# POISSON–TRANSMUTED LINDLEY DISTRIBUTION

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## ABSTRACT

The main purpose of this paper is to introduce a new discrete compound distribution, namely Poisson Transmuted Lindley distribution (PTL) which offers a more flexible model for analyzing some types of countable data. The proposed distribution is accommodate unimodel, bathtub as well as decreasing failure rates. Most of the statistical and reliability measures are derived. For the estimation purposes the method of moment and maximum likelihood methods are studied for PTL. Simulation studies are conducted to investigate the performance of the maximum likelihood estimators. A real life application for PTL is introduced to test its goodness of fit and examine its performance compared with some other distributions.

**Keywords:** Transmuted Lindley distribution; Poisson distribution; Compound distributions

## 1. INTRODUCTION

A random variable $X$ is said to have transmuted distribution if its cumulative distribution function (cdf) is given by

$$F_T(x) = (1+c)G(x) - cG(x)^2; \qquad |c| \le 1$$

where G(x) is the cdf of the base distribution; Shaw and Buckly [8]. Merovci [5] studied the transmuted Lindley distribution as an extension to Lindley distribution, firstly introduced by Lindley [4]. The first discretization of Lindley distribution was made by Sankaran [6] who derived a new compound distribution using Lindley and Poisson distribution. The transmuted Lindley distribution can be used for analyzing more complex data and it generalizes some of the widely used distributions. The probability density function of the transmuted Lindley distribution is given by:

$$f(x) = \frac{\theta^2}{(1+\theta)}(1+x)e^{-\theta x}\left(1 - c + 2c\frac{1+\theta+\theta x}{\theta+1}e^{-\theta x}\right), \quad x, \theta > 0, |c| \le 1$$

In the present study, a mathematical formulation of Poisson transmuted Lindley distribution (PTL) and some of its properties was derived. The paper is organized as follows: Sections 2 and 3 are devoted to study the probability mass function and its behavior, moments, hazard and survival functions, size biased, length biased and zero truncated Poisson Transmuted Lindley distributions. Both maximum likelihood estimators (MLE) and method of moment's estimators (MME) are discussed in Section 4. The simulation schemes for PTL parameters are presented in in Section 5. Finally, in Section 6, two real life applications are introduced to illustrate the PTL performance.

## 2. STATISTICAL MEASURES

In this Section, some important statistical measures of the PTL will be derived.

## 2.1 Probability mass function

PTL is a result of the compound between Poisson distribution with parameter $\lambda$ and transmuted Lindley distribution with two parameters $c, \theta$, as follows;

$$f(x; c, \theta) = \int_0^\infty \frac{e^{-2\theta\lambda}\theta^2(1+\lambda)(-(-1+c)e^{\theta\lambda}(1+\theta)+2c(1+\theta+\theta\lambda))}{(1+\theta)^2} \times \frac{e^{-\lambda}\lambda^x}{x!}\, d\lambda$$

From the above equation, the probability mass function of PTL is given by

$$f(x; c, \theta) = \frac{\theta^2((1-c)(2+x+\theta)(1+2\theta)^{1+x} + 2c(1+\theta)^{1+x}(2+x+(11+x(7+x))\theta+4(3+x)\theta^2+4\theta^2))}{(1+\theta)^{1+x}(1+2\theta)^{1+x}x!}$$

$$= \frac{1}{(1+\theta)^2}\theta^2\left(\frac{(1-c)(2+x+\theta)}{(1+\theta)^x} + \frac{2c(1+\theta)(2+x+(11+x(7+x))\theta+4(3+x)\theta^2+4\theta^2)}{(1+2\theta)^{2+x}}\right) \quad (2.1)$$

where, $x = 0, 1,, \dots;\ \theta > 0, -1 < c < 1$

The cumulative distribution function of PTL is defined by

$$F(x) = \left(1 + \frac{(-1+c)(1+\theta(3+x+\theta))}{(1+\theta)^{1+x}(1+2\theta)^{1+x}}\right) - c\frac{(1+\theta(8+x^2\theta+x(2+\theta)(1+4\theta)+\theta(21+4\theta(4+\theta)))))}{(1+\theta)^{-1-x}} \quad (2.2)$$

