Convergence Theorems ofIterative Schemes For Nonexpansive Mappings

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ABSTRACT

In this paper, we give atype of iterative scheme for sequence ofnonexpansive mappings and we study the strongly convergence of these schemes in real Hilbert space to common fixed point which is also a solution of a variational inequality.Also there are some consequent of this results in convex analysis

Keywords: maximal monotone; strongly convergence; variational inequality; nonexpansive mapping.

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Functional Analysis

INTRODUCTIONAND PRELIMINARI

Let X be a Hilbert space, $\emptyset \neq C$ be a convex closed subset of X and A be amultivalued mapping with domain $D(A)$ and range $R(A)$.The mapping A is called monotone mapping if the following inequality hold

$$< x_1 - x_2 , y_1 - y_1 > \geq 0, \forall x_i \in D(A), \quad \forall y_i \in R(A).$$

Also, any mapping A is called maximal monotone mapping of A if the graph $G(T)$ of T is not properly contained in the graph of any other monotone mapping ,where

$G(T) = \{(u, v) \in X \times X ; u \in A(x)\}$.Monotone mappings play a crucial role in

modern nonlinear analysis and optimization ,see the books [1,2,3,4,5]

The single valued nonexpansiveself- mapping onCis defined as: $J_{r_n} = (I + r_n A)^{-1}(x)$,and is called resolvent mapping on C,where $< r_n >$ be asequence of positive real numbers. In[6]Moudafi, studied the strong convergence of both thefollowing iterative schemes in Hilbert space

$$x_t = tf(x_t) + (1-t)T_{x_t} \qquad \text{as } t \to \infty \qquad (1)$$

$$x_{n+1} = \alpha_n f(x_n) + (1-t)T_{x_n} \qquad \text{as } n \to \infty \qquad (2)$$

where fbe a contraction mapping , T is nonexpansive mapping and$< \alpha_n >$ be a sequence in (0,1).In this paper we study the stronglyconvergenceof common fixed point of sequence of nonexpansive mapping which is also a solution of varational inequality,

$$< (I - f_n)\acute{x}, x - \acute{x} > \leq 0, \qquad x \in A^{-1}(0)$$

Now, We recall some definitions and lemmas which will used in the proofs:

Definition1. [6]and [7]

1- A mapping $T : C \to X$ is called Lipchitz continuous with constant $\alpha > 0$

$|| Tx - Ty|| \leq \alpha || x - y||$, for any $x, y \in C$

2- I f $\alpha \in (0,1) \Rightarrow$ T is called contraction mapping .

3- If $\alpha = 1 \Rightarrow$ T is called nonexpensive mapping.

Definition2. [6]and [7]A mapping $T : C \to X$ is called

1.firmly nonexpensive mapping if for any$x, y in C$ then,

$$|| Tx - Ty||^2 + ||(I - T)x - (I - T) y||^2 \leq ||x - y ||^2$$

2.strongly nonexpansive mapping if it is nonexpensive and for any $<x_n>$ and

$\langle y_n \rangle$are sequences in C such that$\langle x_n - y_n \rangle$ is bounded and $||x_n - y_n || - || T x_n - T y_n || \to 0$ it follows that $(x_n - y_n) - (Tx_n - Ty_n) \to 0$.

Note that Both firmly nonexpansive and strongly nonexpansive imply nonexpansive.

Theorem3.[7] If T be a mapping from X into X , then the following are equivalent

1- T is firmly nonexpensive
2- $(I-T)$ is firmly nonexpensive
3- $(2I-T)$ is nonexpensive
4- $\|Tx-Ty\| \leq < x-y, Tx-Ty >$ for all $x, y in X$
5- $0 \leq < x-y, Tx-Ty >$ for all $x, y in X$

Lemma4.[8] If X be a real Hilbert space , $\emptyset \neq C$ be a convex closed in X and T be anonexpansive mapping with $F(T) \neq \emptyset$. suppose that $<x_n>$ converge weakly to x If $(I-T)x_n \to y$ then $(I-T)x = y$.

