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# ALGEBRAIC PROOF IV FERMAT'S LAST THEOREM

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**Abstract:-** The special case $z^4 = x^4 + y^4$ is impossible [1]. In view of this fact, it is only necessary to prove, if $x, y, z,$ are relatively prime positive integers, $\pi$ is an odd prime, $z^\pi \neq x^\pi + y^\pi$ (In this article, the symbol $\pi$ will represent an odd prime). Also, a new proof is given that $z^4 = x^4 + y^4$ is impossible.

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**Theorem 1** *If* $z^\pi = x^\pi + y^\pi$, *then if* $z \not\equiv 0\,(mod\,\pi)(y \not\equiv 0\,(mod\,\pi))[x \not\equiv 0\,(mod\,\pi)]$,

1. $(x+y)^\pi - z^\pi \equiv 0\,(mod\,\pi^2),((z-x)^\pi - y^\pi \equiv 0\,(mod\,\pi^2)),[(z-y)^\pi - x^\pi \equiv 0\,(mod\,\pi^2)];$
2. $(x+y)^\pi - z^\pi \not\equiv 0\,(mod\,\pi^3),((z-x)^\pi - y^\pi \not\equiv 0\,(mod\,\pi^3)),[(z-y)^\pi - x^\pi \not\equiv 0\,(mod\,\pi^3)];$
3. $x \equiv 0\,(mod\,\pi)(y \equiv 0\,(mod\,\pi))[z \equiv 0\,(mod\,\pi)].$

Theorem 1 is arrived at through the following three lemmas.

**Lemma 1** *If* $z^\pi = x^\pi + y^\pi$, *then* $x+y-z \equiv 0\,(mod\,\pi)(z-x-y \equiv 0\,(mod\,\pi)).$

**Proof.** It is obvious that
$(x+y)^\pi - z^\pi \equiv 0\,(mod\,\pi),(z-y)^\pi - x^\pi \equiv 0\,(mod\,\pi),(z-x)^\pi - y^\pi \equiv 0\,(mod\,\pi)$

$$(x+y)^\pi - z^\pi = (x+y-z+z)^\pi - z^\pi = \sum_0^{\pi-1} C(\pi,k)(x+y-z)^{\pi-k} z^k;$$

$$(1)\;\; (x+y)^\pi - z^\pi - (x+y-z)^\pi = \sum_1^{\pi-1} C(\pi,k)(x+y-z)^{\pi-k} z^k.$$

$$(z-x)^\pi - y^\pi = (z-x-y+y)^\pi - y^\pi = \sum_0^{\pi-1} C(\pi,k)(z-x-y)^{\pi-k} y^k;$$

$$(2)\;\; (z-x)^\pi - y^\pi - (z-x-y)^\pi = \sum_1^{\pi-1} C(\pi,k)(z-x-y)^{\pi-k} y^k.$$

$$(z-y)^\pi - x^\pi = (z-x-y+x)^\pi - x^\pi = \sum_0^{\pi-1} C(\pi,k)(z-x-y)^{\pi-k} x^k;$$

$$(3)\;\; (z-y)^\pi - x^\pi - (z-x-y)^\pi = \sum_1^{\pi-1} C(\pi,k)(z-x-y)^{\pi-k} x^k.$$

**Lemma 2** *If* $z^\pi = x^\pi + y^\pi$, *and* $z \not\equiv 0\,(mod\,\pi)(y \not\equiv 0\,(mod\,\pi))[x \not\equiv 0\,(mod\,\pi)],$ *then*
$(x+y)^\pi - z^\pi \equiv 0\,(mod\,\pi^2)((z-x)^\pi - y^\pi \equiv 0\,(mod\,\pi^2))[(z-y)^\pi - x^\pi \equiv 0\,(mod\,\pi^2)].$

**Proof.** See proof of Lemma 1.

**Lemma 3** *If* $z^\pi = x^\pi + y^\pi$, *and*
$(x+y)^\pi - z^\pi \equiv 0\,(mod\,\pi^3),((z-x)^\pi - y^\pi \equiv 0\,(mod\,\pi^3)),[(z-y)^\pi - x^\pi \equiv 0\,(mod\,\pi^3)],$ *then*
$z \equiv 0\,(mod\,\pi);(y \equiv 0\,(mod\,\pi));[x \equiv 0\,(mod\,\pi)],$ *If*
$(x+y)^\pi - z^\pi \equiv 0\,(mod\,\pi^2)((z-x)^\pi - y^\pi \equiv 0\,(mod\,\pi^2)),[(z-y)^\pi - x^\pi \equiv 0\,(mod\,\pi^2)],$ *then*
$z \equiv 0\,(mod\,\pi);(y \equiv 0\,(mod\,\pi));[x \equiv 0\,(mod\,\pi)],$