The next figure illustrates the behavior of the probability mass function of PTL for different values of $c$ and $\theta$.
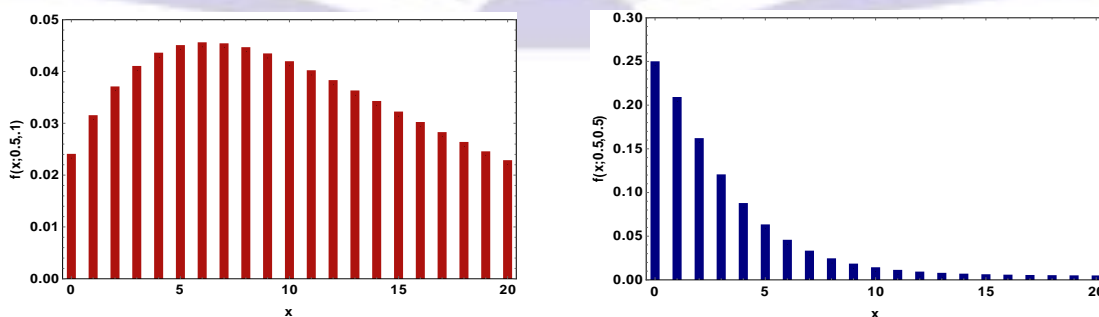


**Fig. (1):** p.m.f. of PTL at different parameter values

## 2.2 Moments and related measures

The $r^{th}$ moments, the moment generating function, the characteristic function and coefficient of variation besides the Skewness and kurtosis of PTL are derived as follows

## Moments about the origin (raw moments)

The first four moments about the origin for PTL are

The Mean $(\mu) = \acute{\mu}_1 = \dfrac{4(1+\theta)(2+\theta)-c(3+2\theta(3+\theta))}{4\theta(1+\theta)^2}$

$$\acute{\mu}_2 = \frac{4(1+\theta)(6+\theta(4+\theta))-c(15+\theta(27+2\theta(6+\theta)))}{4\theta^2(1+\theta)^2}$$

$$\acute{\mu}_3 = \frac{4(1+\theta)(2+\theta)(12+\theta(6+\theta))-c(75+2\theta(75+\theta(48+\theta(12+\theta))))}{4\theta^3(1+\theta)^2}$$

$$\acute{\mu}_4 = \frac{1}{4\theta^4(1+\theta)^2}(4(1+\theta)(120+\theta(168+\theta(78+\theta(16+\theta)))) - c(420+\theta(990+\theta(825+\theta(297+2\theta(24+\theta))))))$$

- **Central moments.**

The central moments of PTL are defined as follows

The variance

$$(\sigma^2) = \mu_2 = \frac{-c^2(3+2\theta(3+\theta))^2+16(1+\theta)^2\big(2+\theta(6+\theta(4+\theta))\big)-4c(1+\theta)(3+\theta(12+\theta(19+2\theta(5+\theta))))}{16\theta^2(1+\theta)^4}$$

$$\mu_3 = -\frac{1}{32\theta^3(1+\theta)^6}(c^3(3+2\theta(3+\theta))^3 - 32(1+\theta)^3(2+\theta)(2+\theta(1+\theta)(8+\theta(4+\theta))) + 8c(1+\theta)^2(3+\theta(21+2\theta(3+\theta(3+\theta))(10+\theta(5+\theta)))) + 6c^2(1+\theta)(3+2\theta(3+\theta))(3+\theta(12+\theta(19+2\theta(5+\theta)))))$$

$$\mu_4 = -\frac{1}{256\theta^4(1+\theta)^8}(3c^4(3+2\theta(3+\theta))^4 + 24c^3(1+\theta)(3+2\theta(3+\theta))^2(3+\theta(12+\theta(19+2\theta(5+\theta)))) + 32c^2(1+\theta)^2(3+2\theta(3+\theta))(24+\theta(132+\theta(276+\theta(297+2\theta(77+\theta(19+2\theta)))))) - 256(1+\theta)^4(24+\theta(144+\theta(338+\theta(406+\theta(258+\theta(87+\theta(15+\theta)))))) + 64c(1+\theta)^3(36+\theta(270+\theta(849+\theta(1446+\theta(1450+\theta(822+\theta(251+2\theta(19+\theta))))))))))$$