Lemma5. [9] be a sequence of nonnegative real number such that $>a_n<$ If; $n \geq 0 a_{n+1} \leq (1-\gamma_n) a_n + \gamma_n S_n$

Where $<S_n>$ be a sequence in the real number and $<\gamma_n>$ be a sequence in (0,1) such that $\sum |S_n| < \infty$ and $0 \geq \lim_{n\to\infty} \sup a_n/\gamma_n$ .Then $a_n \to 0$ as $n \to \infty$.

Lemma6. [10] Let $\emptyset \neq C$ convex closed in C and T be a multivalued nonexpansive mapping. If x_n convergence weakly to p and $\| x_n - T_{x_n} \| \to 0$.Then $p \in F(T)$.

2.MAIN RESULTS Let X be a real Hilbert space and C be a nonempty convex closed subset of X. Denote by :

- $\mathcal{F}$ is the class of the sequence $< f_n >$ of mappings on C such that

$$\|f_n(x_n) - f_{n-1}(x_{n-1})\| \leq \|f_{n-1}(x_n) - f_{n-1}(x_{n-1})\|$$

- T_t be a mapping on C such that : $T_t(x) = tf_n(x) + (1-t)J_{rn}(x) \,;\, t > 0$

Now, we give the following definition.

Definition 2.1. Let $< T_n >$ be a sequence of mappings on C, then $p \in C$ is called asymptotic common fixed point of $< T_n >$ if there exist a sequence $< x_n >$ in C converges weakly to p and $\lim_{n\to\infty}\|x_n - T_n(x_n)\| = 0$.

In this paper, we study the strong convergence of types of iterative schemes in real Hilbert space.

Remark 2.2. If $< f_n >$ be a sequence of nonexpansive mappings then T_t is also nonexpansive.

Proof For all $x, y \in C$,

$$\|T_t(x) - T_t(y)\| \leq t\|f_n(x) - f_n(y)\| + (t-1)\|J_{rn}(x) - J_{rn}(y)\|$$
$$\leq t\|x-y\| + (1-t)\|x-y\|$$
$$\leq \|x-y\|$$

Theorem 2.3 Let A be a maximal multivalued mapping , $< f_n >$ be a sequence of bounded and contraction mappings on C and $A^{-1}(0) \neq \varphi$. Then $< x_t >$ converges strongly to the point $\tilde{x}$, where $\tilde{x} = p_E(f_n(\tilde{x}))$ or $\tilde{x}$ is the unique solution of variation of variational inequality. $< (I - f_n)\tilde{x}, x - \tilde{x} > \geq 0$, $x \in E = A^{-1}(0)$.

Proof Let $p \in A^{-1}(0)$

$\|x_t - p\| \leq t\|f_n(x_t) - p\| + (1-t)\|J_{rn}(x_t) - p\|$

$\leq t\|f_n(x_t) - p\| + (1-t)\|x_t - p\|$

$t\|x_t - p\| \leq t\|f_n(x_t) - p\|$

$\|x_t - p\| \leq \|f_n(x_t) - f_n(p)\| + \|f_n(p) - p\|$

$\leq \alpha\|x_t - p\| + \|f_n(p) - p\| \,;\, \alpha = \max\{\alpha_i, i \in N\} \,;\, 0 < \alpha < 1$

$\|x_t - p\| \leq \dfrac{1}{1-\alpha}\|f_n(p) - p\|$

But $< f_n >$ is bounded sequence, and hence $< x_t >$ is bounded sequence, So $< J_{nr} >$ also bounded.

$\|x_t - J_{rn}x_t\| = \|tf_n(x_t) + (1-t)J_{rn}(X_t) - J_{rn}(x_t)\|$

$= t\|f_n(x_t) - J_{rn}(x_t)\| \to 0$ as $t \to 0$

Since $< x_t >$ is bounded then there exists a subsequence $< x_{tn} >$ of $< x_t >$ such that $x_{tn} \rightharpoonup \tilde{x}$.

