Traveling Solitary Wave Solutions for the Symmetric Regularized Long-Wave Equation

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ABSTRACT

In this paper, we employ the extended tanh function method to find the exact traveling wave solutions involving parameters of the symmetric regularized long- wave equation. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. These studies reveal that the symmetric regularized long-wave equation has a rich variety of solutions.

Keywords

The extended tanh function method; The symmetric regularized long-wave equation; Traveling wave solutions; Solitary wave solutions.

INTRODUCTION

The nonlinear partial differential equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, Optics, Plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, tanh – sech method [2]-[4], extended tanh - method [5]-[7], sine - cosine method [8]-[10], modified simple equation method [11, 12], the exp $\left(exp\left(-\phi(\xi)\right)\right)$ −expansion method [13]-[15], the extended exp $\left(exp\left(-\phi(\xi)\right)\right)$ − expansion method [16], $\left(\frac{G'}{G}\right)$ −expansion method [17]-[19], Jacobi elliptic function method [20]- [23] and so on.

The objective of this article is to apply The extended tanh function method for finding the exact traveling wave solution of the symmetric regularized long- wave equation Which describe shallow water waves and plasma drift waves. The rest of this paper is organized as follows: In Section **2**, we give the description of the extended Jacobi elliptic function expansion method In Section **3**, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section **4**, conclusions are given.

Description of method

Consider the following nonlinear evolution equation

$$P(u,u_t,u_x,u_{tt},u_{xx},\ldots\ldots)=0, \tag{2.1}$$

Since, P is a polynomial in $u(x,t)$ and its partial derivatives. In the following, we give the main steps of this method.

Step 1. We use the traveling wave solution in the form

$$u(x,t)=u(\xi), \qquad \xi=x-ct, \tag{2.2}$$

Where c is a positive constant, to reduce Eq. (2.1) to the following ODE:

$$p(u,u',u'',u''',\ldots\ldots)=0, \tag{2.3}$$

Where P is a polynomial in u (ξ) and its total derivatives, while $u'=\dfrac{du}{d\xi}$.

Step 2. Suppose that the solution of ODE (2.3) can be expressed

$$u(\xi)=a_0+\sum_{i=0}^{M}\left(a_i\varphi^i+b_i\varphi^{-1}\right), \tag{2.4}$$

where a_i,b_i are arbitrary constants to be determined, such that $a_m\neq 0\ or\ b_m\neq 0$, while φ satisfies the Riccati equation

$$\varphi'=b+\varphi^2, \tag{2.5}$$

where b is a constant. Eq.(2.5) admits several types of solutions according to

Case 1. If $b<0,$ then

$$\varphi=-\sqrt{-b}\,\tanh\left(\sqrt{-b}\,\xi\right),or\ \varphi=-\sqrt{-b}\,\coth\left(\sqrt{-b}\,\xi\right). \tag{2.6}$$

Case 2. If $b>0,$ then

$$\varphi=\sqrt{b}\,\tan\left(\sqrt{b}\,\xi\right),or\ \varphi=\sqrt{b}\,\cot\left(\sqrt{b}\,\xi\right). \tag{2.7}$$

Case 3. If $b = 0$, then

$$\varphi = -\frac{1}{\xi}.$$

(2.8)

Step 3. Determine the positive integer m in Eq.(1.4) by balancing the highest order derivatives and the nonlinear terms.

Step 4. Substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $\left(\varphi^i, i = 0, \pm 1, \pm 2, \pm 3, ...\right)$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i and b_i.

Step 5. Substituting these values and the solutions of Eq.(2.5) into Eq.(2.4) we obtain the exact solutions of Eq.(2.1).

The SRLW equation

Here, we will apply the extended tanh function method described in Sec.**2** to find the exact traveling wave solutions and then the solitary wave solutions The SRLW equation [24].