The CV of PTL is:

$$CV = \frac{\sigma}{\mu} = \frac{\theta(1+\theta)^2\sqrt{\dfrac{c^2(3+2\theta(3+\theta))^2-16(1+\theta)^2(2+\theta(6+\theta(4+\theta)))+4c(1+\theta)(3+\theta(12+\theta(19+2\theta(5+\theta))))}{\theta^2(1+\theta)^4}}}{4(1+\theta)(2+\theta)-c(3+2\theta(3+\theta))}$$

The moment-generating function $M_x(t)$ and of PTL can be derived as:

$$M_x(t) = \sum_{x=0}^{\infty} e^{xt} f(x) = \frac{\theta^2\left(-\dfrac{4c\theta}{(-1+e^t-2\theta)^3}-\dfrac{2c(1+\theta)}{-1+e^t-2\theta}+\dfrac{(-1+c)(1+\theta)}{-1+e^t-\theta}+\dfrac{(-1+c)(1+\theta)}{(1-e^t+\theta)^2}+\dfrac{2c(1+2\theta)}{(-1+e^t-2\theta)^2}\right)}{(1+\theta)^2}$$

$$\varphi(t) = \sum_{x=0}^{\infty} e^{ixt} f(x) = \frac{\theta^2\left(-\dfrac{4c\theta}{(-1+e^{it}-2\theta)^3}-\dfrac{2c(1+\theta)}{-1+e^{it}-2\theta}+\dfrac{(-1+c)(1+\theta)}{-1+e^{it}-\theta}+\dfrac{(-1+c)(1+\theta)}{(1-e^{it}+\theta)^2}+\dfrac{2c(1+2\theta)}{(-1+e^{it}-2\theta)^2}\right)}{(1+\theta)^2}$$

- **Measures of Skewness and Kurtosis.**

The Skewness $\beta_1$ is a measure of the asymmetry of the probability distribution of a real valued random variable about its mean. The Kurtosis is any measure of the peakedness of the probability distribution of a real-valued random variable. In a similar way to the concept of Skewness, kurtosis $\beta_2$ is a descriptor of the shape of a probability distribution just as Skewness. The Skewness, kurtosis can be written as follows:

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = -(4(c^3(3+2\theta(3+\theta))^3 - 32(1+\theta)^3(2+\theta)(2+\theta(1+\theta)(8+\theta(4+\theta))) + 8c(1+\theta)^2(3+\theta(21+2\theta(3+\theta(3+\theta))(10+\theta(5+\theta)))) + 6c^2(1+\theta)(3+2\theta(3+\theta))(3+\theta(12+\theta(19+2\theta(5+\theta)))))^2)/(c^2(3+2\theta(3+\theta))^2 - 16(1+\theta)^2(2+\theta(6+\theta(4+\theta))) + 4c(1+\theta)(3+\theta(12+\theta(19+2\theta(5+\theta)))))^3$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = -(3c^4(3+2\theta(3+\theta))^4 + 24c^3(1+\theta)(3+2\theta(3+\theta))^2(3+\theta(12+\theta(19+2\theta(5+\theta)))) + 32c^2(1+\theta)^2(3+2\theta(3+\theta))(24+\theta(132+\theta(276+\theta(297+2\theta(77+\theta(19+2\theta)))))) - 256(1+\theta)^4(24+\theta(144+\theta(338+\theta(406+\theta(258+\theta(87+\theta(15+\theta)))))) + 64c(1+\theta)^3(36+\theta(270+\theta(849+\theta(1446+\theta(1450+\theta(822+\theta(251+2\theta(19+\theta)))))))))/(c^2(3+2\theta(3+\theta))^2 - 16(1+\theta)^2(2+\theta(6+\theta(4+\theta))) + 4c(1+\theta)(3+\theta(12+\theta(19+2\theta(5+\theta)))))^2$$

## 3. RELIABILITY MEASURES

### 3.1 Survival and Hazard functions

The survival function is defined as the probability that a subject survives to time $t$ without experiencing the event of interest. The survival function $S(t)$ for PTL is derived as:

$$S(t) = \frac{(1-c)(1+\theta(3+x+\theta))}{(1+\theta)^{3+x}} + \frac{c(1+\theta(8+x^2\theta+x(2+\theta)(1+4\theta)+\theta(21+4\theta(4+\theta))))}{(1+2\theta)^{3+x}(1+\theta)^2}$$

where $x = 0,1,,\ldots;\ \theta > 0, -1 < c < 1$

The hazard rate function is the total number of failures within an item population, divided by the total time expended by that population, during a particular measurement interval under stated conditions.