By lemma (1.4), we get $\tilde{x} \in A^{-1}(0)$

Now, since $x_t - \tilde{x} = t(f_n(x_t) - \tilde{x}) + (1-t)(J_{rn}(x_t) - \tilde{x})$,

$\|x_t - \tilde{x}\|^2 = t < f_n(x_t) - \tilde{x}, x_t - \tilde{x} > +(1-t) < J_{rn}(x_t) - \tilde{x}, x_t - \tilde{x} >$

$\leq t < f_n(x_t) - \tilde{x}, x_t - \tilde{x} > +\|x_t - \tilde{x}\|^2$

$\|x_t - \tilde{x}\|^2 \leq < f_n(x_t) - \tilde{x}, x_t - \tilde{x} >$

$$\leq < f_n(x_t) - f_n(\tilde{x}), x_t - \tilde{x} > + < f_n(\tilde{x}) - \tilde{x}, x_t - \tilde{x} >$$

$\leq \alpha\|x_t - \tilde{x}\|^2 + < f_n(\tilde{x}) - \tilde{x}, x_t - \tilde{x} >$;

$\alpha = \sup\{\alpha_i, i \in N\}$ such that $0 < \alpha < 1$

$$\|x_t - \tilde{x}\|^2 \leq \frac{1}{1-\alpha} < f_n(\tilde{x}) - \tilde{x}, x_t - \tilde{x} >$$

And hence, $\|x_{tn} - \tilde{x}\|^2 \leq \frac{1}{1-\alpha} < f_n(\tilde{x}) - \tilde{x}, x_{tn} - \tilde{x} >$

But $x_{tn} \rightharpoonup \tilde{x}$, then as $n \to \infty$ we get

$< f_n(\tilde{x}) - \tilde{x}, x_{tn} - \tilde{x} > \to \infty$ and hence , $\|x_t - \tilde{x}\| \to 0$

Now, to prove that $\tilde{x}$ is unique solves of the variational inequality.

Since, $x_t = tf_n(x_t) + (1-t)J_{rn}x_t \Rightarrow (I - f_n)(x_t) = -\left(\frac{1-t}{t}\right)(I - J_{rn})(x_t)$

And for all $z \in A^{-1}(0)$

$$< (I - f_n)(x_t), x_t - z > = -\left(\frac{1-t}{t}\right) < (I - J_{rn})(x_t), x_t - z >$$

$$= -\left(\frac{1-t}{t}\right) < (I - J_{rn})(x_t) - (I - J_{rn})(z), x_t - z >$$

≤ 0 as $(I - J_{rn})$ is monotone.

Therefore, $\tilde{x}$ is a solution of variational inequality

$$< (I - f_n)(x_t), x_t - z > \leq 0 \quad, \forall z \in A^{-1}(0)$$

To prove the uniqueness, suppose that

$x_{tn} \to \hat{x} \in E = A^{-1}(0)$ and $\hat{x}$ is solution of variational inequality

$$< (I - f_n)(\tilde{x}), \tilde{x} - \hat{x} > \leq 0 \tag{3}$$

Interchange $\tilde{x}$ and $\hat{x}$

$$< (I - f_n)(\hat{x_t}), \hat{x} - \tilde{x} > \leq 0 \tag{4}$$

Adding up (3) and (4) we have

$$< \tilde{x} - \hat{x}, (I - f_n)(\tilde{x}), (I - f_n)(\hat{x}) > \leq 0$$

By lemma (1.5), we get $\tilde{x} = \hat{x}$

corollary 2.4. Let A be a maximal multivalued mapping and $< T_n >$ be a sequence of firmly non expansive. If the scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f_n(x_n) + \beta_n T_n^{\alpha_n}(x_n) + (1 - \gamma_n)(x_n)$$

Where $< f_n >, < \alpha_n >, < \gamma_n > and < \beta_n >$ as in theorem (2.3) and

$$T_n^{\alpha_n}(x_n) = (1 - \alpha_n)x_n + \alpha_n T_n(x_n)$$

Then $< x_n >$ converges strongly to an asymptotic common fixed point of $T_n^{\alpha_n}, \forall n \in N$.