Consider the SRLW equation is in the form

$$v_{tt} - v_{xx} + \left(\frac{v^2}{2}\right)_{xt} - v_{xxtt} = 0,$$

(3.1)

by using the transformation $v(\xi) = v(x, t)$ $since\ \xi = x + kt.$ Where k is arbitrary constant to be determined later, we get

$$(k^2 - 1)v'' - k\left(\frac{v^2}{2}\right)'' - k^2 v'''' = 0.$$

(3.2)

By integration Eq.(3.2) twice with negligence of integral constant, we get

$$(k^2 - 1)v - \frac{k}{2}v^2 - k^2 v'' = 0.$$

(3.3)

Balancing v'' $and\ v^2 \Rightarrow m = 2$, so that, we assume the solution of Eq.(3.3) be in the form

$$v(\xi) = a_0 + a_1\phi + a_2\phi^2 + \frac{b_1}{\phi} + \frac{b_2}{\phi^2}.$$

(3.4)

Substituting Eq.(3.4) and it's derivatives into Eq.(3.3) and collecting the coefficients of $\phi^i, i = 0, \pm 1, \pm 2, ...$ and set it to zero we obtain the system of equation

$$\begin{cases} \frac{-1}{2}ka_2^2 - 6k^2 a_2 = 0, \\ -ka_1 a_2 - 2k^2 a_1 = 0, \\ (k^2 - 1)a_2 - \frac{1}{2}k(a_1^2 + 2a_0 a_2) - 8k^2 a_2 b = 0, \\ (k^2 - 1)a_1 - \frac{1}{2}k(2a_0 a_1 + 2a_2 b_1) - 2k^2 a_1 b = 0, \\ (k^2 - 1)a_0 - \frac{1}{2}k(a_0^2 + 2a_2 b_2 + 2a_1 b_1) - k^2(2b_2 + 2a_2 b^2) = 0, \\ (k^2 - 1)b_1 - \frac{1}{2}k(2a_0 b_1 + 2a_1 b_2) - 2k^2 b_1 b = 0, \\ (k^2 - 1)b_2 - \frac{1}{2}k(b_1^2 + 2a_0 b_2) - 8k^2 b_2 b = 0, \\ -kb_1 b_2 - 2k^2 b_1 b^2 = 0, \\ -\frac{1}{2}kb_2^2 - 6k^2 b_2 b^2 = 0. \end{cases}$$

(3.5)

Solving above system by using Maple 16, we get

Case 1.

$$b = -\frac{1}{4}\frac{k^2-1}{k^2}, a_0 = 3\frac{k^2-1}{k}, a_1 = 0, a_2 = -12k, b_1 = 0, b_2 = 0.$$

Case 2.

$$b = \frac{1}{4}\frac{k^2-1}{k^2}, a_0 = \frac{1-k^2}{k}, a_1 = 0, a_2 = -12k, b_1 = 0, b_2 = 0.$$

Case 3.

$$b = -\frac{1}{16}\frac{k^2-1}{k^2}, a_0 = \frac{3}{2}\frac{k^2-1}{k}, a_1 = 0, a_2 = -12k, b_1 = 0, b_2 = -\frac{3}{64}\frac{k^4-2k^2+1}{k^3}.$$

Case 4.

$$b = \frac{1}{16}\frac{k^2-1}{k^2}, a_0 = \frac{1}{2}\frac{k^2-1}{k}, a_1 = 0, a_2 = -12k, b_1 = 0, b_2 = -\frac{3}{64}\frac{k^4-2k^2+1}{k^3}.$$

Case 5.

$$b = -\frac{1}{4}\frac{k^2-1}{k^2}, a_0 = 3\frac{k^2-1}{k}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -\frac{3}{4}\frac{(k^2-1)^2}{k^3}.$$

Case 6.

$$b = \frac{1}{4}\frac{k^2-1}{k^2}, a_0 = -\frac{k^2-1}{k}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = \frac{-3}{4}\frac{(k^2-1)^2}{k^3}.$$

So that, we will study each case and get the exact traveling wave solution and also the solitary wave solutions for Eq. (3.3).

For Case 1.