For PTL, the hazard rate function is

$$h(t) = \frac{(1+\theta(3+x+\theta))(-(-1+c)(1+\theta)^{-x}(2+x+\theta))}{(1-c)(1+\theta)^{3+x}} + \frac{2c(1+\theta)(2+x+(11+x(7+x))\theta+4(3+x)\theta^2+4\theta^3))}{(1+2\theta)^{3+x}} + \frac{c(1+\theta(8+x^2\theta+x(2+\theta)(1+4\theta)+\theta(21+4\theta(4+\theta))))}{(1+2\theta)^{3+x}(1+\theta)^2}$$

### 3.2 Size and length biased distribution

Let $X$ be a discrete random variable having PTL with p.m.f given in (2.1). The size –biased function of is defined by

$$f_{SB} = \frac{-(x^\theta(1+\theta)^{-x}(1+2\theta)^{-x}(-(-1+c)(2+x+\theta)(1+2\theta)^{2+x}+2c(1+\theta)^{1+x}(2+x+(11+x(7+x))\theta+4(3+x)\theta^2+4\theta^2)))}{((-1+c)(1+2\theta)^3 H_1(x,\theta,0)+(-1+c)(2+\theta)(1+2\theta)^3 H_4(x,\theta,0)-2c(1+\theta)(\theta H_2(x,\theta,0)+(1+\theta(7+4\theta))H_3(x,\theta,0)+(2+\theta(11+4\theta(3+\theta)))H_4(x,\theta,0)))}$$

where,
$$H_1(x,\theta,0) = \text{HurwitzLerchPhi}\left[\frac{1}{1+\theta}, -1-\beta, 0\right]$$

$$H_2(x,\theta,0) = \text{HurwitzLerchPhi}\left[\frac{1}{1+2\theta}, -2-\beta, 0\right]$$

$$H_3(x,\theta,0) = \text{HurwitzLerchPhi}\left[\frac{1}{1+2\theta}, -1-\beta, 0\right]$$

$$H_4(x,\theta,0) = \text{HurwitzLerchPhi}\left[\frac{1}{1+2\theta}, -\beta, 0\right]$$

and HurwitzLerchPhi is the Hurwitz–Lerch transcendent function defined as an analytic continuation of $\Phi(z,s,a) = \sum_{k=0}^{\infty} z^k(k+a)^{-s}$.

The length -biased distribution is a special case of the size-biased distribution at $\beta = 1$ and is derived as

$$f_{LB} = \frac{4x\theta^2(1+\theta)^{-1-x}(1+2\theta)^{-3-x}((-1+c)(2+x+\theta)(1+2\theta)^{2+x}-2c(1+\theta)^{1+x}(2+x+(11+x(7+x))\theta+4(3+x)\theta^2+4\theta^2))}{-4(1+\theta)(2+\theta)+c(3+2\theta(3+\theta))}$$

### 3.3. Zero truncated Poisson Transmuted Lindley Distribution

Ghitany et al. [1] introduced the zero truncated Poisson Lindley distribution. Zero-truncated Poisson Transmuted Lindley distribution (ZTPTL) is used to model countable data for which the zeroes cannot occur or doesn't recorded. This case occurs in many applications for example; a study of length of hospital stays in days; the length of hospital stay is recorded as a minimum of at least one day. The zero-truncated Poisson transmuted Lindley distribution (ZTPTL) p.m.f. is defined by

$$P(x;c,\theta) = \frac{\theta^2(1+\theta)^{-x}(1+2\theta)^{-x}\left((-1+c)(2+x+\theta)(1+2\theta)^{3+x}-2c(1+\theta)^{1+x}(2+x+(11+x(7+x))\theta+4(3+x)\theta^2+4\theta^2)\right)}{-1+\theta\left(-9+\theta\left(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta)))\right)\right)}$$