Proof For any $x, y \in X$

$\|T_n^{\alpha_n}(x) - T_n^{\alpha_n}(y)\| \leq (1 - \alpha_n)\|x - y\| + \alpha_n\|T_n(x) - T_n(y)\|$

$$\leq (1 - \alpha_n)\|x - y\| + \alpha_n\|x - y\|$$

$$= \|x - y\|$$

Therefore, $< T_n^{\alpha_n} >$ is a sequence of nonexpansive. Then by theorem (2.3) we get the result.

Theorem 2.5. Let A be a maximal monotone multivalued mapping, $< f_n >$ be a sequence of contraction mapping on C and $< T_n >$ be a sequence of nonexpansive mapping on $C, < f_n >$ and $< T_n >$ lines in $\mathcal{F}$such that $A^{-1}(0) \cap (\cap F(f_n) \cap (\cap F(T_n)) \neq \varphi$..If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f_n(x_n) + \beta_n T_n(x_n) + (1-\gamma_n)J_{rn}(x_n)$$

Where $< \alpha_n >$ and $< \beta_n >$ are decreasing sequences in $[0,1)$ converges to 0,such that

$\sum_{n=0}^{\infty} \alpha_n = \infty$ and $\alpha_n + \beta_n + (1-\gamma_n) = 1$.

$\frac{1}{2} \leq \alpha_n + \beta_n < 1$ and$\sum_{n=0}^{\infty}(\|f_n(x_n)\| + \|J_{r_n}(x_n)\|) < \infty$.Then the iterative scheme $< x_n >$ converges strongly to an asymptotic common fixed point of $T_n, \forall\, n \in N$

ProofLet $p \in A^{-1}(0) \cap (\cap F(T_n)) \cap (\cap F(f_n))$

$$\|x_{n+1} - p\| \leq \alpha_n \|f_n(x_n) - p\| + \beta_n \|T_n(x_n) - p\| + (1-\gamma_n)\|J_{rn}(x_n) - p\|$$

$$\leq \alpha\alpha_n \|x_n - p\| + \beta_n \|x_n - p\| + (1-\gamma_n)\|x_n - p\|$$

where $\alpha = \sup\{\alpha_i, i \in N\}$ and $0 < \alpha < 1$

$\|x_{n+1} - p\| \leq (\alpha_n + \beta_n + (1-\gamma_n))\|x_n - p\|$

$\|x_{n+1} - p\| \leq \|x_n - p\| \Rightarrow < x_n >$ is bounded sequence, So $< f_n >, < T_n >$ and $< J_{rn} >$ also bounded.

Now, since $< f_n >$ and $< T_n >$ lies in $\mathcal{F}$.Therefore,

$$\begin{aligned}
\|x_{n+1} - x_n\| &\leq \alpha_{n-1}\|f_{n-1}(x_n) - f_{n-1}(x_{n-1})\| \\
&+\beta_{n-1}\|T_{n-1}(x_n) - T_{n-1}(x_{n-1})\| + \\
&(1-(\alpha_{n-1} + \beta_{n-1}))\|J_{r_n}(x_n) - J_{r_{n-1}}(x_{n-1})\| \\
&\leq \alpha_{n-1}\alpha\|x_n - x_{n-1}\| + \beta_{n-1}\|x_n - x_{n-1}\| \\
&+(1-(\alpha_{n-1} + \beta_{n-1}))\|J_{r_n}(x_n) - J_{r_{n-1}}(x_{n-1})\| \\
&\leq (\alpha_{n-1}\alpha + \beta_{n-1})\|x_n - x_{n-1}\| + (1-(\alpha_{n-1} + \beta_{n-1}))\|J_{r_n}(x_n) - J_{r_{n-1}}(x_{n-1})\| \\
&+(1-(\alpha_{n-1} + \beta_{n-1}))\|J_{r_n}(x_n) - J_{r_{n-1}}(x_{n-1})\| \\
&\leq (\alpha_{n-1}\alpha + \beta_{n-1})\|x_n - x_{n-1}\| + \\
&(1-(\alpha_{n-1} + \beta_{n-1}))\|J_{r_n}(x_n) - J_{r_{n-1}}(x_{n-1})\| \\
&+(\alpha_{n-1} + \beta_{n-1})\|J_{r_n}(x_n) - J_{r_{n-1}}(x_{n-1})\|
\end{aligned}$$

