if and only if$\mu_A(x+y,q) \leq (\mu_A(x,q) \vee \mu_A(y,q)$, $\mu_A(xy,q) \leq (\mu_A(x,q) \vee \mu_A(y,q)$ for all x and y in R.

Proof: It is trivial.

2.6 Theorem: If A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .),then $H = \{x/x \in R : \mu_A(x,q) = 0\}$ is either empty or is a subhemiring of R.

Proof: It is trivial.

2.7 Theorem: Let A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .).If $\mu_A(x+y,q) = 1$, then either $\mu_A(x,q) = 1$ or $\mu_A(y,q) = 1$, for all x and y in R.

Proof: It is trivial.

2.8 Theorem: Let A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .),then the pseudo anti (Q,L)-fuzzy coset $(aA)^p$ is anti (Q,L)-fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A is an anti (Q,L)-fuzzy subhemiring of a hemiring R. For every x and y in R, we have

$((a\,\mu_A)^p)(x+y,q) \le p(a)\left(\mu_A(x,q) \vee \mu_A(y,q)\right) \in \left(p(a)\mu_A(x,q) \vee p(a)\mu_A(y,q)\right) = (a\,\mu_A)^p(x,q) \vee (a\,\mu_A)^p(y,q)$. Therefore, $((a\,\mu_A)^p)(x+y,q) \le \left((a\,\mu_A)^p(x,q) \vee (a\,\mu_A)^p(y,q)\right)$.Now, $((a\,\mu_A)^p)(xy,q) \le p(a)\left(\mu_A(x,q) \vee \mu_A(y,q)\right) = \left(p(a)\mu_A(x,q) \vee p(a)\mu_A(y,q)\right) = \left((a\,\mu_A)^p(x,q) \vee (a\,\mu_A)^p(y,q)\right)$.

Therefore, $((a\,\mu_A)^p)(xy,q) \le \left((a\,\mu_A)^p(x,q) \vee (a\,\mu_A)^p(y,q)\right)$.Hence $(aA)^P$ is an anti (Q,L)-fuzzy subhemiring of a hemiring R .

2.9 Theorem: Let (R, +,.) and (R', +, .) be any two hemirings. The homomorphic image of an anti (Q, L)-fuzzy subhemiring of R is an anti (Q, L)-fuzzy subhemiring of R'.

Proof: Let $f: R \to R'$ be a homomorphism.Then ,$f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$ for all x and y in R. Let V= f(A), where A is an anti (Q,L) fuzzy subhemiring of R. Now, for $f(x)$, $f(y)$ in R', $\mu_V((f(x),q)+(f(y),q)) \le \mu_A(x+y,q) \le (\mu_A(x,q) \vee \mu_A(y,q)$,which implies that $\mu_V(f(x)+(f(y),q)) \le (\mu_V\left((f(x),q) \vee \mu_V(f(y),q)\right)$.Again, $\mu_V((f(x),q)(f(y),q)) \le \mu_A(xy,q) \le (\mu_A(x,q) \vee \mu_A(y,q)$, which implies that $\mu_V((f(x)(f(y)),q)) \le \left(\mu_V(f(x),q) \vee \mu_V(f(y),q)\right)$. Hence V is an anti (Q,L)-fuzzy subhemiring of hemiring R'.

2.10 Theorem: Let (R, +, .) and (R', +,.) be any two hemirings. The homomorphic preimage of an anti (Q, L)-fuzzy subhemiring of R' is an anti (Q, L)-fuzzy subhemiring of R.

Proof: Let $f: R \to R'$ be a homomorphism.Then,$f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R .Let V= f(A) where V is an anti (Q,L) fuzzy subhemiring of R'. Now,

for x,y in R, $\mu_A(x+y,q) = \mu_V((f(x)+f(y)),q)) \le (\mu_V(f(x),q) \vee \mu_V(f(y),q) = \left(\mu_A(x,q) \vee \mu_A(y,q)\right)$which implies that $\mu_A((x+y,q)) \le \left(\mu_A(x,q) \vee \mu_A(y,q).\right)$ Again,$\mu_A((xy,q)) = \mu_V((f(x),q)(f(y),q) \le (\mu_V(f(x),q) \vee \mu_V(f(y),q) = (\mu_A(x,q) \vee \mu_A(y,q)$ which implies that $\mu_A((xy,q)) \le \left(\mu_A(x,q) \vee \mu_A(y,q)\right)$. Hence A is an anti (Q, L)-fuzzy subhemiring of hemiring R.

2.11 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. The anti-homomorphic image of an anti (Q,L)-fuzzy subhemiring of R is an anti (Q,L)-fuzzy subhemiring of R'.

Proof: Let $f: R \to R'$ be a anti- homomorphism.Then,$f(x+y) = f(y)+f(x)$ and $f(xy) = f(y)f(x)$,for all x and y in R. Let V= f(A) where A is an anti (Q,L)- fuzzy subhemiring of R.

Now, for $f(x)$,$f(y)$ in R', $\mu_V((f(x)+f(y)),q)) \le \mu_A(y+x,q) \le (\mu_A(y,q) \vee \mu_A(x,q) = (\mu_A(x,q) \vee \mu_A(y,q)$, which implies that $\mu_V((f(x)+f(y)),q)) \le (\mu_V\left((f(x),q) \vee \mu_V(f(y),q)\right)$. Again, $\mu_V((f(x),q)(f(y),q)) \le \mu_A(yx,q) \le \left(\mu_A(y,q) \vee \mu_A(x,q)\right) = (\mu_A(x,q) \vee \mu_A(y,q)$ which implies that $\mu_V((f(x)f(y)),q)) \le (\mu_V\left((f(x),q) \vee \mu_V(f(y),q)\right)$. Hence V is an anti (Q, L)-fuzzy subhemiring of hemiring R'.