$where\ x = 0,1,,\ldots;\ \theta > 0, -1 < c < 1$

The cumulative distribution function of ZTPTL is:

$$F(x) = ((1+\theta)^{-x}(1+2\theta)^{-x}(c(1+\theta)^{1+x}(1+\theta(8+x^2\theta+x(2+\theta)(1+4\theta)+\theta(21+4\theta(4+\theta))))+(1+2\theta)^x(-(-1+c)(1+2\theta)^3(1+\theta(3+x+\theta))+(1+\theta)^x(-1+\theta(-9+\theta(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta)))))))))/(-1+\theta(-9+\theta(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta))))))$$

The first four moments about the origin for ZTPWLD are:

The Mean $(\mu) = \acute{\mu}_1 = \dfrac{(1+\theta)(1+2\theta)^2(-4(1+\theta)(2+\theta)+c(3+2\theta(3+\theta)))}{4\theta(-1+\theta(-9+\theta(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta))))))}$

$$\acute{\mu}_2 = \frac{(1+\theta)(1+2\theta)^2(-4(1+\theta)(6+\theta(4+\theta))+c(15+\theta(27+2\theta(6+\theta))))}{4\theta^2(-1+\theta(-9+\theta(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta))))))}$$

$$\acute\mu_3 = \frac{(1+\theta)(1+2\theta)^3(-4(1+\theta)(2+\theta)(12+\theta(6+\theta))+c(75+2\theta(75+\theta(48+\theta(12+\theta)))))}{4\theta^3(-1+\theta(-9+\theta(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta)))))))}$$

$$\acute\mu_4 = \frac{(1+\theta)(1+2\theta)^3(-4(1+\theta)(120+\theta(168+\theta(78+\theta(16+\theta))))+c(420+\theta(990+\theta(825+\theta(297+2\theta(24+\theta))))))}{4\theta^4(-1+\theta(-9+\theta(-31-2\theta(25+2\theta(9+2\theta))+c(2+\theta(13+4\theta(4+\theta)))))))}$$

The variance of ZTPTL is given by:

$$\mu_2 = ((1+\theta)(1+2\theta)^3(16(1+\theta)(1+2\theta)^3(2+\theta(10+\theta(6+\theta)))) + c^2(-9+\theta(-99+2\theta(-171+2\theta(-48+\theta(158+$$
$$\theta(225+4\theta(23+3\theta)))))))) - 4c(3+\theta(36+\theta(208+\theta(719+2\theta(608+\theta(481+4\theta(41+5\theta)))))))))/(16(\theta+9\theta^2+(31-$$
$$2c)\theta^3 + (50-13c)\theta^4 + 4(9-4c)\theta^5 - 4(-2+c)\theta^6)^2)$$

## 4. PARAMETER ESTIMATION

### 4.1 Method of Moments

Let $x_1, \ldots \ldots \ldots, x_n$ be a random sample of size $n$ from PTL, the method of moment estimates (MME), for $c$ $and$ $\theta$ are given by solving the following two equations simultaneously

$$\frac{4(1+\theta)(2+\theta)-c(3+2\theta(3+\theta))}{4\theta(1+\theta)^2} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\frac{4(1+\theta)(6+\theta(4+\theta))-c(15+\theta(27+2\theta(6+\theta)))}{4\theta^2(1+\theta)^2} = \frac{\sum_{i=1}^n x_i^2}{n}$$

### 4.2 Maximum-Likelihood Estimation

Let $x_1, \ldots \ldots \ldots, x_n$ be a random sample with size n from PTL distribution, the log –likelihood function is given by

$$ln\ell(c,\theta) = 2n\text{Log}[\theta] - 3n\text{Log}[1+\theta] + \sum_{i=1}^n \text{Log}[(1-c)(1+\theta)^{-x_i}(2+\theta+x_i) + 2c(1+\theta)(1+2\theta)^{-3-x_i}(2+4\theta^3 + x_i + 4\theta^2(3+x_i) + \theta(11+x_i(7+x_i)))]$$