And hence, $\|x_{n-1} - x_n\| \to 0 \quad \text{as} \quad n \to \infty$ (5)

$$\begin{aligned}
\|x_n - T_n x_n\| &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - T_n x_n\| \\
&< \|x_{n+1} - x_n\| + \alpha_n\|f_n(x_n)\| + 2\beta_n\|f_n(x_n)\| + \\
&(\alpha_n + \beta_n)\|J_r(x_n)\|
\end{aligned}$$

But $< f_n >$ and $< J_n >$ are bounded and by (5), we get

$\|x_n - T_n x_n\| \to 0 \quad \text{as} \quad n \to \infty$ (6)

Since $< x_n >$ is bounded sequence then there exists as a subsequence $< x_{nk} >$ of $< x_n >$ such that $x_{nk} \rightharpoonup \tilde{x}$.

By equation (6) and by using lemma (1.6) we get $\tilde{x} \in \cap F(T_n)$

$$\begin{aligned}
\|x_{n-1} - \tilde{x}\| &\leq \alpha_n\|f_n(x_n) - \tilde{x}\| + \beta_n\|T_n(x_n) - \tilde{x}\| + (1-\gamma_n)\|J_{r_n}(x_n) - \tilde{x}\| \\
&\leq \alpha_n\|f_n(x_n) - \tilde{x}\| + (1-(1-\gamma_n) + \alpha_n)\|x_n - \tilde{x}\| + \\
&(\alpha_n + \beta_n)\|J_{r_n}(x_n) - \tilde{x}\| \\
&= (1-\alpha_n)\|x_n - \tilde{x}\| + \alpha_n\|f_n(x_n) - \tilde{x}\| + \\
&(\alpha_n + \beta_n)\|J_{r_n}(x_n) - \tilde{x}\|
\end{aligned}$$

By lemma (1.5), we get, $\|x_n - \tilde{x}\| \to 0 \quad as \quad n \to \infty$.And hence $< x_n >$ converges strongly to an asymptotic fixed point of $T_n, \forall n \in N$.

Corollary2.6. Let A be a maximal monotone multivalued mapping, f be a contraction self-mapping on CandT be a non-expansive self- mapping on C such that $A^{-1}(0) \cap (F(f) \cap (F(T)) \neq \varphi$ and f and T lines in $\mathcal{F}$ .If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n T(x_n) + (1 - \gamma_n) J_{rn}(x_n)$$

Where $< \alpha_n >$ and $< \beta_n >$ are decreasing sequences in $[0,1)$ converges to 0,such that

1. $\sum_{n=0}^{\infty} \alpha_n = \infty$ and $\alpha_n + \beta_n + (1 - \gamma_n) = 1$.
2. $\frac{1}{2} \leq \alpha_n + \beta_n < 1$ and$\sum_{n=0}^{\infty} \|f(x_n)\| + \|J_{r_n}(x_n)\| < \infty$.Then the iterative scheme $< x_n >$ converges strongly to an asymptotic common fixed point of $T_n, \forall\, n \in N$

Corollary2.7. Let A be a maximal monotone multivalued mapping, f be a contraction mapping on C and T be a non-expansive mapping on C such that $A^{-1}(0) \cap (\cap F(f) \cap (\cap F(T)) \neq \varphi$ and f and T lines in $\mathcal{F}$.If the scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n T_{nT}(x_n) + (1 - \gamma_n) J_{rn}(x_n)$$

Where $< \alpha_n >$ and $< \beta_n >$ are decreasing sequences in $[0,1)$ converges to 0,such that

1. $\sum_{n=0}^{\infty} \alpha_n = \infty$ and $\alpha_n + \beta_n + (1 - \gamma_n) = 1$.

2.$\frac{1}{2} \leq \alpha_n + \beta_n < 1$ and$\sum_{n=0}^{\infty} \|f(x_n)\| + \|J_{r_n}(x_n)\| < \infty$.Then the iterative scheme $< x_n >$ converges strongly to an asymptotic fixed point of $T_n, \forall\, n \in N$.