The exact traveling wave solution is in the form:

$$v(\xi) = 3\frac{k^2-1}{k} - 12k\,\phi^2. \qquad (3.6)$$

The solitary wave solution is in the form:

Case i. If $b < 0$, we get

$$v(\xi) = 3\frac{k^2-1}{k} - 12k\left(-\sqrt{-b}\,tanh(\sqrt{-b}\,\xi)\right)^2.$$

Or

$$v(\xi) = 3\frac{k^2-1}{k} - 12k\left(-\sqrt{-b}\,coth(\sqrt{-b}\,\xi)\right)^2.$$

Case ii. If $b > 0$, we get

$$v(\xi) = 3\frac{k^2-1}{k} - 12k\left(\sqrt{b}\,tan(\sqrt{b}\,\xi)\right)^2.$$

Or

$$v(\xi) = 3\frac{k^2-1}{k} - 12k\left(\sqrt{b}\,cot(\sqrt{b}\,\xi)\right)^2.$$

Case iii. If $b = 0$, we get

$$v(\xi) = 3\frac{k^2 - 1}{k} - 12k\left(\frac{1}{\xi}\right)^2.$$

For Case 2.

The exact traveling wave solution is in the form:

$$v(\xi) = \frac{k^2 - 1}{k} - 12k\,\phi^2. \tag{3.7}$$

The solitary wave solution is in the form:

Case i. If $b < 0$, we get

$$v(\xi) = \frac{k^2 - 1}{k} - 12k\left(-\sqrt{-b}\,\tanh(\sqrt{-b}\,\xi)\right)^2.$$

Or

$$v(\xi) = \frac{k^2 - 1}{k} - 12k\left(-\sqrt{-b}\,\coth(\sqrt{-b}\,\xi)\right)^2.$$

Case ii. If $b > 0$, we get

$$v(\xi) = \frac{k^2 - 1}{k} - 12k\left(\sqrt{b}\,\tan(\sqrt{b}\,\xi)\right)^2.$$

Or

$$v(\xi) = \frac{k^2 - 1}{k} - 12k\left(\sqrt{b}\,\cot(\sqrt{b}\,\xi)\right)^2.$$

Case iii. If $b = 0$, we get

$$v(\xi) = \frac{k^2 - 1}{k} - 12k\left(\frac{1}{\xi}\right)^2.$$

For Case 3.

The exact traveling wave solution is in the form:

$$v(\xi) = \frac{3}{2}\frac{k^2 - 1}{k} - 12k\,\phi^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\phi^2}. \tag{3.8}$$

The solitary wave solution is in the form:

Case i. If $b < 0$, we get

$$v(\xi) = \frac{3}{2}\frac{k^2 - 1}{k} - 12k\left(-\sqrt{-b}\,\tanh(\sqrt{-b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(-\sqrt{-b}\,\tanh(\sqrt{-b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = \frac{3}{2}\frac{k^2 - 1}{k} - 12k\left(-\sqrt{-b}\,\coth(\sqrt{-b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(-\sqrt{-b}\,\coth(\sqrt{-b}\,\xi)\right)^2}.$$

Case ii. If $b > 0$, we get

$$v(\xi) = \frac{3}{2}\frac{k^2 - 1}{k} - 12k\left(\sqrt{b}\,tan(\sqrt{b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(\sqrt{b}\,tan(\sqrt{b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = \frac{3}{2}\frac{k^2 - 1}{k} - 12k\left(\sqrt{b}\,cot(\sqrt{b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(\sqrt{b}\,cot(\sqrt{b}\,\xi)\right)^2}.$$

Case iii. If $b = 0$, we get

$$v(\xi) = \frac{3}{2}\frac{k^2 - 1}{k} - 12k\left(\frac{1}{\xi}\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(\frac{1}{\xi}\right)^2}.$$

For Case 4.