The partial derivatives of the log-likelihood for the parameters $c, \theta$ can be expressed by the following equations

$$\frac{\partial \text{Ln}\ell(c,\theta)}{\partial\theta} = \frac{2n}{\theta} - \frac{3n}{1+\theta}$$
$$+ \sum_{i=1}^n ((1-c)(1+\theta)^{-x_i} - (1-c)(1+\theta)^{-1-x_i}x_i(2+\theta+x_i) + 2c(1+\theta)(1+2\theta)^{-3-x_i}(11+12\theta^2 + 8\theta(3 + x_i) + x_i(7+x_i)) + 2c(1+2\theta)^{-3-x_i}(2+4\theta^3 + x_i + 4\theta^2(3+x_i) + \theta(11+x_i(7+x_i))) + 4c(1 + \theta)(1+2\theta)^{-4-x_i}(-3-x_i)(2+4\theta^3 + x_i + 4\theta^2(3+x_i) + \theta(11+x_i(7+x_i))))/((1-c)(1+\theta)^{-x_i}(2+\theta + x_i) + 2c(1+\theta)(1+2\theta)^{-3-x_i}(2+4\theta^3 + x_i + 4\theta^2(3+x_i) + \theta(11+x_i(7+x_i))))$$

$$\frac{\partial \text{Ln}\ell(c,\theta)}{\partial c} = \sum_{i=1}^n \frac{-(1+\theta)^{-x_i}(2+\theta+x_i)+2(1+\theta)(1+2\theta)^{-3-x_i}(2+4\theta^3+x_i+4\theta^2(3+x_i)+\theta(11+x_i(7+x_i)))}{(1-c)(1+\theta)^{-x_i}(2+\theta+x_i)+2c(1+\theta)(1+2\theta)^{-3-x_i}(2+4\theta^3+x_i+4\theta^2(3+x_i)+\theta(11+x_i(7+x_i)))}$$

The MLEs can be obtained by equating the derivatives of the log likelihood function to zero and solving for the parameters $c$ and $\theta$. For PTL, the solution of the nonlinear equations has no closed form and some numerical methods are needed for the solution.

## 5. SIMULATION STUDY

In this section, a complete simulation study for PTL will be made by generating 1000 samples for each pair of the parameters $c, \theta$ at different sample sizes. The simulation nodes was chosen at $(c, \theta) = (0.01,1)$ $and$ $(0.5, 0.15)$ at different sample sizes $n = 20, 30, 40, 50, 60, 70, 80, 90, 100$. The next tables presents the average bias and the MSE of the estimates at different values of $n$. It is observed that bias and the MSEs decreases while the sample size increases.

**Table 1. Bias and MSE for the parameters $c$ and $\theta$**

| N | $c = 0.01$ | | $\theta = 1$ | |
|---|---|---|---|---|
| | MSE | Bias | MSE | Bias |
| 20 | 0.0533177 | 0.0457183 | 0.0000053 | 0.0003818 |
| 30 | 0.0462968 | 0.0392166 | 0.0000032 | 0.0003649 |
| 40 | 0.0382659 | 0.0376281 | 0.0000027 | 0.0003596 |
| 50 | 0.0368369 | 0.0335041 | 0.0000024 | 0.0003566 |
| 60 | 0.0283886 | 0.0288943 | 0.0000020 | 0.0002763 |
| 70 | 0.0274950 | 0.0236517 | 0.0000016 | 0.0002501 |
| 80 | 0.0265418 | 0.0218903 | 0.0000015 | 0.0002671 |
| 90 | 0.0265085 | 0.0204329 | 0.0000015 | 0.0002158 |
| 100 | 0.0205565 | 0.0101958 | 0.0000012 | 0.0001562 |

| N | $c = 0.5$ | | $\theta = 0.15$ | |
|---|---|---|---|---|
| | MSE | Bias | MSE | Bias |
| 20 | 0.6419421 | 0.4336126 | 0.0070023 | 0.0434446 |
| 30 | 0.4418301 | 0.3302858 | 0.0038014 | 0.0342961 |
| 40 | 0.3502343 | 0.2998311 | 0.0028893 | 0.0309144 |
| 50 | 0.2890053 | 0.2684835 | 0.0024620 | 0.0289807 |
| 60 | 0.2101090 | 0.2150965 | 0.0018194 | 0.0241833 |
| 70 | 0.2034110 | 0.2140964 | 0.0018056 | 0.0236621 |
| 80 | 0.1818890 | 0.1990123 | 0.0015371 | 0.0215177 |
| 90 | 0.1510736 | 0.1693524 | 0.0012808 | 0.0197241 |
| 100 | 0.1370095 | 0.1509415 | 0.0011042 | 0.0176460 |