Corollary2.8. Let A be a maximal monotone multivalued mapping, $f$be a sequence of contraction mapping on C and T be a sequence of non-expansive mapping on C such that $A^{-1}(0) \cap (\cap F(f) \cap (\cap F(T)) \neq \varphi$ and f and T lines in $\mathcal{F}$.If the scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n T(x_n) + (1 - \gamma_n) J_{rn}(x_n)$$

Where $< \alpha_n >$ and $< \beta_n >$ are decreasing sequences in $[0,1)$ converges to 0,such that

1.$\sum_{n=0}^{\infty} \alpha_n = \infty$ and $\alpha_n + \beta_n + (1 - \gamma_n) = 1$.

2.$\frac{1}{2} \leq \alpha_n + \beta_n < 1$ and$\sum_{n=0}^{\infty} \|f(x_n)\| + \|J_{r_n}(x_n)\| < \infty$.Then the iterative scheme $< x_n >$ converges strongly to an asymptotic fixed point of $T_n, \forall\, n \in N$.

Corollary 2.9. Let A be a maximal multivalued mapping and $< T_n >$ be a sequence of nonexpansive. If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f_n(x_n) + \beta_n {T_n}^{\alpha_n}(x_n) + (1 - \gamma_n) J_{r_n}(x_n)$$

Where $< f_n >, < \alpha_n >$ *and* $< \beta_n >$ as in theorem (2.5) and

$${T_n}^{\alpha_n}(x_n) = (1 - \alpha_n)x_n + \alpha_n T_n(x_n)$$

Then $< x_n >$ converges strongly to common asymptotic fixed point of ${T_n}^{\alpha_n}, \forall n \in N$.

ProofFor any $x, y \in X$

$$\|{T_n}^{\alpha_n}(x) - {T_n}^{\alpha_n}(y)\| \leq (1 - \alpha_n)\|x - y\| + \alpha_n \|T_n(x) - T_n(y)\|$$

$$\leq (1 - \alpha_n)\|x - y\| + \alpha_n \|x - y\| = \|x - y\|$$

Therefore, $< {T_n}^{\alpha_n} >$ is a sequence of nonexpansive. Then by theorem (2.5) we get the result.

Corollary2.10. Let A be a maximal multivalued mapping and $< T_n >$ be a sequence of strongly nonexpansive. If the scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f_n(x_n) + \beta_n {T_n}^{\alpha_n}(x_n) + (1 - \gamma_n) J_{r_n}(x_n)$$

Where $< f_n >, < \alpha_n >$ *and* $< \beta_n >$ as in theorem (2.5) and

$${T_n}^{\alpha_n}(x_n) = (1 - \alpha_n)x_n + \alpha_n T_n(x_n)$$

Then $< x_n >$ converges strongly to common asymptotic fixed point of ${T_n}^{\alpha_n}, \forall n \in N$

Corollary 2.11. Let A be a maximal multivalued mapping and $< T_n >$ be a sequence of firmly nonexpansive. If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f_n(x_n) + \beta_n {T_n}^{\alpha_n}(x_n) + (1 - \gamma_n) J_{r_n}(x_n)$$

Where $< f_n >, < \alpha_n > and < \beta_n >$ as in theorem (2. 5) and

$$T_n^{\alpha_n}(x_n) = (1-\alpha_n)x_n + \alpha_n T_n(x_n)$$

Then $< x_n >$ converges strongly to common asymptotic fixedpoint of $T_n^{\alpha_n}, \forall n \in N$.

Corollary2.12. Let A be a maximal multivalued mapping and $T: C \to C$ be a nonexpansive mapping . If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n T^{\alpha_n}(x_n) + (1-\gamma_n) J_{r_n}(x_n)$$

Where $< f_n >, < \alpha_n > and < \beta_n >$ as in theorem (2. 5) and

$$T^{\alpha_n}(x_n) = (1-\alpha_n)x_n + \alpha_n T(x_n)$$

Then $< x_n >$ converges strongly to asymptotic fixed point of $T^{\alpha_n}, \forall n \in N$