**Proof.** The first assertion comes from the equations in Lemma 2 and the Unique Factorizaeion Theorem. For the second assertion, if two of the equivalences $x+y \equiv 0\,(mod\,\pi), z-x \equiv 0\,(mod\,\pi), z-y \equiv 0\,(mod\,\pi)$ hold the proof is complete by the equivalence $x+y-z.$ Assume $x+y \not\equiv 0\,(mod\,\pi), z-x \not\equiv 0\,(mod\,\pi),$ then

$$(x+y)^\pi - z^\pi$$

$$= (x+y)((x+y)^{\pi-1} - z^\pi (x+y)^{-1})$$

$$\equiv 0\,(mod\,\pi^2)$$

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$$= (x+y)((x+y)^{\pi-1} - z^{\pi-1} + z^{\pi-1} - z^{\pi}(x+y)^{-1}) \equiv 0 \, (mod \, \pi^2);$$

$$(x+y)^{\pi-1} - z^{\pi-1} \equiv 0 \, (mod \, \pi);$$

because of the equation

$$(x+y)^{\pi-1} - z^{\pi-1} = (x+y-z)\sum_{0}^{\pi-2}(x+y)^{\pi-1-k} z^k;$$

if

$$(1) \;\; (x+y)^{\pi-1} - z^{\pi-1} \equiv 0 \, (mod \, \pi^2),$$

Compare this with

$$(2) \;\; (x+y)^{\pi} - z^{\pi} \equiv 0 \, (mod \, \pi^2),$$

multiplying through (1) by $x+y$ and subtracting (1) from (2) gives

$$z^{\pi-1}(x+y-z)) \equiv 0 \, (mod \, \pi^2);$$

$$z^{\pi-1} \equiv 0 \, (mod \, \pi);$$

thus $z^{\pi-1} \equiv 0 \, (mod \, \pi), z \equiv 0 \, (mod \, \pi).$

If

$$(x+y^{\pi-1} - z^{\pi-1} \not\equiv 0 \, (mod \, \pi^2),$$

then

$$(x+y)\mu\pi \equiv 0 \, (mod \, \pi^2), (\mu, \pi) = 1,$$

so

$$x+y \equiv 0 \, (mod \, \pi),$$

a contradiction.

To show that $y \equiv 0 \, (mod \, \pi).$

$$(z-x)^{\pi} - y^{\pi} = (z-x)^{\pi} - (z-x)y^{\pi}(z-x)^{-1}$$

$$= (z-x)((z-x)^{\pi-1} - y^{\pi-1} + y^{\pi-1} - y^{\pi}(z-x)^{-1}) \equiv 0 \, (mod \, \pi^2);$$

because of the equation

$$(z-x)^{\pi-1} - y^{\pi-1} = (z-x-y)\sum_{0}^{\pi-2}(z-x)^{\pi-1-k} y^k,$$

$$(z-x)^{\pi-1} - y^{\pi-1} \equiv 0 \, (mod \, \pi),;$$

if $((z-x)^{\pi-1} - y^{\pi-1} \equiv 0 \, (mod \, \pi^2),$ this leads to

$$(1) \;\; (z-x)^{\pi} - y^{\pi} \equiv 0 \, (mod \, \pi^2),$$

$$(2) \;\; (z-x)^{\pi-1} - y^{\pi-1} \equiv 0 \, (mod \, \pi^2);$$

multiplying through (2) by $z-x$ and subtracting from (1) gives

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$$y^{\pi-1}(z-x-y)\equiv 0\,(mod\,\pi^2);$$

$$y^{\pi-1}\equiv 0\,(mod\,\pi);$$

thus $y\equiv 0\,(mod\,\pi)$.

If

$$(z-x)^{\pi-1}-y^{\pi-1}\not\equiv 0\,(mod\,\pi),$$

then

$$(x+y)\mu\pi\equiv 0\,(mod\,\pi^2),(\mu,\pi)=1,$$

so

$$z-x\equiv 0\,(mod\,\pi),$$

a contradiction.

The paper is finished by producting a new proof of the following theorem.

**Theorem 2** *If* $x,y,z$ *are relatively prime positive integers, then* $z^4\neq x^4+y^4$.

Following the proofs above, several equivalences arise:

**Proof.**

1. $z^2(x+y-z)\equiv 0\,(mod\,4)$;
2. $y^2(z-x-y)\equiv 0\,(mod\,4)$;
3. $x+y\equiv 0\,(mod\,2)$;
4. $z-x\equiv 0\,(mod\,2)$.

this leads to $z\equiv 0\,(mod\,2), y\equiv 0\,(mod\,2)$.

## REFERENCES

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