## 6. APPLICATION

Two real data sets based on a portfolio of automobile insurance claims are considered. These data were taken from a sample of 298 automobile liability policies (Klugman et al., [3]) for the first example and, from a sample of 7842 automobile liability policies that appeared in Seal [7] for the second case. To make a comparison with the proposed distribution PTL, the expected frequencies are provided for Poisson (Po), negative binomial (NB) and DGL (Góméz-Déniz and Calderín [2]) distributions obtained after estimating parameters by maximum likelihood. The obtained results are summarized in tables 2 and 3. The results showed greater Log-likelihood for PTL compared with other distributions. We can conclude that PTL provides better fits than some other known distributions.

**Table 2. Fitted estimates for the first data set**

| Claim number | Observed frequencies | Expected frequencies | | | |
|---|---|---|---|---|---|
| | | Po | NB | DGL | PTL |
| 0 | 99 | 54 | 95.85 | 96.65 | 98.29 |
| 1 | 65 | 92.24 | 75.83 | 74.29 | 72.81 |
| 2 | 57 | 78.77 | 50.35 | 50.23 | 49.34 |
| 3 | 35 | 44.85 | 31.29 | 31.71 | 31.57 |
| 4 | 20 | 19.15 | 18.79 | 19.17 | 19.39 |
| 5 | 10 | 8.95 | 11.04 | 11.26 | 11.55 |
| 6 | 4 | ------ | 6.39 | 6.47 | 6.70 |
| ≥7 | 8 | ------ | 8.14 | 7.95 | 8.21 |
| $\hat{c}$ | ------ | ------ | 1.473 | 0.501 | -0.2875 |
| $\hat{\theta}$ | ------ | 1.708 | 0.463 | 0.695 | 0.9947 |
| - Log-likelihood | ------ | 577.002 | 528.769 | 528.619 | 528.595 |
| $\chi^2$ | ------ | 151.73 | 3.14 | 2.64 | 0.4772 |
| p-value | ------ | 0.000 | 0.533 | 0.618 | 0.8105 |

**Table (3): Fitted estimates for the second data set**

| Claim number | Observed frequencies | Expected frequencies | | | |
|---|---|---|---|---|---|
| | | Po | NB | DGL | PTL |
| 0 | 5147 | 4783.18 | 5147.94 | 5149.94 | 5160.36 |
| 1 | 1859 | 2364.75 | 1859.84 | 1852.33 | 1839.62 |
| 2 | 595 | 584.55 | 586.33 | 591.902 | 590.32 |
| 3 | 167 | 96.33 | 175.85 | 177.28 | 178.77 |
| 4 | 54 | 11.90 | 51.39 | 50.96 | 52.24 |
| 5 | 14 | 1.26 | 14.78 | 14.24 | 14.91 |
| ≥6 | 5 | ------ | 5.82 | 5.32 | 5.863 |
| $\hat{c}$ | ------ | ------ | 0.730 | 0.239 | -0.3601 |
| $\hat{\theta}$ | ------ | 0.494 | 1.341 | 0.621 | 2.9699 |
| - Log-likelihood | ------ | 7068.77 | 7429.60 | 7429.85 | 7426.83 |
| $\chi^2$ | ------ | 809.59 | 0.75 | 0.90 | 0.0916 |
| p-value | ------ | 0.000 | 0.944 | 0.923 | 0.9784 |

## CONCLUSION

A new two-parameter distribution, called Poisson Transmuted Lindley distribution (PTL) is proposed and investigated through most statistical and reliability measures. The new distribution has a property that its hazard rate has unimodel, bathtub as well as decreasing shapes. Simulation studies provides less bias and mean square error of the maximum likelihood estimators as the sample size increases. The proposed distribution PTL provides the better fit model and more flexibility.

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