2.12 Theorem: Let (R, +,.) and (R', +, .) be any two hemirings. The anti-homomorphic preimage of an anti (Q, L)-fuzzy subhemiring of R' is an anti (Q,L)-fuzzy subhemiring of R.

Proof: Let $V = f(A)$ where V is an anti (Q,L) – fuzzy subhemiring of R'. Let x and y in R.Then $\mu_A((x+y,q)) = \mu_V((f(x)+f(y)),q) \le (\mu_V(f(y),q) \vee \mu_V(f(x),q) = (\mu_A(x,q) \vee \mu_A(y,q)$,which implies that $\mu_A((x+y,q)) \le (\mu_A(x,q) \vee \mu_A(y,q))$. Again, $\mu_A((xy,q)) = \mu_V((f(x)f(y)),q) \le (\mu_V(f(y),q) \vee \mu_V(f(x),q) = (\mu_A(x,q) \vee \mu_A(y,q)$, which implies that $\mu_A((xy,q)) \le (\mu_A(x,q) \vee \mu_A(y,q)$. Hence A is an anti (Q, L)-fuzzy subhemiring of hemiring R.

In the following Theorem∘ is the composition operation of functions:

2.13 Theorem: Let A be an anti (Q, L)-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then A∘ f is an anti (Q, L)-fuzzy subhemiring of R.

Proof: Let x and y in R .Then we have, $(\mu_A \circ f)((x+y),q)) = \mu_A((f(x)+f(y)),q) \le (\mu_A(f(x),q) \vee \mu_A(f(y),q) \le$

$(\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$. which implies that $(\mu_A \circ f)((x+y),q)) \leq (\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$. And, $(\mu_A \circ f)((xy,q)) = \mu_A((f(x),q)(f(y),q) \leq (\mu_A(f(x),q) \vee \mu_A(f(y),q) \leq (\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$, which implies that $(\mu_A \circ f)((xy,q)) \leq (\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$. Therefore A∘f is an anti(Q, L)-fuzzy subhemiring of hemiring R.

2.14 Theorem: Let A be an anti (Q, L)-fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then A∘ f is an anti (Q,L)-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $(\mu_A \circ f)((x+y,q)) = \mu_A(f(y)+f(x),q) \leq (\mu_A(f(x),q) \vee \mu_A(f(y),q) \leq (\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$. which implies that $(\mu_A \circ f)((x+y,q)) \leq ((\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q))$. And, $(\mu_A \circ f)((xy),q)) = \mu_A((f(y)\,f(x)),q) \leq (\mu_A(f(x),q) \vee \mu_A(f(y),q) \leq (\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$ which implies that $(\mu_A \circ f)((xy,q)) \leq (\mu_A \circ f)(x,q) \vee (\mu_A \circ f)(y,q)$. Therefore A∘f is an anti (Q, L)-fuzzy subhemiring of hemiring R.

REFERENCE

[1]. Akram.M and K.H.Dar 2007 On anti fuzzy left h-ideals in hemi rings, International Mathematical Forum, 2(46); 2295-2304,.

[2]. Anitha.N and Arjunan.K, 2010 Homomorphism in Intuitionistic fuzzy subhemirings of a hemi ring, International J.of.Math.Sci &Engg.Appls. (IJMSEA),'Vol.4 (V); 165-172,.

[3]. Anthony.J.M and H.Sherwood, 1979 Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69; 124-130,.

[4]. Asok kumer Ray, 1999 On product of fuzzy subgroups, fuzzy sets and systems, 105; 181-183,.

[5]. Biswas.R, 1990 Fuzzy subgroups and anti-fuzzy subgroups, fuzzy sets and systems, 35; 121-124.

[6]. Palaniappan.N&K.Arjunan, 2008 The homomorphism, anti homomorphism of a fuzzy and an anti-fuzzy ideals of a ring, Varahmihir Journal of Mathematical Sciences,6(1);181-006,.

[7]. Palaniappan.N &K.Arjunan , 2007 Operation on fuzzy and anti fuzzy ideals, Antartical J.Math, 4(1); 59-64, 2007.

[8]. Palaniappan.N &K.Arjunan, 2007 Some properties of intuitionistic fuzzy subgroups, Acta Ciencia Indica, VolXXXIIII (2); 321-328,.

[9]. Rajesh Kumar, 1993 Fuzzy Algebra, University of Delhi Publication Division,Volume 1,.

[10].Vasantha Kandasamy.W.B., 2003 Smarandache fuzzy algebra, American research press, Rehoboth,.

[11] .Xueling MA.Jianming ZHAN, 2007 on fuzzy h-ideals of hemi rings, journal of Systems science & Complexity, 20; 470-478.

[12].Zadeh.L.A, 1965 Fuzzy sets Information and control, 8; 338-353,.

[13].Osman Kazanci, Sultan Yamark and Serife Yilmaz, 2007.On intuitionistic Q-fuzzy R-subgroups of near rings, International mathematical forum, 2(59):2899-2910.

[14]. Solairaju.A and R.Nagarajan, 2008.Q-fuzzy left R-subgroups of near rings w.r.t T-norms, Antarctica journal of mathematics, 5:1-2.

[15]. Solairaju.A and R.Nagarajan, 2009.A new structure and construction of Q-fuzzy groups, Advances in fuzzy mathematics, Volume4 (1):23-29.