Corollary2.13. Let A be a maximal multivalued mapping and $T: C \to C$ be a strongly nonexpansive. If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n T^{\alpha_n}(x_n) + (1-\gamma_n) J_{r_n}(x_n)$$

Where $< f_n >, < \alpha_n > and < \beta_n >$ as in theorem (2. 5) and

$$T^{\alpha_n}(x_n) = (1-\alpha_n)x_n + \alpha_n T(x_n)$$

Then $< x_n >$ converges strongly to asymptotic fixed point of $T^{\alpha_n}, \forall n \in N$

Corollary2.14. Let A be a maximal multivalued mapping and $T: C \to C$ be a firmly nonexpansive. If the iterative scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f(x_n) + \beta_n T^{\alpha_n}(x_n) + (1-\gamma_n) J_{r_n}(x_n)$$

Where $< f_n >, < \alpha_n > and < \beta_n >$ as in theorem (2. 5) and

$$T^{\alpha_n}(x_n) = (1-\alpha_n)x_n + \alpha_n T(x_n)$$

Then $< x_n >$ converges strongly to asymptotic fixed point of $T^{\alpha_n}, \forall n \in N$

3.APPLICATIONS

Let X be a real Hilbert space and C be a nonempty closed convex of X. If f be a proper lower semi continuous convex mapping of X into $(-\infty, \infty]$ then the sub differential ∂f of f is:

$$\partial f(x) = \{z \in X; f(y) \geq f(x) + \langle z, y-x \rangle, \forall y \in X\}, \forall x \in X.$$

Rockefeller [11] proved that ∂f is maximal monotone multivalued mapping .we recall the normal cone $N_c(x)$ of C at x is define as:

$$N_c(x) = \{z \in X; \langle z, y-x \rangle \leq 0, \forall y \in C\}$$

And the indicator mapping of C is define as:

$i_c: X \to (-\infty, \infty]$ such that $i_c(x) = \begin{cases} 0 & \text{if } x \in C \\ \infty & \text{if } x \in C \end{cases}$

i_c is proper lower semicontinuous convex mapping, ∂i_c is maximal monotone and $\partial i_c(x) = N_c(x)$.Now,we introduced application for the results presented in this paper

Corollary 3.1. If f be a proper lower semicontinuous convex mapping of X into $(-\infty, \infty]$, $< f_n >$ be a sequence of bounded and contraction mappings on C and $(\partial f)^{-1} \neq \varphi$. Then $< x_t >$ converges strongly to the point $\tilde{x}$, where $\tilde{x} = p_E(f_n(\tilde{x}))$ or $\tilde{x}$ is the unique solution of variation of variational inequality.

$< (I - f_n)\tilde{x}, x - \tilde{x} > \geq 0$, $x \in E = (\partial f)^{-1}$.

Corollary3.2. If f be a proper lower semi continuous convex mapping of X into $(-\infty, \infty]$, $< f_n >$ be a sequence of contraction mapping on C and $< T_n >$ be a sequence of firmly nonexpansive mapping on C such that $(\partial f)^{-1} \cap (\cap F(f_n)) \cap (\cap F(T_n)) \neq \varphi$

$< f_n >$ and $< T_n >$ lines in $\mathcal{F}$.If the scheme $< x_n >$ is defined as:

$$x_{n+1} = \alpha_n f_n(x_n) + \beta_n T_n(x_n) + (1-\gamma_n) J_{rn}(x_n)$$

Where $< \alpha_n >$ and $< \beta_n >$ are decreasing sequences in [0,1) converges to 0, such that

1. $\sum_{n=0}^{\infty} \alpha_n = \infty$ and $\alpha_n + \beta_n + (1-\gamma_n) = 1$.

2. $\frac{1}{2} \leq \alpha_n + \beta_n < 1$ and$\sum_{n=0}^{\infty} \|f_n(x_n)\| + \|J_{r_n}(x_n)\| < \infty$.Then the iterative scheme $< x_n >$ converges strongly to an asymptotic common fixed point of $T_n, \forall n \in N$. Then the iterative scheme $< x_n >$ converges strongly

to common asymptotic fixed point of $T_n, \forall n \in N$.

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