The exact traveling wave solution is in the form:

$$v(\xi) = \frac{1}{2}\frac{k^2 - 1}{k} - 12k\,\phi^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\phi^2}.$$ (3.9)

The solitary wave solution is in the form:

Case i. If $b < 0$, we get

$$v(\xi) = \frac{1}{2}\frac{k^2 - 1}{k} - 12k\left(-\sqrt{-b}\,tanh(\sqrt{-b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(-\sqrt{-b}\,tanh(\sqrt{-b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = \frac{1}{2}\frac{k^2 - 1}{k} - 12k\left(-\sqrt{-b}\,coth(\sqrt{-b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(-\sqrt{-b}\,coth(\sqrt{-b}\,\xi)\right)^2}.$$

Case ii. If $b > 0$, we get

$$v(\xi) = \frac{1}{2}\frac{k^2 - 1}{k} - 12k\left(\sqrt{b}\,tan(\sqrt{b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(\sqrt{b}\,tan(\sqrt{b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = \frac{1}{2}\frac{k^2 - 1}{k} - 12k\left(\sqrt{b}\,cot(\sqrt{b}\,\xi)\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(\sqrt{b}\,cot(\sqrt{b}\,\xi)\right)^2}.$$

Case iii. If $b = 0$, we get

$$v(\xi) = \frac{1}{2}\frac{k^2 - 1}{k} - 12k\left(\frac{1}{\xi}\right)^2 - \frac{3}{64}\frac{k^4 - 2k^2 + 1}{k^3}\frac{1}{\left(\frac{1}{\xi}\right)^2}.$$

For Case 5.

The exact traveling wave solution is in the form:

$$v(\xi) = 3\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\phi^2}.$$ (3.10)

The solitary wave solution is in the form:

Case i. If $b < 0$, we get

$$v(\xi) = 3\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(-\sqrt{-b}\,\tanh(\sqrt{-b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = 3\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(-\sqrt{-b}\,\coth(\sqrt{-b}\,\xi)\right)^2}.$$

Case ii. If $b > 0$, we get

$$v(\xi) = 3\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(\sqrt{b}\,\tan(\sqrt{b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = 3\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(\sqrt{b}\,\cot(\sqrt{b}\,\xi)\right)^2}.$$

Case iii. If $b = 0$, we get

$$v(\xi) = 3\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(\frac{1}{\xi}\right)^2}.$$

For Case 6.

The exact traveling wave solution is in the form:

$$v(\xi) = -\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\phi^2}.$$ (3.11)

The solitary wave solution is in the form:

Case i. If $b < 0$, we get

$$v(\xi) = -\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(-\sqrt{-b}\,\tanh(\sqrt{-b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = -\frac{k^2-1}{k} - \frac{3}{4}\frac{(k^2-1)^2}{k^3}\frac{1}{\left(-\sqrt{-b}\,\coth(\sqrt{-b}\,\xi)\right)^2}.$$

Case ii. If $b > 0$, we get

$$v(\xi) = -\frac{k^2 - 1}{k} - \frac{3}{4}\frac{(k^2 - 1)^2}{k^3}\frac{1}{\left(\sqrt{b}\,tan(\sqrt{b}\,\xi)\right)^2}.$$

Or

$$v(\xi) = -\frac{k^2 - 1}{k} - \frac{3}{4}\frac{(k^2 - 1)^2}{k^3}\frac{1}{\left(\sqrt{b}\,cot(\sqrt{b}\,\xi)\right)^2}.$$

Case iii. If $b = 0$, we get

$$v(\xi) = -\frac{k^2 - 1}{k} - \frac{3}{4}\frac{(k^2 - 1)^2}{k^3}\frac{1}{\left(\frac{1}{\xi}\right)^2}.$$

Remark:

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

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Conclusion

The extended tanh function method has been applied in this paper to find the exact traveling wave solutions and then the solitary wave solutions of the symmetric regularized long-wave equation. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the symmetric regularized long-wave equation are new and different from those obtained in [28]. The obtained exact solutions can be used as benchmarks against the numerical simulations in theoretical physics and fluid mechanics.

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